Tidal resonances in EMRIs

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Work in collaboration with Scott Hughes & Huan Yang [arXiv:1905.00030]

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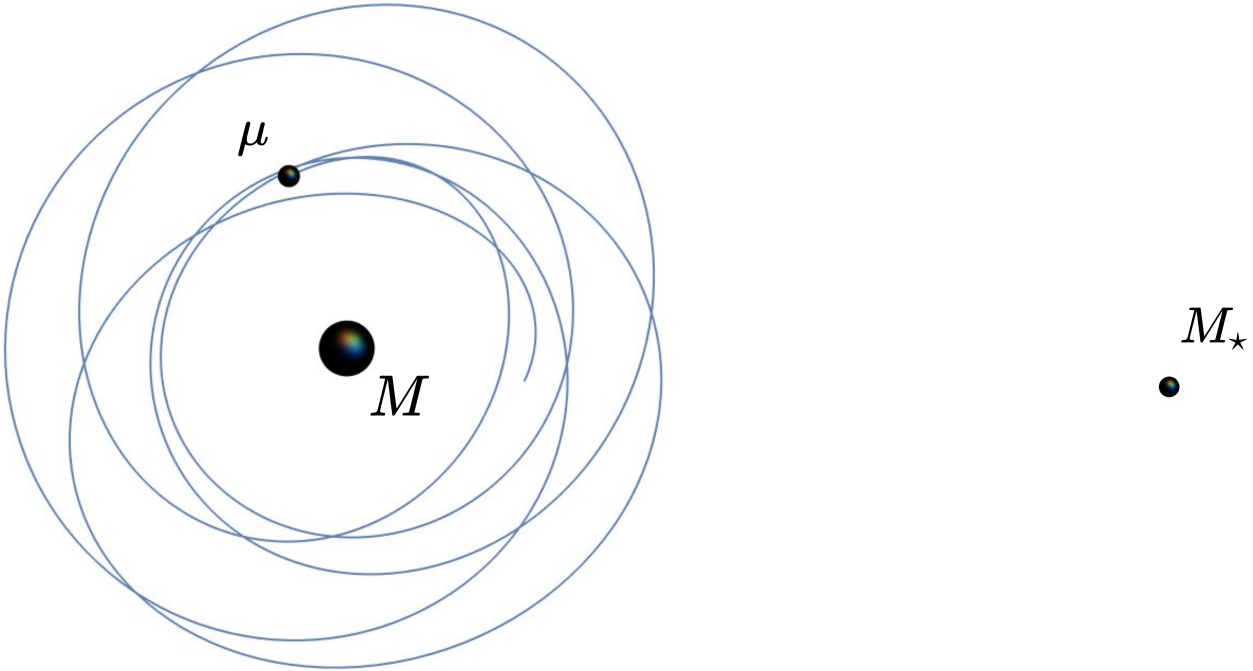
Disclaimer

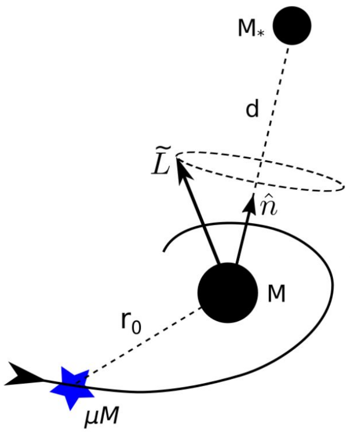
Luis Lehner quoting someone else:

> You are an expert when you have ≥2 paper on the topic.

EMRIs as isolated systems?



Origen story



$$\sim \frac{M_\star}{a^3}$$

A 10^8 Msun BH at 0.1 pc (their fiducial perturber) has essentially the same tide as a 10 Msun object at 0.00046 parsecs (~135 AU).

Where is the tidal perturber?

Expected merger rate

$$\tau \sim 3 \left(\frac{M}{10^6 M_\odot}\right)^{-0.19} \text{ Myr}$$

Assume: merger driven by GW emission

$$\tau \sim \frac{\dot{a}}{a} \sim \frac{a^4}{M_\star M^2}$$

$$\Rightarrow \quad a \sim 4.3 \text{ AU} \left(\frac{M_\star}{10 M_\odot}\right)^{1/4} \left(\frac{M}{M_{\rm SgrA*}}\right)^{0.45}$$

Action-angle variables

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \epsilon\, g^{(1)}_{i,\mathrm{td}}(q_\phi, q_\theta, q_r, \mathbf{J}) + \eta\, g^{(1)}_{k,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)$$

$$\epsilon = M_\star M^2/a^3$$

$$\frac{dJ_i}{d\tau} = \epsilon\, G^{(1)}_{i,\mathrm{td}}(q_\phi, q_\theta, q_r, \mathbf{J}) + \eta\, G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)$$

Tidal resonance

$$G_i^{(1)}(q_\phi, q_\theta, q_r, \mathbf{J}) = \sum_{m,k,n} G_{i,mkn}^{(1)}(\mathbf{J}) e^{i(mq_\phi + kq_\theta + nq_r)}$$

Resonance condition $\omega_{mkn} := m\omega_\phi + k\omega_\theta + n\omega_r = 0$

> more general condition than for transient resonance

> *also* occurs for low eccentricity orbits

Metric of tidally perturbed Kerr

$h_{\mu\nu}$ from [Gonzales + Yunes, 2005]

> Teukolsky equation + metric reconstruction

> Assumes tidal field is static

> Takes as input $\mathcal{E}_{ij}$

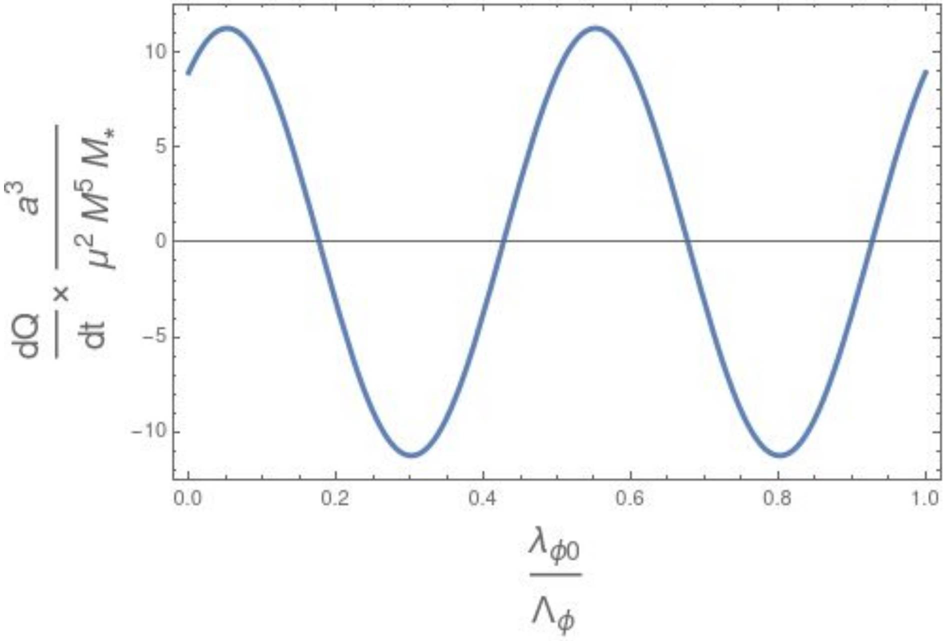
> Caveat: only takes into account m=±1 and m=±2

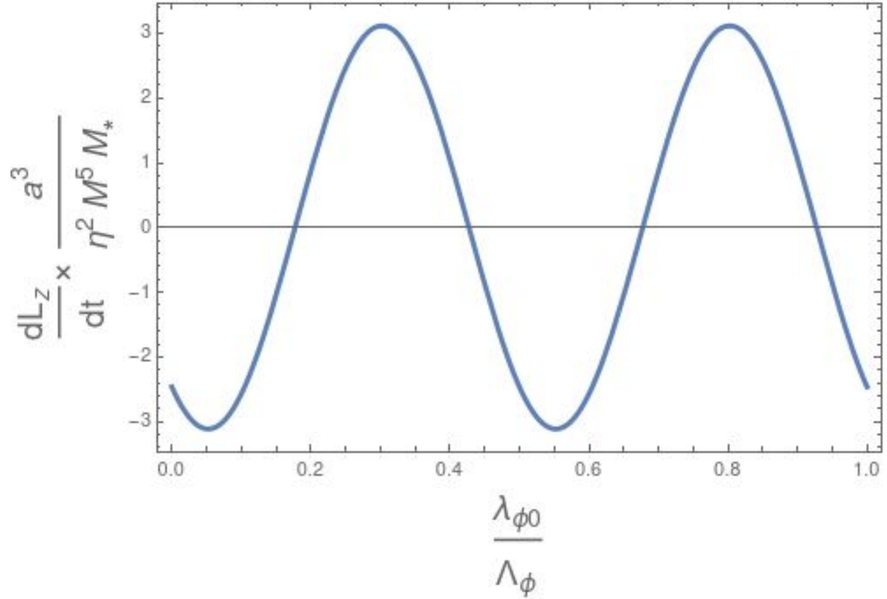
Sample evolution with m=-2,k=2,n=1

$$\left\langle G_i^{(1)}(q_\phi, q_\theta, q_r, \mathbf{J}) \right\rangle \approx G_{i,-2,2,1}^{(1)}(\mathbf{J}) e^{-2iq_{\phi 0}} + \text{cc}\,.$$

a	r_{min}	r_{max}	θ_{min}	$\dot{Q}_{-2,2,1}$	$\dot{L}_{z\,-2,2,1}$
0.7	3.5	5.1628033	$\pi/3$	$1.66 + 2.27i$	$-0.35 - 0.47i$
0.9	3	6.6159726	$\pi/4$	$6.60 + 7.70i$	$-1.72 - 2.01i$
0.99	3	5.3718120	$\pi/4$	$4.46 + 3.43i$	$-1.23 - 0.95i$

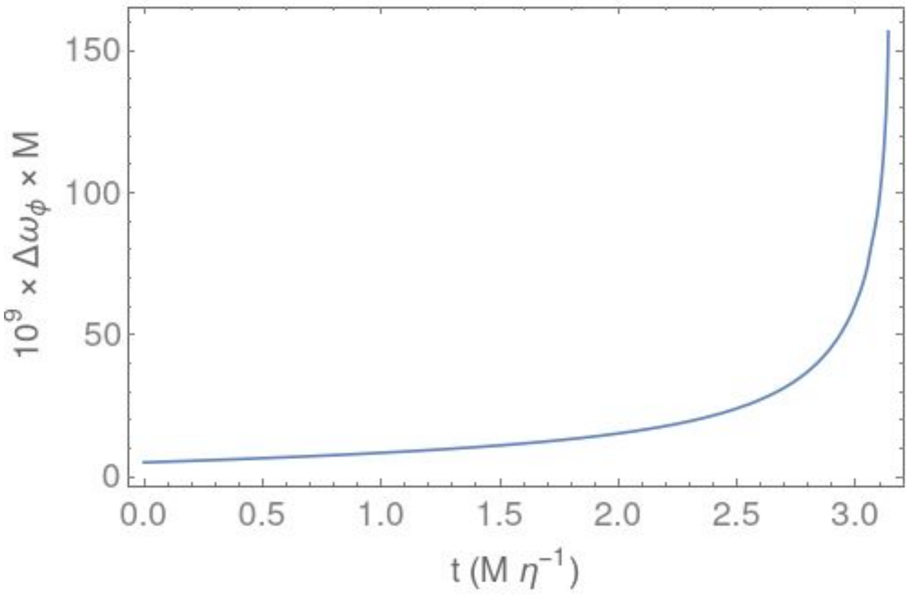
Rate of change depends on the phase





Compare two orbits using Numerical Kludge

$$\{E, Q, L_z\} \rightarrow \omega_\phi^{(1)} \quad \text{versus} \quad \{E, Q+\Delta Q, L_z+\Delta L_z\} \rightarrow \omega_\phi^{(2)}$$

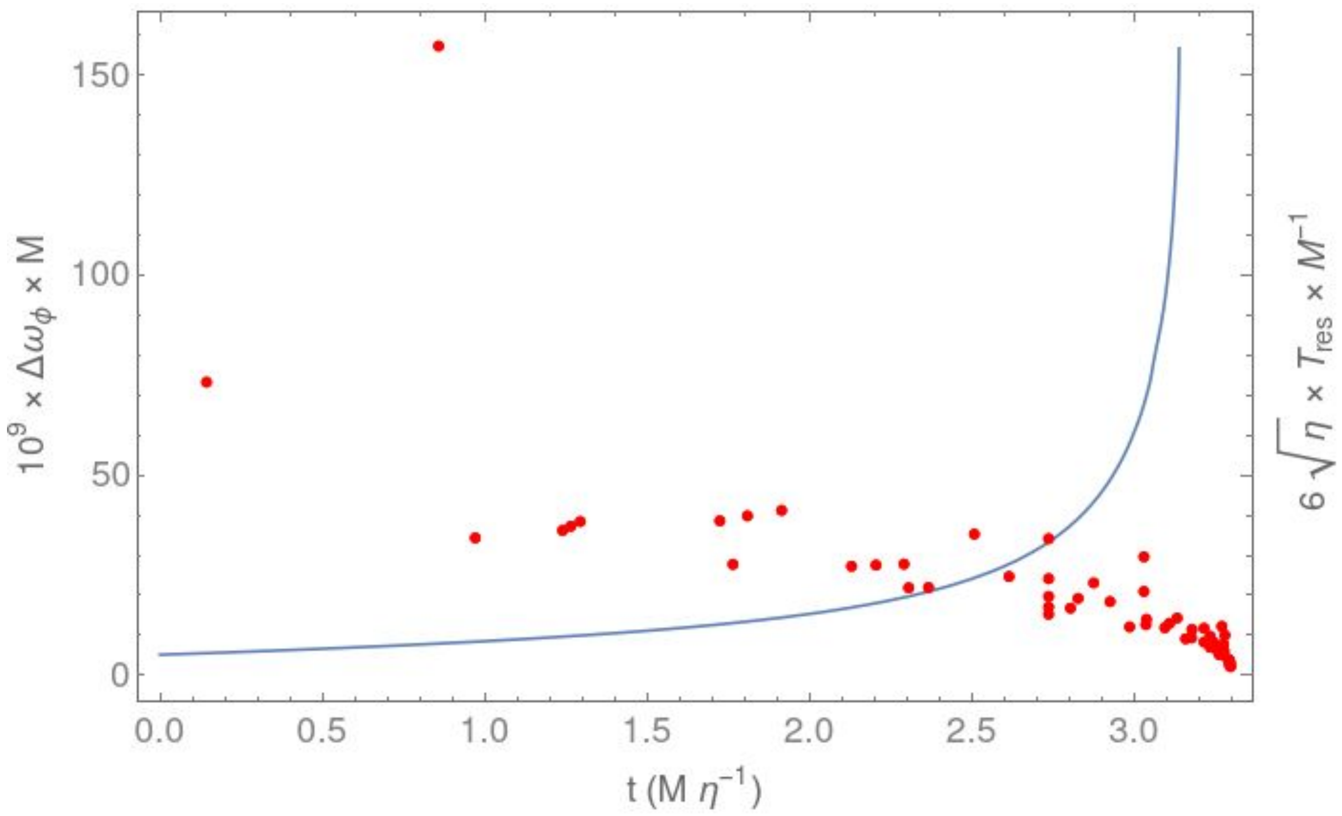


Influence on phase

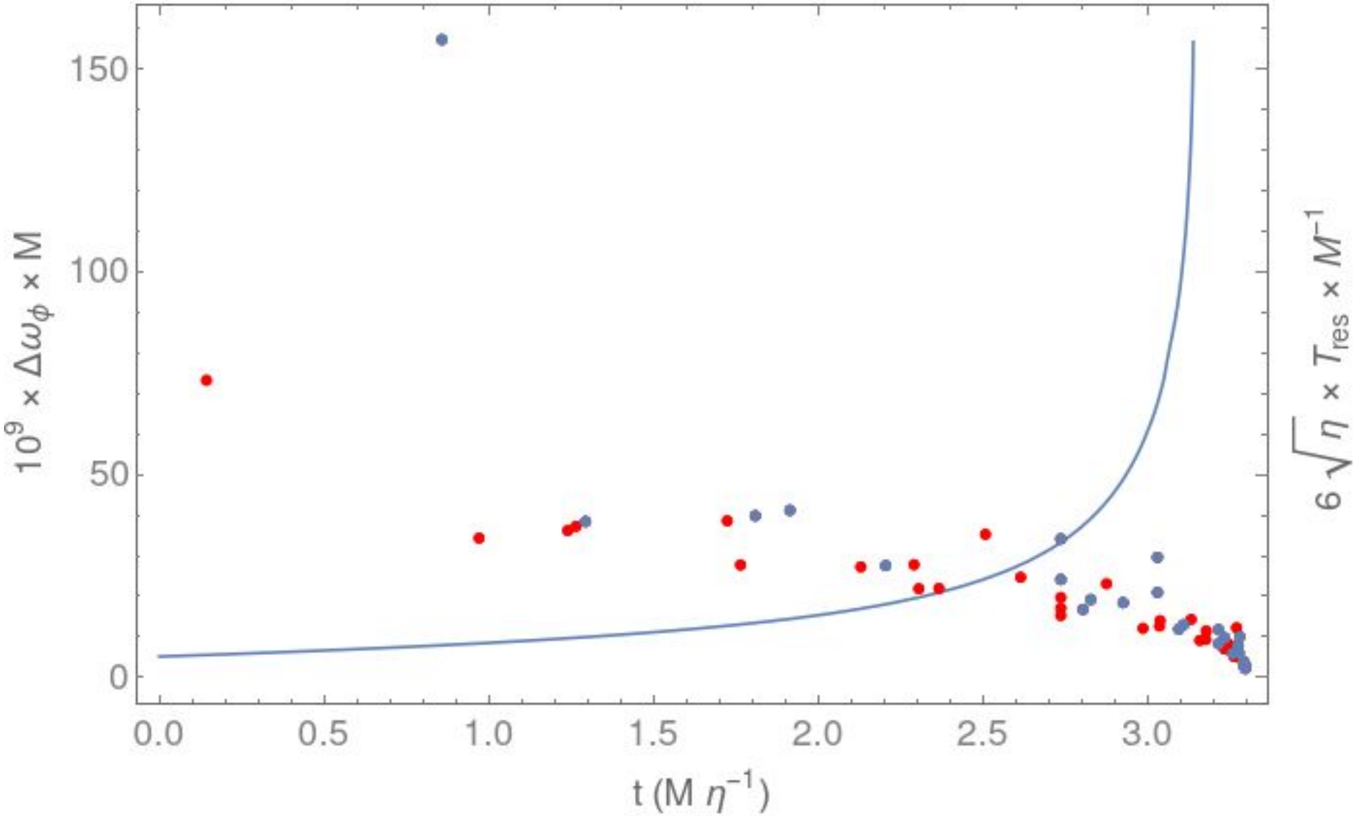
$$\Delta\Psi := \int_0^{T_{\rm plunge}} 2\Delta\omega_\phi dt$$

$$= 1.4 \left(\frac{\mu}{10 M_\odot}\right)^{-\frac{1}{2}} \left(\frac{M}{M_{\rm SgrA^*}}\right)^{\frac{7}{2}} \left(\frac{M_*}{10\, M_\odot}\right) \left(\frac{a}{4.3\,{\rm AU}}\right)^{-3}$$

Many resonances



Many resonances



Discussion