Black hole scattering in post-Minkowskian (classical and quantum) gravity

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Solving the relativistic two-body problem

- Numerical relativity
- Analytic approximation schemes:

Limit	Perturbation theory	Natural for
Newtonian gravity $c \to \infty$	post-Newtonian $\frac{m_1}{m_2} \sim 1, \qquad \frac{Gm}{rc^2} \sim \frac{v^2}{c^2} \ll 1$	bound orbits
special relativity $G \to 0$	post-Minkowskian $\frac{m_1}{m_2} \sim 1, \qquad \frac{Gm}{rc^2} \ll \frac{v^2}{c^2} \sim 1$	scattering
test-body motion in a stationary background $\frac{m_1}{m_2} \to 0$	post-test-body ("self-force") $\frac{m_1}{m_2} \ll 1, \qquad \frac{Gm}{rc^2} \sim \frac{v^2}{c^2} \sim 1$	both

post-Newtonian (PN) vs. post-Minkowskian (PM)

		0PN	1PN	2PN	3PN	4PN‡	S5PN‡	6PN‡	…
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	v^{14}	
1PM:	tree	$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	v^{12}/r	
2PM:	1-loop		$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	v^{10}/r^2	
3PM:	2-loop			$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	v^8/r^3	
4PM:	3-loop				$1/r^4$	v^2/r^4	v^4/r^4	v^6/r^4	
5PM:	4-loop					$1/r^5$	v^2/r^5	v^4/r^5	
6PM:	5-loop						S$1/r^6$	v^2/r^6	
…								…	

$$1 \to mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{Gm}{rc^2}.$$

current PN results. current PM results. matching overlap. unknown.

(monopole) conservative dynamics. ‡mod tails. mod logs. Sstatic.

PM binary-black-hole conservative dynamics: results

	spin^0	spin^1	spin$^{2,3,\ldots}$
1PM	('70s, ...) Bicak+ 0807. Foffa 1309. Damour 1609.	*G–AH-H-H Bini, Damour 1709.	*G–AH-H-H JV 1709.
2PM	Westpfahl '85 *G–AH-H-H *Damour 1710. *Bjerrum-Bohr+ 1806. *Cheung+ 1808.	*G–AH-H-H Bini, Damour 1805.	*G–AH-H-H JV+ 1812. *Guevara+ 1812. *Chung+ 1812. –
3PM	*Bern+ 1901.	–	–

*Amplitudes (see also Antonelli+ 1901. on 3PM EOB)

G–AH-H-H: Guevara 1706., using Arkani-Hamed, Huang & Huang 1709.

... and many more related papers in the past few years ...

Spin effects in BBH conservative dynamics

PN order: 0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
N	1PN		2PN		3PN		4PN		5PN		
		LO SO		NLO SO		NNLO SO		NNNLO SO			
			LO S^2		NLO S^2		NNLO S^2		NNNLO S^2		
						LO S^3		NLO S^3		NNLO S^3	
							LO S^4		NLO S^4		
										LO S^5	
											LO S^6

need up to

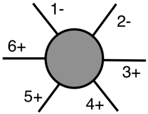
- 1PM / tree
- 2PM / 1-loop
- 3PM / 2-loop
- 4PM / 3-loop
- 5PM / 4-loop
- 6PM / 5-loop

New from Amplitudes: Guevara, Ochirov, JV 1812.†
Chung, Huang, Kim, Lee 1812.‡

using minimally coupled massive spinor-helicity amplitudes from Arkani-Hamed, Huang, Huang 1709.

Lessons from Scattering Amplitudes

- Big long results often collapse into super-simple ones
——when reorganized in the proper way——e.g.:
  - Parke-Taylor MHV six-gluon amplitude ('86):



$$= 220 \text{ Feynman diagrams} = \frac{\langle 12\rangle^4}{\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 63\rangle}$$

- Work (only?) with **observable gauge-invariant** quantities:
("the S-matrix approach")
  - On-shell amplitudes
  - Scattering angles (encode same info as Hamiltonians)
- **"Bootstrapping"**: amplitudes are surprisingly tightly constrained by general principles such as symmetries, unitarity, locality...
——sometimes you can just write down the answer (or a parametrization)
- Using good variables: geometrical quantities; spinor-helicity/twistors, ...
- Unitarity methods; soft theorems; double copy: GR = $(\text{QCD})^2$; ...

A nonspinning [or aligned-spin] PN Hamiltonian

$$H(R,\, P_R,\, L\,;\, m_1,\, m_2)$$
$$\Big[\text{or } H(R,\, P_R,\, L\,;\, m_1,\, m_2,\, S_1,\, S_2)\Big]$$

:

$$\dot{R} = \frac{\partial H}{\partial P_R}, \qquad \dot{\phi} = \frac{\partial H}{\partial L},$$
$$\dot{P}_R = -\frac{\partial H}{\partial R}, \qquad \dot{L} = -\frac{\partial H}{\partial \phi} = 0.$$

with $\mu^2 \bar{P}^2 = P^2 = P_R^2 + \dfrac{L^2}{R^2}$, $\qquad \mu = \nu M = \dfrac{m_1 m_2}{M}$, $\qquad M = m_1 + m_2$,
in an isotropic gauge:

$$\begin{aligned}\frac{H}{\mu} &= \frac{Mc^2}{\mu} + \frac{\bar{P}^2}{2} - \frac{GM}{R} \\ &\quad + \frac{1}{c^2}\left(-\frac{1+\nu}{8}\bar{P}^4 - \frac{3-\nu}{2}\bar{P}^2\frac{GM}{R} + \frac{1-\nu}{2}\frac{G^2M^2}{R^2}\right) \\ &\quad + \frac{1}{c^4}\left(\frac{1+\nu+\nu^2}{16}\bar{P}^6 + \frac{5+5\nu-3\nu^2}{8}\bar{P}^4\frac{GM}{R}\right. \\ &\qquad \left. + \frac{10-\nu+3\nu^2}{4}\bar{P}^2\frac{G^2M^2}{R^2} - \frac{1+2\nu^2}{4}\frac{G^3M^3}{R^3}\right) + \dots\end{aligned}$$

The gauge-invariant scattering angle χ

- Solve $E = H(R, P_R, L)$ for $P_R = P_R(R, E, L)$,
  then find the com-frame scattering angle χ from [Damour 1710.]

$$\pi + \chi(E, L) = -\int_{\infty}^{\infty} dR \, \frac{\partial}{\partial L} P_R(R, E, L).$$

- New variables from SR @ ∞: relative velocity v, impact parameter b,

$$E^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \qquad \gamma = \frac{1}{\sqrt{1 - v^2}}, \qquad L = \frac{m_1 m_2}{E} \gamma v b.$$

- $\chi(v, b)$, including the (local-in-time) Hamiltonian through 4PN,

$$\frac{M}{E}\chi = 2\frac{GM}{v^2 b}\Big(1+v^2+O(v^{10})\Big)+\frac{3\pi}{4}\left(\frac{GM}{v^2 b}\right)^2\Big(4v^2+v^4+O(v^{10})\Big)+O(G^3),$$

  encodes the complete gauge-invariant info. of the Hamiltonian.

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encodes the complete gauge-invariant info. of the PN-PM Hamiltonian.

- According to Westpfahl '85, [after a large gauge-dependent mess]

$$\frac{M}{E}\chi = 2\frac{GM}{v^2 b}(1+v^2) + \frac{3\pi}{4}\left(\frac{GM}{v^2 b}\right)^2 (4v^2+v^4) + O(G^3).$$

The gauge-invariant scattering angle χ

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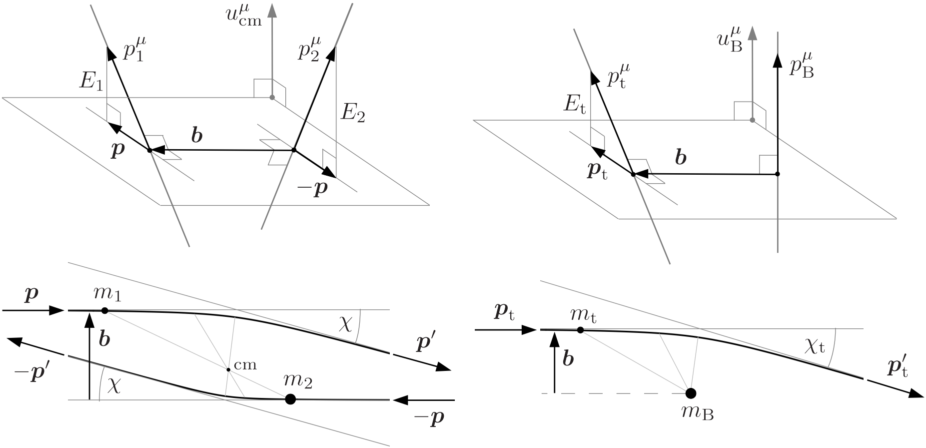
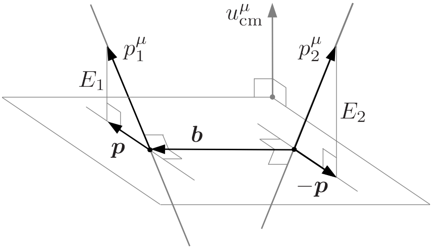
- The Schwarzschild result,

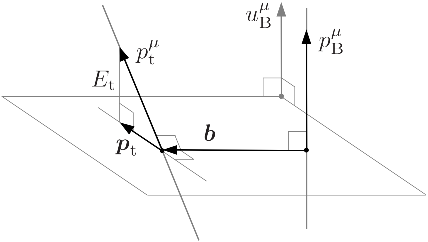
$$\chi_{\rm Schw} = 2\frac{GM}{v^2 b}(1+v^2)+\frac{3\pi}{4}\left(\frac{GM}{v^2 b}\right)^2(4v^2+v^4)+O(G^3).$$

Two-body vs. test-body



Two-body vs. test-body





$$\gamma = \frac{p_1 \cdot p_2}{m_1 m_2},$$

$$E = E_1 + E_2 = \sqrt{m_1^2 + m_2^2 + 2 m_1 m_2 \gamma}$$

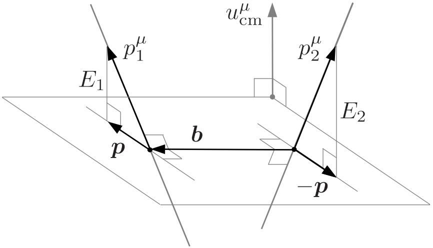
$$\gamma_{\rm t} = \frac{p_{\rm t} \cdot p_{\rm B}}{m_{\rm t} m_{\rm B}},$$

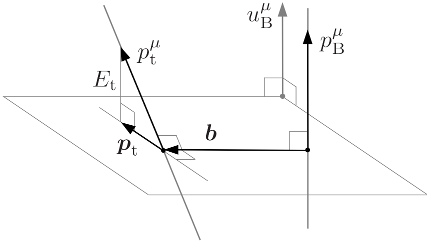
$$E_{\rm t} = m_{\rm t} \gamma_{\rm t}$$

- Identify $\gamma = \gamma_t$,
- "Newtonian EOB" mass maps: $m_{\rm t} = \dfrac{m_1 m_2}{m_1 + m_2}, \quad m_{\rm B} = m_1 + m_2,$

$\Rightarrow$ EOB energy map: $E_t = \dfrac{E^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}$.

Two-body vs. test-body





$$E = E_1 + E_2 = \sqrt{m_1^2 + m_2^2 + 2m_1 m_2 \gamma} \qquad E_{\rm t} = m_{\rm t}\gamma_{\rm t}$$

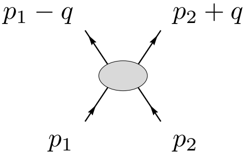
- Identify $\gamma = \gamma_t, \quad m_{\rm t} = \dfrac{m_1 m_2}{m_1 + m_2}, \quad m_{\rm B} = m_1 + m_2,$

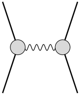
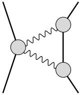
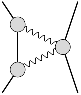
$\Rightarrow$ EOB energy map: $E_t = \dfrac{E^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$

——Applying the energy map to the Schw. geodesic Ham.,
you get a two-body Ham. correct at 1PM.
[Damour 1609., Bini-Damour 1709.]

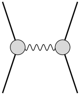
The scattering angle from amplitudes

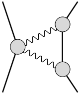
The relevant classical part of the amplitude
for two scalars exchanging gravitons:

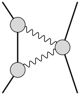


$= \mathcal{M} =$  $+$  $+$  $+ \mathcal{O}(G^3),$

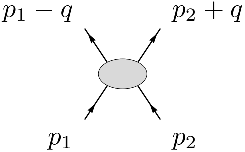
with $\quad p_1 \cdot p_2 = \gamma = (1 - v^2)^{-1/2}$ and $\quad p_1 \cdot q = p_2 \cdot q = 0,$

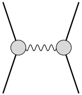

$$= \frac{16\pi G (m_1 m_2 \gamma)^2}{\hbar |\boldsymbol{q}|^2}(1 + v^2),$$

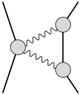
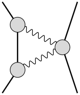

$$= \frac{6\pi^2 G^2 (m_1 m_2 \gamma)^2}{\hbar^2 |\boldsymbol{q}|}(4 + v^2) m_2,$$


$$= \frac{6\pi^2 G^2 (m_1 m_2 \gamma)^2}{\hbar^2 |\boldsymbol{q}|}(4 + v^2) m_1.$$

The scattering angle from amplitudes

$\mathcal{M} =$ 

$= \Big[$  $= \dfrac{16\pi G(m_1 m_2 \gamma)^2}{\hbar |\boldsymbol{q}|^2}(1+v^2) \Big]$

$+ \Big[$  $+$  $= \dfrac{6\pi^2 G^2 (m_1 m_2 \gamma)^2}{\hbar^2 |\boldsymbol{q}|}(4+v^2)\Big(m_2 + m_1\Big) \Big]$

Scattering angle from the eikonal approximation [Bjerrum-Bohr+ 1806.],

$$\chi = -\frac{\hbar E}{4v^2}\frac{\partial}{\partial b}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}/\hbar}\frac{\mathcal{M}(\boldsymbol{q})}{(m_1 m_2 \gamma)^2} + \mathcal{O}(G^3)$$

$$= 2\frac{GE}{b}\left(\frac{1+v^2}{v^2} + \frac{3\pi}{8}\frac{G(m_2+m_1)}{b}\frac{4+v^2}{v^2}\right) + O(G^3),$$

matching Westpfahl ’85. (test-body limit: $m_1 \to 0$, $E \to m_2$)

Adding spin: classical approaches

- **Spin-orbit** (linear-in-spin, dipole) effects, which are universal, can be determined from the **metric for monopolar bodies**, via the **parallel transport map** along the monopolar trajectories, (assuming of the existence of a canonical Hamiltonian) (**crucial boost between "covariant" and "canonical" spins**)
  1PM: Bini & Damour 1709.
  2PM: Bini & Damour 1805.
- All-multipole effects for black holes at 1PM $\left[\, \mathcal{I}_\ell + i\mathcal{J}_\ell = m(ia)^\ell \,\right]$ can be determined from assuming existence of an effective action [already in Levi & Steinhoff 1501.] and matching to the linearized Kerr solution,

$$\bar{h}^{\mu\nu} = u^\rho u^{(\mu} \exp(a * \partial)^{\nu)}{}_\rho \frac{4Gm}{r}, \qquad (a * \partial)^\mu{}_\nu = \epsilon^\mu{}_{\nu\rho\sigma} a^\rho \partial^\sigma,$$

(**crucial translation between "covariant" and "canonical" worldlines**)
JV 1709.

special relativistic kinematics at infinity

Adding spin: classical approaches

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  JV 1709.

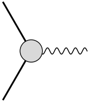
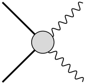
  ——Results reveal remarkable EOB-like maps for BBHs at 1PM:
  aligned-spin scattering angle:

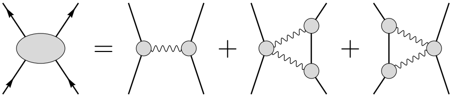
  $$\text{Kerr geod.:} \qquad \chi = \frac{GM}{v^2}\left(\frac{(1+v)^2}{b+a} + \frac{(1-v)^2}{b-a}\right) + O(G^2).$$

  $$\text{BBH:} \qquad \chi = \frac{GE}{v^2}\left(\frac{(1+v)^2}{b+a_1+a_2} + \frac{(1-v)^2}{b-a_1-a_2}\right) + O(G^2).$$

Adding spin: amplitudes approaches

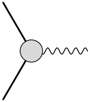
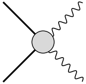
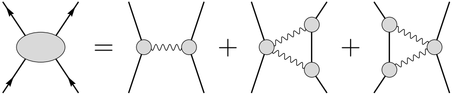
- A key observation (at leading PN orders) from Vaidya 1410.:
  for **minimally coupled** massive spin-s particles exchanging gravitons,
  - spin-0 — (universal) monopole coupling
  - spin-$\frac{1}{2}$ — adds (universal) dipole coupling
  - spin-1 — adds **black-hole** quadrupole coupling
  - spin-2 — adds **black-hole** (octupole and) hexadecapole
- Minimally coupled "amplitudes for arbitrary masses and spins"
  using massive spinor-helicity, Arkani-Hamed, Huang, Huang 1709.,

"3-point":  for any s, "Compton":  for $s \leq 2$.

- Guevara 1706.:  $+\,\mathcal{O}(G^3)$,

  and some matching to the PN potential à la Holstein & Ross 0802.
  (Leading singularity: Cachazo & Guevara 1705.)

Adding spin: amplitudes approaches

- Vaidya 1410.: minimal spin-s gives black-hole 2^{2s}-pole (LO PN, $n \leq 2$)
- Arkani-Hamed, Huang, Huang 1709.:  $\forall\, s$,  $s \leq 2$.
- Guevara 1706.:  $+\,\mathcal{O}(G^3)$.
- Guevara, Ochirov, JV 1812.:
  match to linearized Kerr à la JV 1709.,
  aligned-spin BBH scattering angle: 1PM spin$^\infty$ ($s \to \infty$), 2PM spin^4,
  connect to sub-subleading soft theorem of Cachazo & Strominger 1404.
- Chung, Huang, Kim, Lee 1812.:
  match to effective action of Levi & Steinhoff 1501.,
  contributions to PN potential: LO spin$^\infty$, NLO spin^4,
  discussion of $s > 2$.
- Guevara & Bautista 1903.:
  new perspective on multipoles and matching, double copy, LO radiation.

Adding spin: amplitudes approaches

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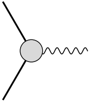
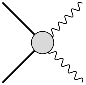
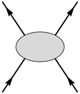
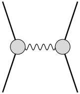
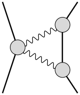
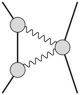
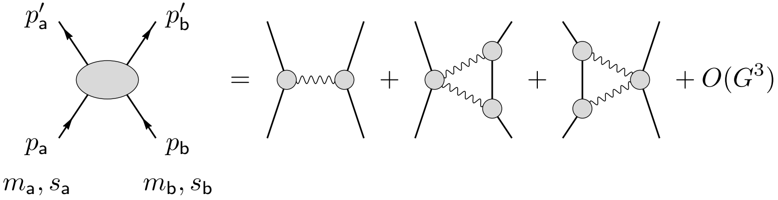
- Arkani-Hamed, Huang, Huang 1709.:  $\forall\, s$,  $s \leq 2$.
- Guevara 1706., Guevara, Ochirov, JV 1812.,
  Chung, Huang, Kim, Lee 1812., Guevara & Bautista 1903.,
  —–aligned-spin scattering angle:

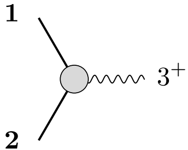
[ $= \chi(v, b, m_1, m_2, a_1, a_2)$] = [ $= GE\, f_1(v, b, a_1 + a_2)$]

+ [ $= G^2 E\, m_2\, f_2(v, b, a_1, a_2)$] + [ $= G^2 E\, m_1\, f_2(v, b, a_2, a_1)$]

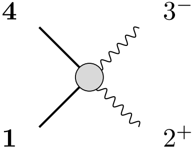
$+\, O(a^5)$ @ 2PM $\quad \Rightarrow$ all obtained from the spinning-test-BH limit

JV, Steinhoff, Buonanno 1812. (to NNLO-PN spin^2)





$$= \frac{1}{m_{\rm P}} \left(\frac{\langle \zeta | p_1 | 3]}{\langle \zeta 3 \rangle} \right)^2 \left(\frac{\langle \mathbf{1}\mathbf{2} \rangle}{m} \right)^{2s}, \qquad \text{for any } s,$$



$$= \frac{-\langle 3 | p_1 | 2]^4}{m_{\rm P}^2 \mathsf{t} (\mathsf{s} - m^2)(\mathsf{u} - m^2)} \left(\frac{\langle \mathbf{4} 3 \rangle [\mathbf{1} 2] + \langle \mathbf{1} 3 \rangle [\mathbf{4} 2]}{\langle 3 | p_1 | 2]} \right)^{2s},$$

for $s \leq 2$.

[Arkani-Hamed+ '17, Guevara '17]

Mathisson-Papapetrou-Dixon dynamics

- Dynamics of an "extended test body", worldline $z(\lambda)$ in background g_{ab},

$$\frac{Dp_a}{d\lambda} + \frac{1}{2}R_{abcd}\dot{z}^b S^{cd} = F_a = -\frac{1}{6}R_{bcde;a}J_2^{bcde} - \frac{1}{12}R_{bcde;fa}J_3^{fbcde} + \dots$$

$$\frac{DS^{ab}}{d\lambda} - 2p^{[a}\dot{z}^{b]} = N^{ab} = \frac{4}{3}R^{[a}{}_{cde}J_2^{b]cde} + \dots$$

- Multipoles $J_n^{abcd\cdots}$ depend on body's internal structure and dynamics
- What should they be for a **"spinning test black hole"** in vacuum**?**
  —an infinitely-small-mass, ultra-super-extremal Kerr
  naked ring singularity of finite radius $a = S/m$? (string worldsheet?)
- Assume only d.o.f.s are z, p, S (others "integrated out") —"minimal MPD"
- Constrain $p_a S^{ab} = 0$, solve for $\dot{z}^a$
- $\Rightarrow$ $J_n^{abcd\cdots}$ built covariantly from only p^a, S^{ab} and $R_{abcd;\dots}(z)$

Couplings in an effective action

- Action approach to minimal MPD: $J_n^{abcd\cdots}(p,S,R)$ all determined by one scalar function $\mathcal{M}^2(u,S,R)$, where $u^a = \dfrac{p^a}{\sqrt{-p^2}}$, such that $p^2 + \mathcal{M}^2 = 0$: dynamical mass shell condition

$$F_a = \frac{p\cdot\dot{z}}{2}\nabla_a^{\text{horizontal}}\log\mathcal{M}^2,$$

$$N^{ab} = p\cdot\dot{z}\left(p^{[a}\frac{\partial}{\partial p_{b]}} + 2S^{[a}{}_c\frac{\partial}{\partial S_{b]c}}\right)\log\mathcal{M}^2.$$

- From matching to (unperturbed) (linearized) Kerr, we know

$$\mathcal{M}^2 = m^2 + 2m^2\left(-\frac{R_{uaua}}{2!} + \frac{R^*_{uaua;a}}{3!} + \frac{R_{uaua;aa}}{4!} - \frac{R^*_{uaua;aaa}}{5!} + \ldots\right)$$

$$\boxed{+\,O(R^2)} \qquad \leftarrow\ : \text{tidal effects?} \qquad\qquad \uparrow\ :\ J_n \sim m\,a^n$$

Couplings in an effective action

- Action approach to minimal MPD: $J_n^{abcd\cdots}(p,S,R)$ all determined by one scalar function $\mathcal{M}^2(u,S,R)$, where $u^a = \dfrac{p^a}{\sqrt{-p^2}}$,
  such that $p^2 + \mathcal{M}^2 = 0$: dynamical mass shell condition

- (Rescaled) spin vector a^a, $S_{ab} = m\,\epsilon_{abcd}u^c a^d$, $u \cdot a = 0$,
  constant bare rest mass m, $\mathcal{M}^2 = m^2 + O(R)$

- From matching to (unperturbed) (linearized) Kerr, we know

$$\mathcal{M}^2 = m^2 + 2m^2\left(-\frac{R_{uaua}}{2!} + \frac{R^*_{uaua;a}}{3!} + \frac{R_{uaua;aa}}{4!} - \frac{R^*_{uaua;aaa}}{5!} + \ldots\right)$$

$\boxed{+\,O(R^2)}$ $\leftarrow$: tidal effects? $\uparrow$: $J_n \sim m\,a^n$

Relevant couplings for a spinning test black hole

- Suppressing indices, $*$'s, u's, dimensionless coefficients,

$$\frac{\mathcal{M}^2}{m^2} = 1 + R\,a^2 + \nabla R\,a^3 + \nabla^2 R\,a^4 + \nabla^3 R\,a^5 + \nabla^4 R\,a^6 + \dots$$

$$\oplus\ R^2 a^4 \quad \oplus\ \nabla R^2 a^5\ \oplus\ \nabla^2 R^2 a^6\ \oplus \dots$$

$$\oplus\ R^3 a^6 \quad \oplus \dots$$

  plus many other $R^{\geq 2}$ terms (with powers of m),

$$\oplus\ (m\nabla)^k\,(a\nabla)^l\,\left(\frac{a}{m}\right)^n\left(m^4 R^2\ \oplus\ m^6 R^3\ \oplus \dots\right),$$

  e.g.: $m^4 R^2$ terms, $k, l, n = 0$: leading adiabatic quadrupolar tidal effects

- Reasonable conjecture?:
  If a spinning test black hole limit exists, it should have only m^0 terms.

Relevant couplings for a spinning test black hole

- PM resums/reorganizes PN,
  reveals new connections to the test-body limit.
- Amplitudes reveal hidden simplicity.
  [reviews: Elvang & Huang 1308.1697, Cheung 1708.03872]
- Black holes correspond to minimal coupling. (how/why??)
- Not discussed here and/or future directions:
  - Radiation!..
  - The high-energy/ultrarelativistic limit...
  - Double copy: " GR = (QCD)^2 "...
  - Precessing spins...
  - Bodies other than black holes...
  - Theories other than GR...