Scalar self-force for generic bound orbits on a Kerr background

Zachary Nasipak[1], Thomas Osborn[2], & Charles R. Evans[1]

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THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

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Motivation

- **Focus:** Self-force calculations on Kerr
- **Difficulties:**
  - Loss of symmetry compared to Schwarzschild
  - Current computational methods inefficient (algebraic-convergence, precision loss, cancellation errors)
- **Goal:** Use a developmental model to improve self-force calculations
- **Approach:** Build scalar self-force for generic bound orbits on Kerr
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Scalar self-force (SSF) problem

Gravitational self-force (GSF)

- Equations of motion

$$\mu\, u^\beta \nabla_\beta u^\alpha = F^\alpha_{\mathrm{GSF}} \sim \mathcal{O}(\mu^2/M^2)$$

- GSF equations

$$F^\alpha_{\mathrm{GSF}} = \mu\, P^{\alpha\beta\gamma\delta}\nabla_\beta h^{\mathrm{R}}_{\gamma\delta}$$

- Field equations

$$\left.\begin{aligned} {}_2\mathcal{O}\psi_0 &= 4\pi\Sigma\hat{T}_0 \\ {}_{-2}\mathcal{O}\rho^{-4}\psi_4 &= 4\pi\Sigma\hat{T}_4 \end{aligned}\right\} \Rightarrow h^{\mathrm{ret}}_{\alpha\beta}$$

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My SSF code

- Written in Mathematica using FD approach
- Performed w/ arbitrary numerical precision
- Computational time of generic orbit
  - ~ 10,000 CPU hours*
- Some modules inherited
  - Credit: Charles R. Evans, Thomas Osburn, Erik Forseth, & Seth Hopper
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Building a generic SSF code

- Construct scalar field

$$\Box_g \Phi^{\text{ret}} = -4\pi\rho \qquad \rho = q\int \delta^{(4)}(x^\mu, z^\mu(\tau))d\tau$$

- Separable in FD $(\omega_{mkn} = m\Omega_\varphi + k\Omega_\theta + n\Omega_r)$

$$\Phi^{\text{ret}} = \frac{1}{\sqrt{r^2+a^2}}\sum_{\hat{l}mkn} X_{\hat{l}mkn}(r)S_{\hat{l}mkn}(\theta)e^{im\varphi}e^{i\omega_{mkn}t}$$

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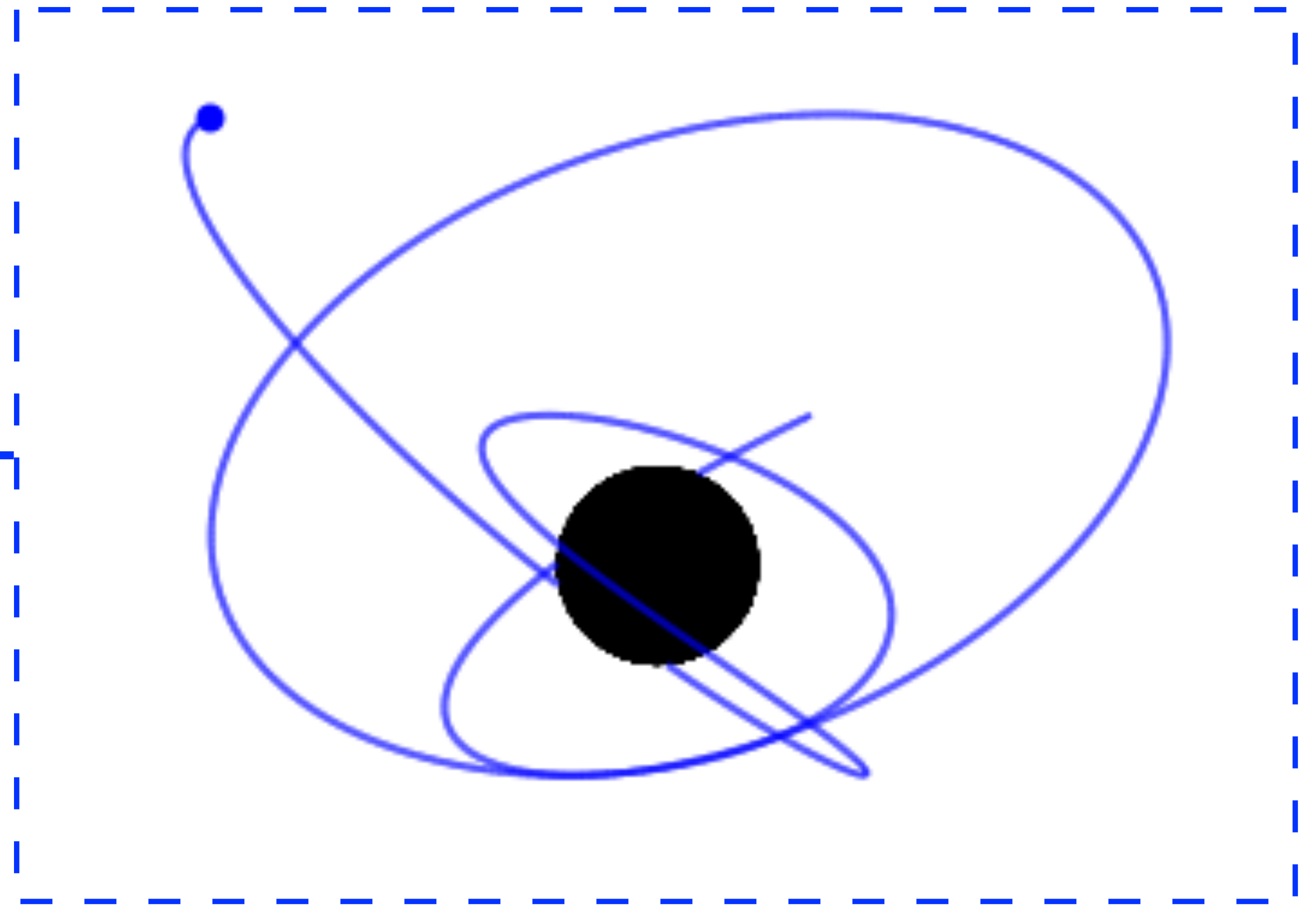

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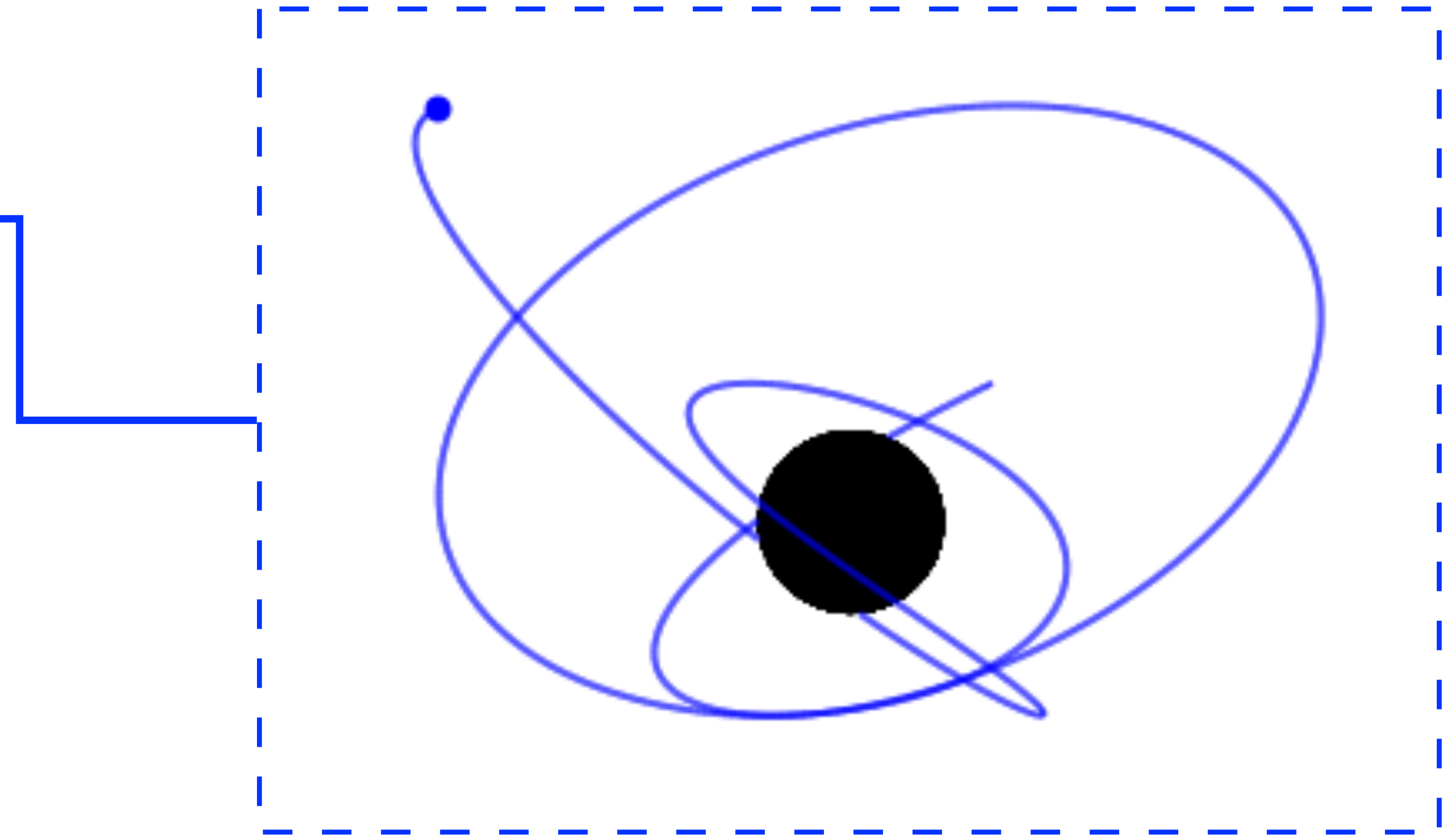

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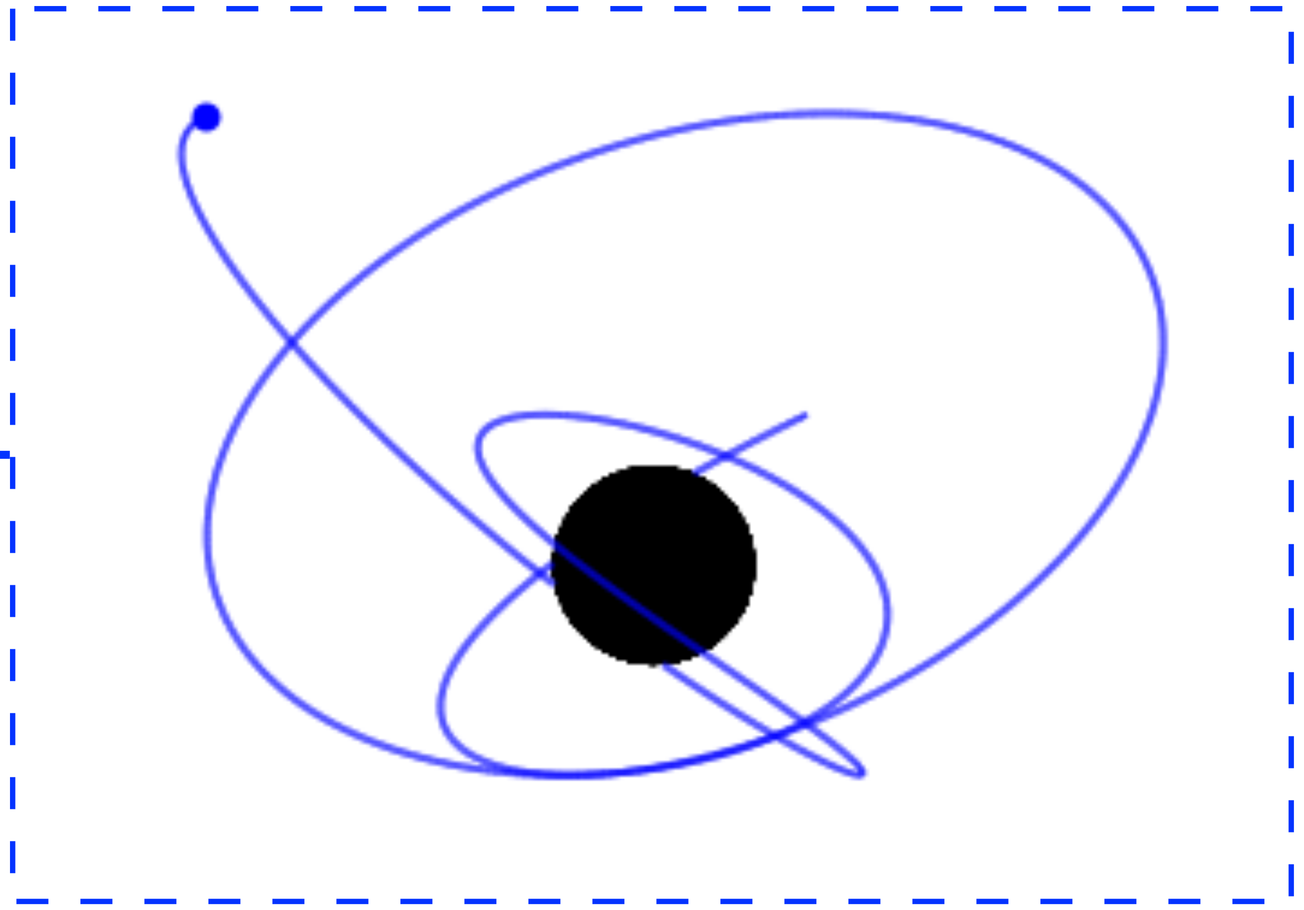

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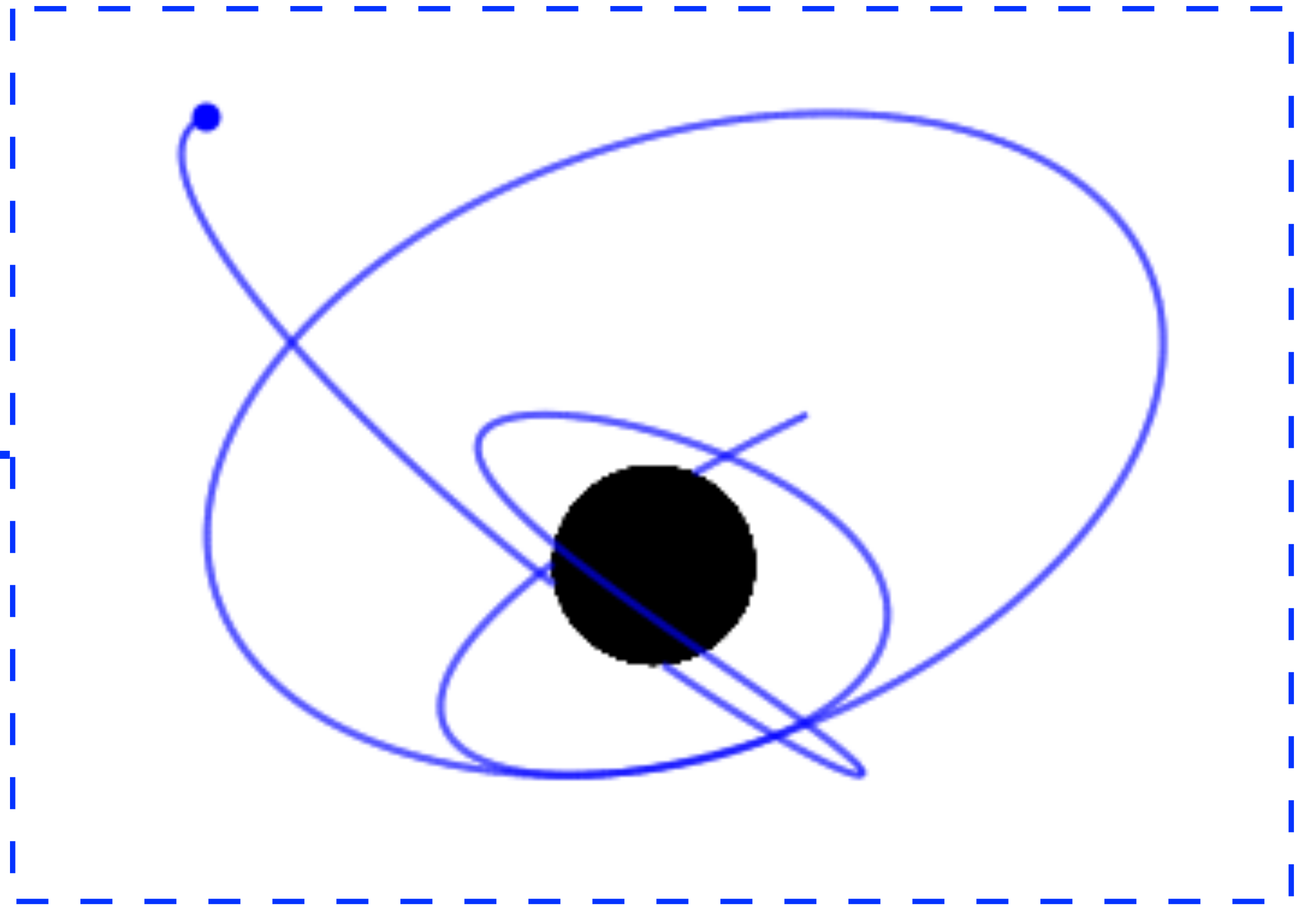

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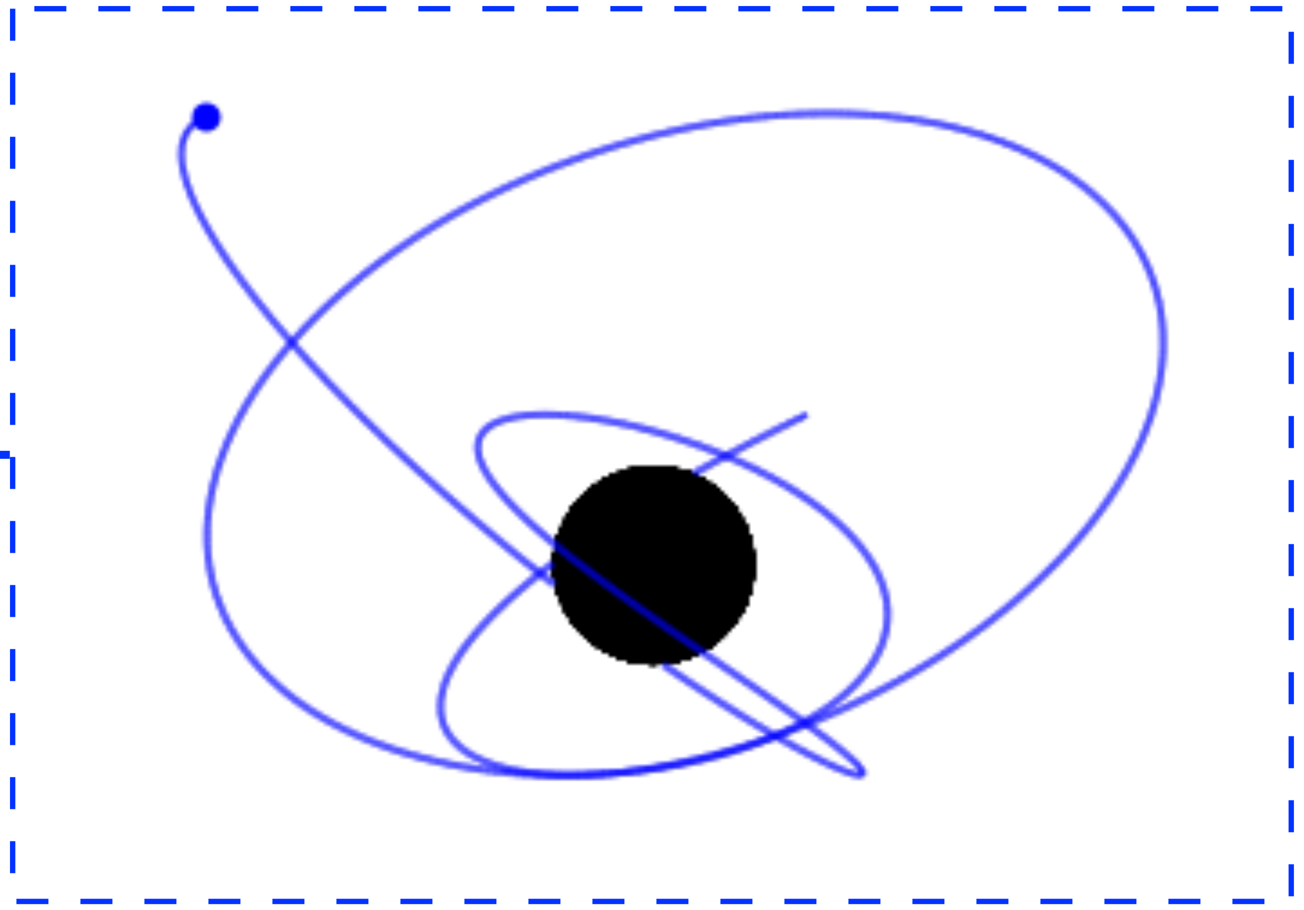

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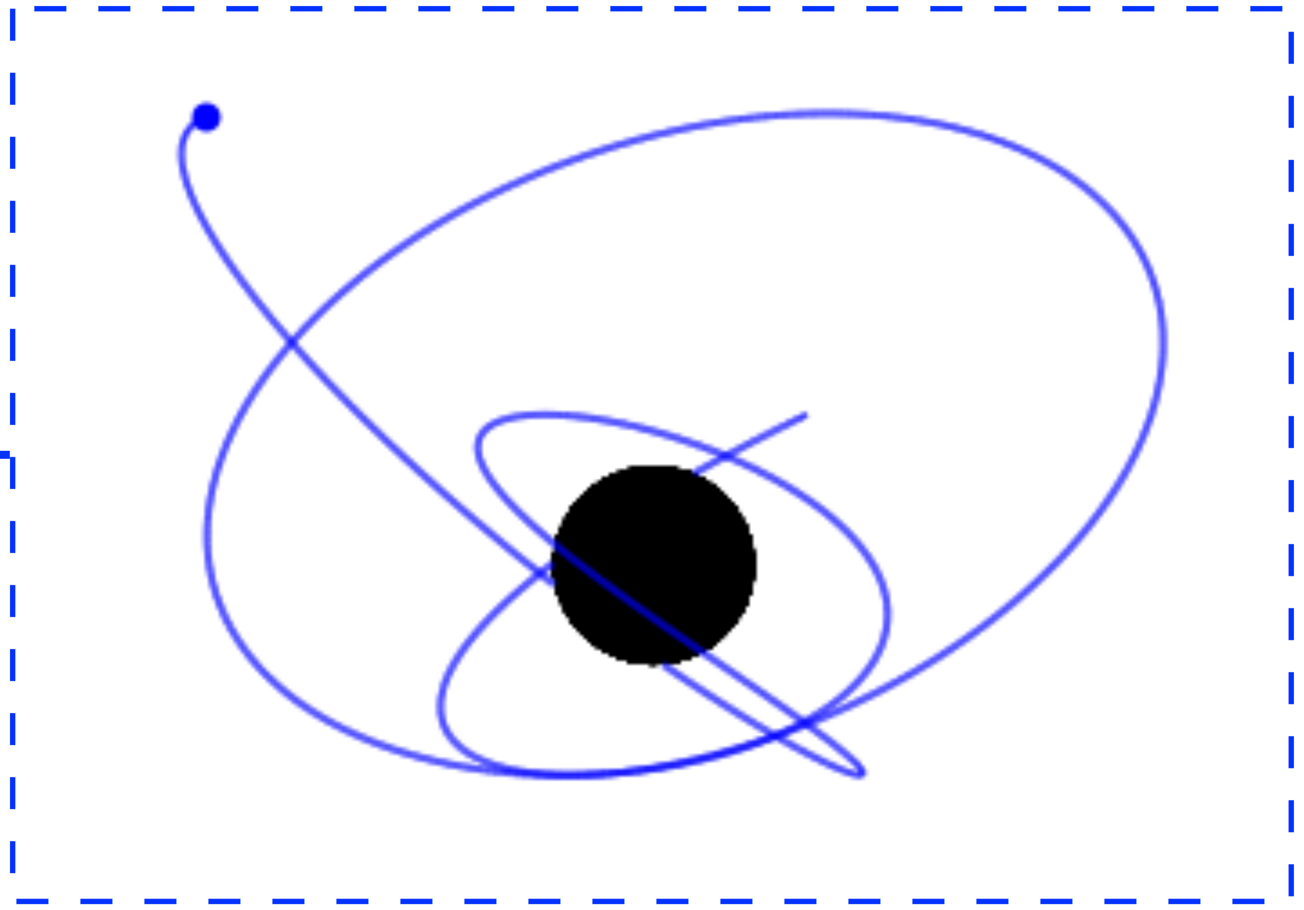

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- Solving the radial ODEs

$$\frac{dr_*}{dr} = \frac{r^2 + a^2}{r^2 - 2Mr + a^2}$$

- Variation of parameters

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Tortoise coordinate

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Generalized Sasaki-Nakamura Eqn

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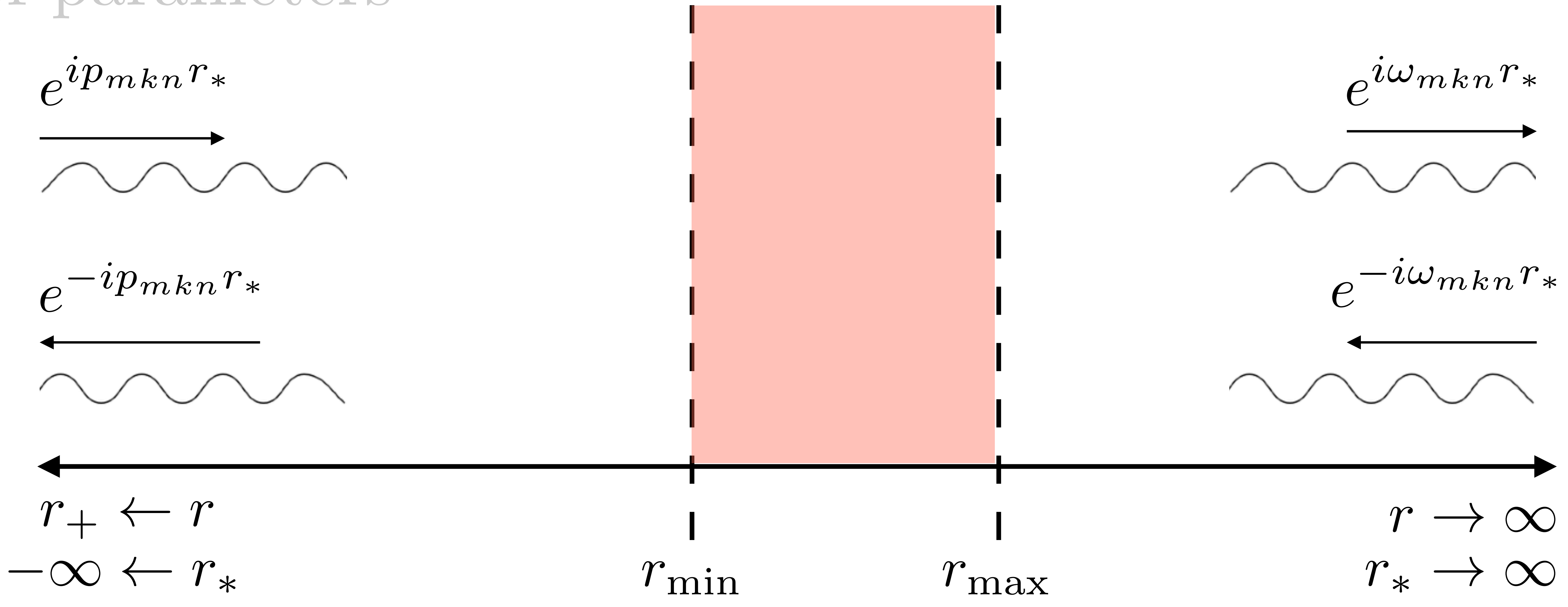

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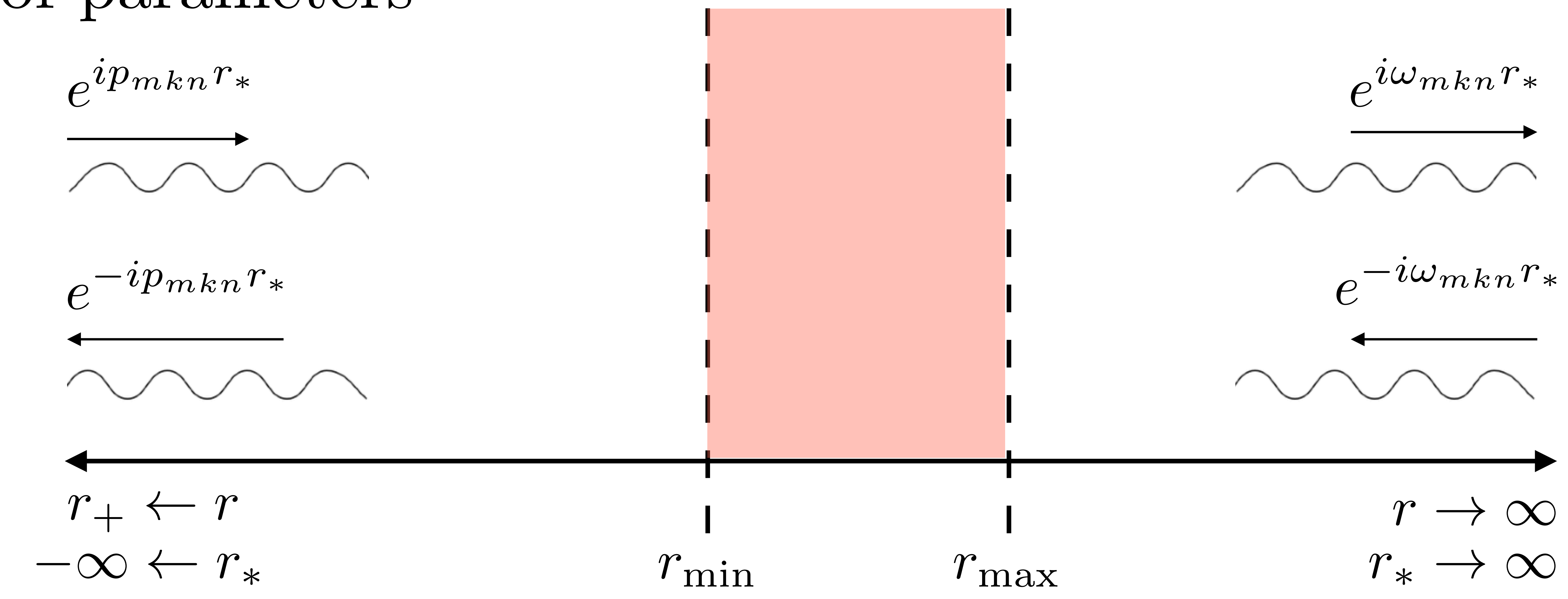

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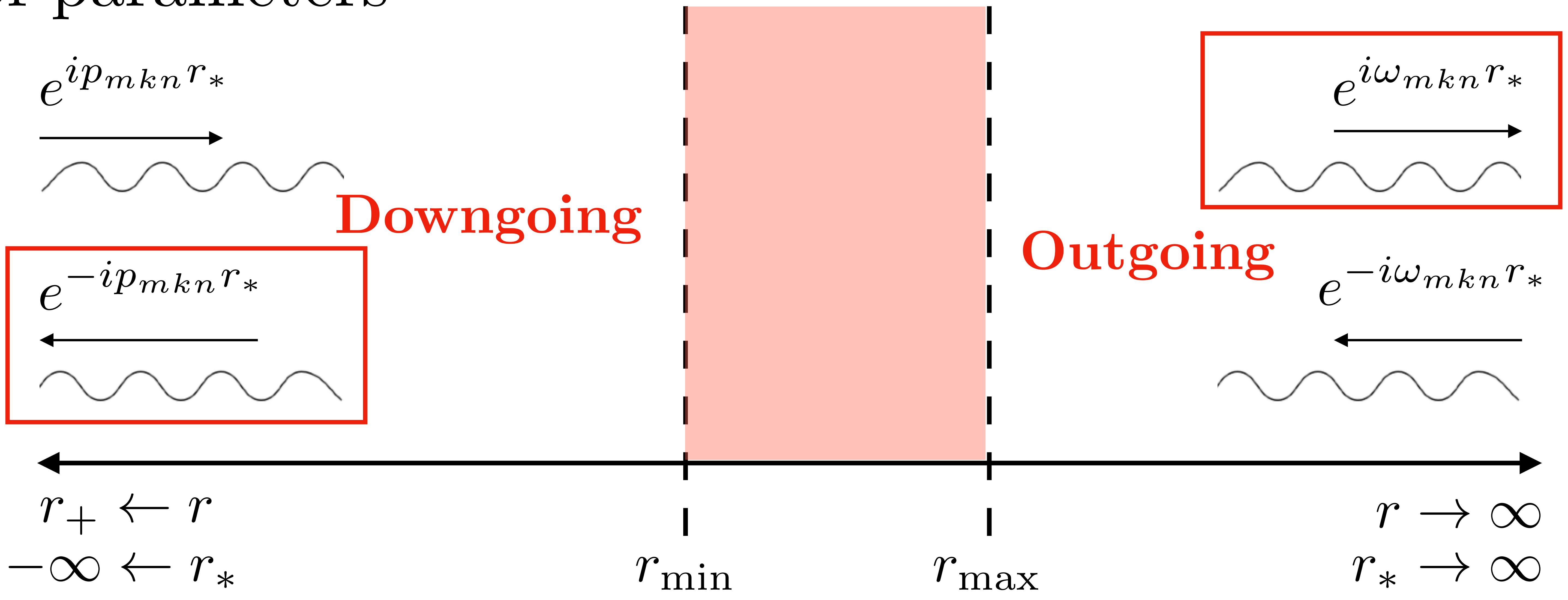

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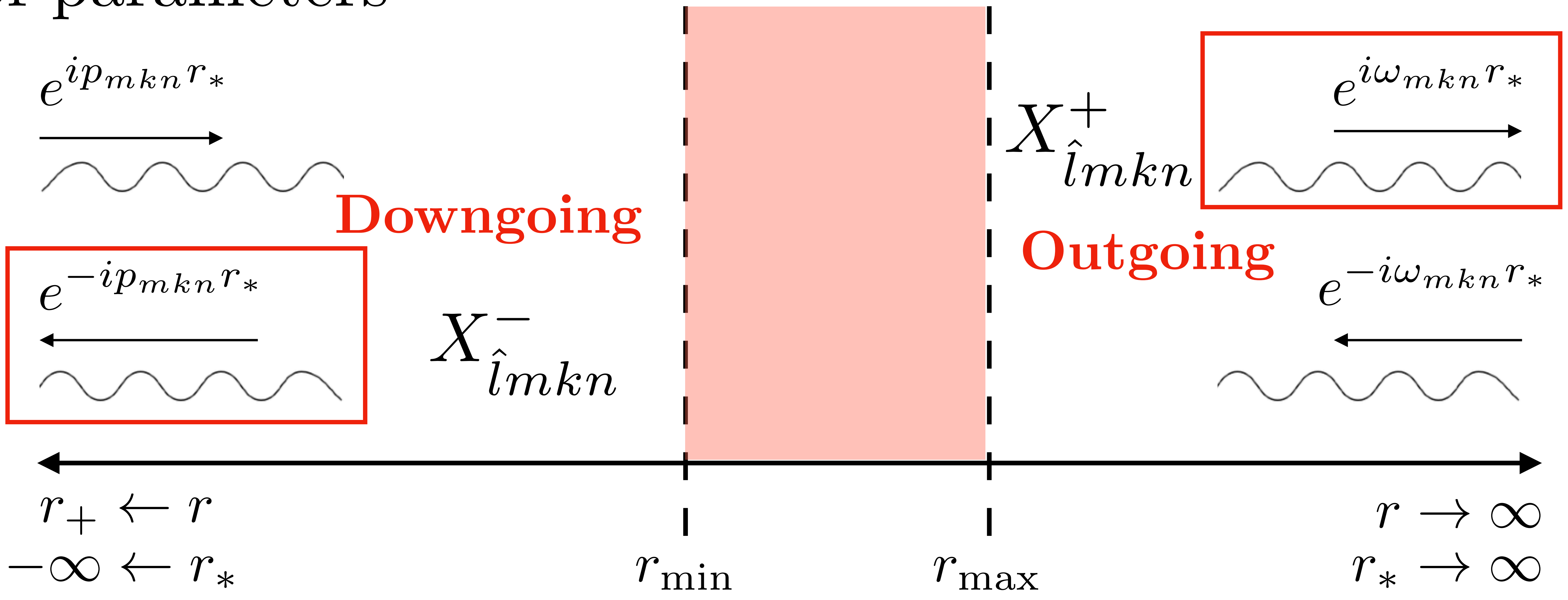

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- Finding homogeneous solutions
  - **Mano-Suzuki-Takasugi (MST)** function expansion formalism
    - Series solution w/ introduction of free parameter ν

$$R^{-}_{\hat{l}mkn}(r) \rightarrow R^{-}_{\hat{l}m}(\omega, r) = \sum_{n=-\infty}^{\infty} f^{\nu}(\omega, x) a_n^{\nu} F(c^{\nu}_{n,1}, c^{\nu}_{n,2}; c^{\nu}_{n,3}; x)$$

$$R^{+}_{\hat{l}mkn}(r) \rightarrow R^{+}_{\hat{l}m}(\omega, r) = \sum_{n=-\infty}^{\infty} g^{\nu}(\omega, z) b_n^{\nu} (2z)^n \Psi(c^{\nu}_{n,4}, c^{\nu}_{n,5}; -2iz)$$

$$c^{\nu}_{n,i} = c^{\nu}_{n,i}(\omega)$$

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Sasaki & Tagoshi (2003) **LRR 6**

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$$c^{\nu}_{n,i} = c^{\nu}_{n,i}(\omega)$$

Building a generic SSF code

- Finding homogeneous solutions
  - **Mano-Suzuki-Takasugi (MST)** function expansion formalism
    - Series solution w/ introduction of free parameter ν

Sasaki & Tagoshi (2003) **LRR 6**

Hypergeometric function

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Irregular confluent hypergeometric function

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ν Renormalized angular momentum

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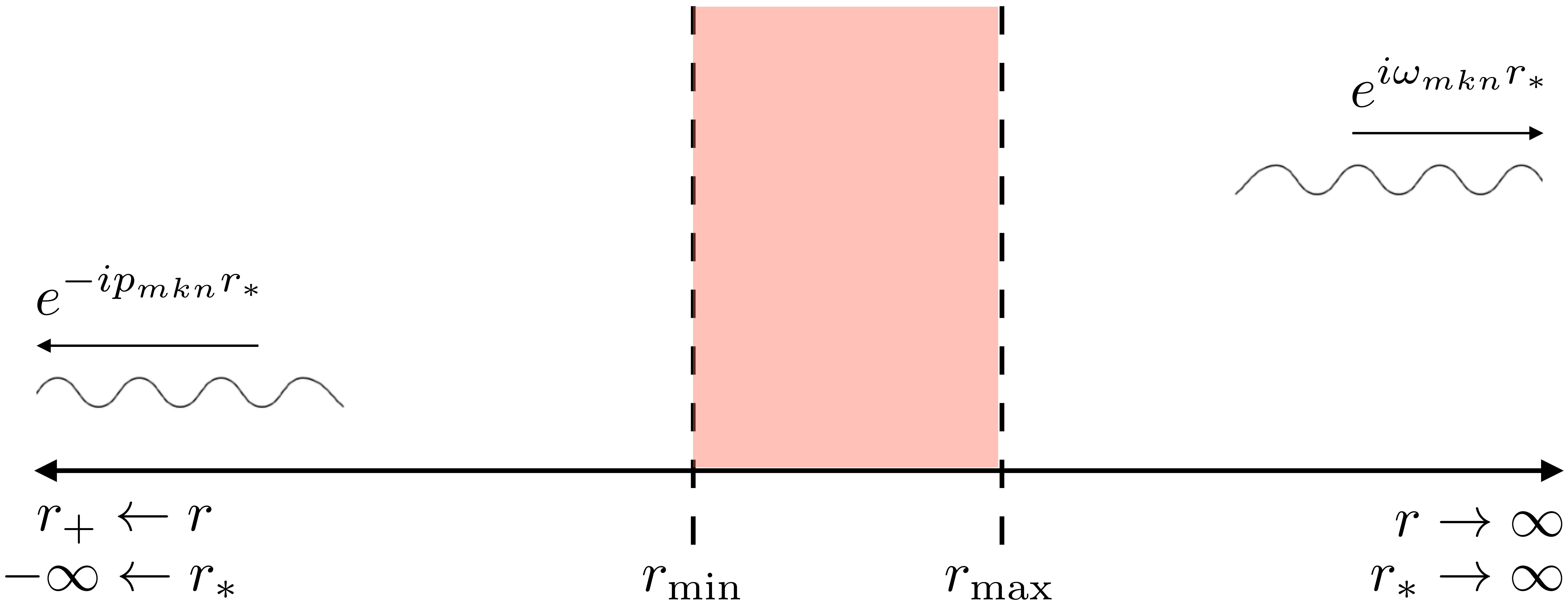
ν Renormalized angular momentum

Black hole monodromy

Building a generic SSF code

- Source integration & TD reconstruction



Building a generic SSF code

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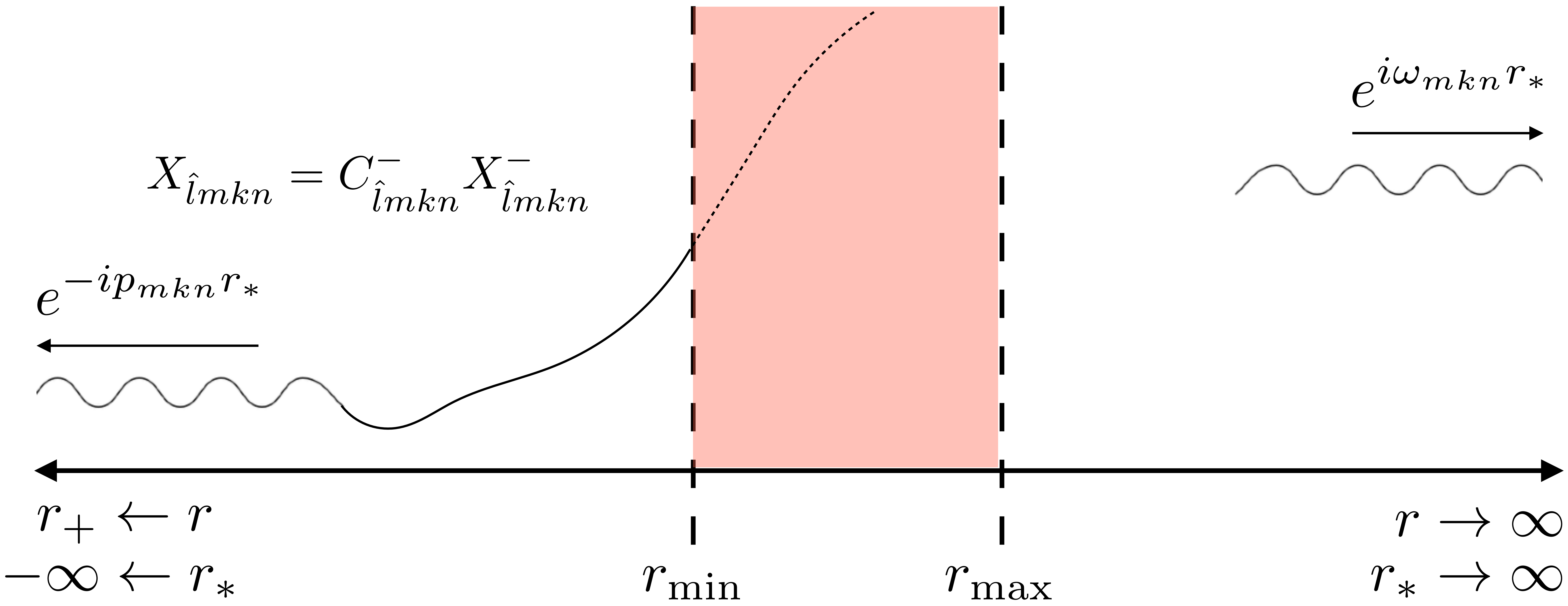

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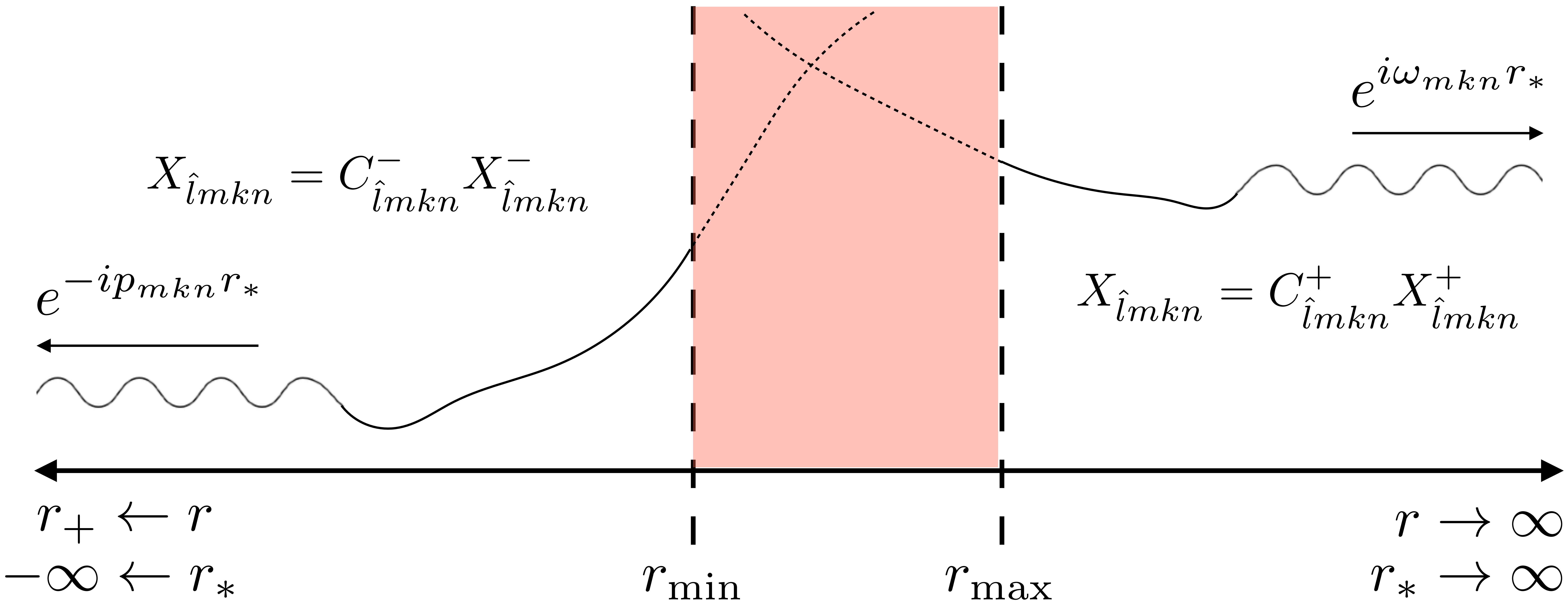
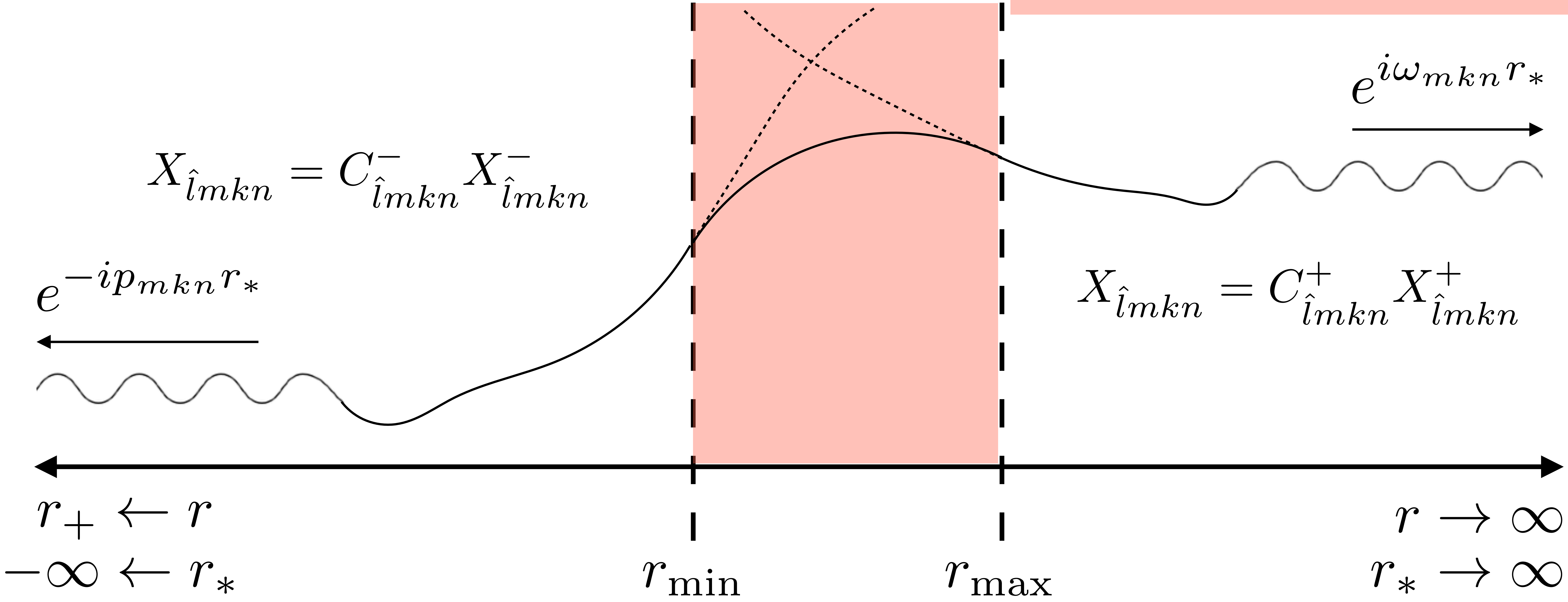


Building a generic SSF code

- Source integration & TD reconstruction

$$X_{\hat{l}mkn} = c^-_{\hat{l}mkn}(r)X^-_{\hat{l}mkn}(r) + c^+_{\hat{l}mkn}(r)X^+_{\hat{l}mkn}(r)$$



THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

Building a generic SSF code

- Source integration & TD reconstruction

2D integral
$\to$ 4 1D
Fourier sums

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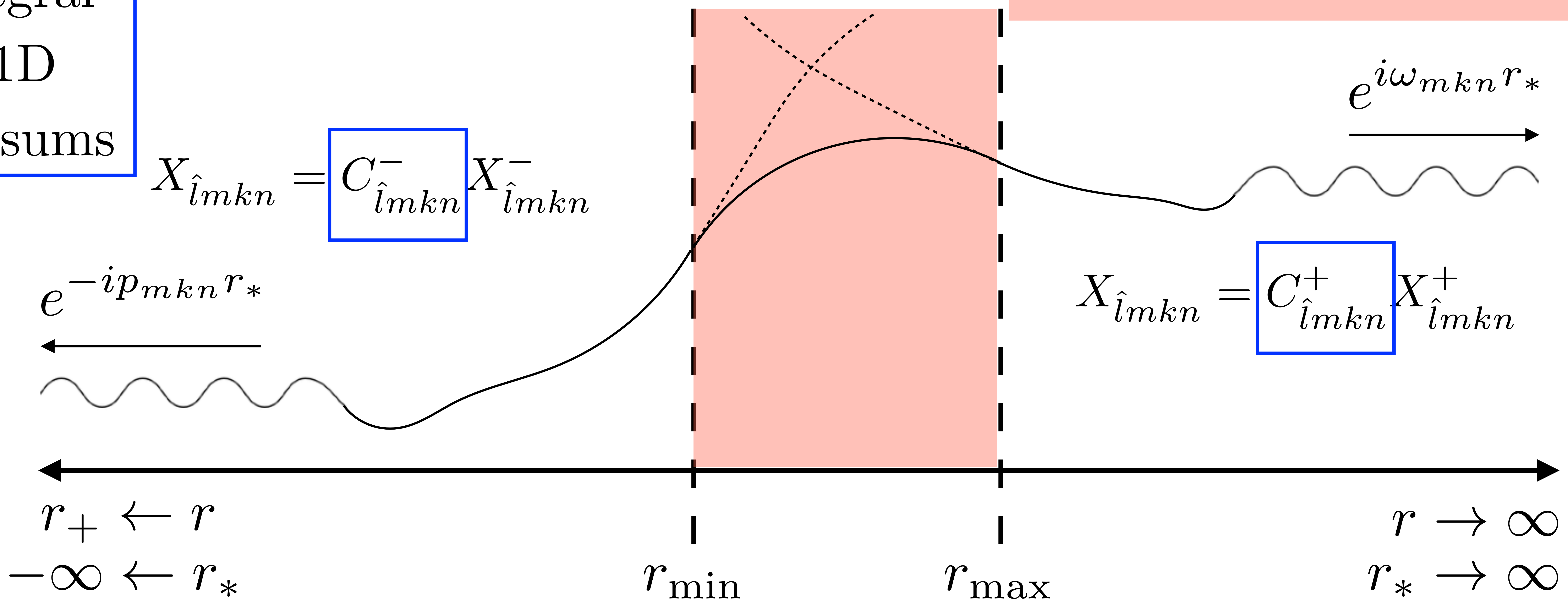
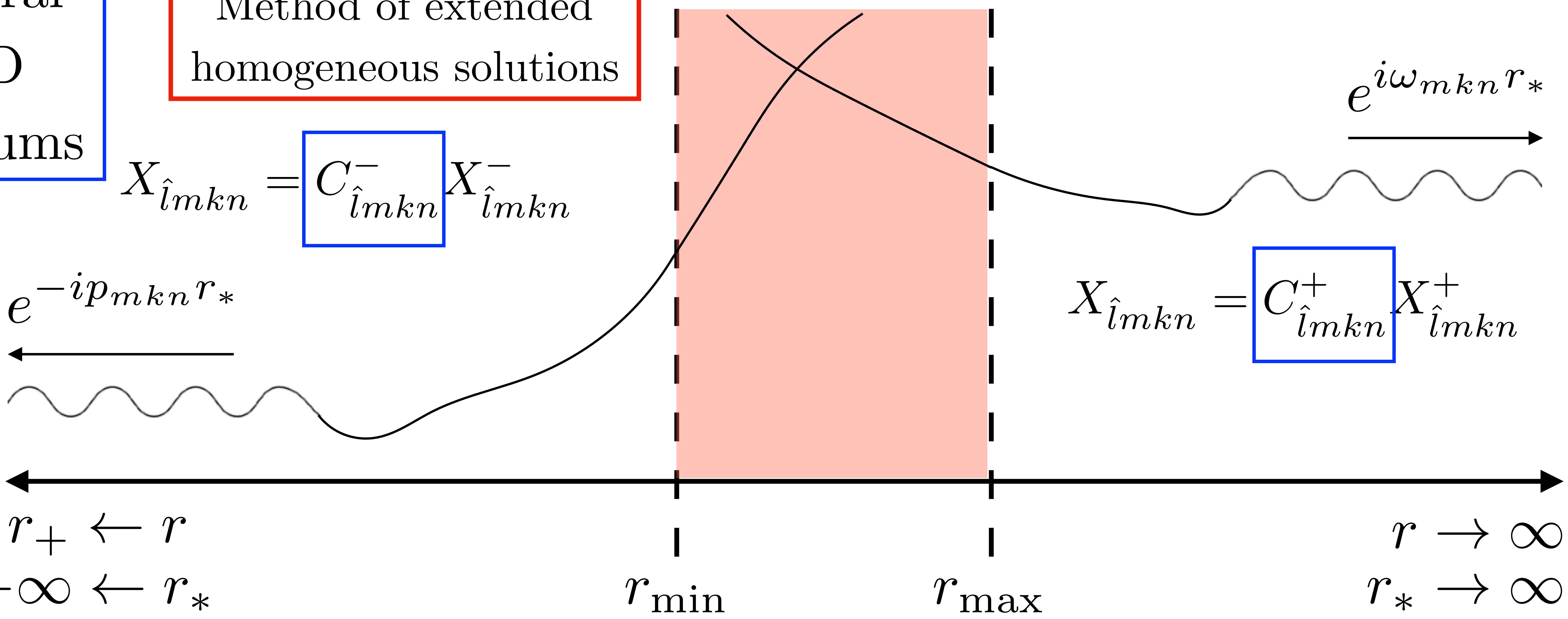


Building a generic SSF code

- Source integration & TD reconstruction

2D integral
→ 4 1D
Fourier sums

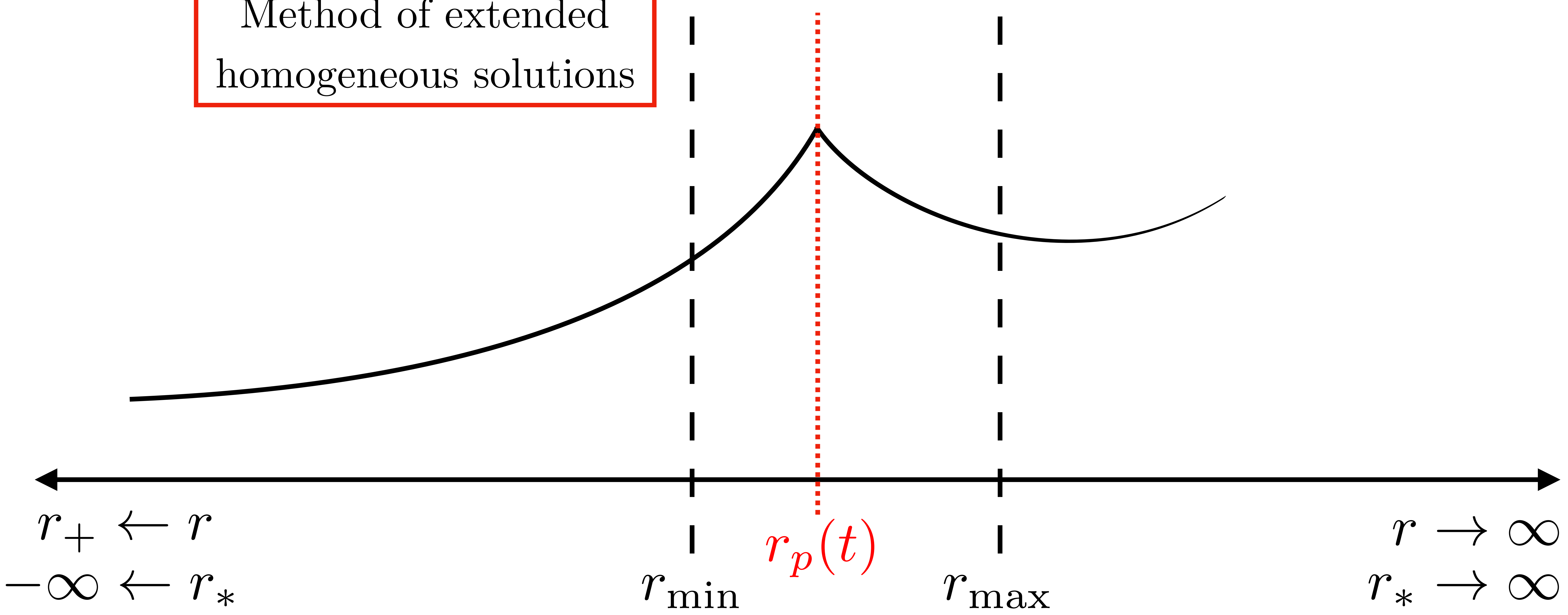
Method of extended
homogeneous solutions



Building a generic SSF code

- Source integration & TD reconstruction

Method of extended homogeneous solutions



Building a generic SSF code

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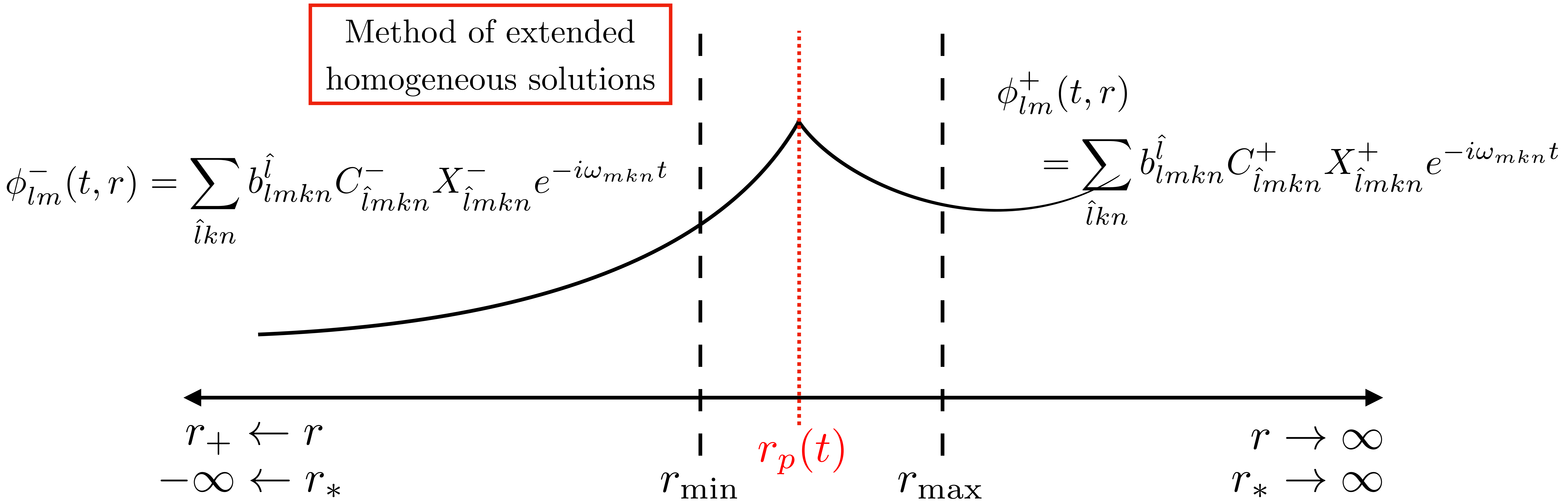


Building a generic SSF code

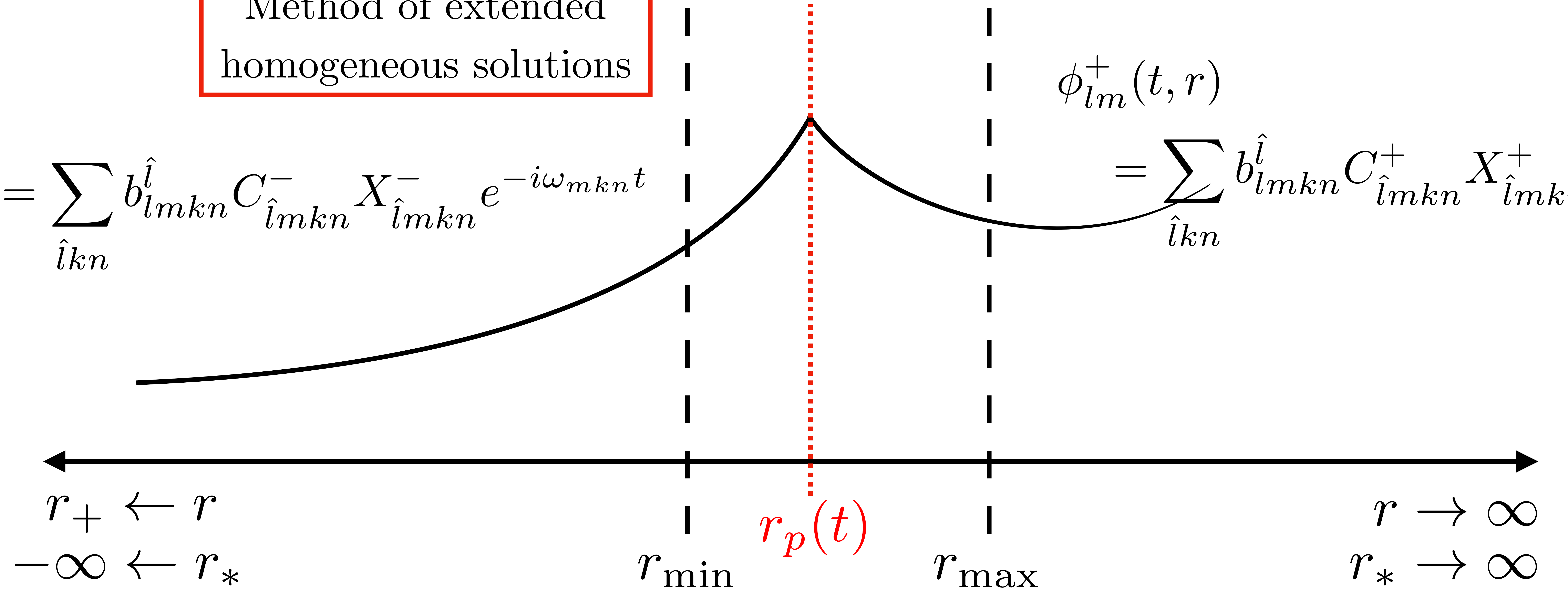
- Source integration & TD reconstruction

$$\Phi^{\text{ret}} = \sum_{lm} \phi^{\pm}_{lm}(t,r) Y_{lm}(\theta,\varphi)$$

Method of extended homogeneous solutions

$$\phi^{-}_{lm}(t,r) = \sum_{\hat{l}kn} b^{\hat{l}}_{lmkn} C^{-}_{\hat{l}mkn} X^{-}_{\hat{l}mkn} e^{-i\omega_{mkn}t}$$

$$\phi^{+}_{lm}(t,r) = \sum_{\hat{l}kn} b^{\hat{l}}_{lmkn} C^{+}_{\hat{l}mkn} X^{+}_{\hat{l}mkn} e^{-i\omega_{mkn}t}$$



Building a generic SSF code

- Mode-sum regularization

$$F_\mu = \sum_{l=0}^{\infty} (F_\mu^{\mathrm{ret},l} - F_\mu^{\mathrm{S},l})$$

$$F_\mu^{\mathrm{S},l} = A_\mu (l + 1/2) + B_\mu + \sum_{n=2}^{\infty} D_\mu^{2n} \left[\prod_{k=1}^{n-1} (2l - 2k + 1)(2l + 2k + 1) \right]^{-1}$$

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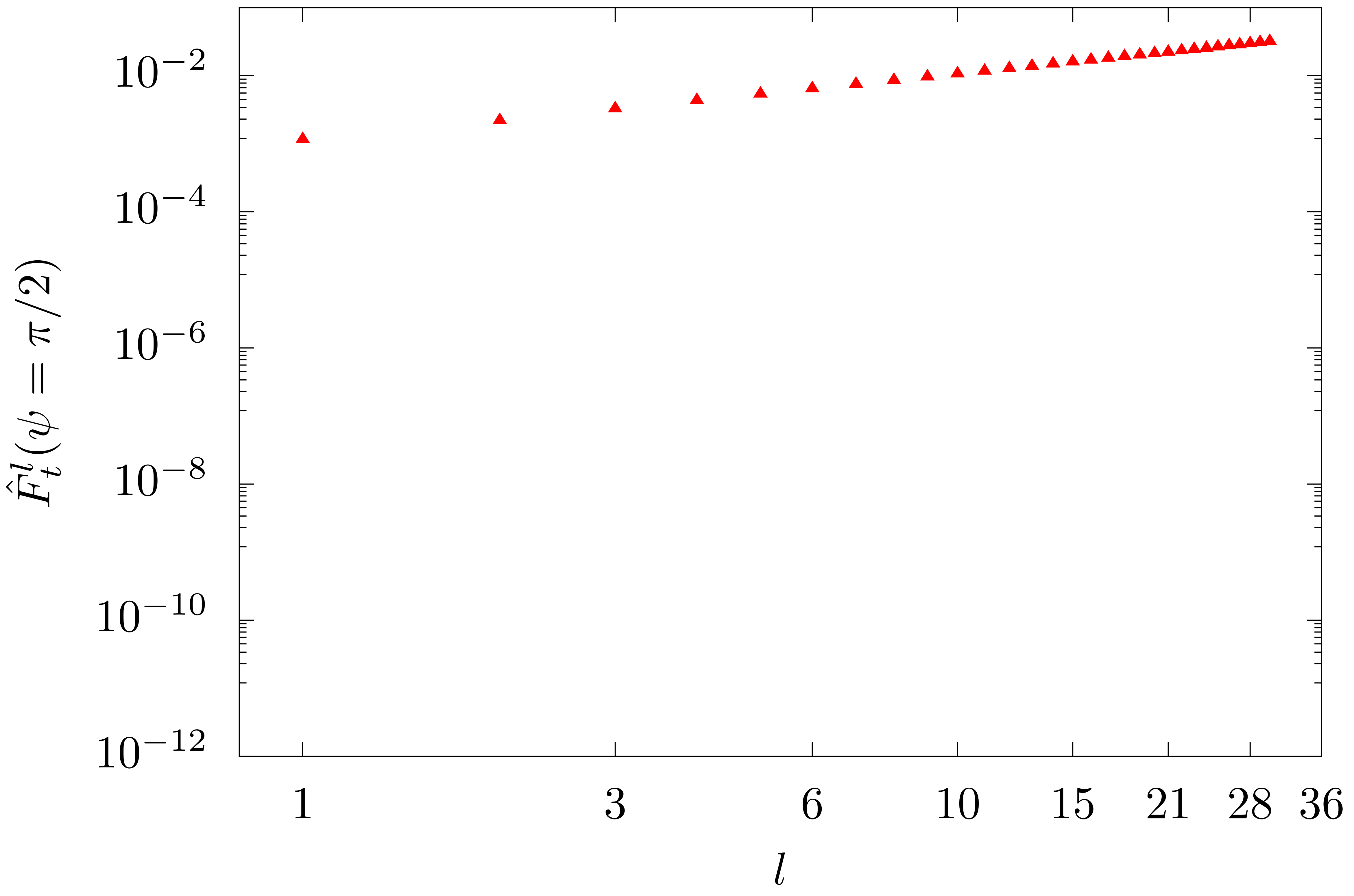

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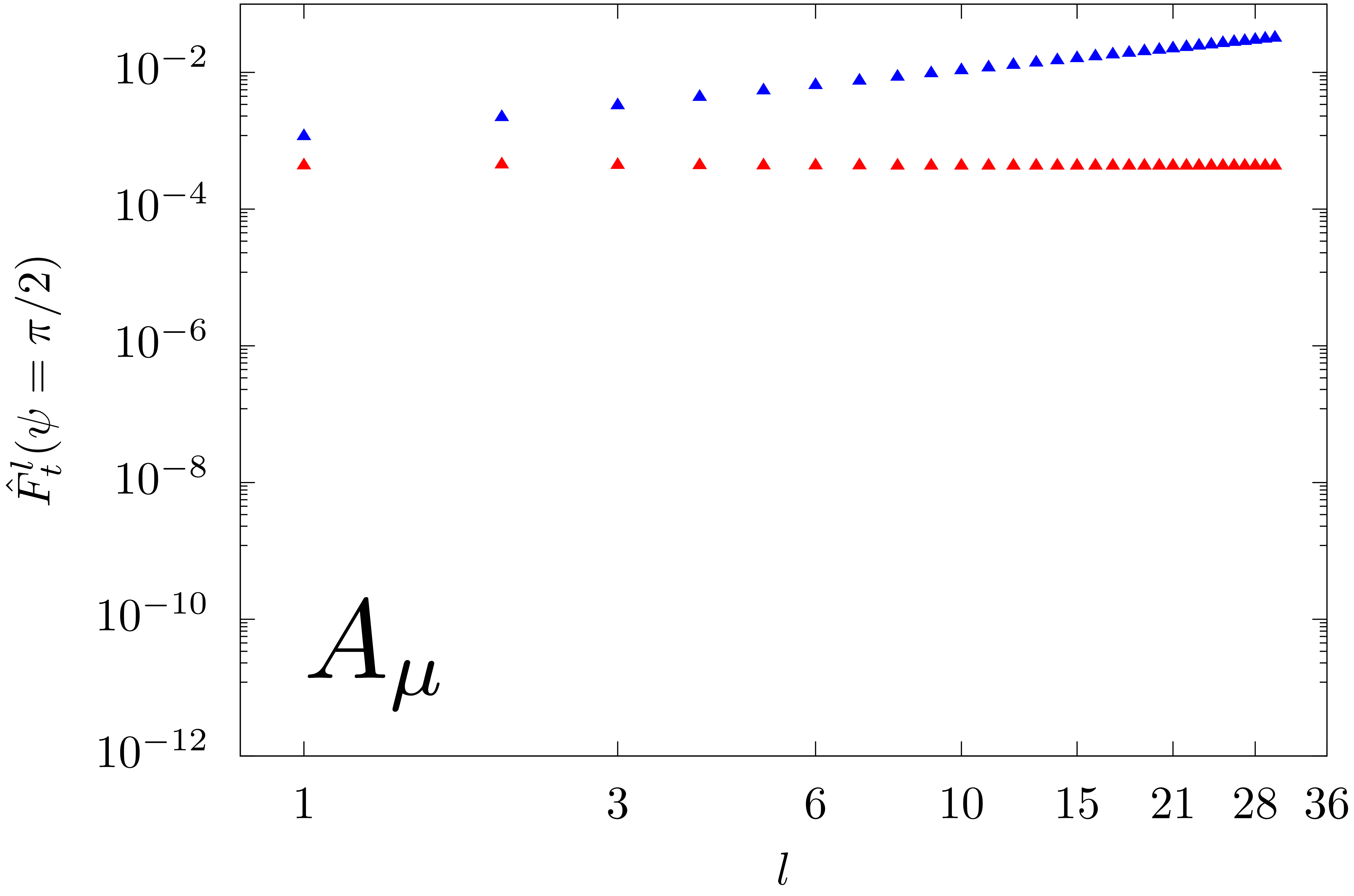

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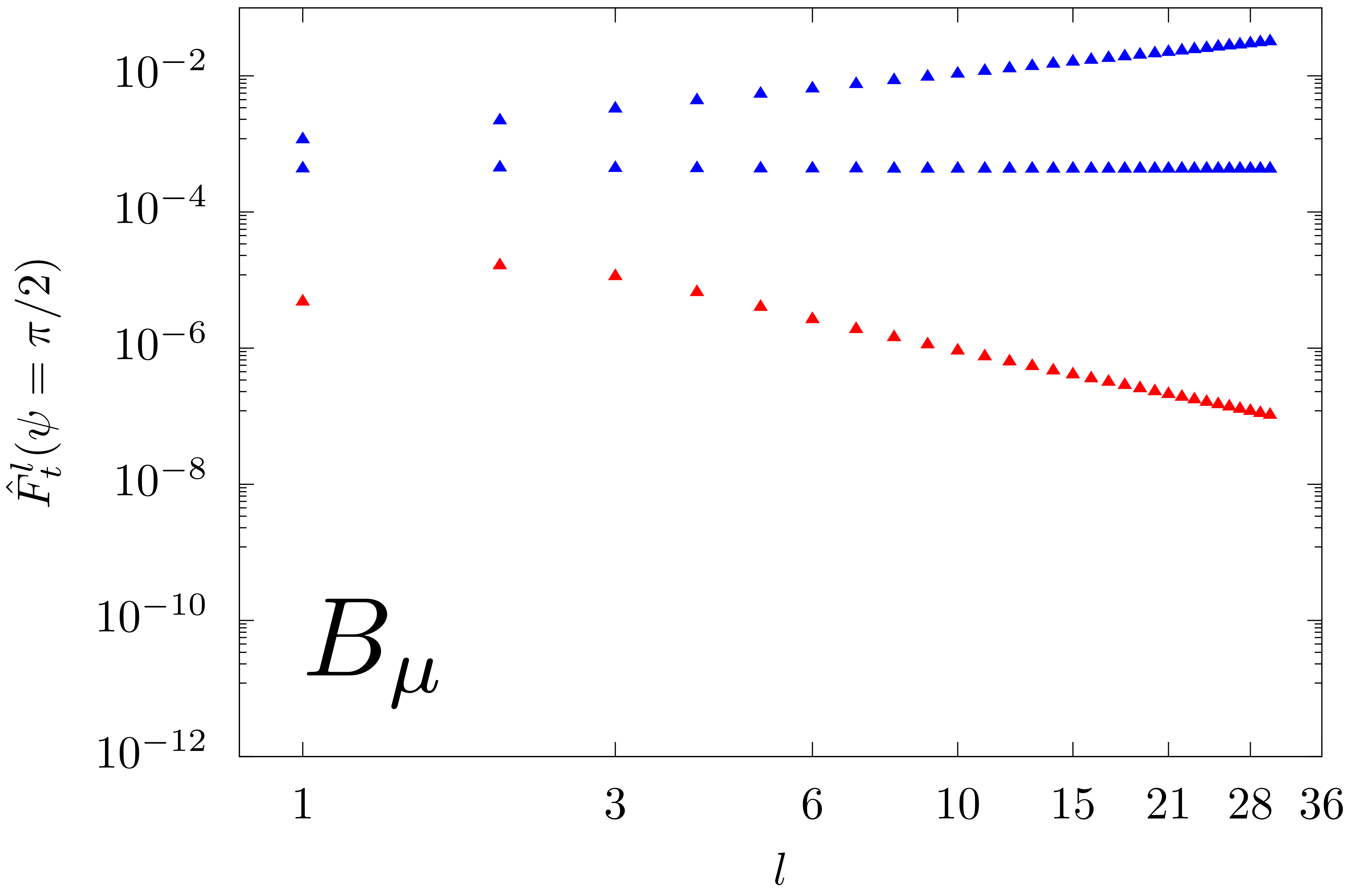

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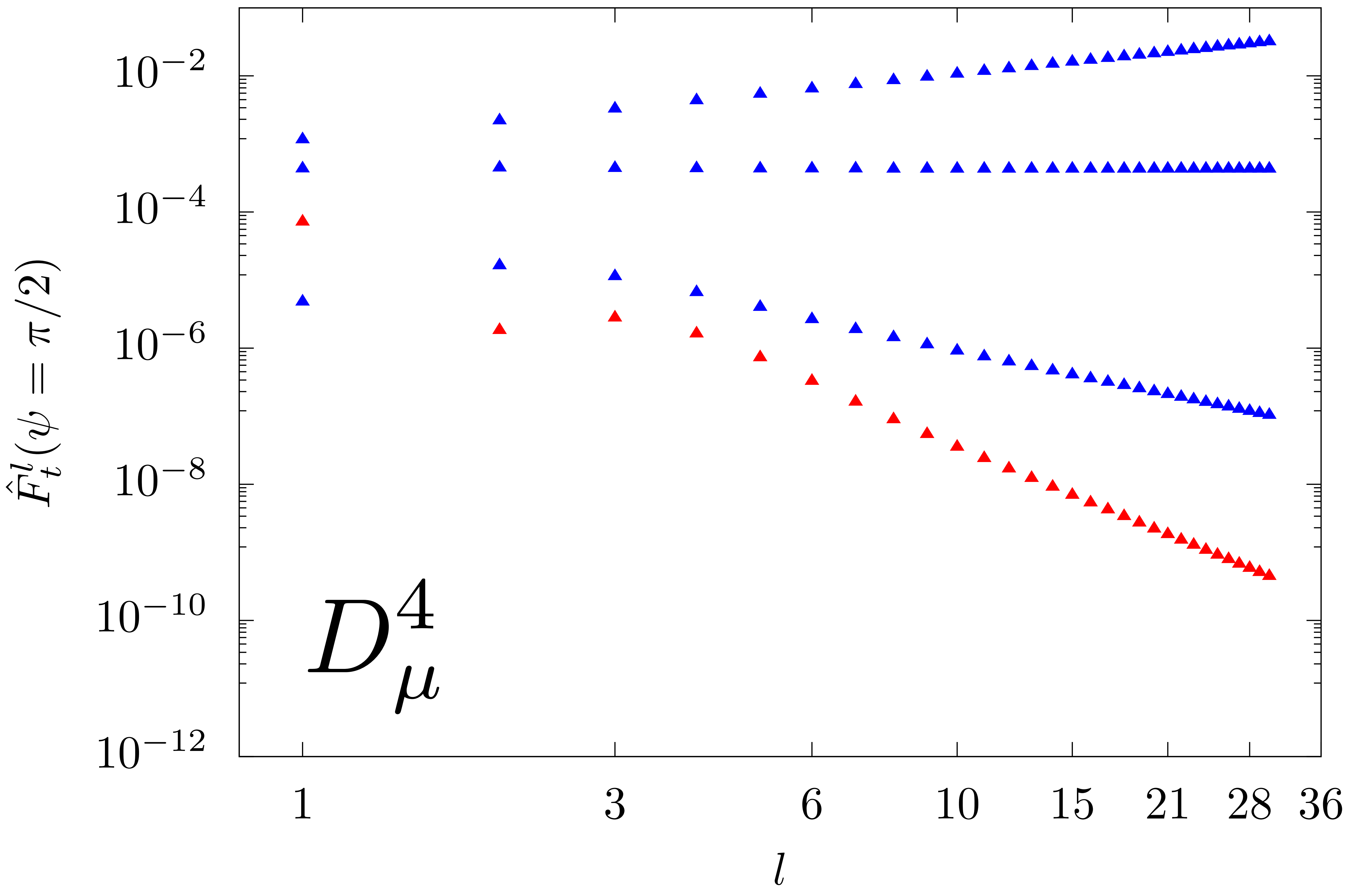

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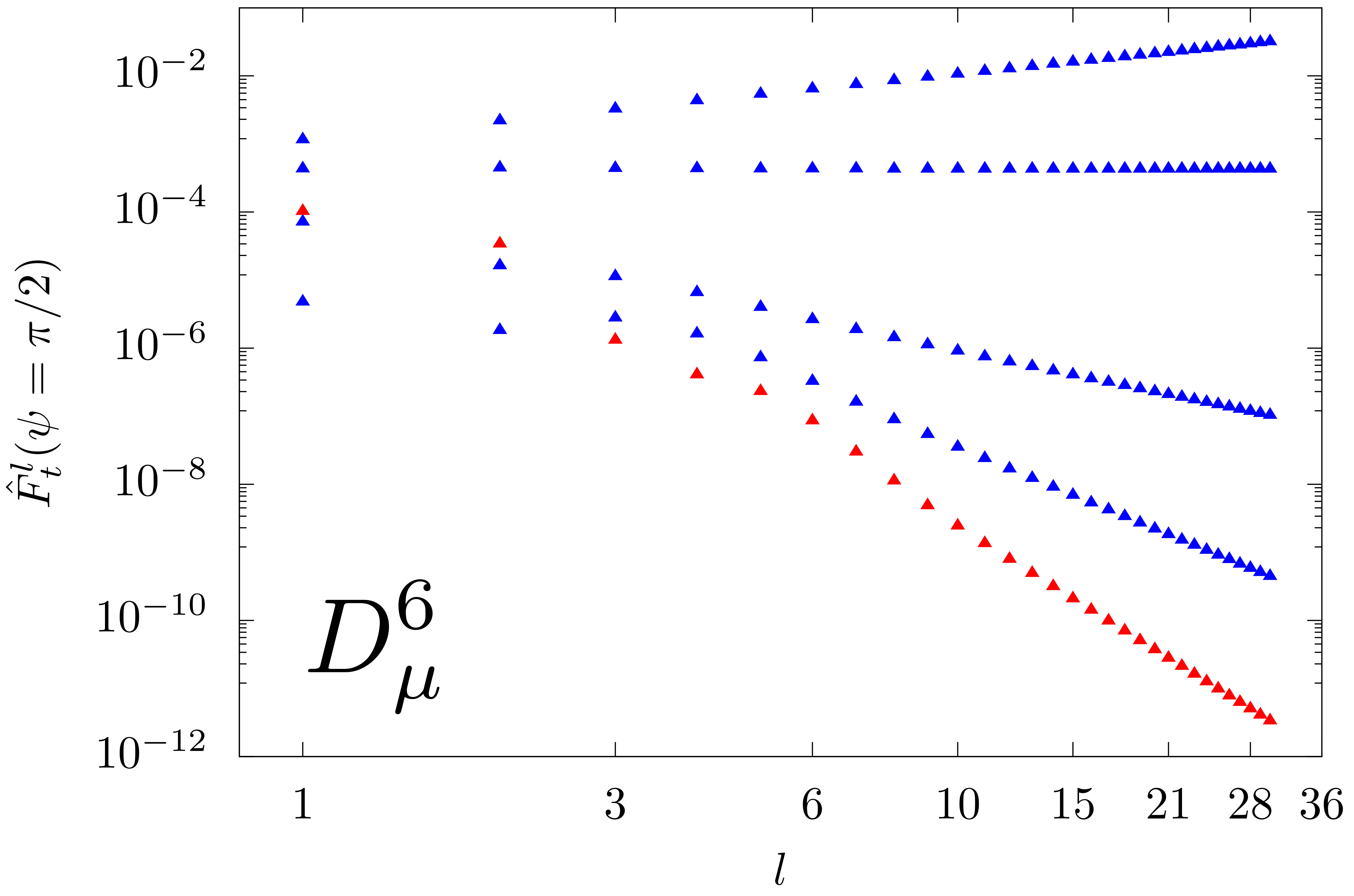

Building a generic SSF code

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Higher-order RPs known for equatorial orbits

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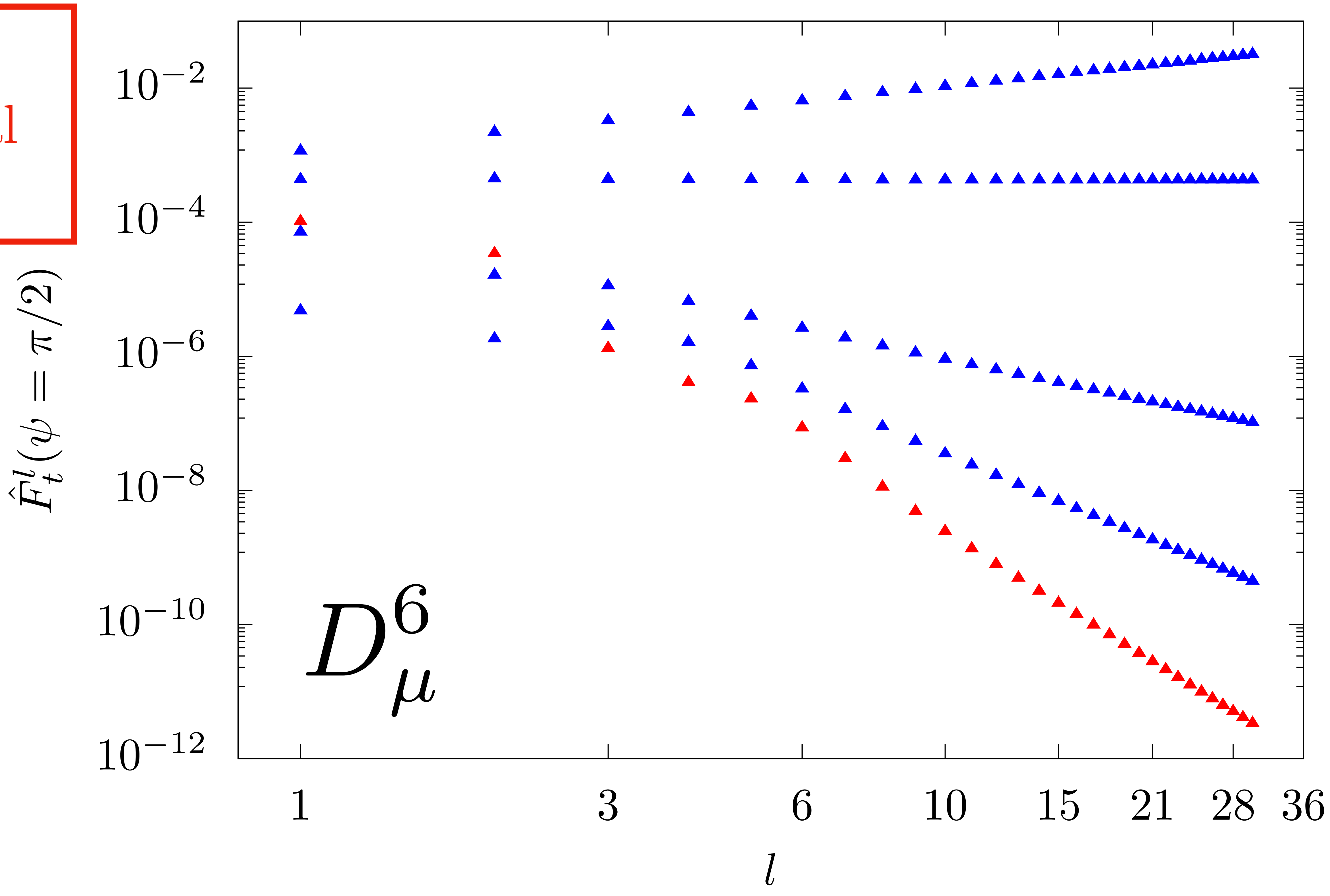

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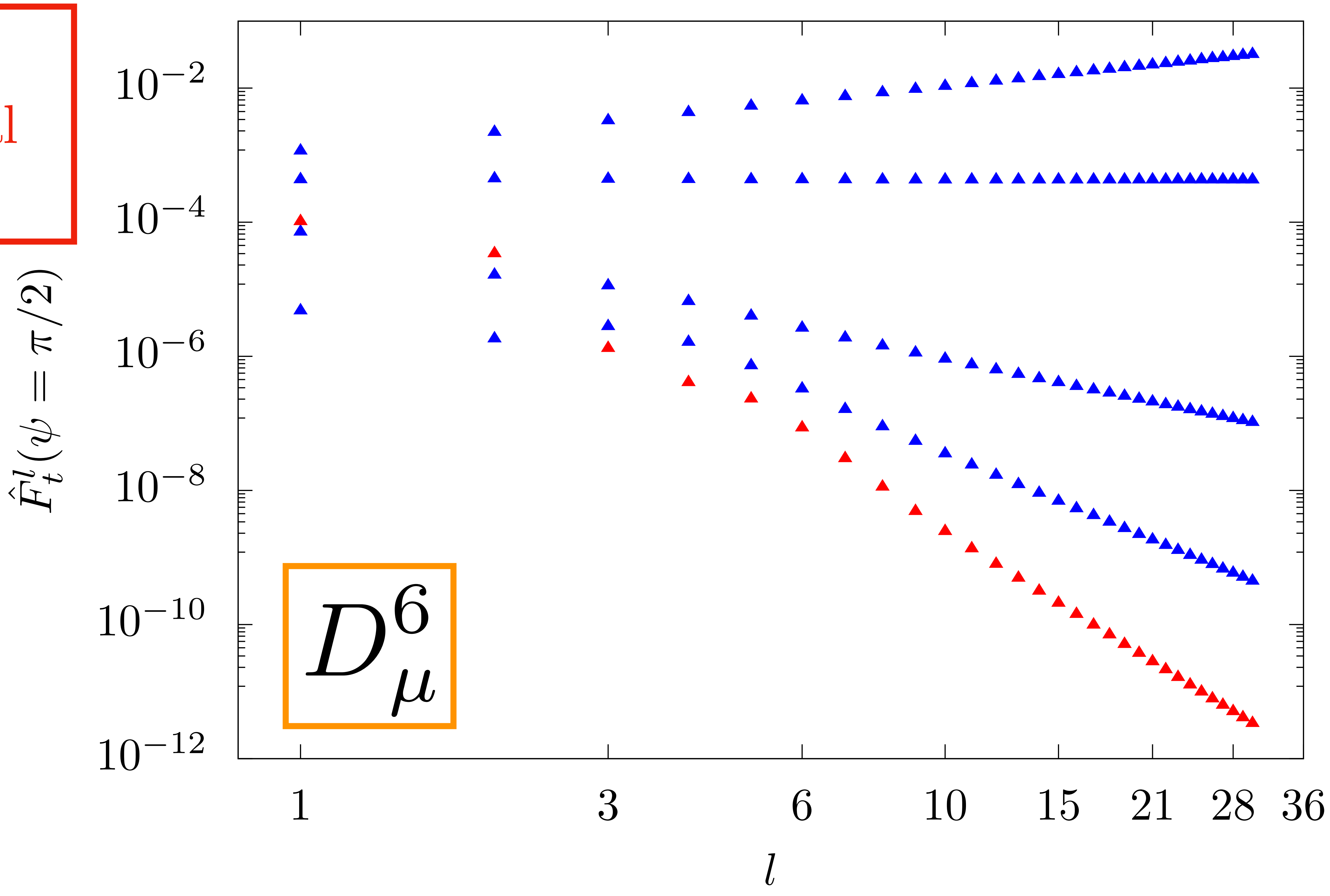
Higher-order parameters for generic orbits?
See Anna Heffernan's talk Tuesday



Schwarzschild test

- **Equatorial v. Inclined**

$p = 10, e = 0.5, \iota = 0, a = 0$

$p = 10, e = 0.5, \iota = \pi/5, a = 0$

Schwarzschild test

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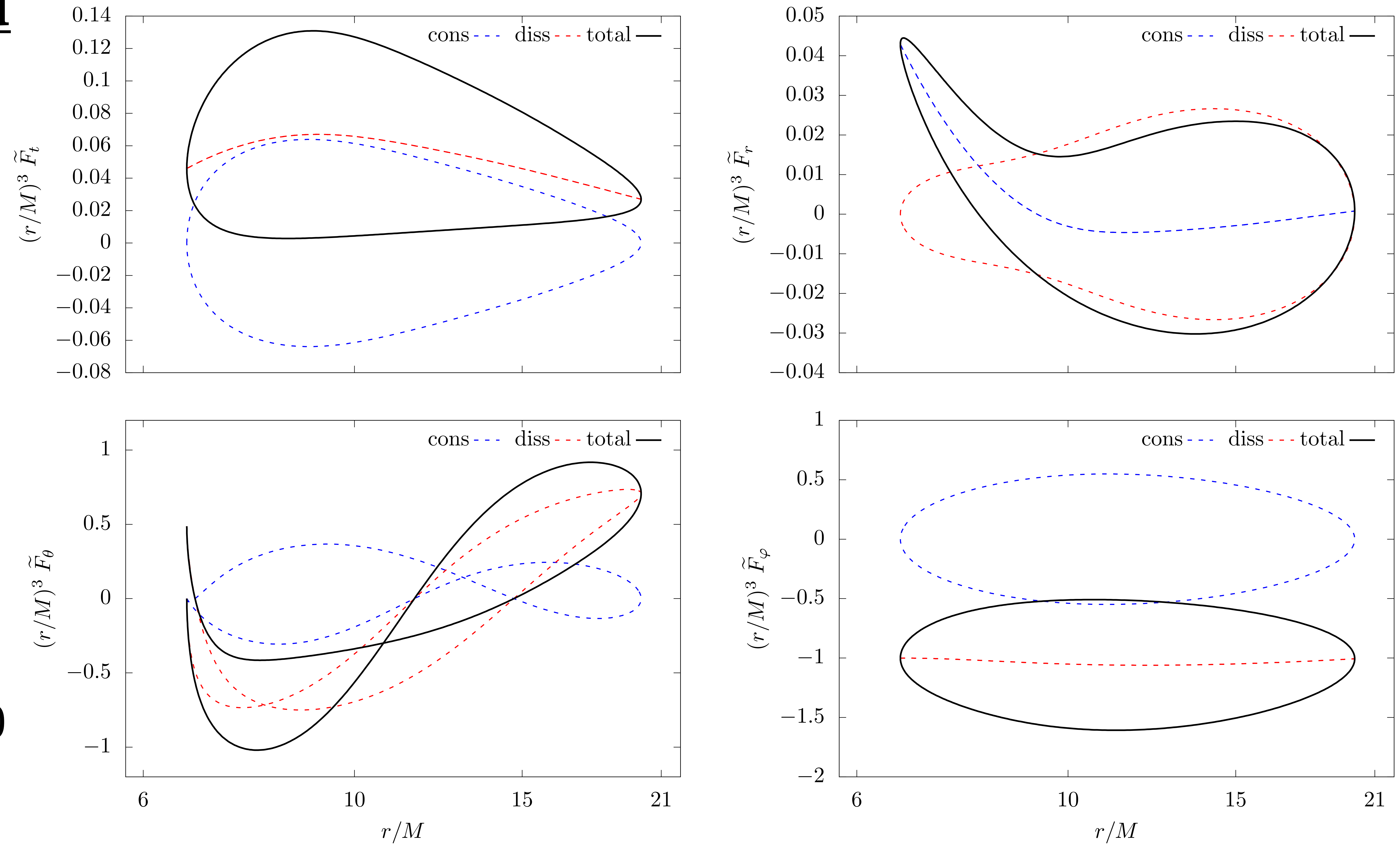

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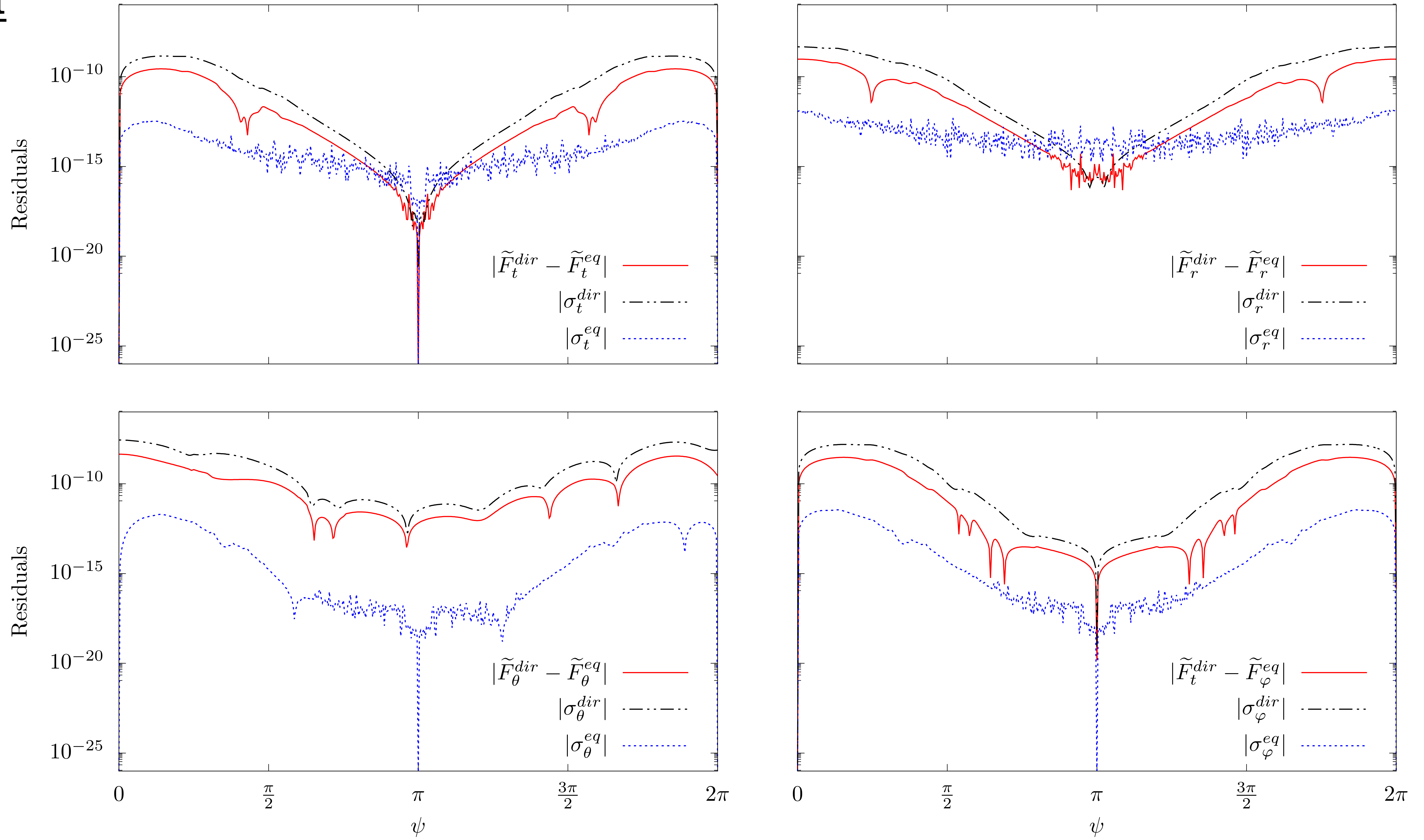
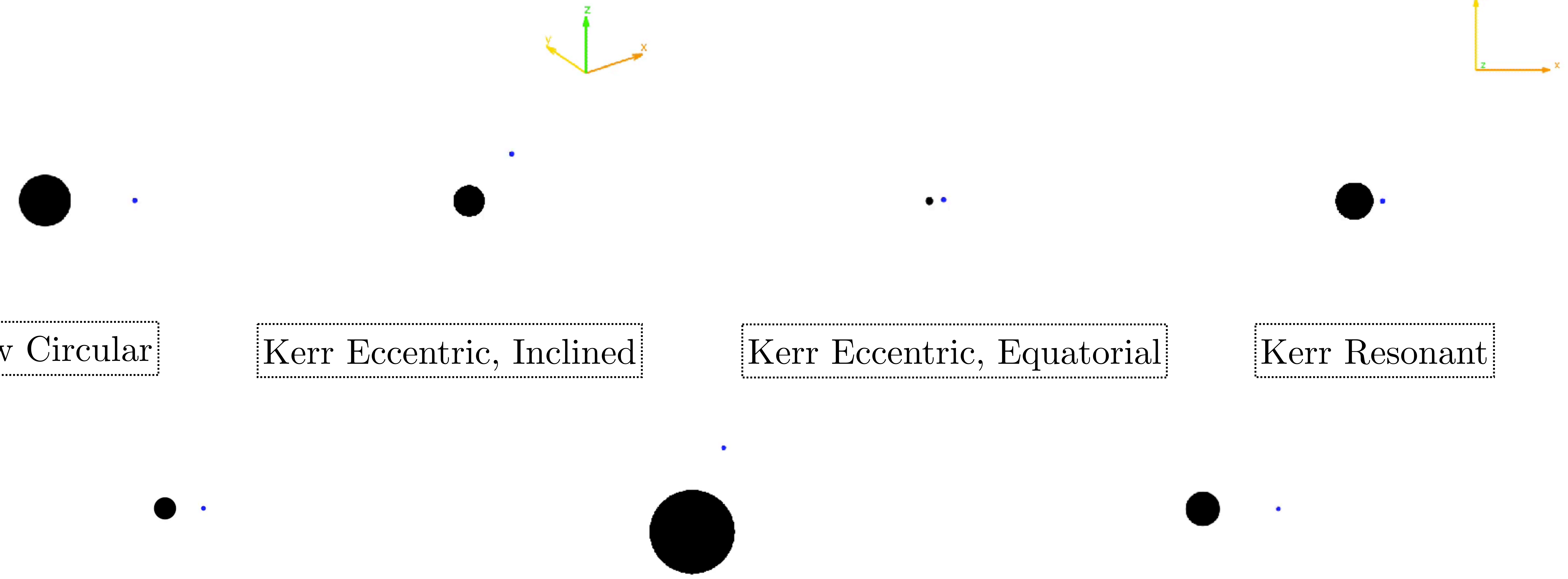


Orbits analyzed by my SSF code



Schw Circular

Kerr Eccentric, Inclined

Kerr Eccentric, Equatorial

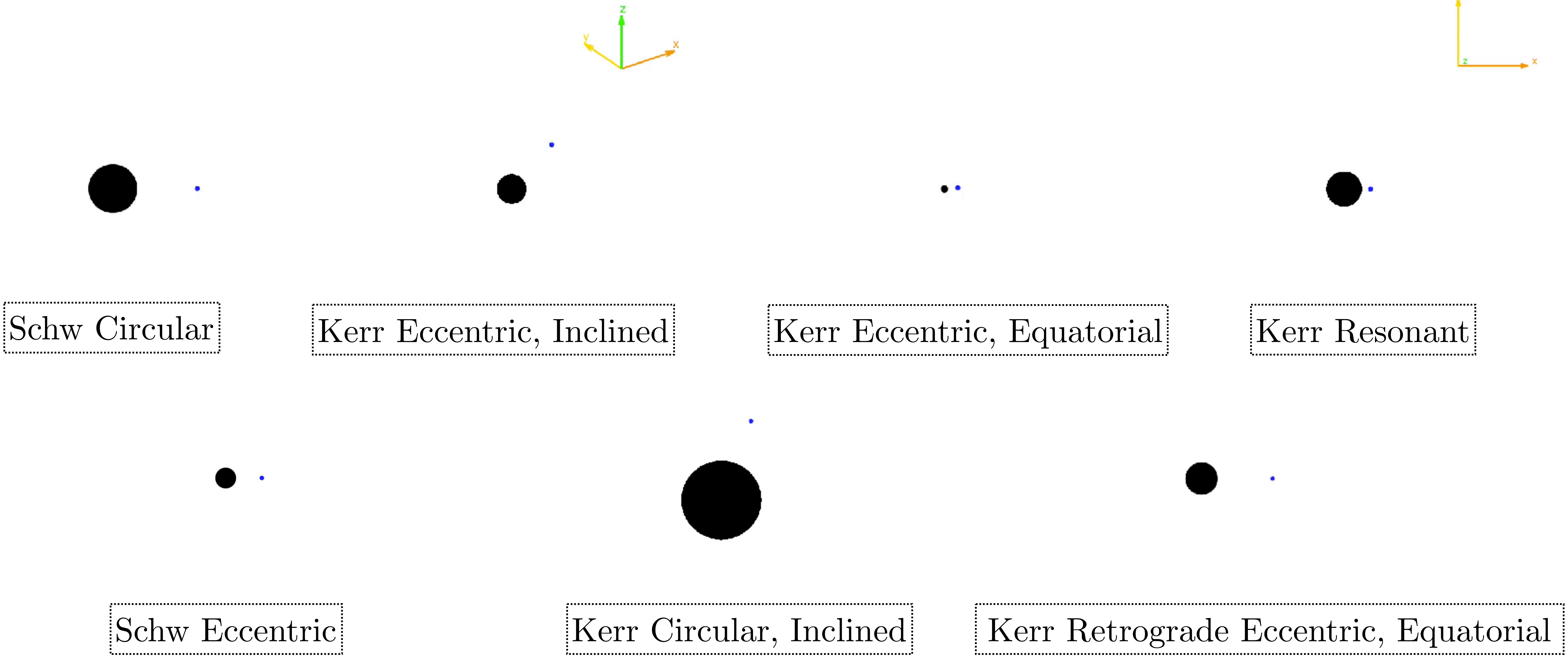
Kerr Resonant

Schw Eccentric

Kerr Circular, Inclined

Kerr Retrograde Eccentric, Equatorial

Orbits analyzed by my SSF code



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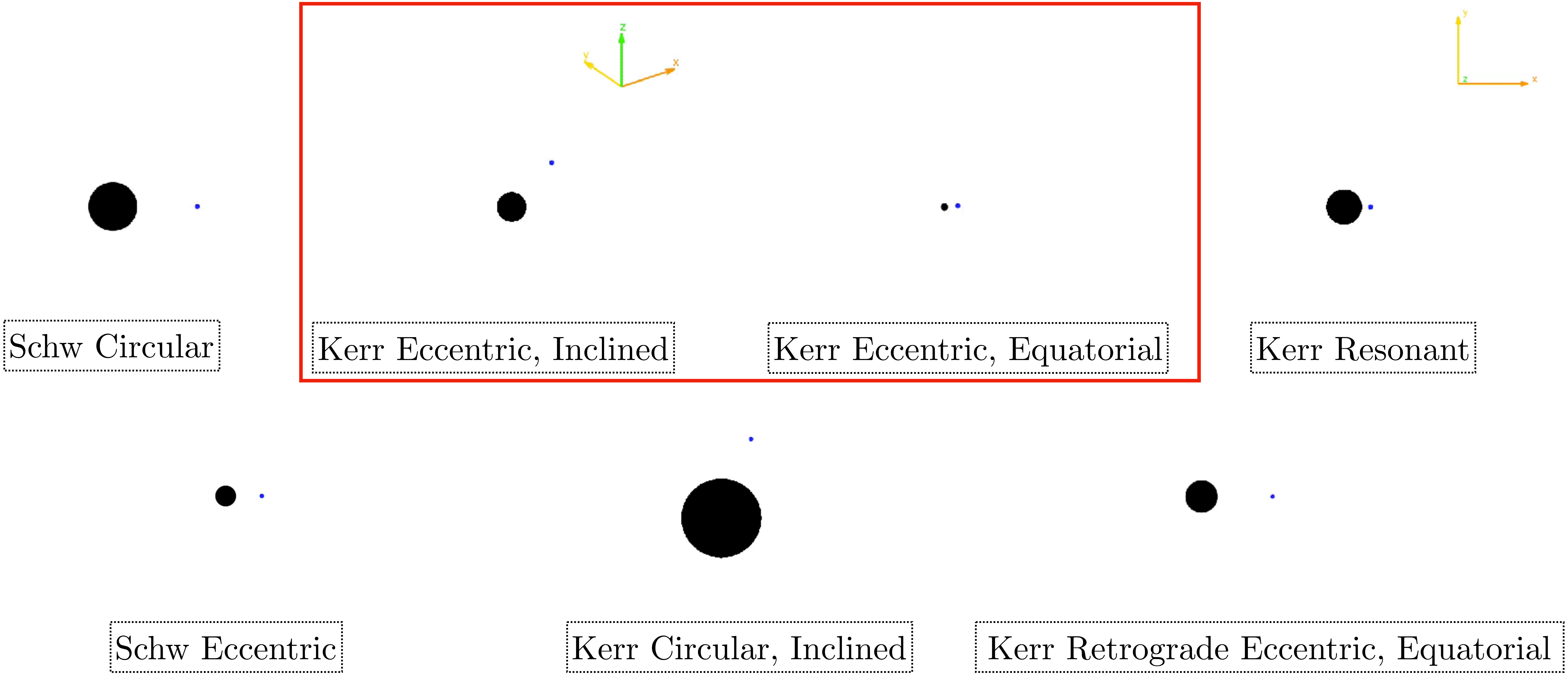

Highly eccentric orbit

$p = 8, e = 0.8, \iota = 0, a = 0.99M$

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[1610.09319]

- Inspired by Thornburg & Wardell (2017): Time-domain, equatorial Kerr SSF code

Highly eccentric orbit

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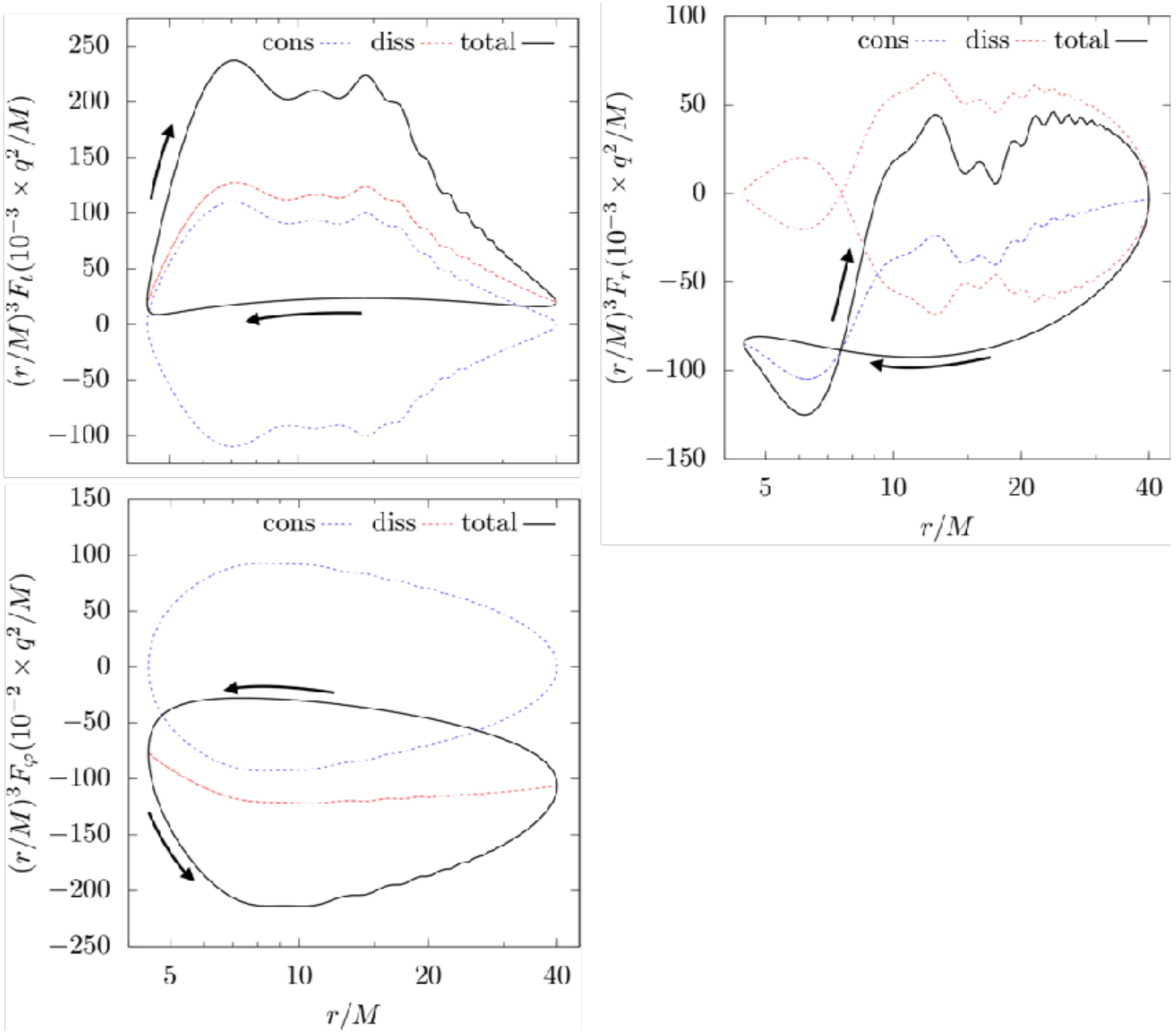

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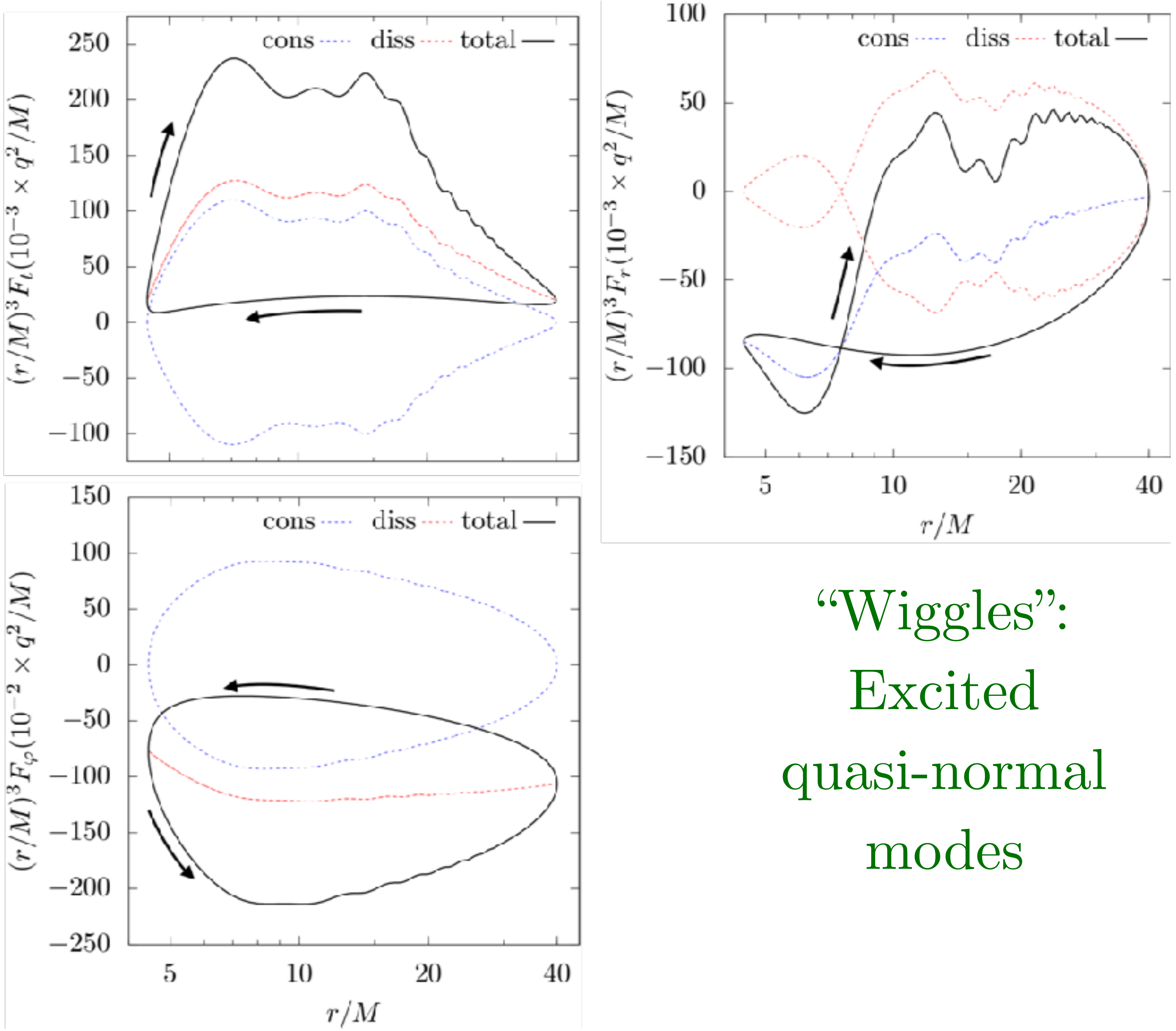
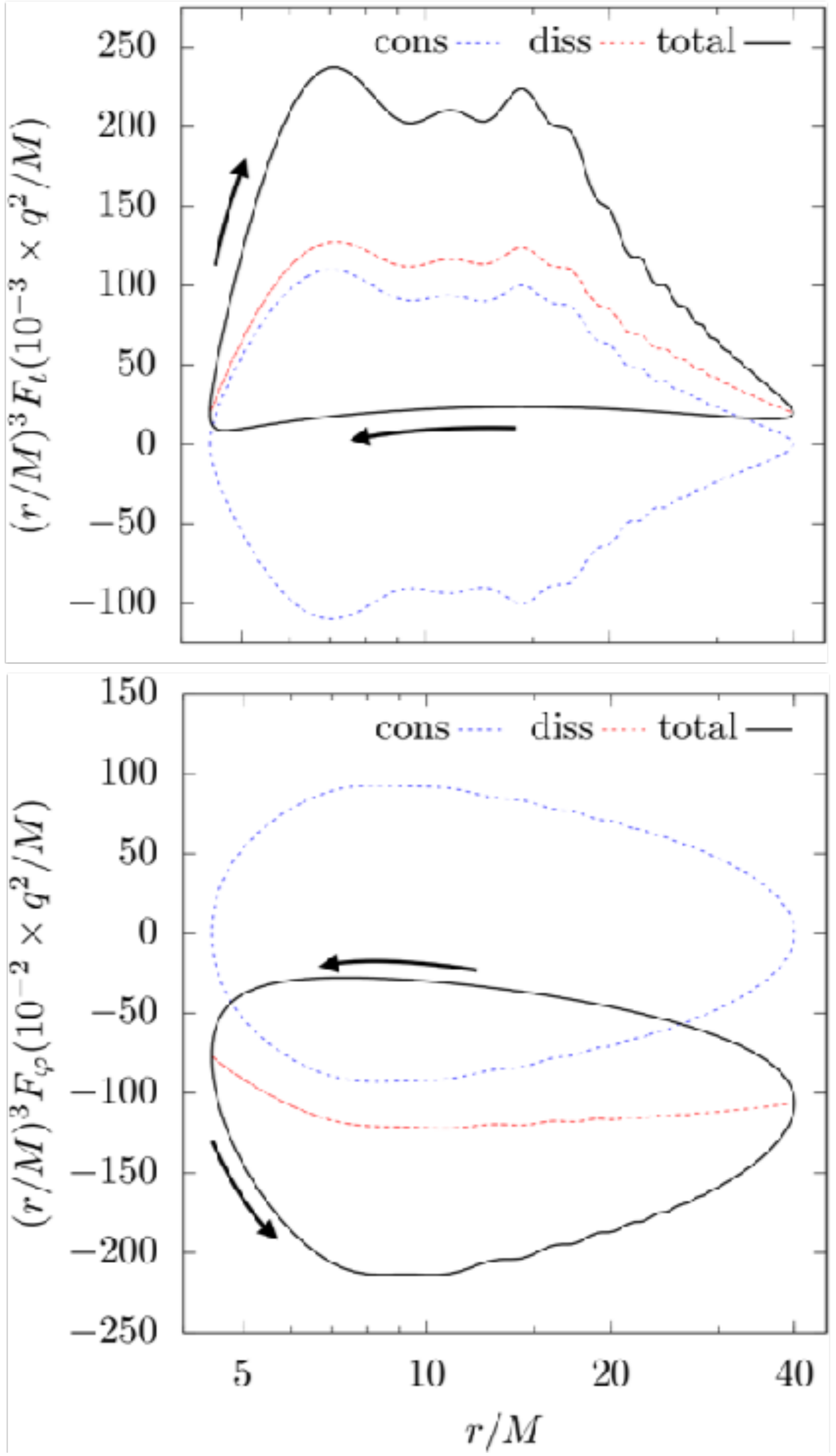


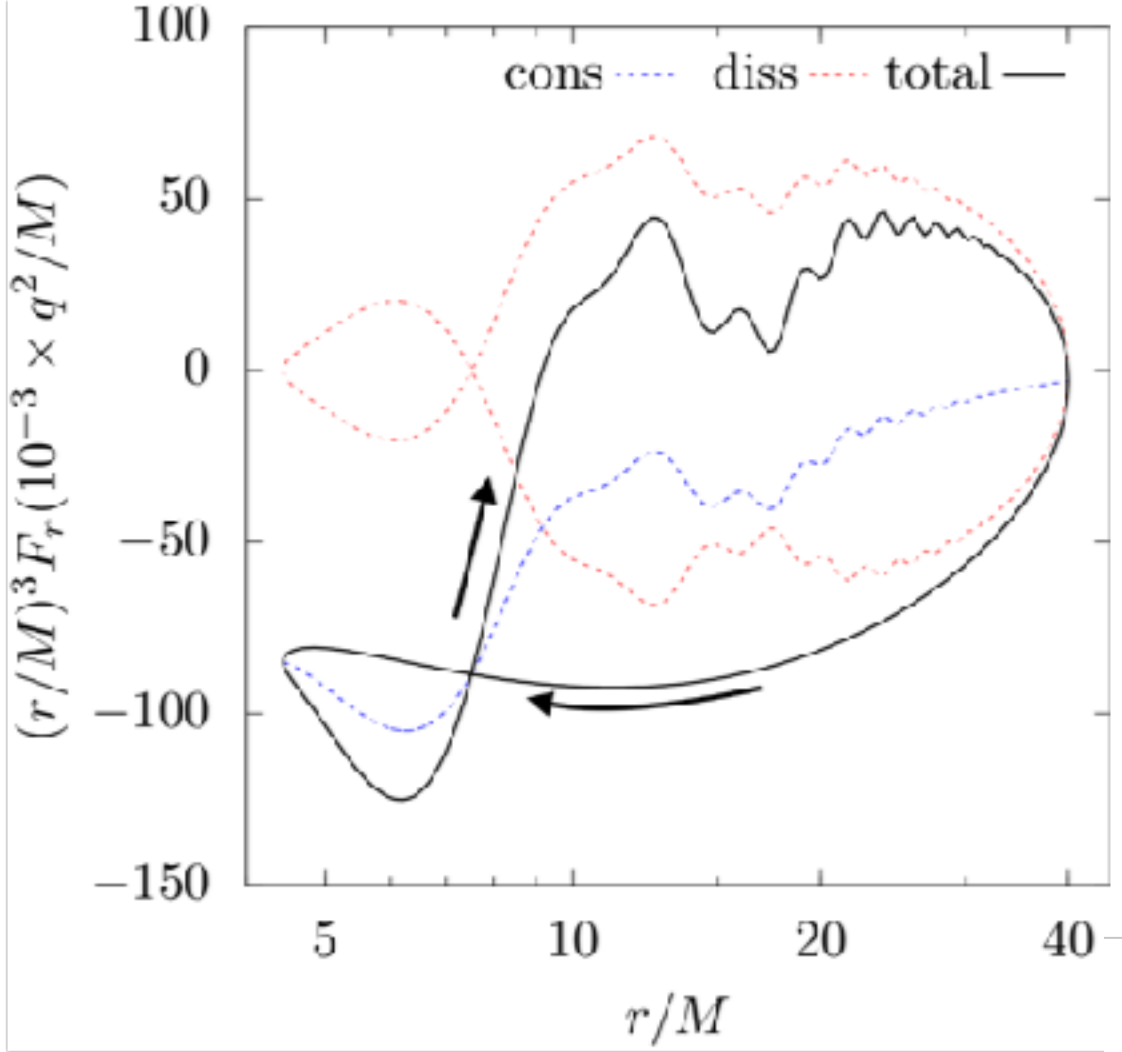
"Wiggles": Excited quasi-normal modes

Highly eccentric orbit

$p = 8, e = 0.8, \iota = 0, a = 0.99M$

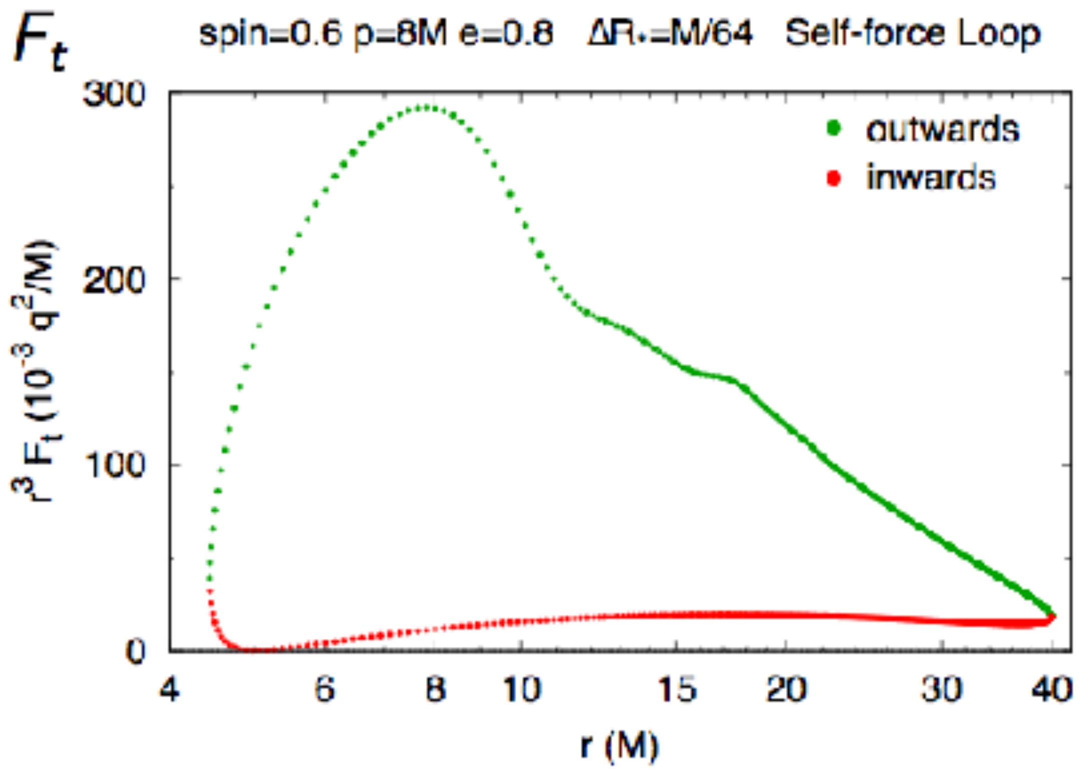
[1610.09319]



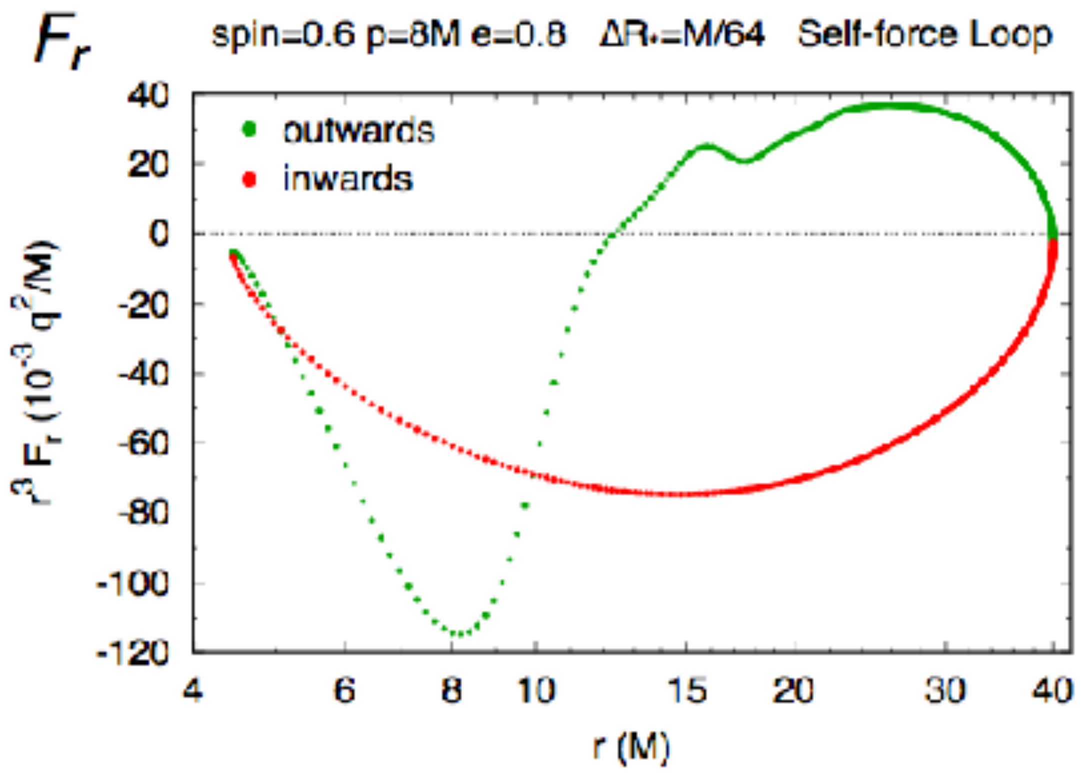


- Inspired by Thornburg & Wardell (2017): Time-domain, equatorial Kerr SSF code

$$p = 8, e = 0.8, \iota = 0, a = 0.6M$$



“Wiggles”: Excited quasi-normal modes



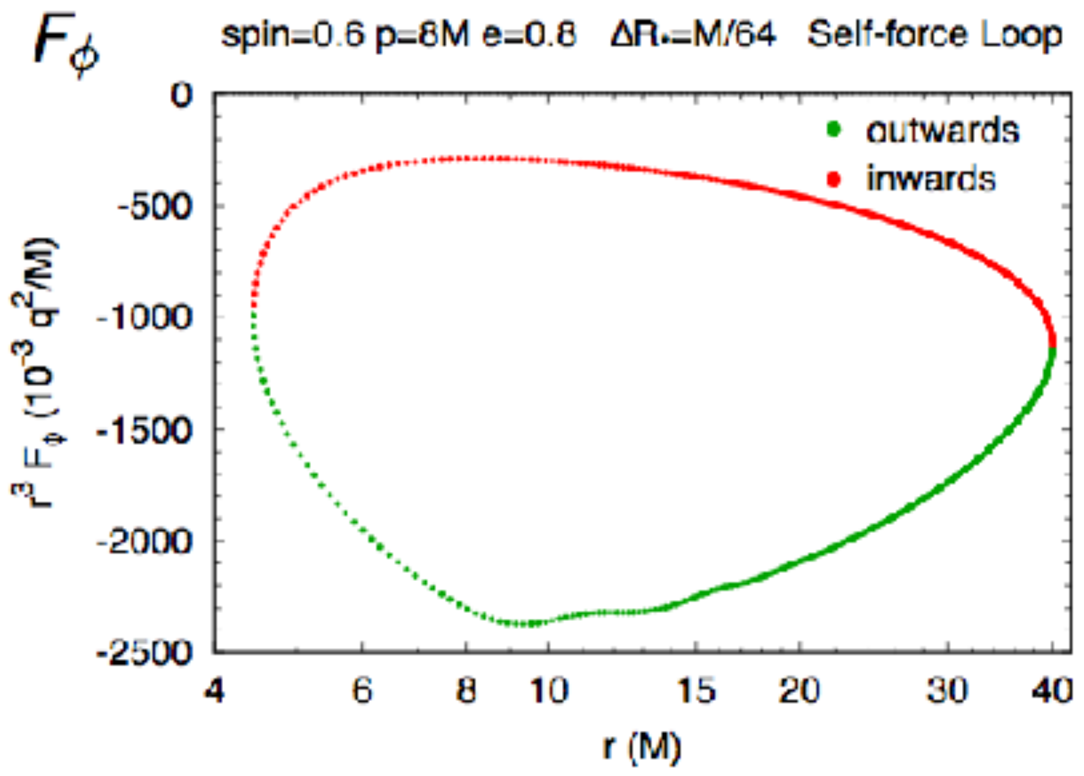


Image Credit: Thornburg 20th Capra

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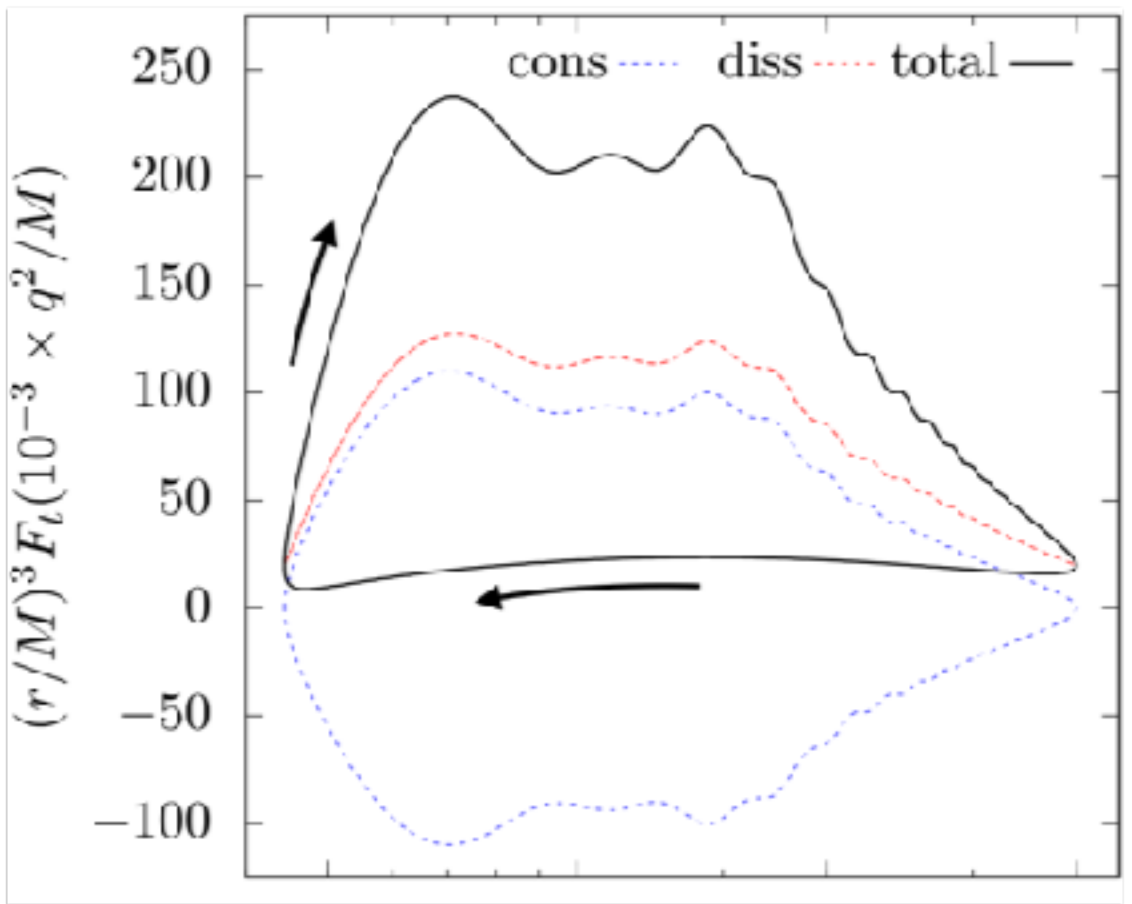
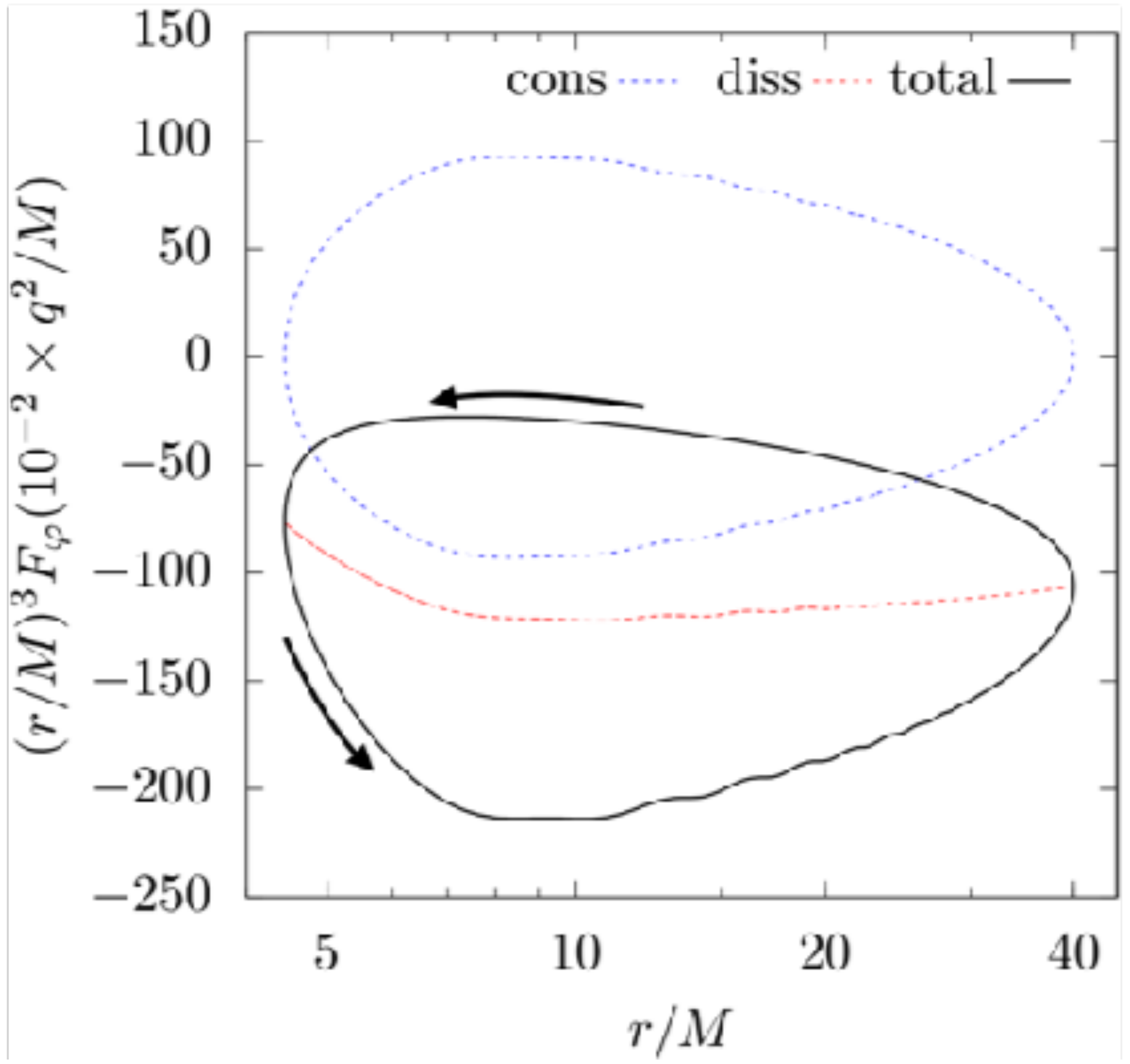
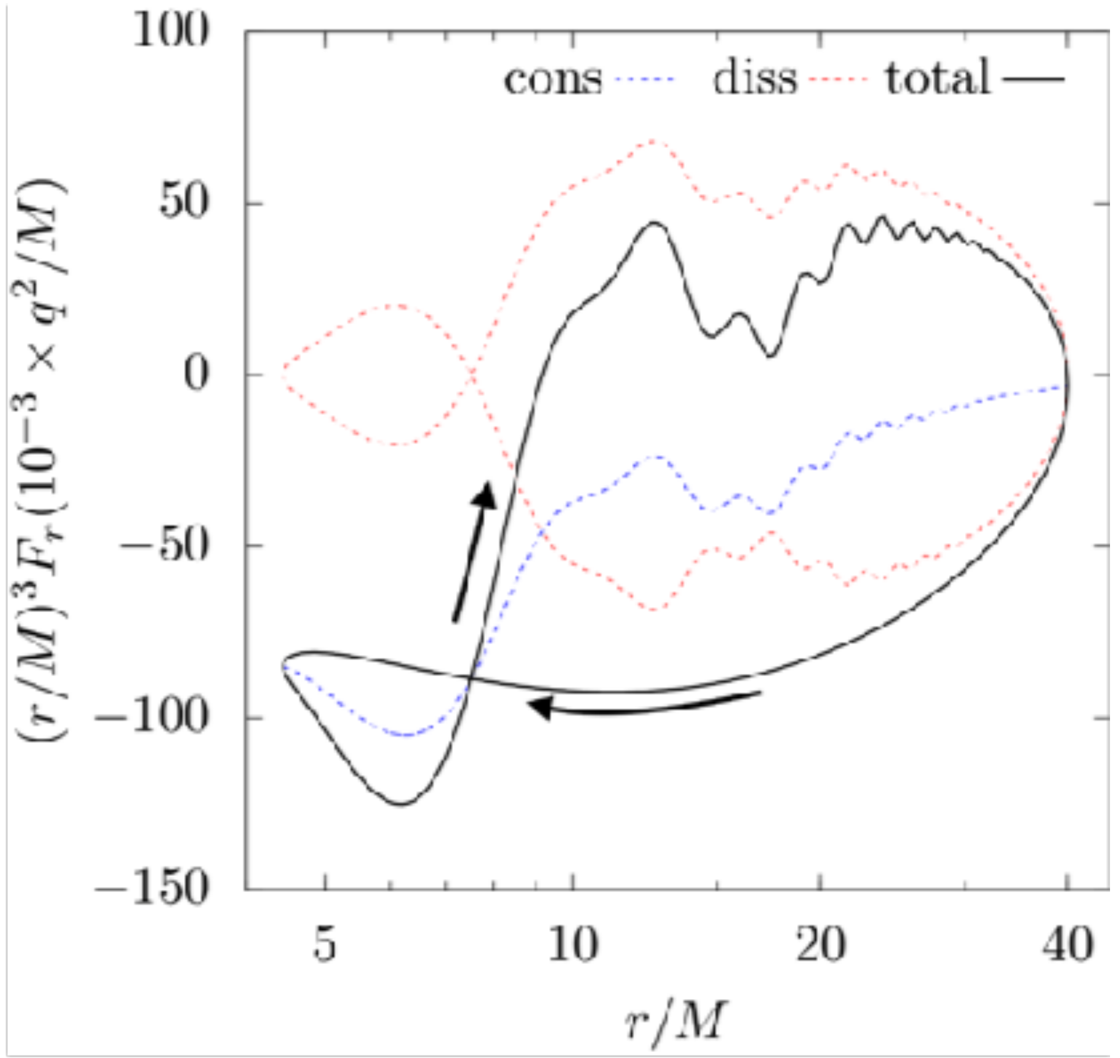
[1610.09319]



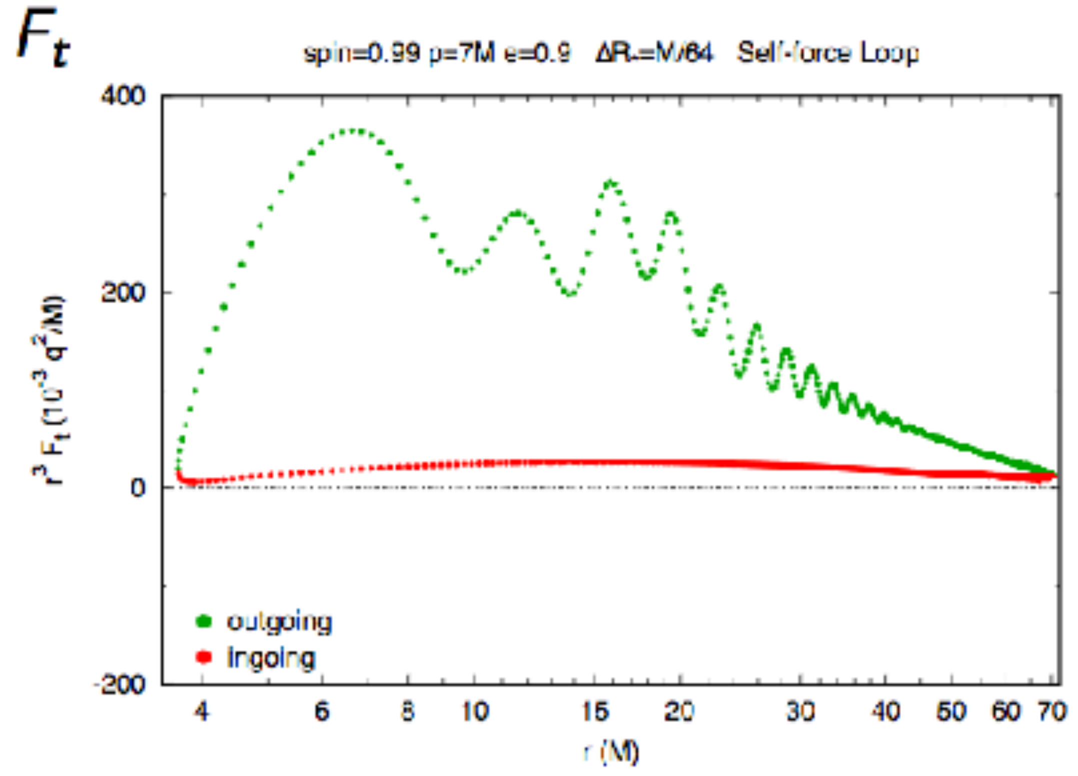


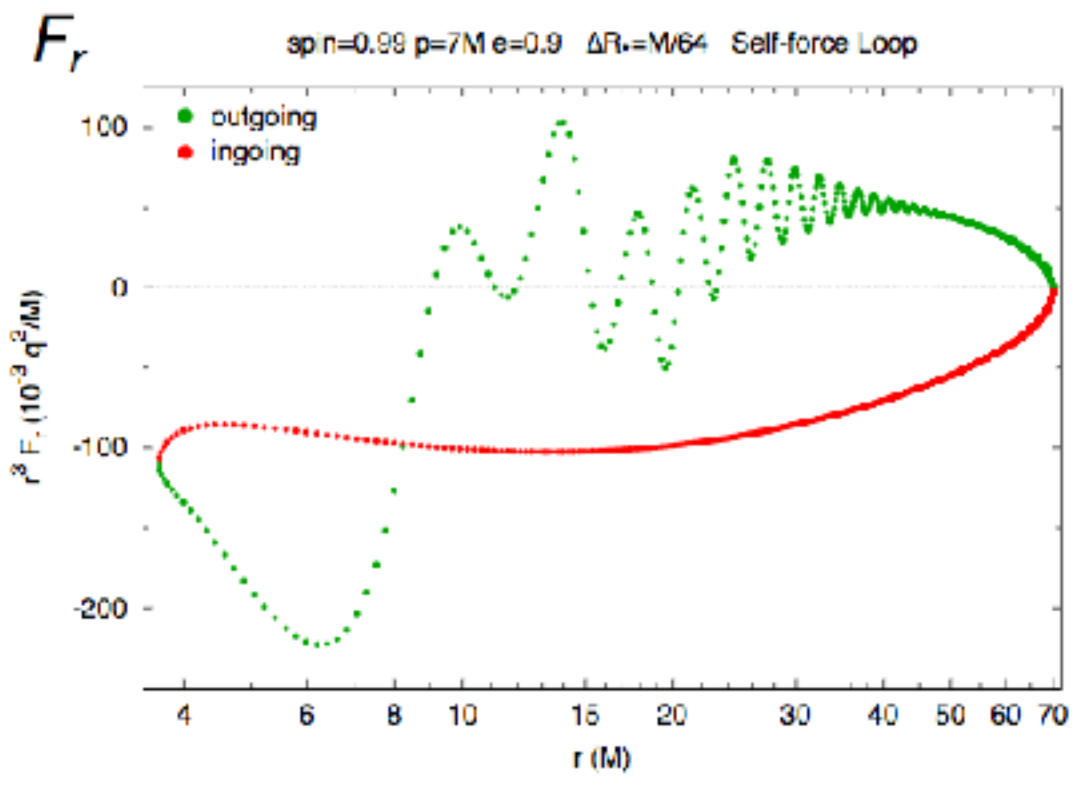


"Wiggles": Excited quasi-normal modes

- Inspired by Thornburg & Wardell (2017): Time-domain, equatorial Kerr SSF code

$p = 7, e = 0.9, \iota = 0, a = 0.99M$





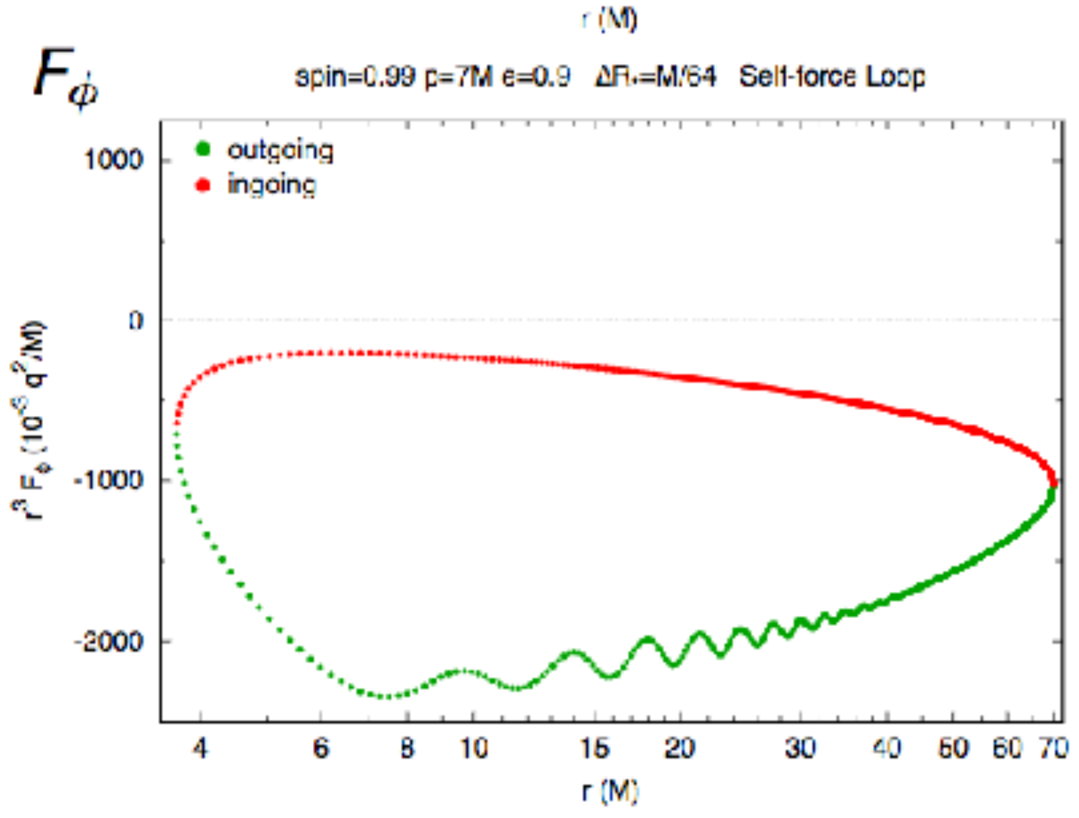


Image Credit: Thornburg 20th Capra

Generic orbit

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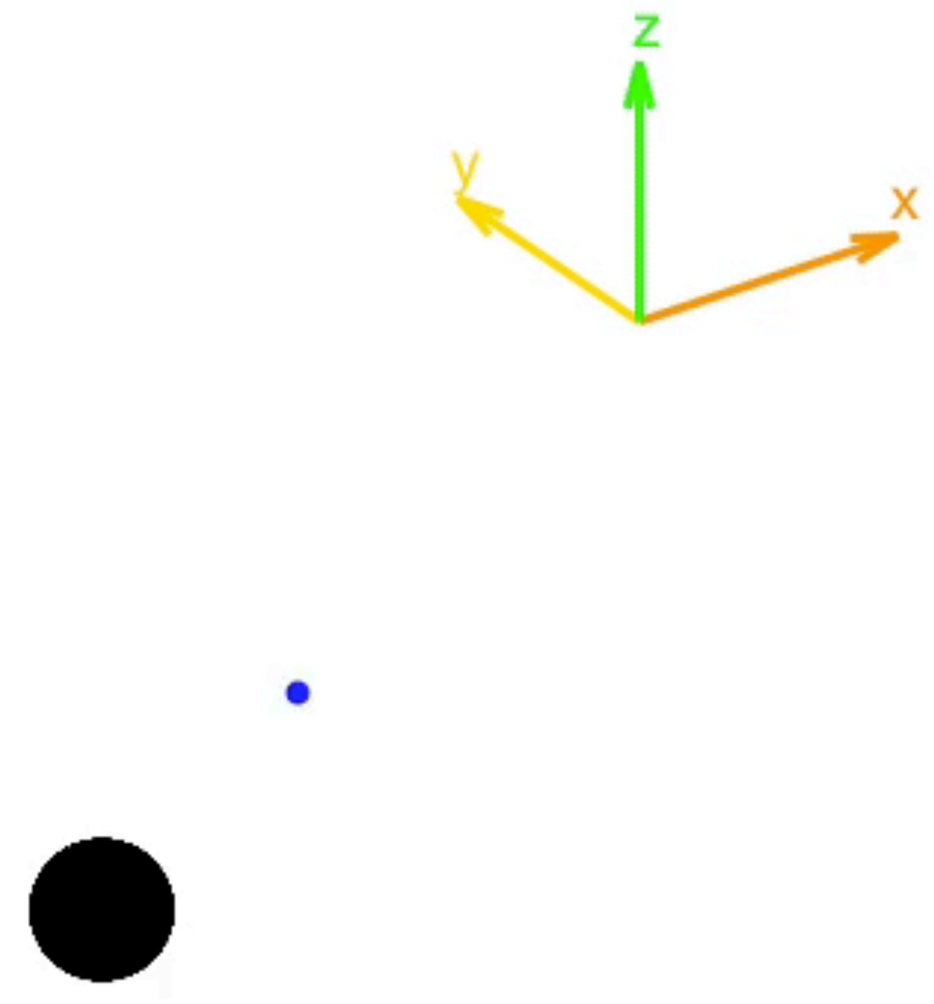

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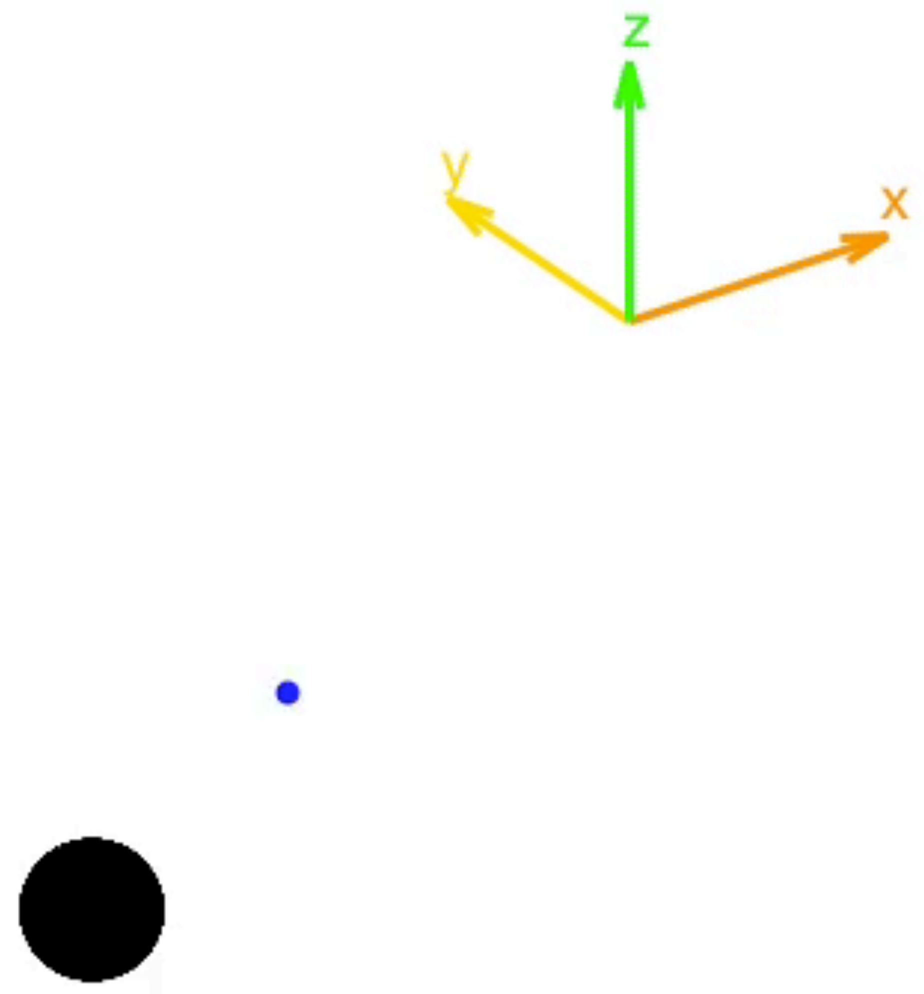

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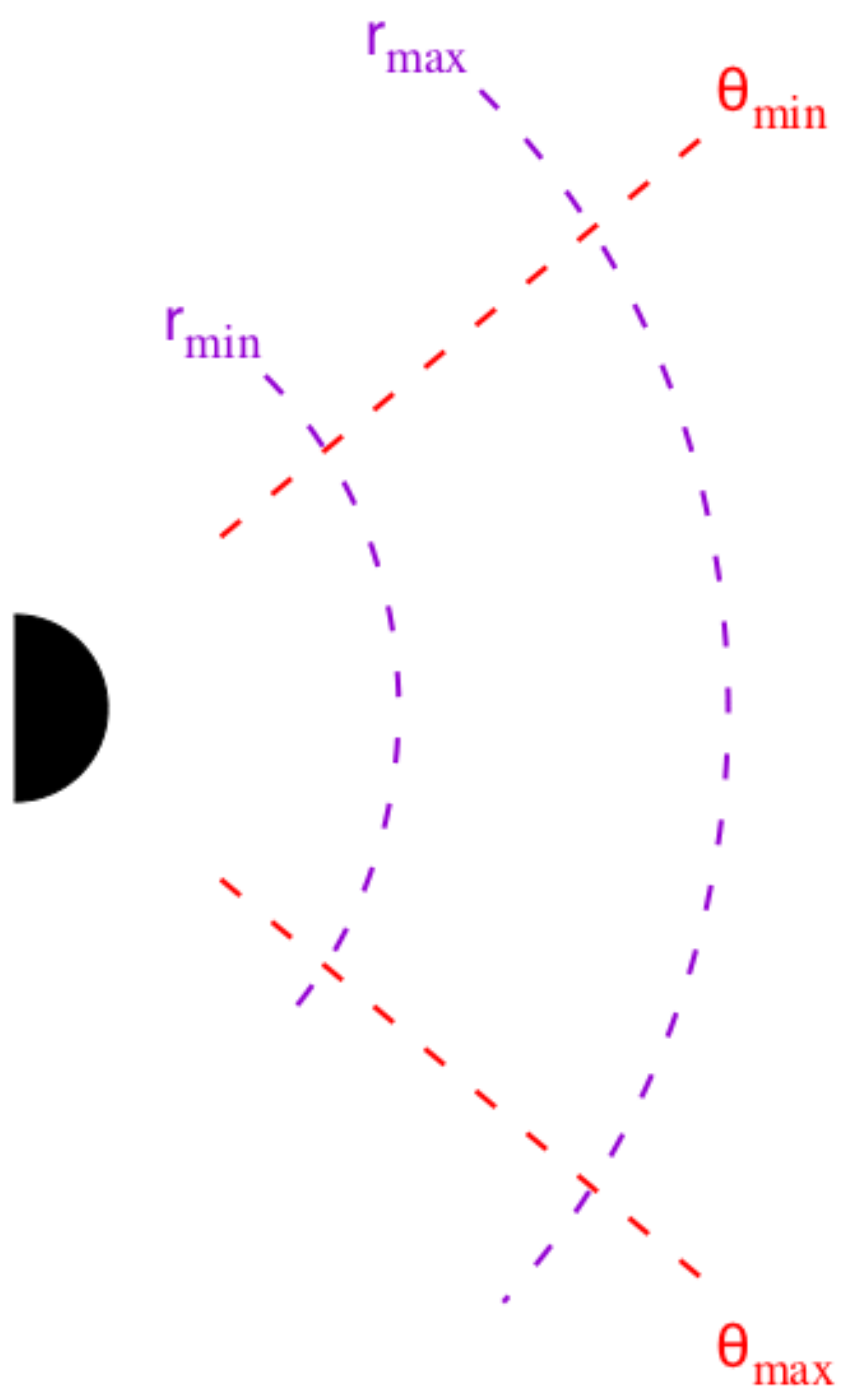

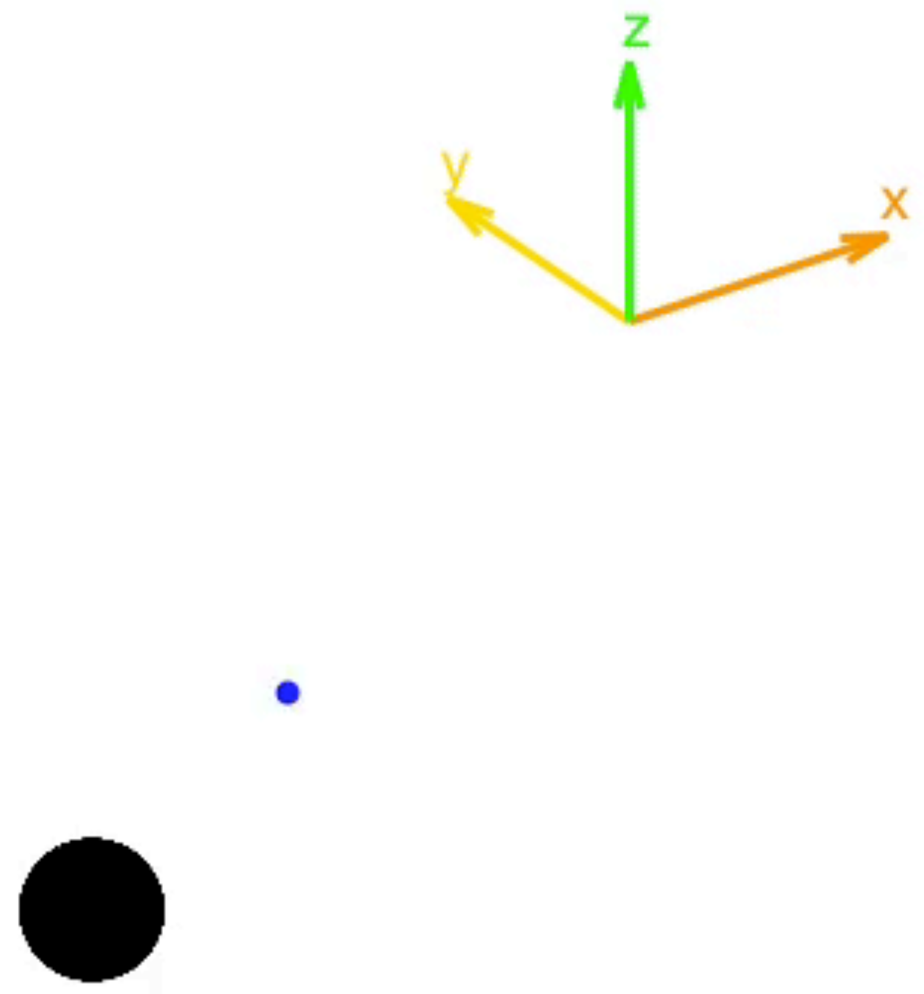

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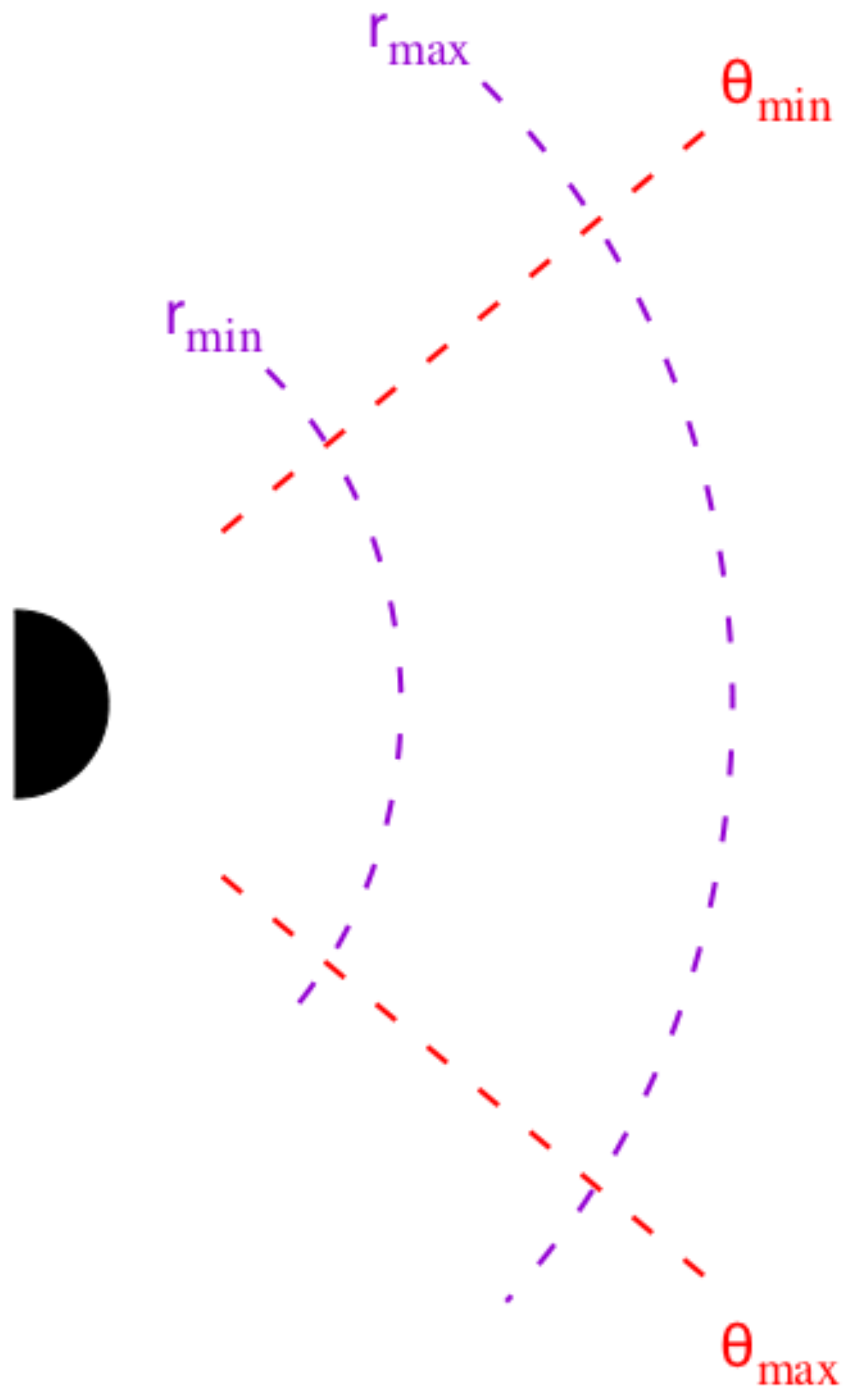

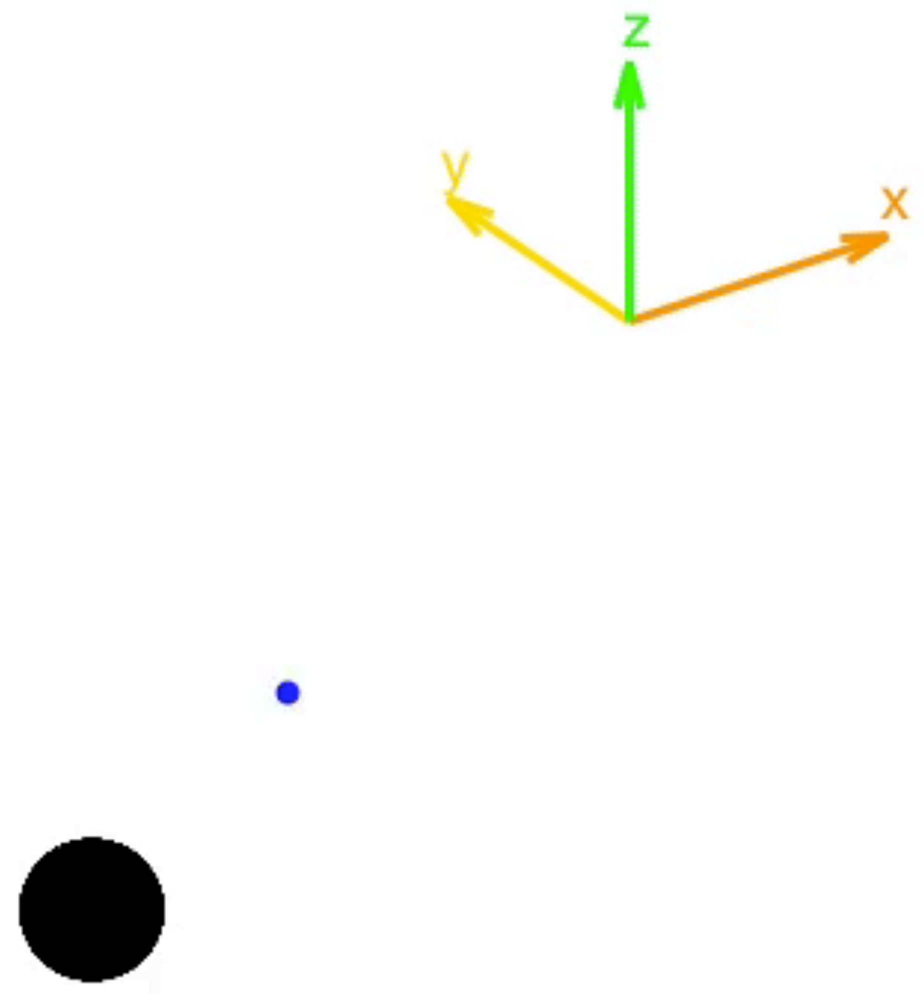

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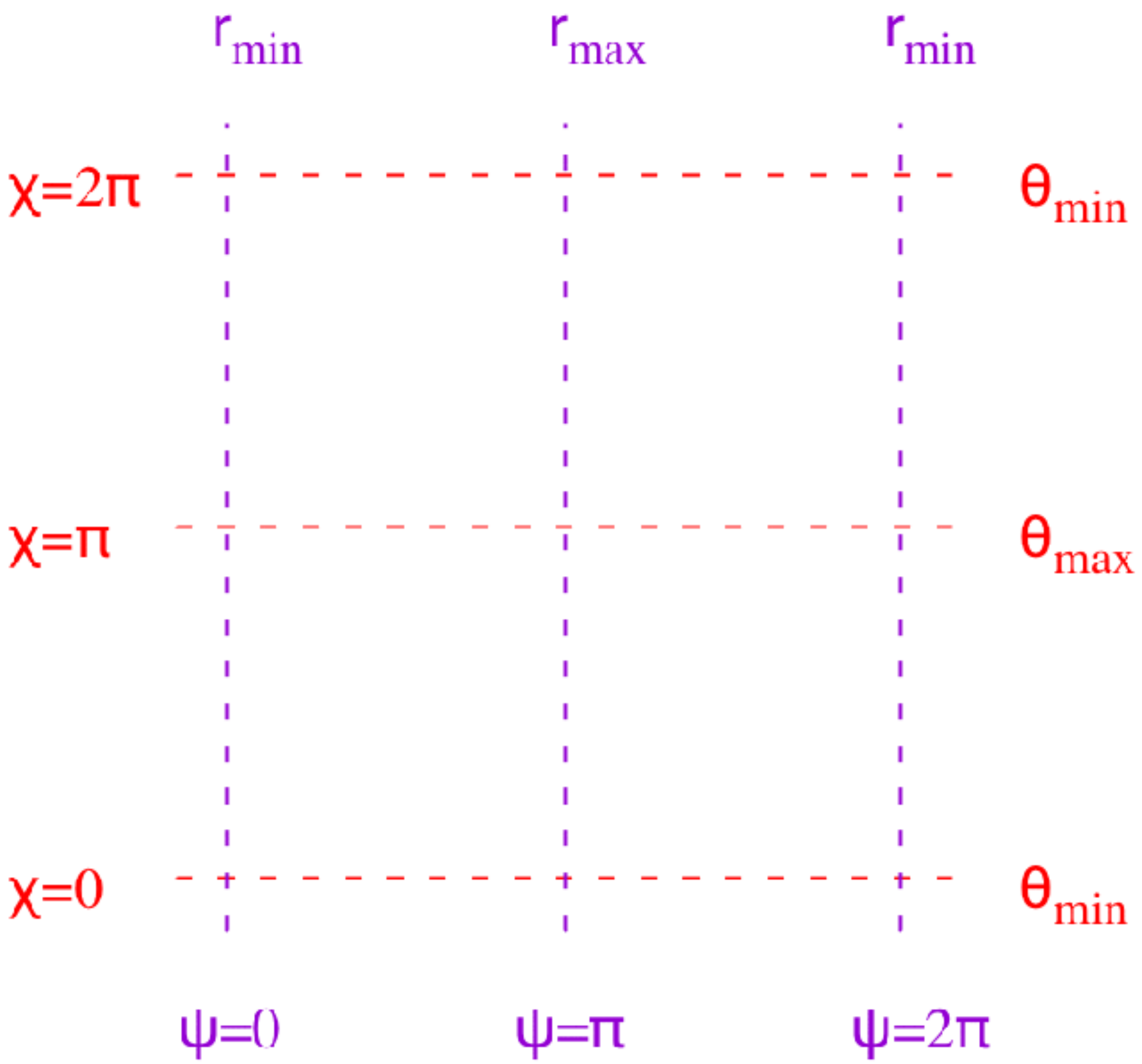

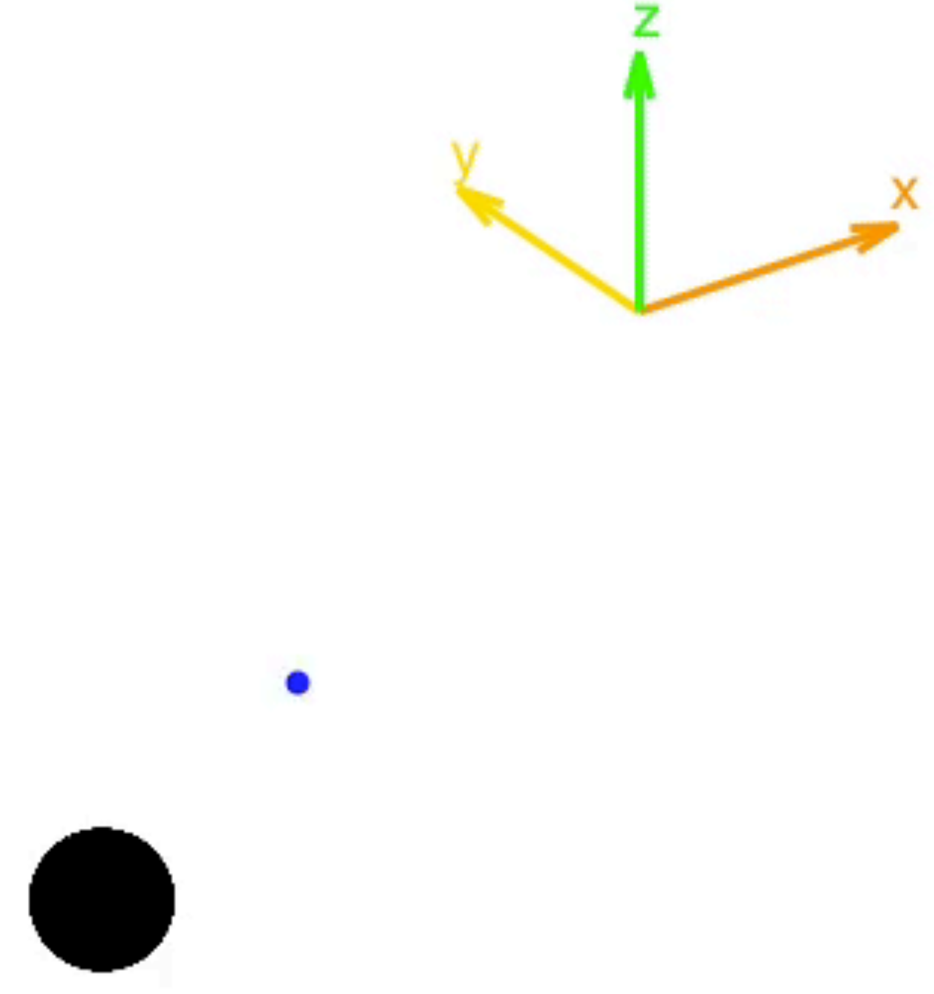

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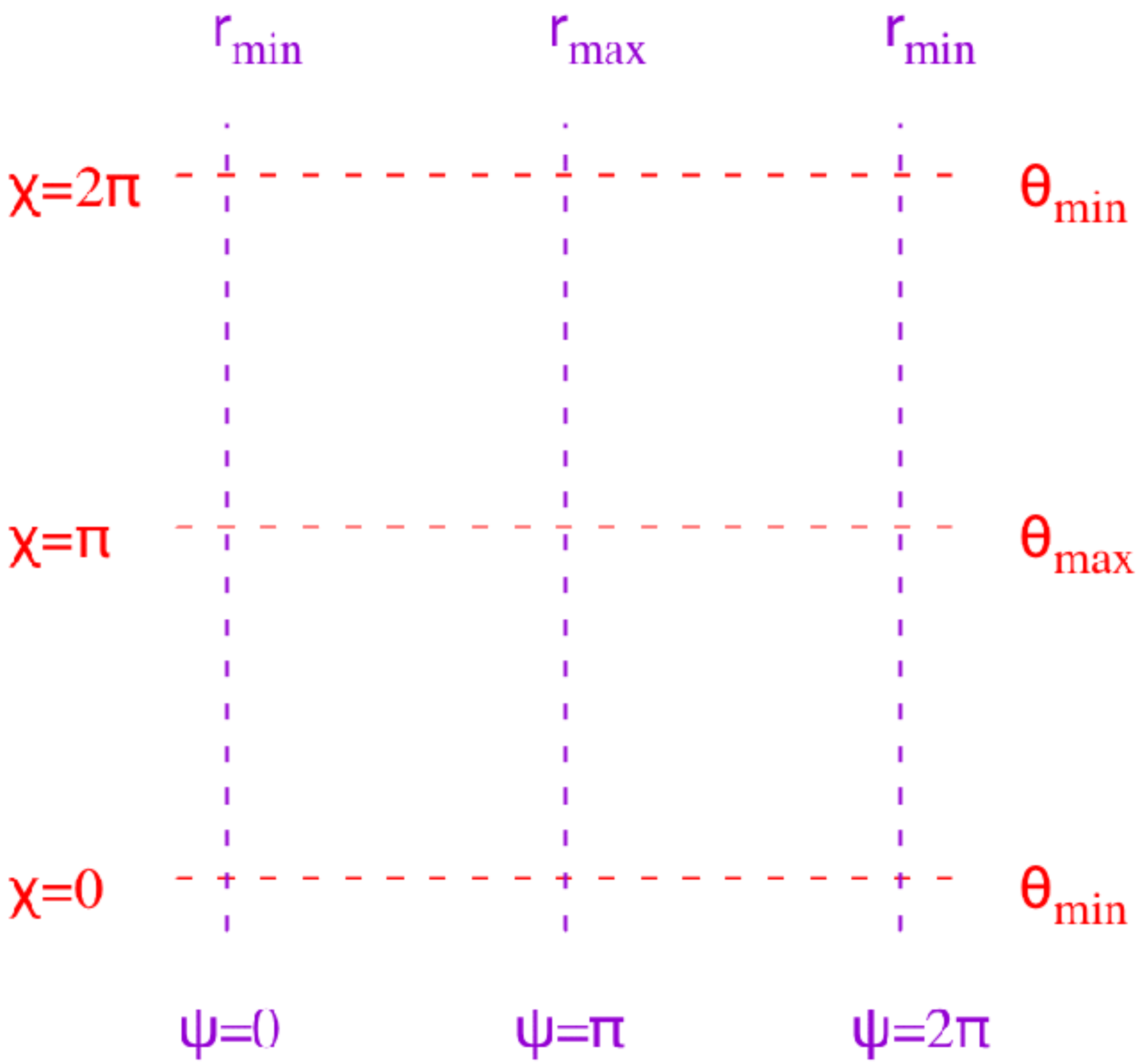

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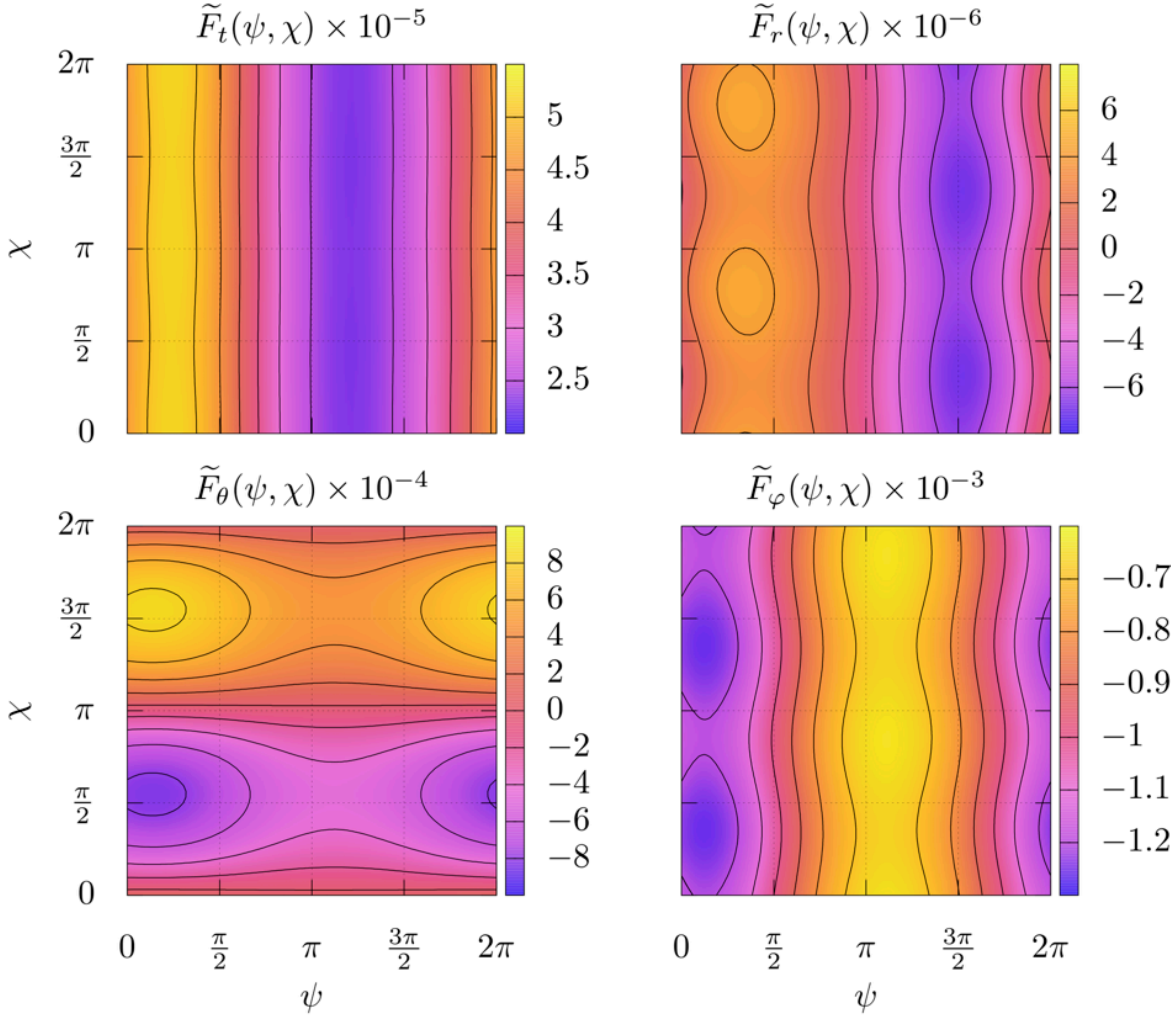

Generic orbit comparison

F_t | F_r | F_θ | F_φ

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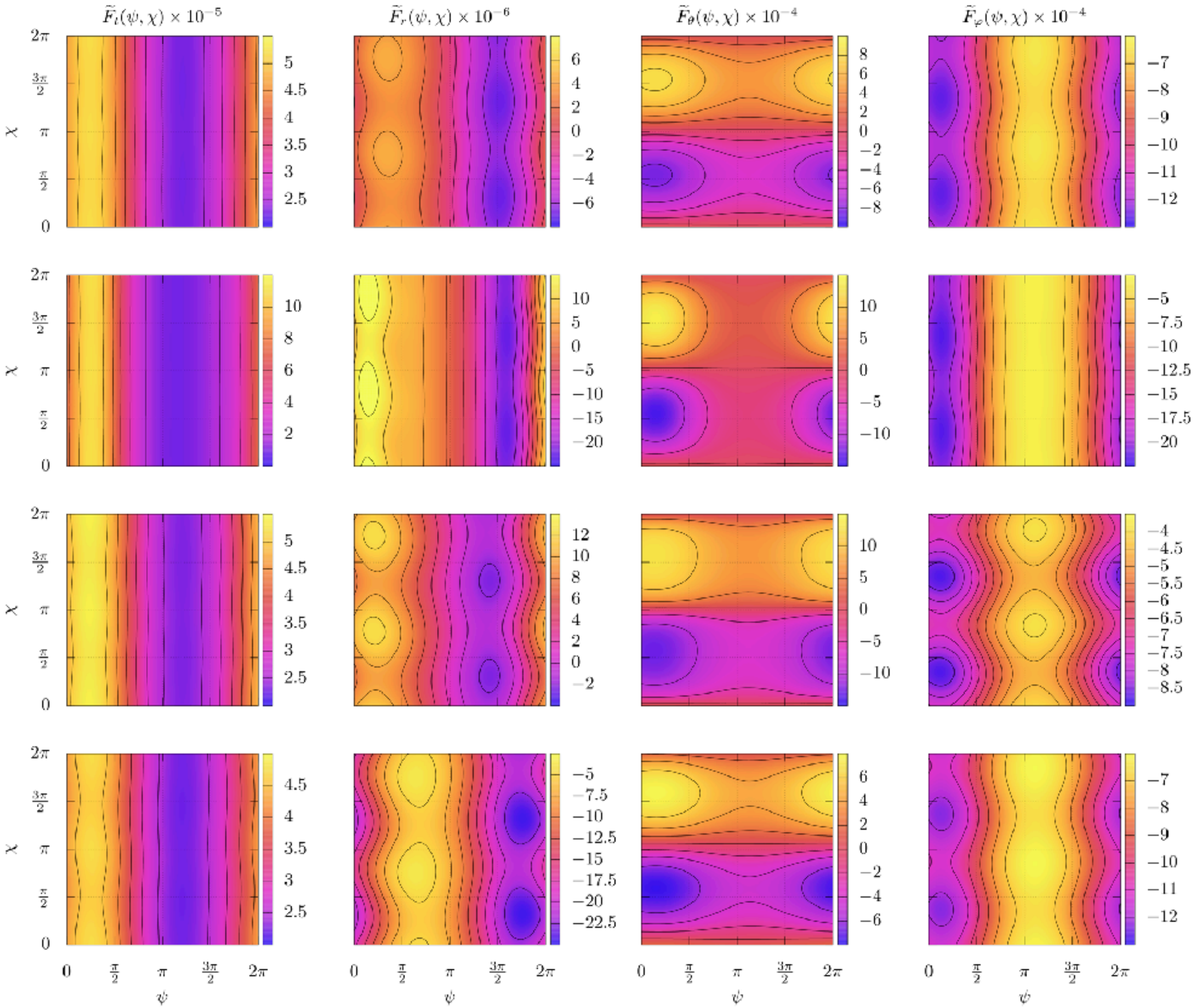

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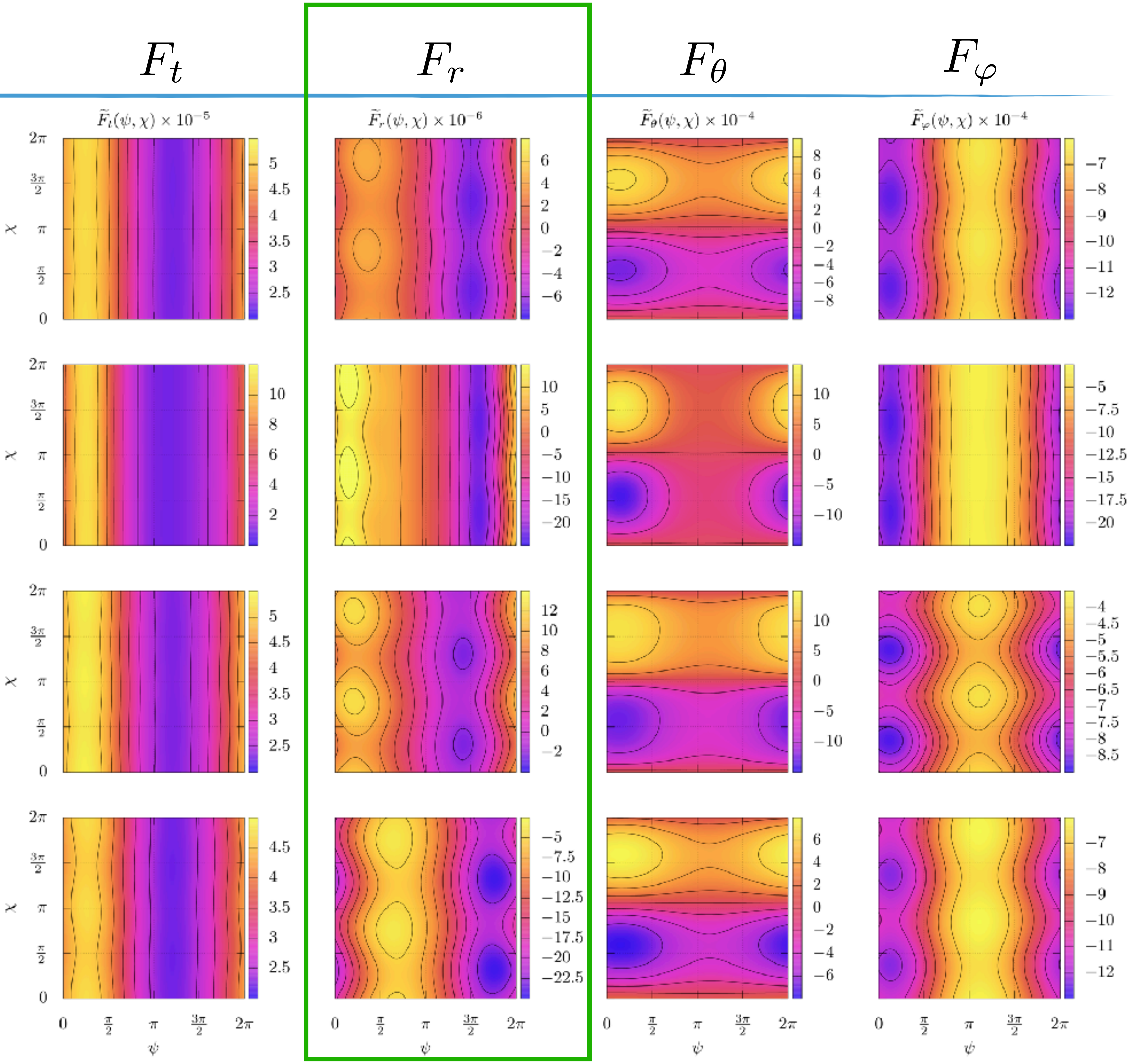

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Generic orbit comparison

- **Radial-component F_r**

$p = 10, e = 0.1, \iota = \pi/5, a = 0.5M$

Baseline $\qquad e = 0.3 \qquad \iota = \pi/3 \qquad a = 0.9M$

Validation tests - fluxes

- **Flux balance**

$$\left\langle \dot{E}^{\mathcal{H}} \right\rangle + \left\langle \dot{E}^{\infty} \right\rangle = \frac{1}{T}\int_0^T \frac{F_t}{u^t}\, dt$$

Radiated energy = Local work

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Radiated angular momentum = Local torque

Radiated energy	Fractional Error
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Angular momentum	Fraction Error
7.56756034×10^{-4}	6×10^{-15}
4.93896206×10^{-4}	4×10^{-14}
6.9840212×10^{-4}	0×10^{-12}
7.281232718×10^{-4}	0×10^{-11}

Validation tests

- **Regularization**

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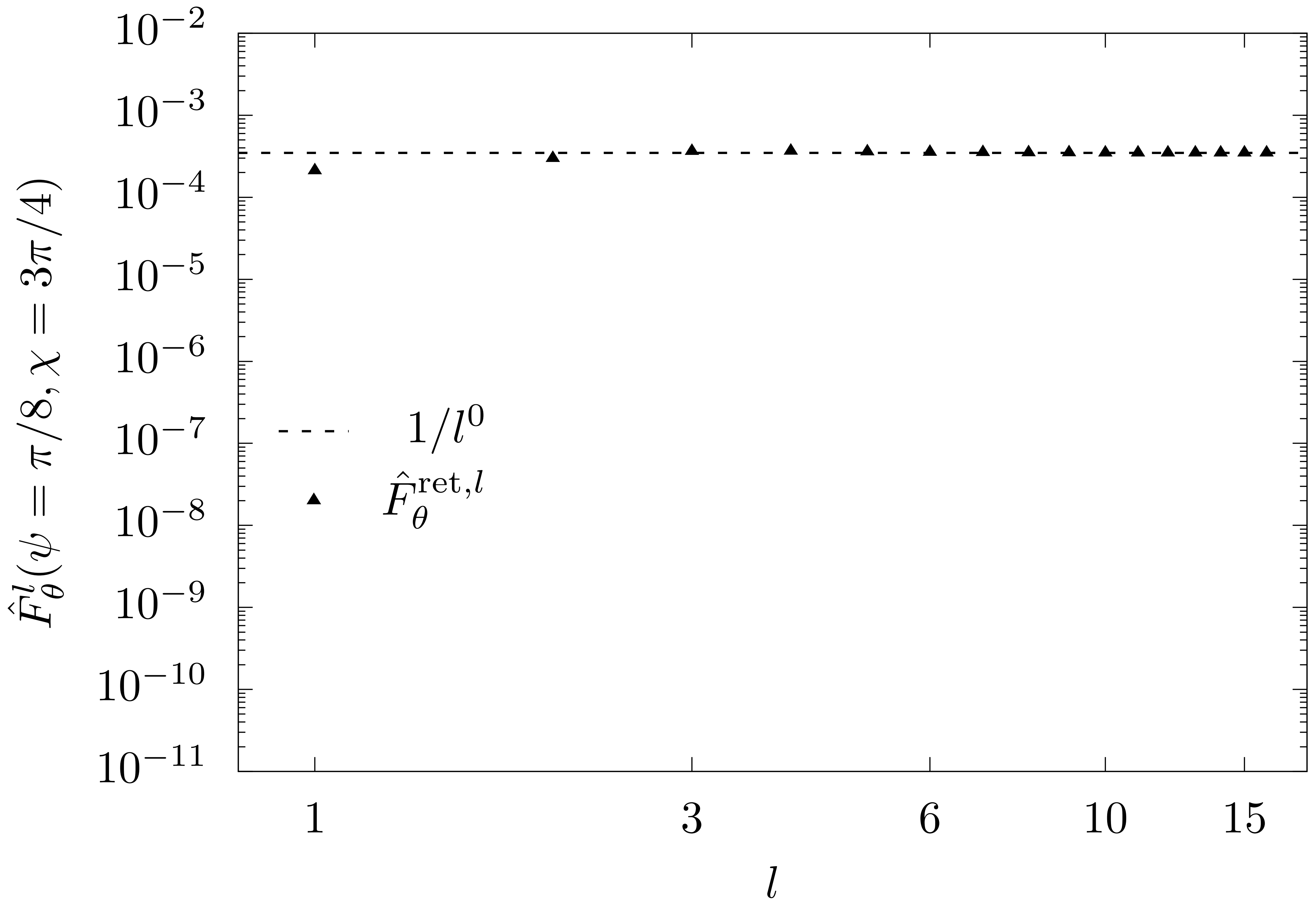

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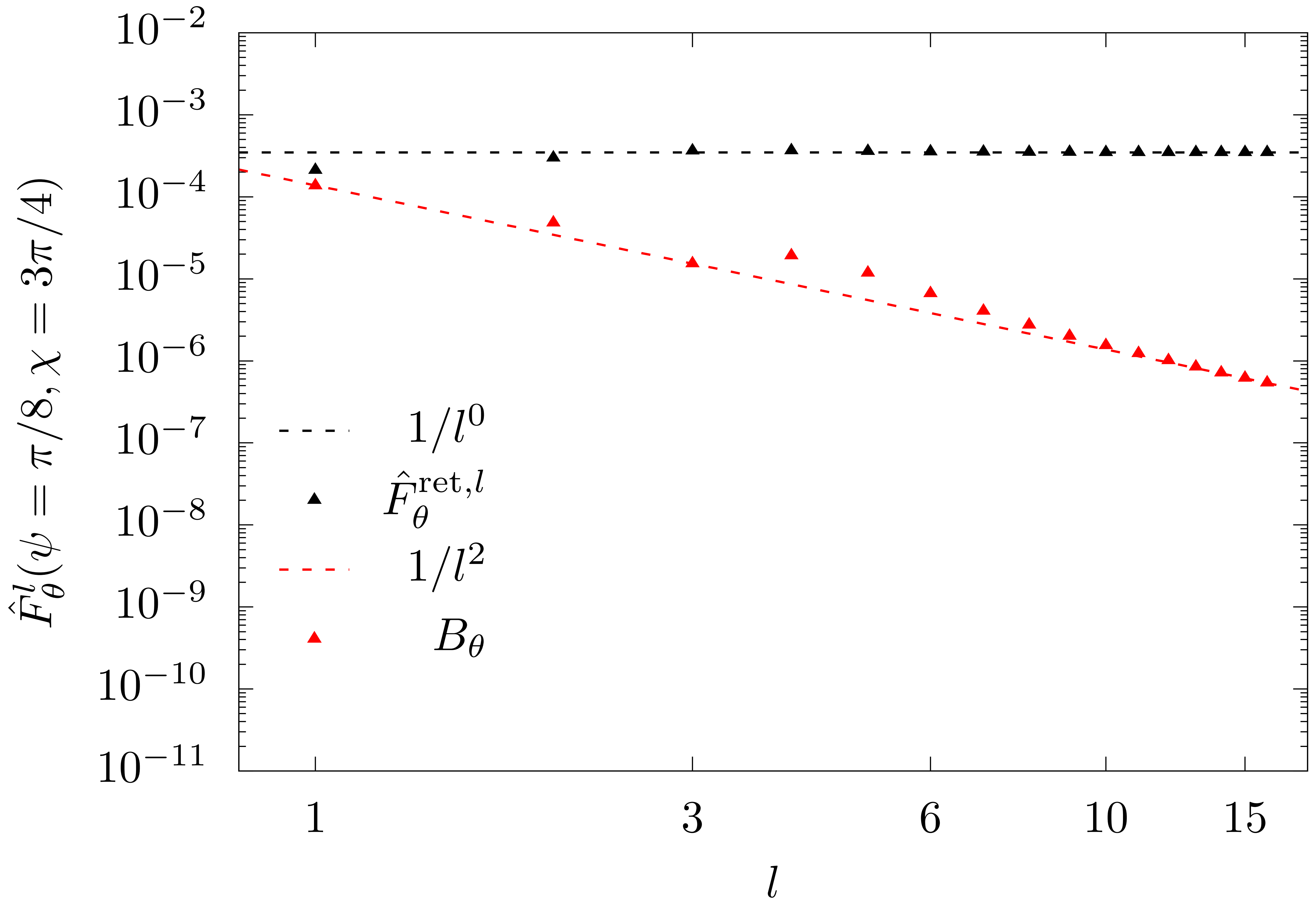

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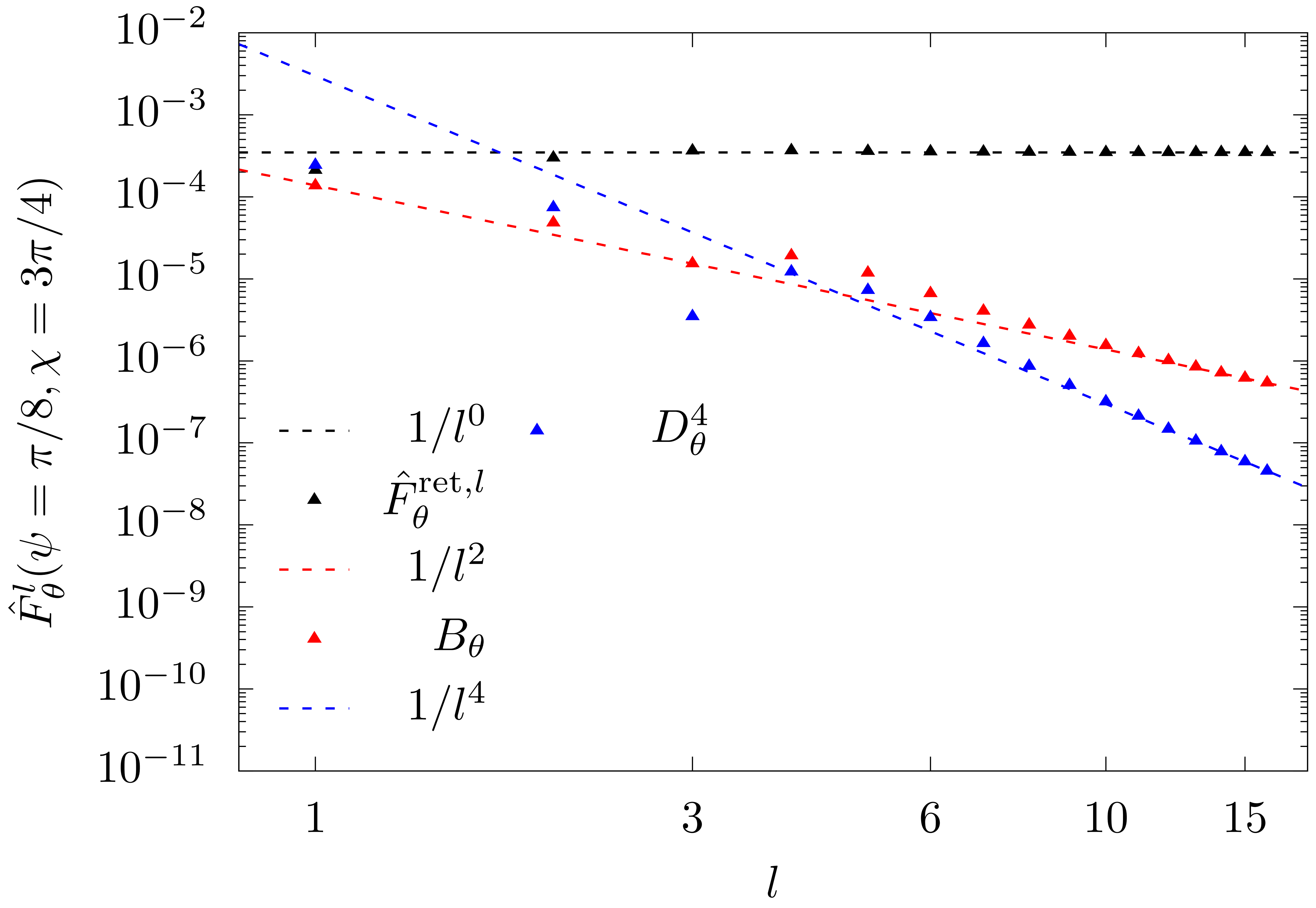

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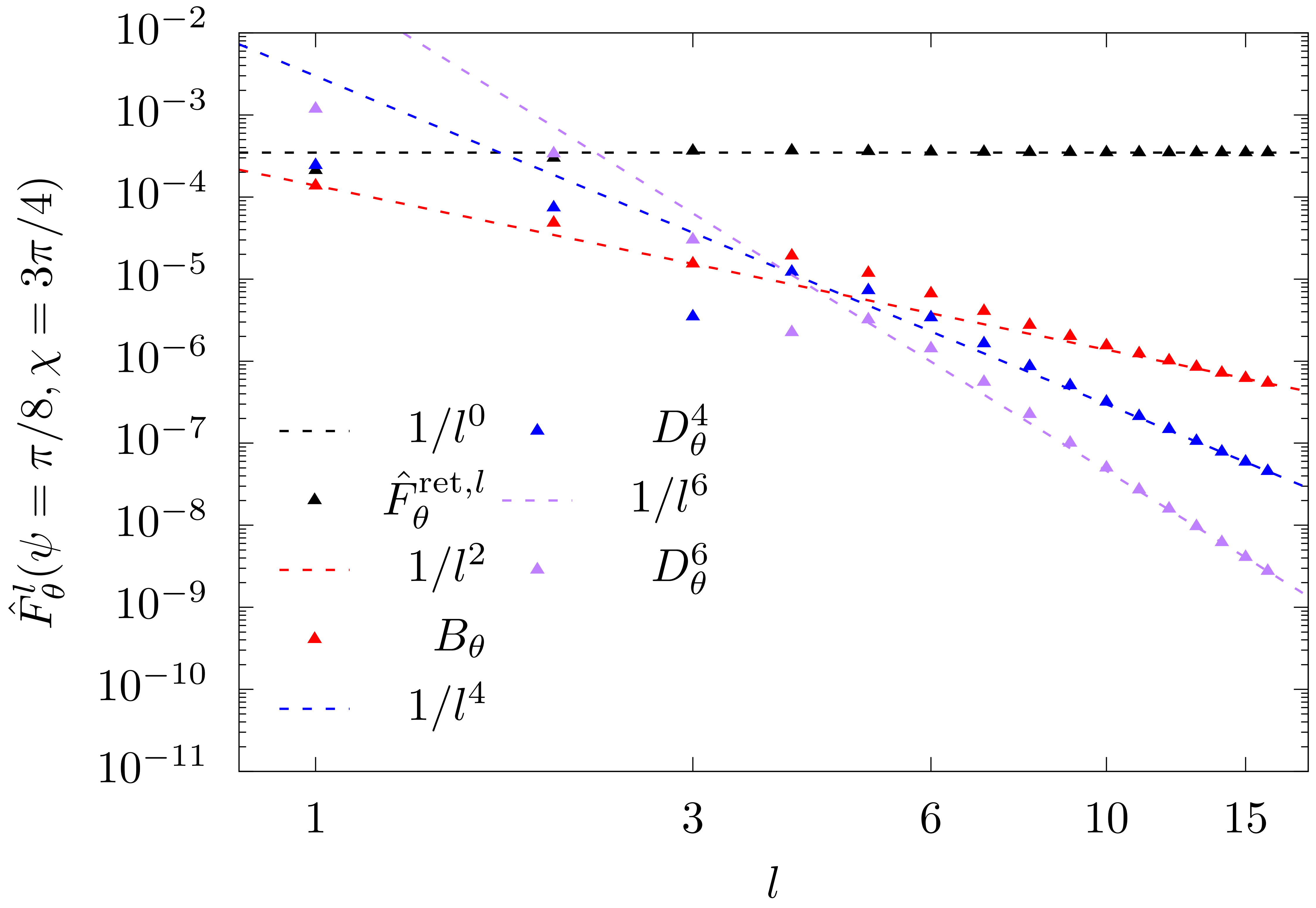

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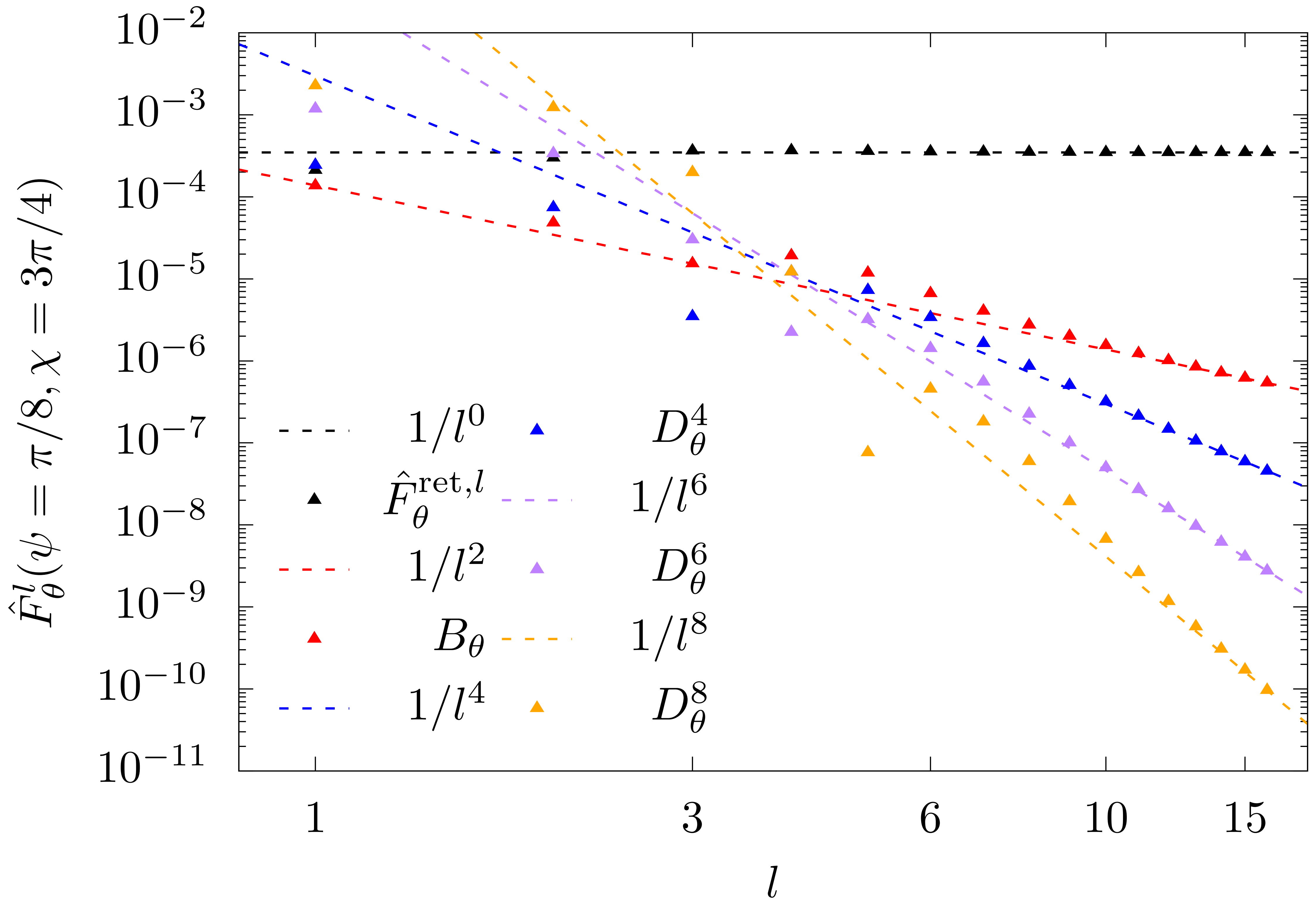

Conclusions

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- Passed latest round of validation tests
  - Successful comparison w/ other results in literature
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- Possibilities moving forward:
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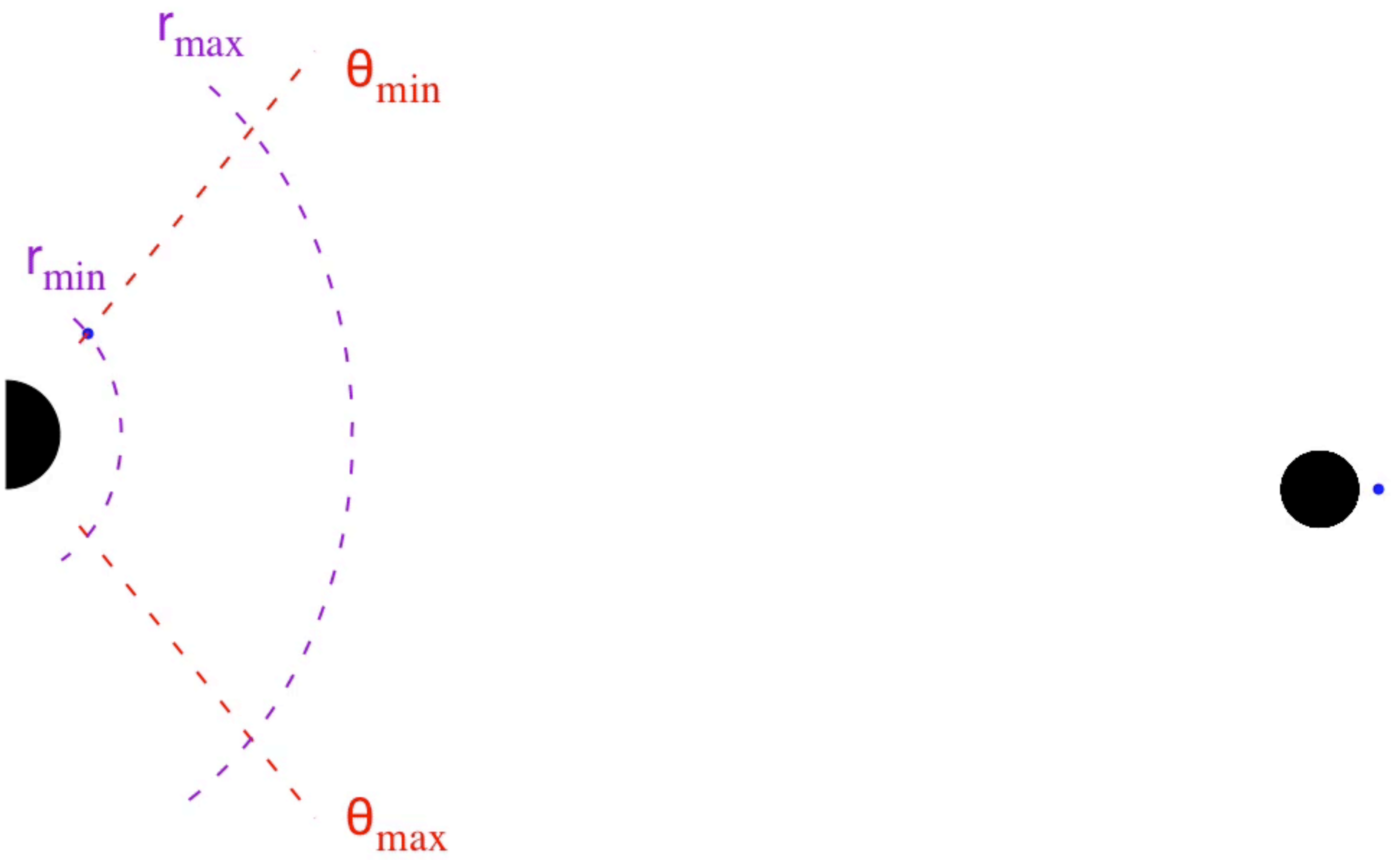
Acknowledgements

- Dr. Charles R. Evans
- Dr. Thomas Osburn
- Black Hole Perturbation Toolkit (bhptoolkit.org)
- National Science Foundation (PHY-1506182)
- North Carolina Space Grant Consortium

Questions?

2:1 resonant orbit $(\Omega_\theta/\Omega_r = 2)$

$p \approx 4.6, e = 0.5, \iota \approx 0.79, a = 0.9M$



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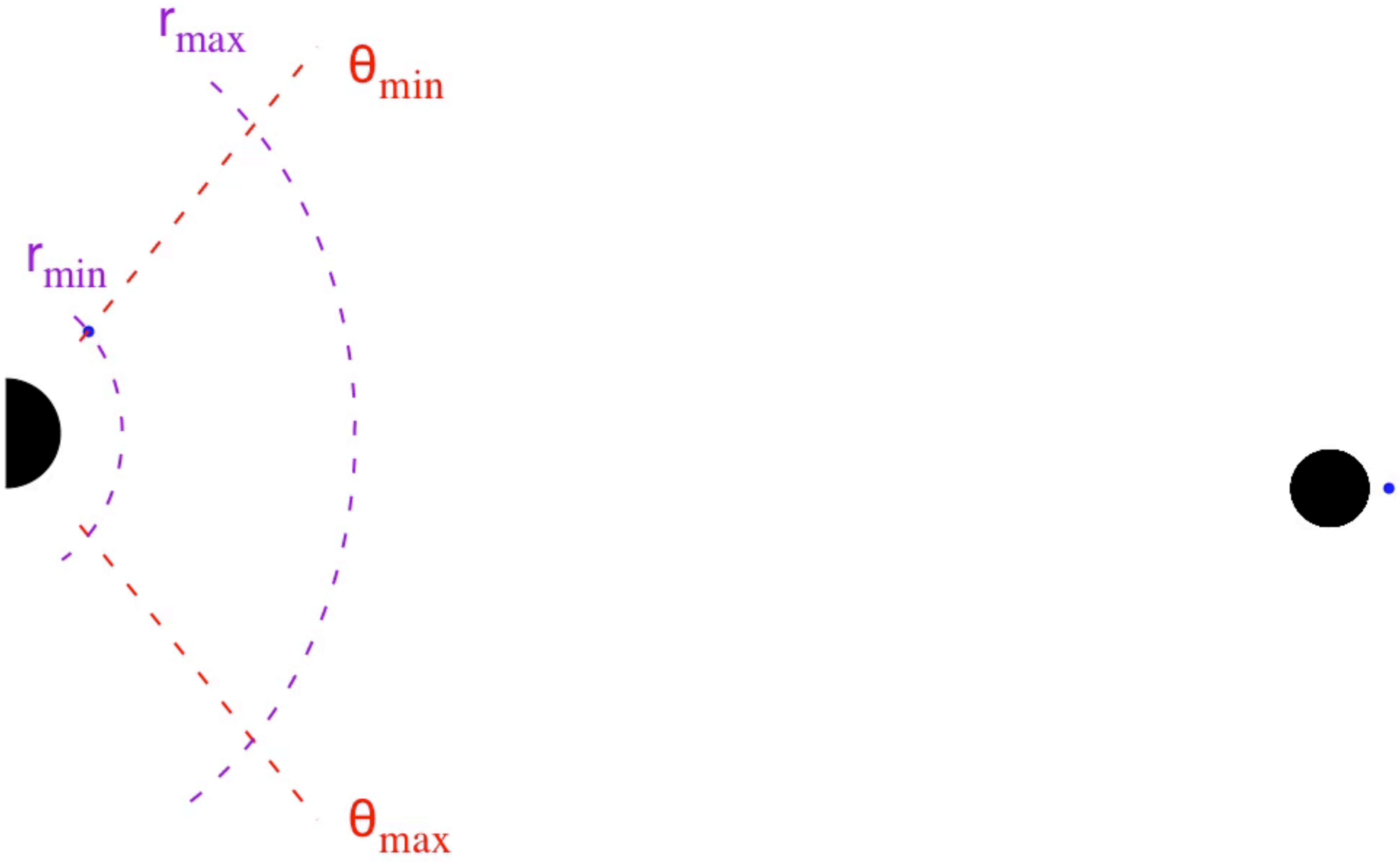

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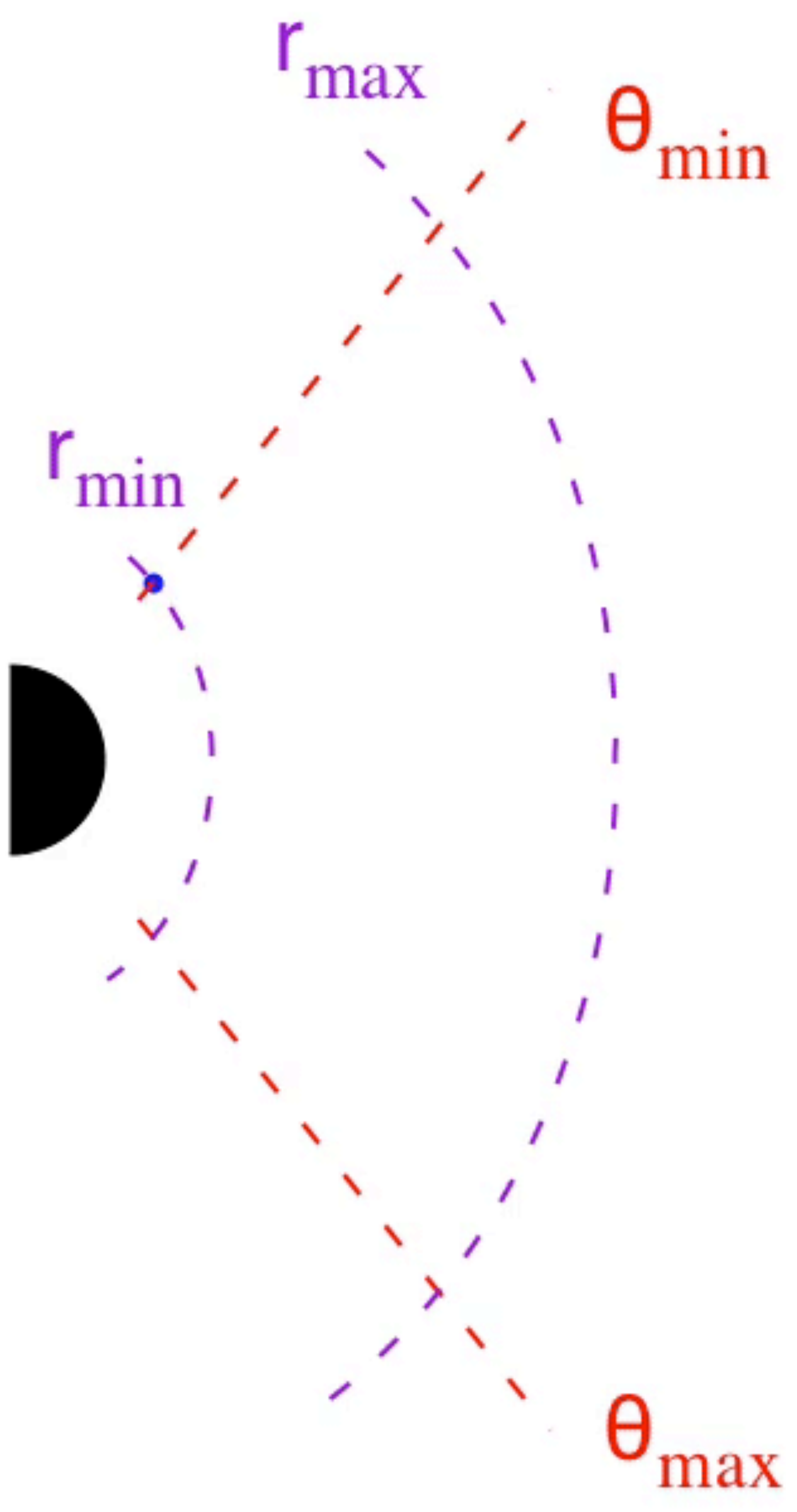

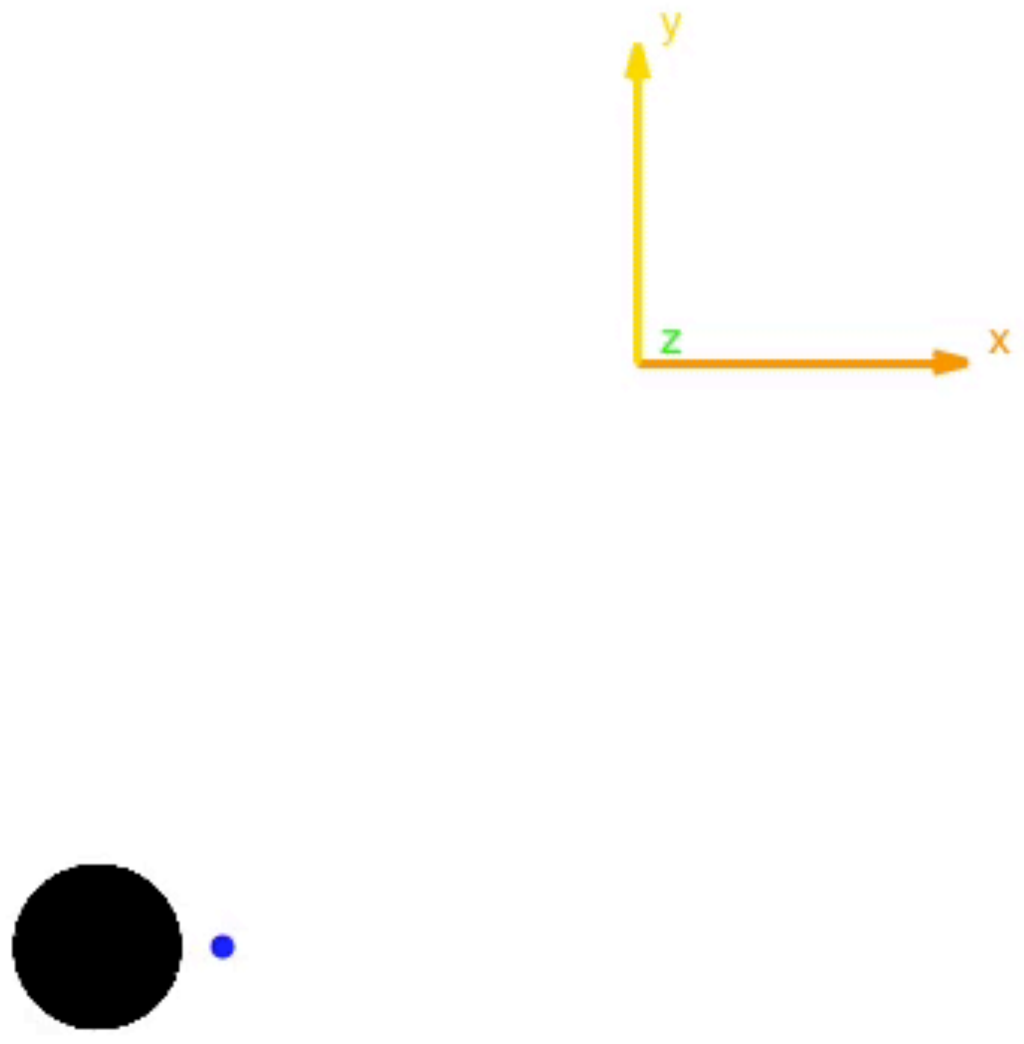

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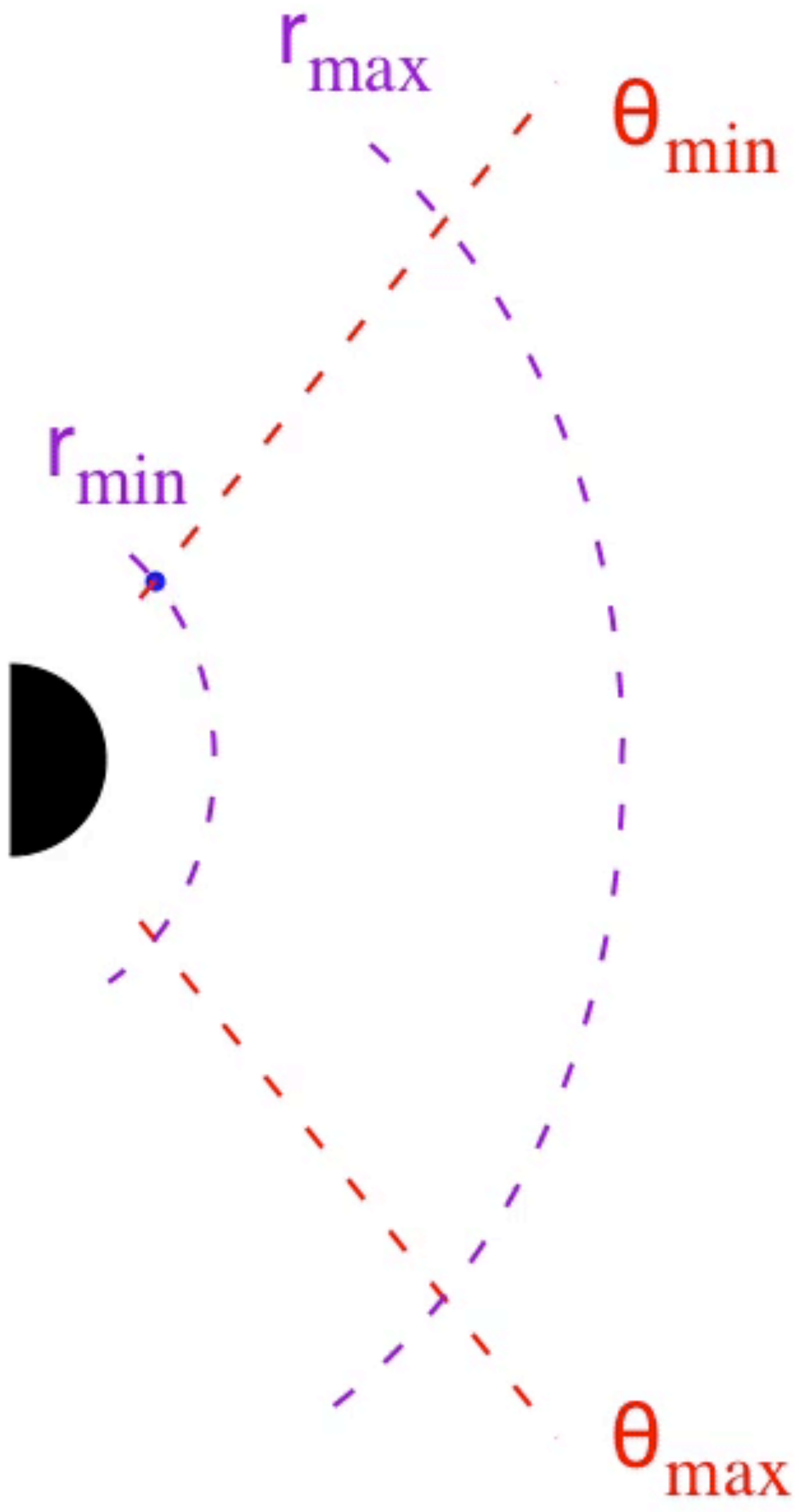
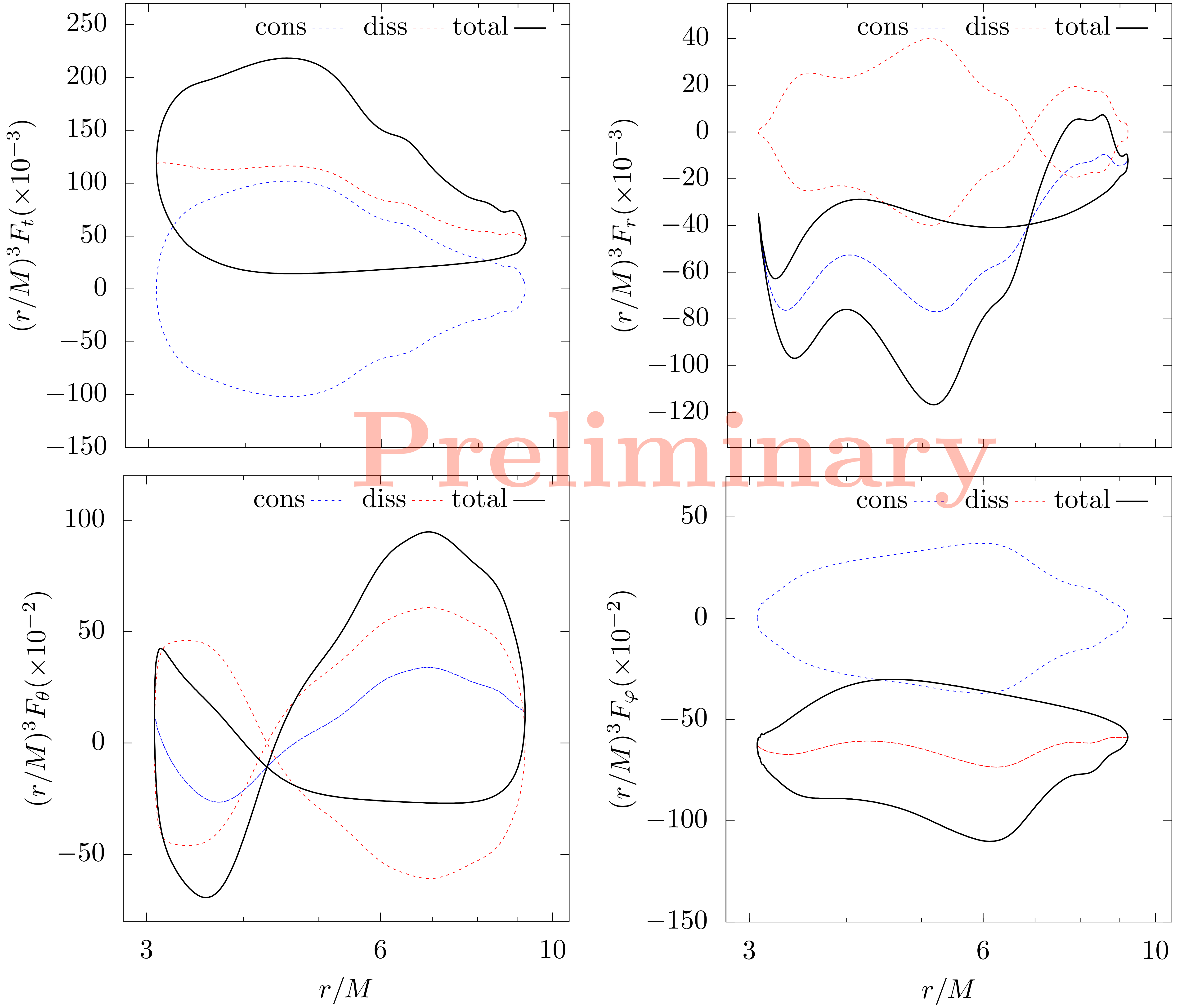
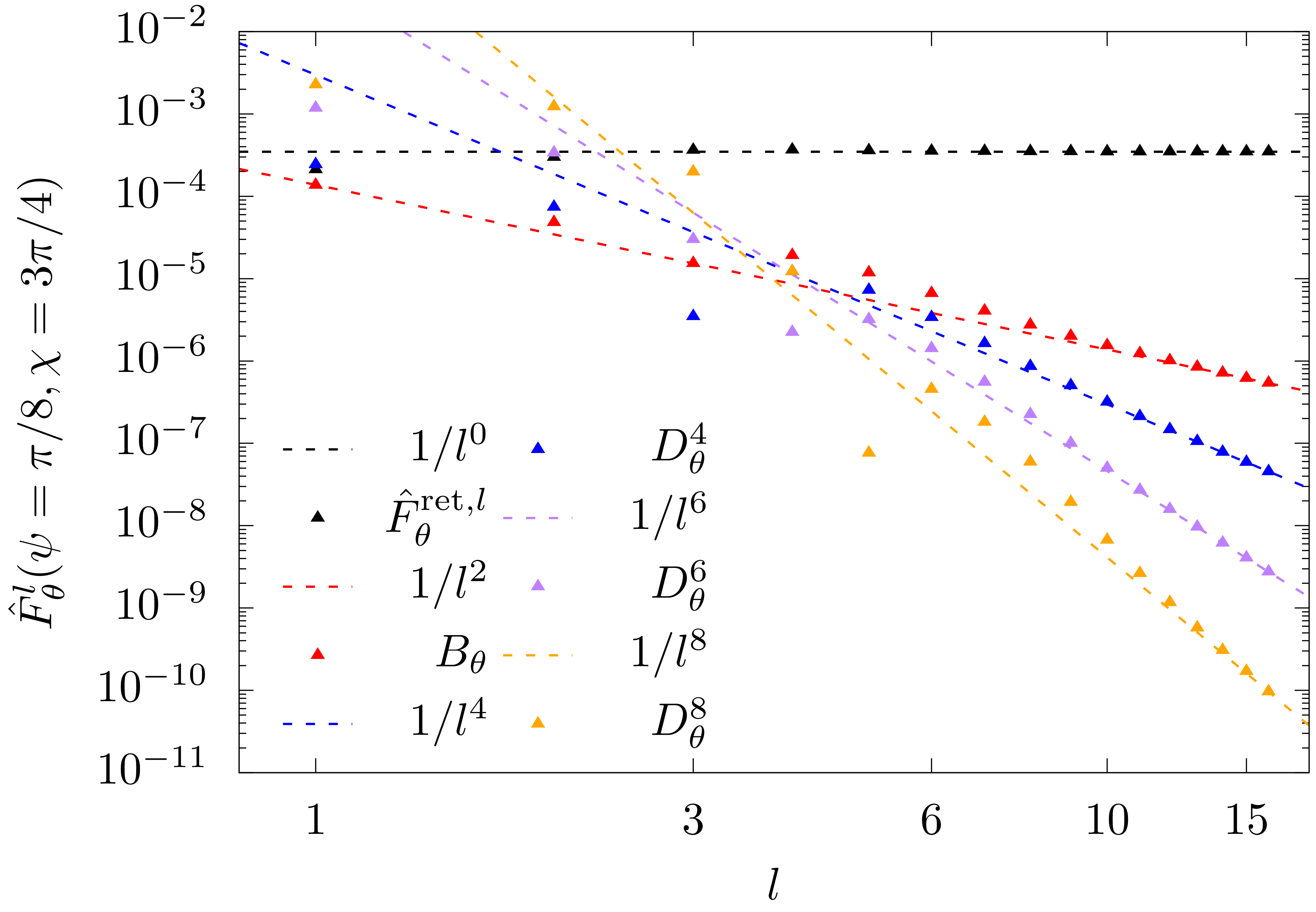




Regularization scheme



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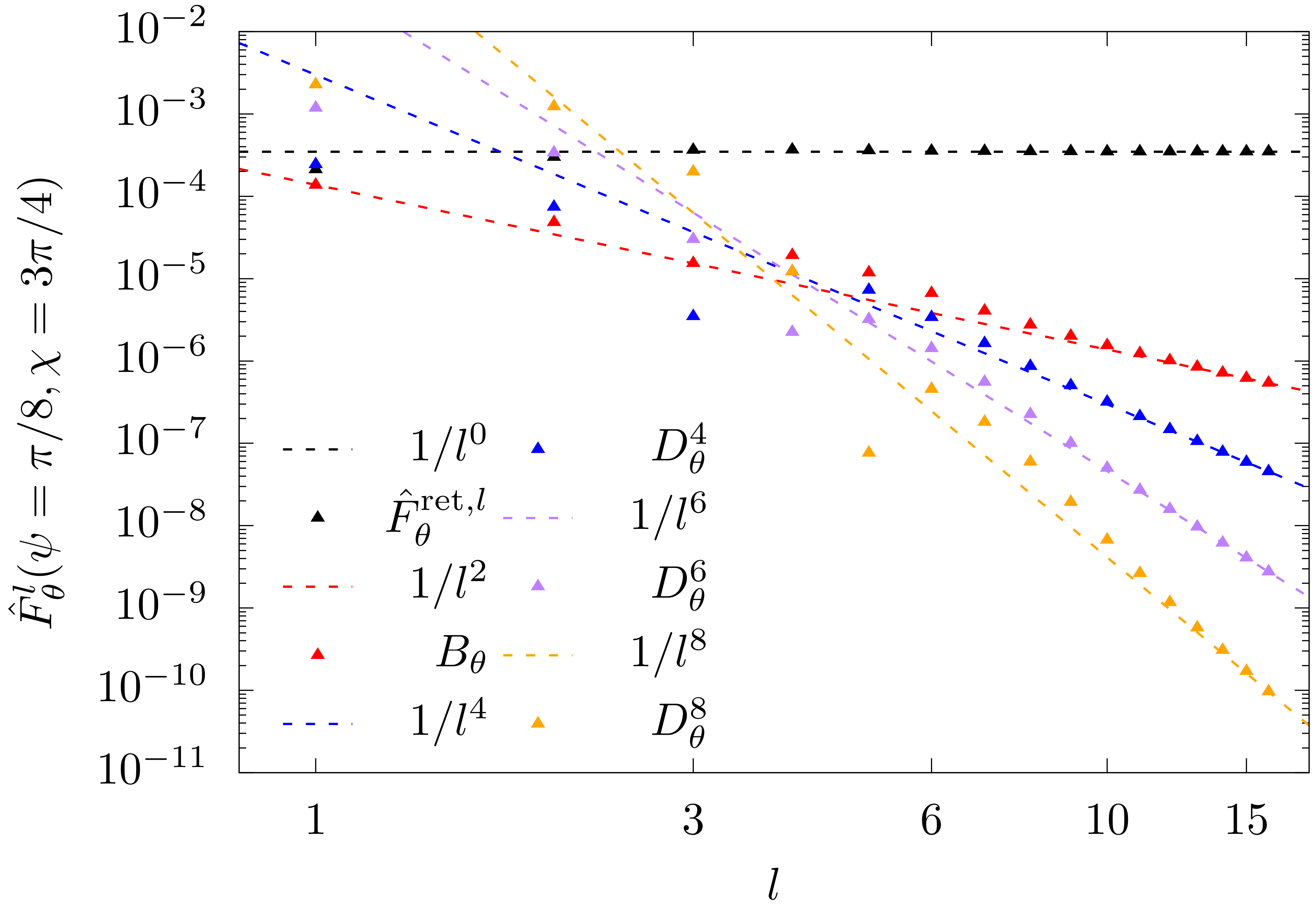

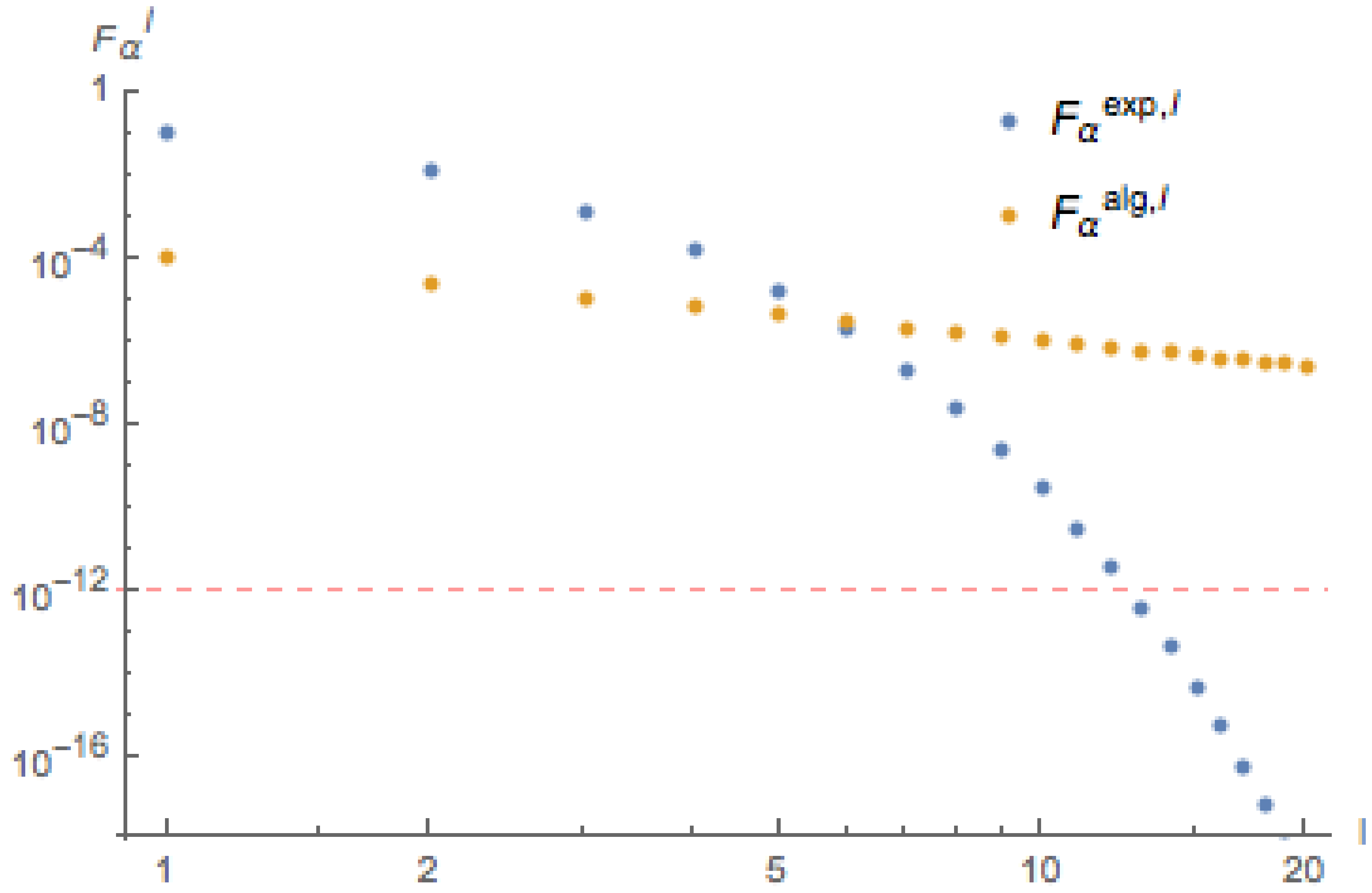

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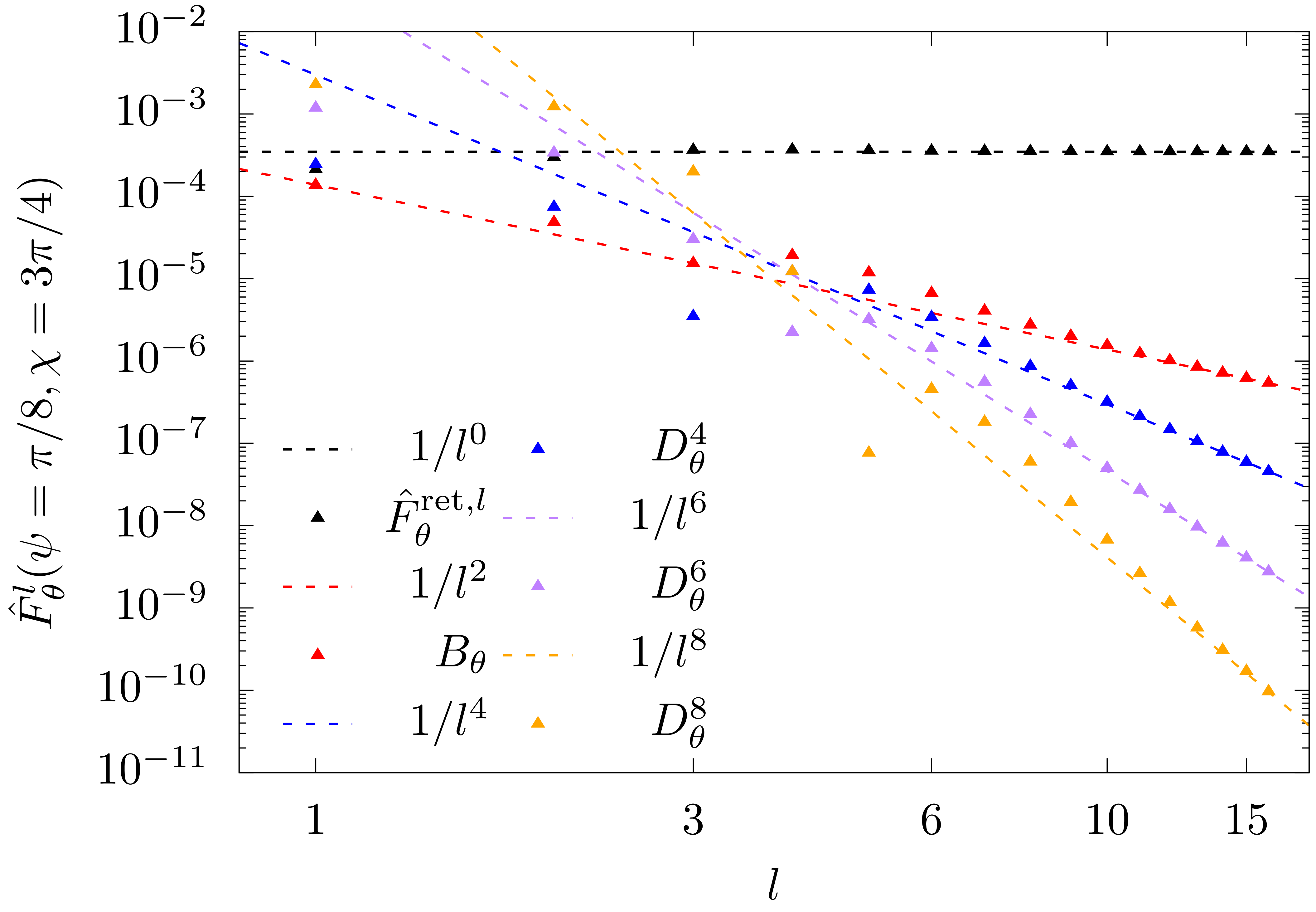

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