# Recent results on tidal deformability of black holes

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# Summary

Tidal deformability and Love numbers of compact objects

New results for AdS-Schwarzschild black holes

GW astronomy:

- possible measurements by present and future gravitational-wave detectors
- tests of GR and (exotic) compact objects in the late inspiral
- first opportunity to look for new physics at the horizon scale

# Tidal deformations and Love numbers

Tidal Love numbers (TLNs) encode the information about the deformability of an object in a tidal environment and depend significantly on the object internal structure and the dynamics of the gravitational field

At first order, the relation between the tidal field and the induced moment is constant

The TLNs of a black hole are precisely zero Fang & Lovelace (2005); Binnington & Poisson (2009); Damour & Nagar (2009) for arbitrary strong fields Gürlebeck (2015)

The TLNs of neutron stars contains information about their EOS Flanagan & Hinderer (2008); Hinderer (2008)

The TLNs enter the gravitational-wave signal as a 5PN correction which adds linearly to the phase of the waveform and (to the leading order) depends on the $l = 2$ polar TLN

Gravitational waveform in the frequency domain $\tilde{h}(f) = \mathcal{A}(f)\, e^{i(\psi_{PP}+\psi_{TD})}$

# Definitions

### Black-hole perturbations

- Regge-Wheeler-Zerilli Regge & Wheeler (1957); Zerilli (1970)
- Kodama-Ishibashi (higher dimensions) Kodama & Ishibashi (2003); Ishibashi & Kodama (2011)
- Kovtun-Starinets (branes) Kovtun & Starinets (2005)

We expand the perturbation in spherical harmonics (separated according to parity)

### Terminology

- polar/electric/even/scalar/longitudinal
- axial/magnetic/odd/vector/transverse

For each sector one obtains a second-order differential equation

To define Love numbers it is sufficient to consider and construct $g_{\mu\nu}$ in the vacuum region external to the body

To compute the Love numbers it is necessary to construct $g_{\mu\nu}$ in the body's interior as well

# Definitions

A compact object immersed in a tidal environment described by the polar (respectively, axial) moments $\mathcal{E}^{lm}$ and $\mathcal{B}^{lm}$

The mass and current multipole moments $M_l$ and $S_l$ of the compact object are deformed

In linear theory, these deformations are proportional to the applied tidal field

$$k_l^E \equiv -\frac{1}{2}\frac{l(l-1)}{R^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{M_l}{\mathcal{E}_{l0}}, \quad k_l^B \equiv -\frac{3}{2}\frac{l(l-1)}{(l+1)R^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{S_l}{\mathcal{B}_{l0}}$$

To extract the tidal field and the induced multipole moments from the solution

- expansion of the metric at large distances in terms of the multipole moments, *e.g.*

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{l\geqslant 2}\left(\frac{2}{r^{l+1}}\left[\sqrt{\frac{4\pi}{2l+1}}M_l Y^{l0} + (l' < l\,\text{pole})\right] - \frac{2}{l(l-1)}r^l\left[\mathcal{E}_l Y^{l0} + (l' < l\,\text{pole})\right]\right)$$

- evaluation of the Riemann tensor in Schwarzschild coordinates

More generically, Love numbers can be defined as the ratio between the normalisble and the non-normalisable solutions

# Other known results

Love numbers for Schwarzschild in higher dimensions are not zero Kol & Smolkin (2012)

Some classes of black holes beyond vacuum GR and exotic compact objects
Mendes & Yang (2017); Cardoso+ (2017); Sennett+ (2017)

Love numbers for neutron stars in $f(R)$ Yazadjiev, Doneva & Kokkotas (2018) and in higher dimensions
Chakravarti+ (2019)

Most of the results are for asymptotically flat spacetimes with some exceptions
Emparan, Fernández-Piqué & Luna (2017)

Notice that some theories introduce extra degrees of freedom (non-minimally) coupled to gravity and the response of black holes to external perturbations is generically richer

Partial results for Kerr black holes Poisson (2015); Pani+ (2015); Landry & Poisson (2015)

An important object was missing: AdS-Schwarzschild black holes

**Setup:** spherically symmetric, static solutions

$$ds^2 = -e^{\psi(r)}\, dt^2 + e^{\lambda(r)}\, dr^2 + r^2\, d\Omega^2\,, \quad e^{\psi(r)} = e^{-\lambda(r)} = 1 - \frac{2M}{r} - \frac{\Lambda}{3}\, r^2$$

Static perturbations in the Regge-Wheeler gauge $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$

We end up with a single equation for each sector that in general we need to numerically integrate

These Love numbers depend on the multipolar number $l$ and on the dimensionless combination $\sqrt{-\Lambda}\, M$

For pure AdS ($M = 0$) the results are analytical

We also have a map between RWZ, KI and KS, for master functions and equations and Love numbers

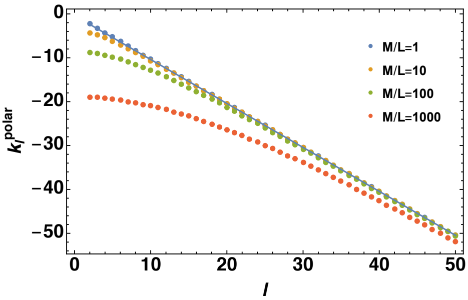
## Polar sector

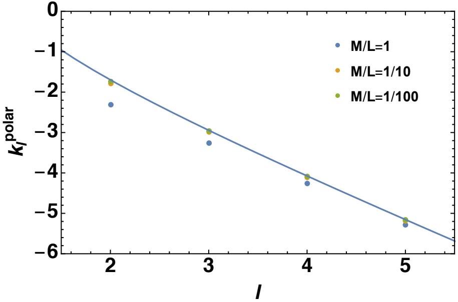
Asymptotically, $H$ is given by a non-normalisable and a normalisable solution,

$$H \sim \frac{C_1^{\rm p}}{r}\left[1 - \frac{3(l^2+l-4)}{2\Lambda r^2} + \dots\right] + \frac{C_2^{\rm p}}{r^2}\left[1 + \dots\right]$$

and we define the dimensionless polar Love numbers as

$$k_{\rm polar} \equiv \frac{1}{L}\frac{C_2^{\rm p}}{C_1^{\rm p}} \xrightarrow{l\to\infty} -l, \quad \text{where } L \equiv \sqrt{-\frac{3}{\Lambda}}$$



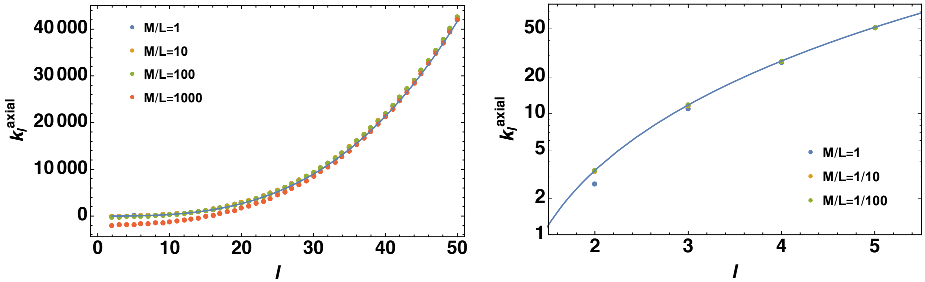



The asymptotic behavior of $h_0$ is

$$h_0 \sim C_1^a r^2 \left[1 + \frac{3(l-1)(l+2)}{2\Lambda r^2} + \dots\right] + \frac{C_2^a}{r}\left[1 + \dots\right]$$

and we define the dimensionless axial Love numbers as

$$k_{\text{axial}} \equiv \frac{1}{L^3}\frac{C_2^a}{C_1^a} \xrightarrow{l\to\infty} \frac{l^3}{3}\,, \quad \text{where } L \equiv \sqrt{-\frac{3}{\Lambda}}$$



# Pure AdS

For pure AdS ($M = 0$) the Love numbers can be computed analytically

$$k_{\text{polar}} = -\frac{2(l-1)(l+2)}{l(l+1)} \frac{\Gamma\left(\frac{l}{2}+1\right)^2}{\Gamma\left(\frac{l+1}{2}\right)^2} \quad \xrightarrow{l\to\infty} \; -l\,, \quad k_{\text{axial}} = \frac{2(l-1)(l+2)}{3} \frac{\Gamma\left(\frac{l}{2}+1\right)^2}{\Gamma\left(\frac{l+1}{2}\right)^2} \quad \xrightarrow{l\to\infty} \; \frac{l^3}{3}$$

In the eikonal limit $l \to \infty$ these results agree with Emparan, Fernández-Piqué and Luna (2017) for large wavenumber $k$

# Tidal Love numbers of ECOs and black holes Cardoso, EF, Maselli, Pani & Raposo (2017)

- Spherically symmetric, static background geometries
- The only surviving tide at large distances is gravitational
- Black hole limit: $\xi \to 0$ or $r_0 \to 2M$, $\xi \equiv r_0/2M - 1$

| | | Tidal Love numbers | | | |
|---|---|---|---|---|---|
| | | $k_2^E$ | $k_3^E$ | $k_2^B$ | $k_3^B$ |
| NS | static, $C \approx 0.2$ | 210 | 1300 | 11 | 70 |
| ECOs | Boson star | 41 | 403 | $-14$ | $-212$ |
| | Wormhole | $\frac{4}{5(8+3\log\xi)}$ | $\frac{8}{105(7+2\log\xi)}$ | $\frac{16}{5(31+12\log\xi)}$ | $\frac{16}{7(209+60\log\xi)}$ |
| | Perfect mirror | $\frac{8}{5(7+3\log\xi)}$ | $\frac{8}{35(10+3\log\xi)}$ | $\frac{32}{5(25+12\log\xi)}$ | $\frac{32}{7(197+60\log\xi)}$ |
| | Gravastar | $\frac{16}{5(23-6\log 2+9\log\xi)}$ | $\frac{16}{35(31-6\log 2+9\log\xi)}$ | $\frac{32}{5(43-12\log 2+18\log\xi)}$ | $\frac{32}{7(307-60\log 2+90\log\xi)}$ |
| BHs | Einstein-Maxwell | 0 | 0 | 0 | 0 |
| | Brans-Dicke | 0 | 0 | 0 | 0 |
| | Chern-Simons | 0 | 0 | $1.1\alpha_{CS}^2/M^4$ | $11.1\alpha_{CS}^2/M^4$? |

For boson stars, in the Newtonian regime, $k_l^E \sim 1/C^{2l+1}$ and $k_l^B \sim -1/C^{2l}$

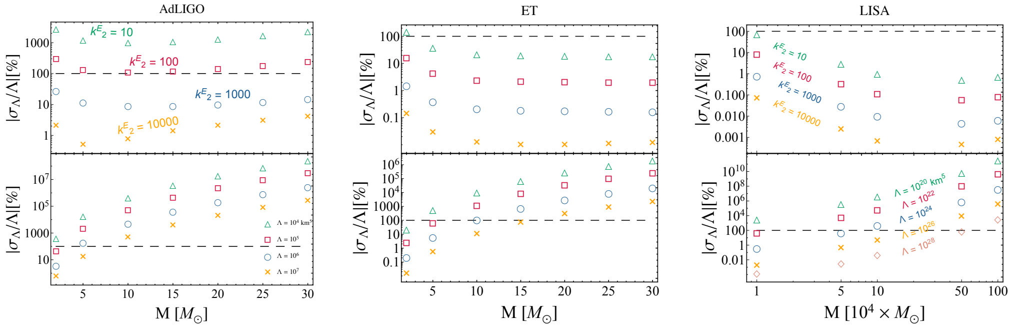
## Detectability: Model-independent tests

Assumptions:

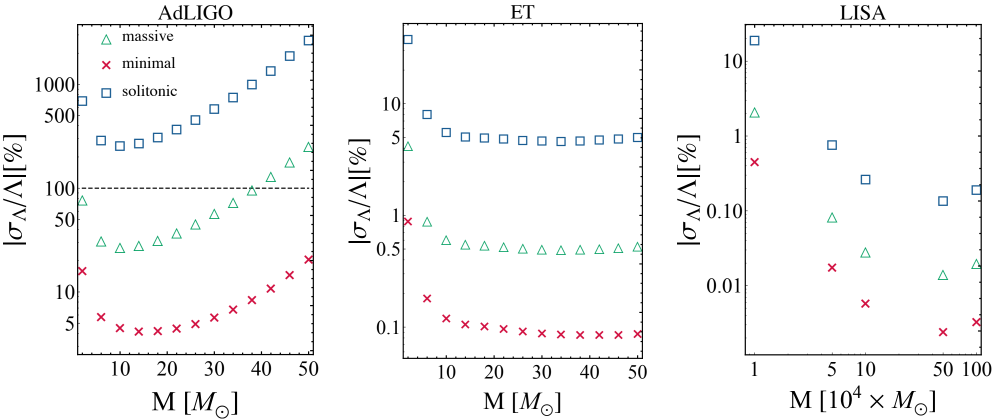
- Equal mass binaries
- For terrestrial interferometers the prototype binary is at $d = 100\,\mathrm{Mpc}$
- For LISA the source is located at $d = 500\,\mathrm{Mpc}$

Tidal average deformability

$$\Lambda = \frac{1}{26}\left[\left(1+\frac{12}{q}\right)\lambda_1 + (1+12q)\lambda_2\right] \qquad \lambda = \frac{2}{3}M^5 k_2^E$$



# Detectability: Boson stars




A more detailed analysis on massive and solitonic boson stars Sennett+ (2017)

# Quantum corrections at the horizon scale?

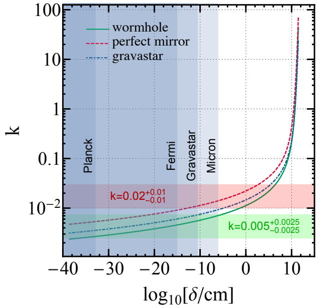
Love numbers of ultracompact objects

$$k_l \sim \left(\log \frac{\delta}{M}\right)^{-1}$$

Not universal, *e.g.* ultracompact anisotropic stars Raposo+ (2019)

Measurement of the Love numbers translate into an estimate of the distance of the ECO surface

$k \sim 0.005$ in order to probe Planckian corrections



# Extensions/What's next?

- Include tidal fields of different nature (electromagnetic, scalar)
  $\rightarrow$ New families of Love numbers
- Include tidal corrections of order $>$ 5PN
- Alternative theories of gravity (in progress)
- $AdS_5$ for holography
- Rotating black holes

# Conclusion & Outlook

- New results for AdS-Schwarzschild black holes (non-zero TLNs)
- Non-vanishing (logarithmically small) Love numbers of ECOs
- Tidal effects can be used to explore
  - Nature of event horizons
  - Existence of ECOs
  - Strong-field behaviour of gravity
- Detectability
  - **aLIGO** ECOs with $C \lesssim 0.2$; constraints on BS models, exclude models with small compactness
  - **ET** ECOs with $C \lesssim 0.35$; can discriminate a BS binary from a BH binary
  - **LISA** supermassive ECOs with $C \lesssim 0.49$; can discriminate a BS binary from a BH binary


## Questions?