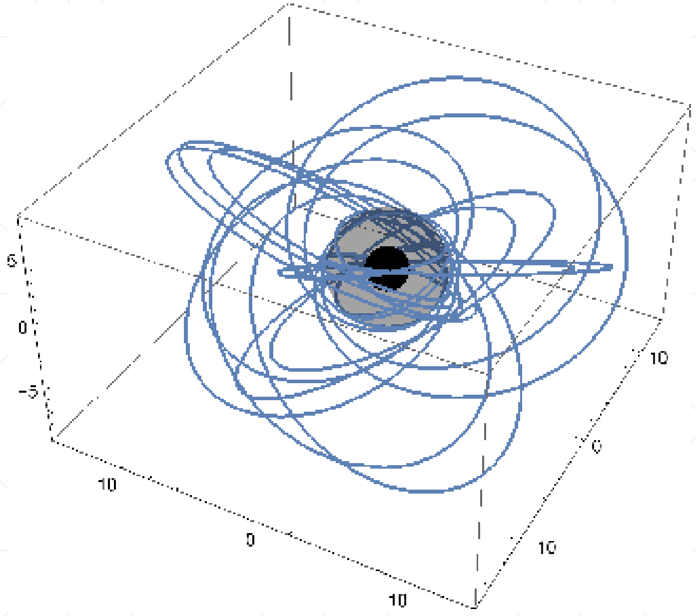


Influence of secondary spin in EMRIs

Vojtěch.Witzany@asu.cas.cz
Astronomical Institute of the Czech Academy of Sciences
20.6.’19, 22nd Capra meeting, Rio de Janeiro, Brasil

Our team @ Prague

Georgios
Lukes-Gerakopoulos
Secondary spin, chaos

Ondřej Kopáček
Time-series, chaos

Lukáš Polcar
Analytical pert. techniques

Ondřej Zelenka,
Secondary spin, Teukolsky wvf.

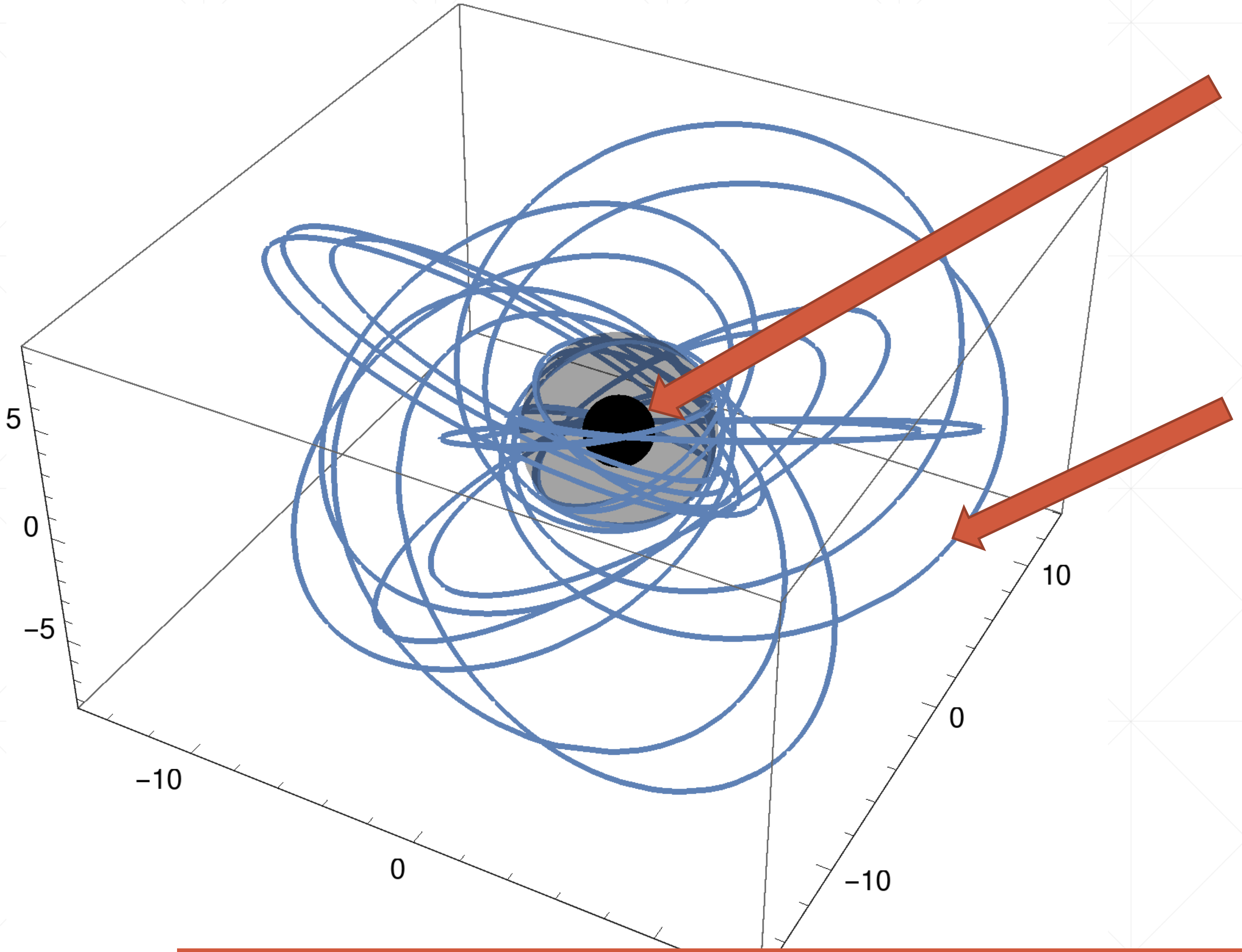
Jiří Svoboda
Czech hardware f. ESA

Petra Suková
Orbital chaos, accretion

Contents

- Finite size of secondary – what we have to include, what we can neglect
- General dynamics of secondary spin
- Resonances, chaos

Finite size effects in secondary – what to include?

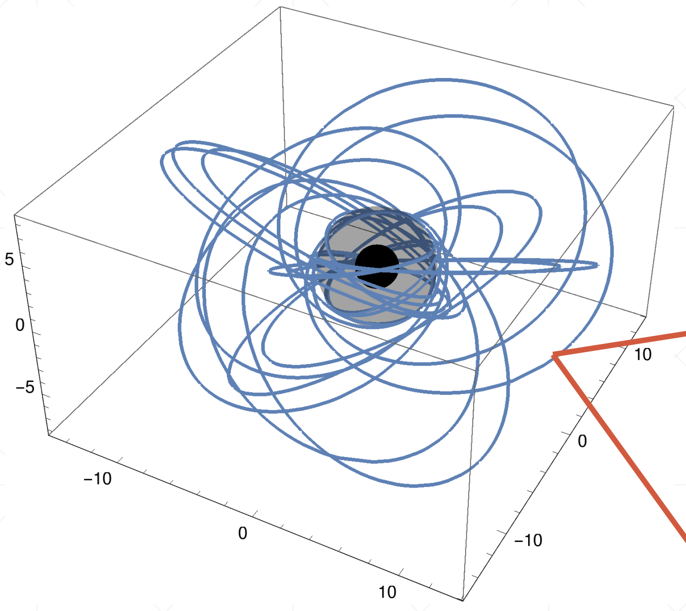


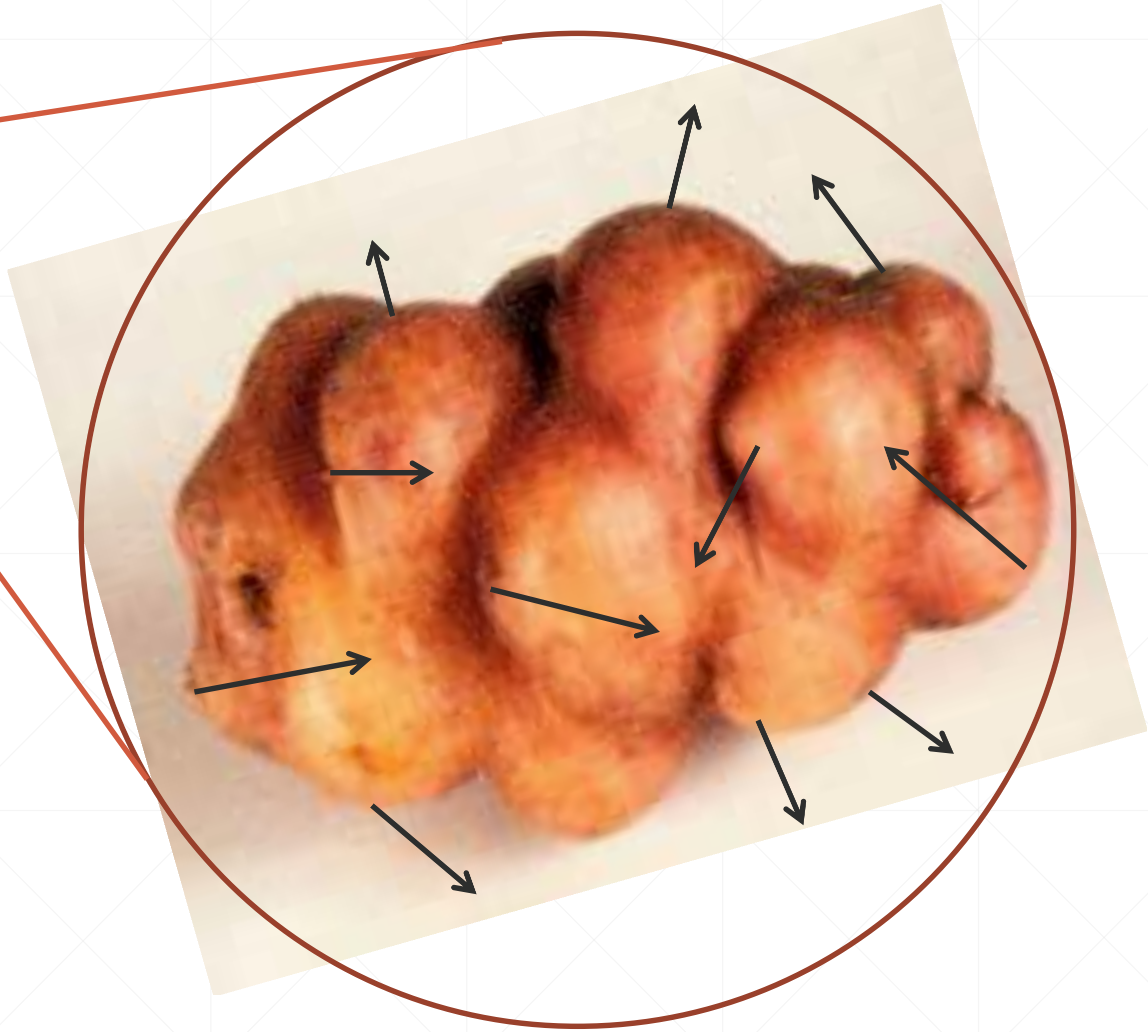
Primary mass: M
Bg variability length:
$R_{\mathrm{c}} \sim \sqrt{r^3/M}$

Secondary mass: μ
Secondary size: R
For black holes, neutron stars $R = \text{few}\,\mu$
For white dw., brown dw., main sequence $R \gg \mu$!!

Self-force – powers of μ/R_{c}
Finite-size – powers of R/R_{c}
(„Finite-time“ – powers of $t_?/T_{\mathrm{orb}} \sim t_?/R_{\mathrm{c}}$*)*

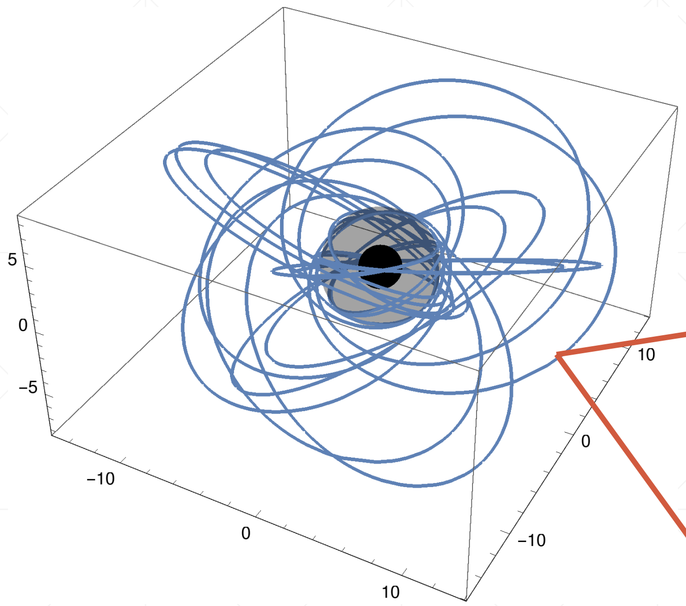
In principle

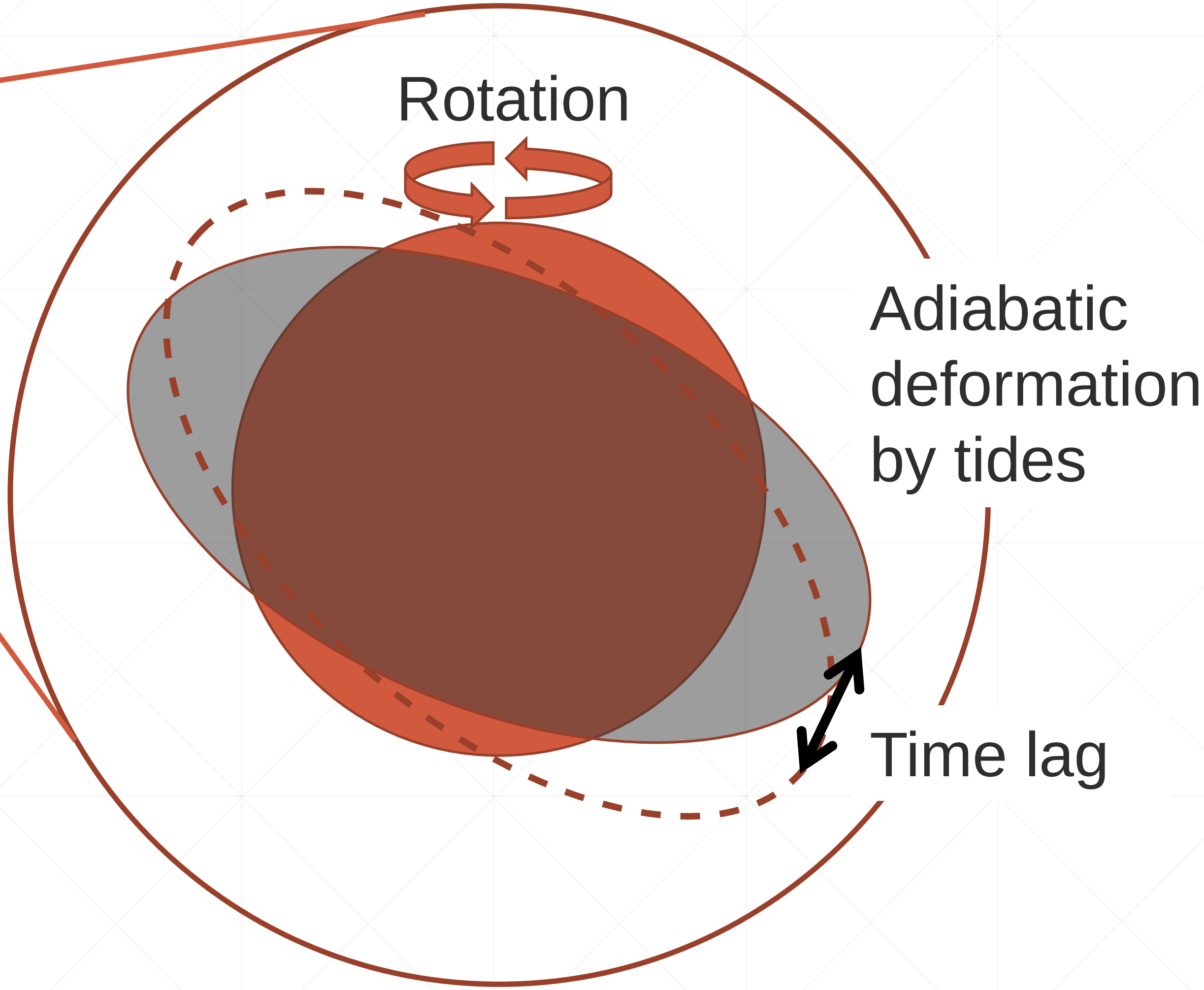




Infinite number of oscillation modes,
infinite number of new degrees of freedom,…

In effect





Only a single new degree of freedom – the orientation of the rotation axis!

Practical model

- Approximately rigid rotation

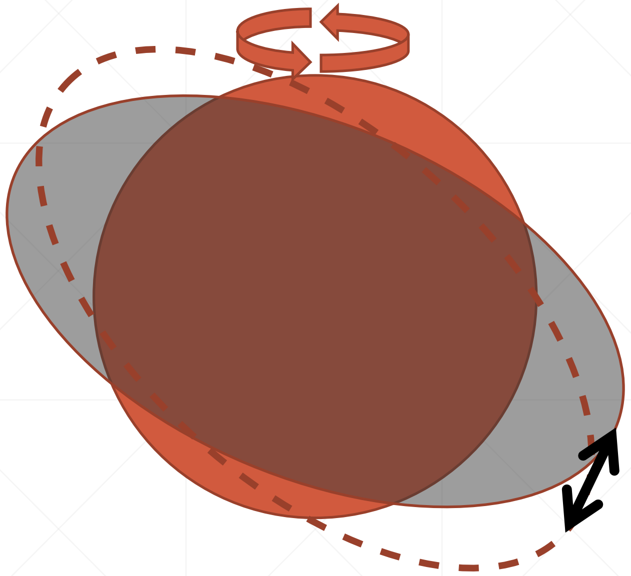
$$\Omega^{\mu\nu}, S^{\mu\nu} = I\,\Omega^{\mu\nu} \approx 2\mu R^2\,\Omega^{\mu\nu}/5$$

- Adiabatic deformation

$$Q^{\mu\nu}_{\mathrm{ad.}} = \frac{k_2 R^5}{2} R^{\mu}{}_{\kappa}{}^{\nu}{}_{\lambda}\,\dot{x}^{\kappa}\dot{x}^{\lambda} + \frac{h_2 R}{\mu^2} S^{\mu\kappa} S_{\kappa}{}^{\nu}$$

- Time lag $\tau_{\mathrm{lag}} \sim \mu\,\bar{\nu}/R$

$$Q^{\mu\nu}_{\mathrm{del.}} = Q^{\mu\nu}_{\mathrm{ad.}} - \tau_{\mathrm{lag}} \frac{\mathrm{D}Q^{\mu\nu}_{\mathrm{ad.}}}{\mathrm{d}\tau} + 2\tau_{\mathrm{lag}} Q^{\kappa(\mu}_{\mathrm{ad.}}\Omega^{\nu)}{}_{\kappa}$$



[Steinhoff & Puetzfeld 12, Vines+ 15, Feraz-Mello+ 08, Gavrilov & Zharkov 77]

Actual EOMs

(Already presupposing certain finer relativistic terms will not matter…)

$$\frac{\mathrm{D}^2 x^\mu}{\mathrm{d}\tau^2} = F^\mu_{\mathrm{GSF}} - \frac{1}{2} R^\mu{}_{\nu\kappa\lambda} \dot{x}^\nu S^{\kappa\lambda} - \frac{1}{6} R_{\nu\kappa\lambda\gamma}{}^{;\mu} J^{\nu\kappa\lambda\gamma}$$

$$\frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau} = \tau^{\mu\nu}_{\mathrm{GSF}} - \frac{4}{3} R^{[\mu}{}_{\kappa\lambda\gamma} J^{\nu]\kappa\lambda\gamma}$$

$$J^{\nu\kappa\lambda\gamma} = -3\, \dot{x}^{[\nu} Q^{\kappa][\lambda} \dot{x}^{\gamma]}$$

With a ***conservative*** *term:* $\dot{x}^\nu \to -\dot{x}^\nu, \Omega^{\mu\nu} \to -\Omega^{\mu\nu}$, *you get the same trajectory evolving backwards!*

With a ***dissipative*** *(irreversible) term you get a different trajectory under reversal!*

Note: If $R \ll R_{\mathrm{c}}$, then *either* the pole-dipole-quadrupole EOM are enough, *or* your body is tidally disrupted.

Weighing the contributions

$$\frac{\delta \ddot{x}_{\mathrm{fin.s.}}}{\delta \ddot{x}_{\mathrm{gsf}}} \sim$$

$$\underbrace{v_{\mathrm{rot}}}_{\text{Spin-curvature}} + \underbrace{k_2 \left(\frac{R}{\mu}\right)^2 \left(\frac{R}{R_c}\right)^4}_{\text{Tidal quadrupole}} + \underbrace{h_2 \left(\frac{R}{\mu}\right)^2 \left(\frac{R}{R_c}\right)^2 v_{\mathrm{rot}}^2}_{\text{Centrifugal quadrupole}} \quad \textbf{Conserv.}$$

$$\underbrace{k_2 \left(\frac{R}{\mu}\right)^2 \left(\frac{R}{R_c}\right)^4 \left(\frac{\tau_{\mathrm{lag}}}{\tau_{\mathrm{orb}}} + \frac{\tau_{\mathrm{lag}}}{\tau_{\mathrm{rot}}}\right)}_{\text{Tidal lag}} + \underbrace{h_2 v_{\mathrm{rot}} \left(\frac{R}{\mu}\right)^2 \left(\frac{R}{R_c}\right)^3 \left[h_2 v_{\mathrm{rot}}^2 + k_2 \left(\frac{R}{R_c}\right)^2\right] \frac{\tau_{\mathrm{lag}}}{\mu}}_{\text{Centrifugal self-lag}} \quad \textbf{Dissip.}$$

[Steinhoff & Puetzfeld 12, Vines+ 15, Feraz-Mello+ 08, Gavrilov & Zharkov 77]

Limits on tidal dissipation

- ***Always*** conserved (leaving out $O(S^2, Q)$, heat in the expressions):

$$E_{\text{tot}} = -\mu u_t + \frac{1}{2}\xi^{(t)}_{\mu;\nu}S^{\mu\nu}$$

$$L_{\text{tot}} = \mu u_\varphi - \frac{1}{2}\xi^{(\varphi)}_{\mu;\nu}S^{\mu\nu}$$

$$\frac{\delta E_{\text{orb}}}{E_{\text{orb}}} \sim \frac{\delta L_{\text{orb}}}{L_{\text{orb}}} \lesssim \frac{\mu}{R_{\text{c}}}$$

- *The real action of tidal dissipation is to transfer angular momentum between orbit and spin!*

TAKEAWAY:

The only finite-size effect we need to worry about is the spin-curvature coupling.

Statement of dynamics

- $\frac{D^2x^\mu}{d\tau^2} = F_{\rm gsf}^{(1,2)\mu} - \frac{1}{2\mu} R^\mu{}_{\nu\kappa\lambda}\dot{x}^\nu S^{\kappa\lambda}, \frac{DS^{\mu\nu}}{d\tau} = \tau_{\rm gsf}^{(1)\mu\nu}$
- $S^{\mu\nu}S^{\kappa\lambda}g_{\mu\kappa}g_{\nu\lambda}$, $S^{\mu\nu}S^{\kappa\lambda}\epsilon_{\mu\nu\kappa\lambda}$ conserved, center-of-mass constraint $S^{\mu\nu}\dot{x}_\nu = 0$ as well
- When the dust settles, only ***two*** dynamical variables in $S^{\mu\nu}$ → a ***single*** degree of freedom (canonical momentum + conjugate coordinate)

[Witzany+ 19, Vines+ 15]

A two-timescale decomposition

	Geodesic	GSF	Spin-curvature
Conservative, orbit evol.	$J_\mathrm{o}(p,e,i)$, $\Omega_\mathrm{o}(J_\mathrm{o})$	$\langle\delta^{\mathrm{gsf1}}\Omega_\mathrm{o}\rangle(J_\mathrm{o},J_\mathrm{s})$ $\delta^{\mathrm{gsf1}}x^\mu$	$\langle\delta^{s}\Omega_\mathrm{o}\rangle(J_\mathrm{o},J_\mathrm{s})$ $\delta^{s}x^\mu$
Dissipative, orbit evol.		$\langle\dot{J}_\mathrm{o}\rangle^{x_\mathrm{geo}}_{\mathrm{gsf1}}(J_\mathrm{o})$, $\langle\delta\dot{J}_\mathrm{o}\rangle^{\delta^{\mathrm{gsf1}}x}_{\mathrm{gsf1}}(J_\mathrm{o})$, $\langle\dot{J}_\mathrm{o}\rangle^{x_\mathrm{geo}}_{\mathrm{gsf2}}(J_\mathrm{o})$	$\langle\delta\dot{J}_\mathrm{o}\rangle^{\delta^{s}x}_{\mathrm{gsf1}}(J_\mathrm{o})$, $\langle\delta\dot{J}_\mathrm{o}\rangle^{\mathrm{Sp.source}}_{\mathrm{gsf1}}(J_\mathrm{o})$
Conservative, spin evol.	$J_\mathrm{s}(p,e,i,S^{\mu\nu})$, $\Omega_\mathrm{s}(J_\mathrm{o},J_\mathrm{s})$	negligible	negligible
Dissipative, spin evol.		$\langle\dot{J}_\mathrm{s}\rangle^{x_\mathrm{geo}}_{\mathrm{gsf1}}(J_\mathrm{o})$	

(Referring to the two-timescale formalism of [Hinderer & Flanagan 08])

A two-timescale decomposition

	Geodesic	GSF	Spin-curvature
Conservative, orbit evol.	$J_o(p,e,i)$, $\Omega_o(J_o)$	$\langle \delta^{gsf1}\Omega_o\rangle(J_o,J_s)$ $\delta^{gsf1}x^\mu$	$\langle \delta^{s}\Omega_o\rangle(J_o,J_s)$ $\delta^{s}x^\mu$
Dissipative, orbit evol.		$\langle \dot{J}_o\rangle^{x_{geo}}_{gsf1}(J_o)$, $\langle \delta\dot{J}_o\rangle^{\delta^{gsf1}x}_{gsf1}(J_o)$, $\langle \dot{J}_o\rangle^{x_{geo}}_{gsf2}(J_o)$,	$\langle \delta\dot{J}_o\rangle^{\delta^{s}x}_{gsf1}(J_o)$, $\langle \delta\dot{J}_o\rangle^{\text{Sp.source}}_{gsf1}(J_o)$
Conservative, spin evol.	$J_s(p,e,i,S^{\mu\nu})$, $\Omega_s(J_o,J_s)$	negligible	negligible
Dissipative, spin evol.		$\langle \dot{J}_s\rangle^{x_{geo}}_{gsf1}(J_o)$	

This talk [Witzany 19], Next talk of Chris Kavanagh

Existing results

- $\langle \delta^s \Omega_o \rangle (J_o, J_s), \langle \delta \dot{J}_o \rangle^{\delta^s x}_{\text{gsf1}} (J_o)$: [Huerta & Gair 11, Huerta+ 12, Burko & Khanna 15, Ruangsri+ 16, Warburton+ 17]
- $\langle \delta \dot{J}_o \rangle^{\delta^s x}_{\text{gsf1}} (J_o), \langle \delta \dot{J}_o \rangle^{\text{Sp.source}}_{\text{gsf1}} (J_o)$: [Harms+ 16, Lukes-Gerakopoulos+ 17]
- $J_s(p, e, i, S^{\mu\nu}), \Omega_s(J_o, J_s)$: [Marck 83, van de Meent 19]

Spin evolution

- Essentially solving parallel transport along geodesics in Kerr
- Start with Killing-Yano tensor $Y_{\mu\nu} = -Y_{\mu\nu}, Y_{\mu\nu;\kappa} = -Y_{\mu\kappa;\nu}$, take geodesic $u^\mu_{geo}, Y^\mu{}_\nu u^\nu_{geo}$ an „angular-momentum vector“, parallel transported!! (Length is $\sqrt{K}$)
- Contract u^μ_{geo} a few more times with KY tensor, orthogonalize for a complete tetrad – the parallel transport wrt this tetrad is separable! [Marck 83, Witzany 19, van de Meent 19]
- ***Take away:*** Projection of spin $S_{\parallel} = S^{\mu\nu} Y^\kappa{}_\gamma u^\gamma\, u^\lambda\, \epsilon_{\mu\nu\kappa\lambda}/2\sqrt{K}$ conserved, rest oscillates (and we know how)

Perturbation on orbit

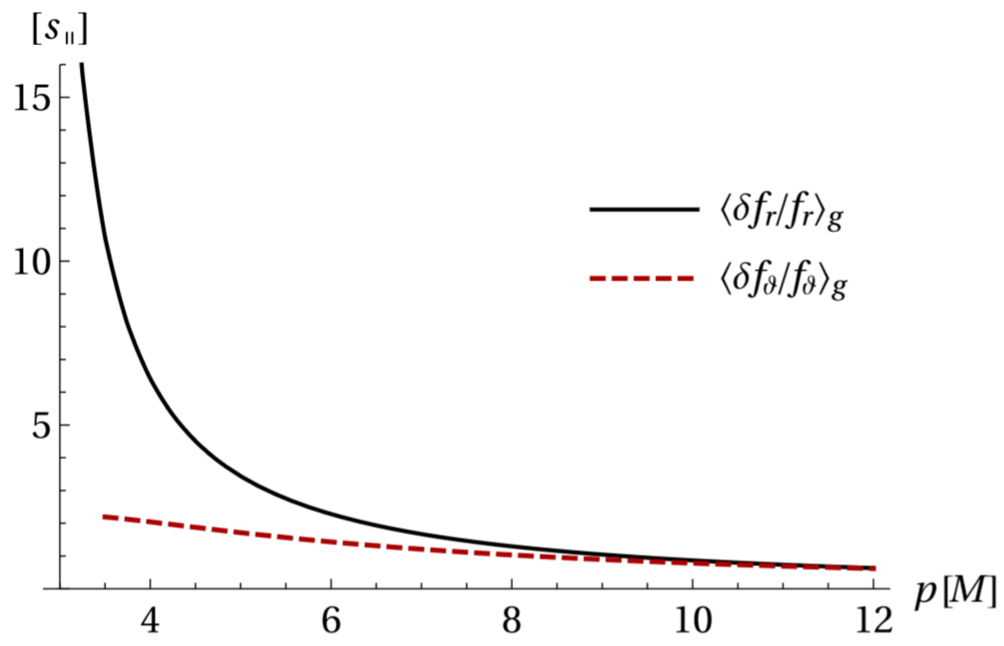
- Hamiltonian formalism $H(x^\mu, U_\mu, S^{\mu\nu})$, you can find canonical coordinates if you choose a tetrad – choose the Marck tetrad
- You have Hamiltonian in canonical coordinates $H(q^i, p_i)$, formulate Hamilton-Jacobi equation for action $W(q^i)$, $H\left(q^i, W_{,q^i}\right) = H_0$, geodesic solution known perturb by spin, *is separable*
- Result: separation constants $K_{so}\ E_{so}, L_{so} = K_g, E_g, L_g + O(S)$, and $S_\parallel$
- EOM reduced to half, but not separable $A = r, \vartheta$

$$\frac{dA}{d\lambda} = \pm\sqrt{w_A\left(A, K_{so}\ E_{so}, L_{so}, S_\parallel\right) - \frac{1}{\mu} e_{0A}\omega_{\mu\nu A}S^{\mu\nu}}$$

(For gory details see [Witzany 19])

Shifts to frequencies

- Separability of the unperturbed problem allows for a *complete* computation of fundamental frequency shifts by a set of closed-form quadratures!
- All of the shifts depend *only* linearly on $S_{\|}$
- Relative frequency shift ~ few $S_{\|}$ as per usual diverges at ISCO



(For gory details see [Witzany 19])

TAKEAWAY:
Frequency shifts due to spin can be computed, you only need to care about parallel component of spin

NOW:
Resonances, chaos

No strong resonances!

Consider perturbed action-angle coordinates:

$$\dot{J}_\alpha = \epsilon \bar{G}_\alpha(J) + \epsilon G_\alpha^{\mathrm{osc}}(J, \psi)$$
$$\dot{\psi}_\alpha = \Omega_\alpha(J) + \epsilon \bar{g}_\alpha(J) + \epsilon g_\alpha^{\mathrm{osc}}(J, \psi)$$

The thickness of resonant layer $\sim \sqrt{\epsilon G_\alpha^{\mathrm{osc}}}$ when it hits$\sim e^{ik^\alpha \psi_\alpha}$ and $k^\alpha \Omega_\alpha = 0$

The perturbative solution of Ham.-Jac. equation implies vars I, ϕ

$$\dot{I}_\alpha = 0 + O(S\^2)$$
$$\dot{\phi}_\alpha = \Omega_\alpha(I) + S_\parallel \bar{g}'_\alpha(I) + S g'^{\mathrm{osc}}_\alpha(I, \phi, S_\parallel/S, \chi_s)$$

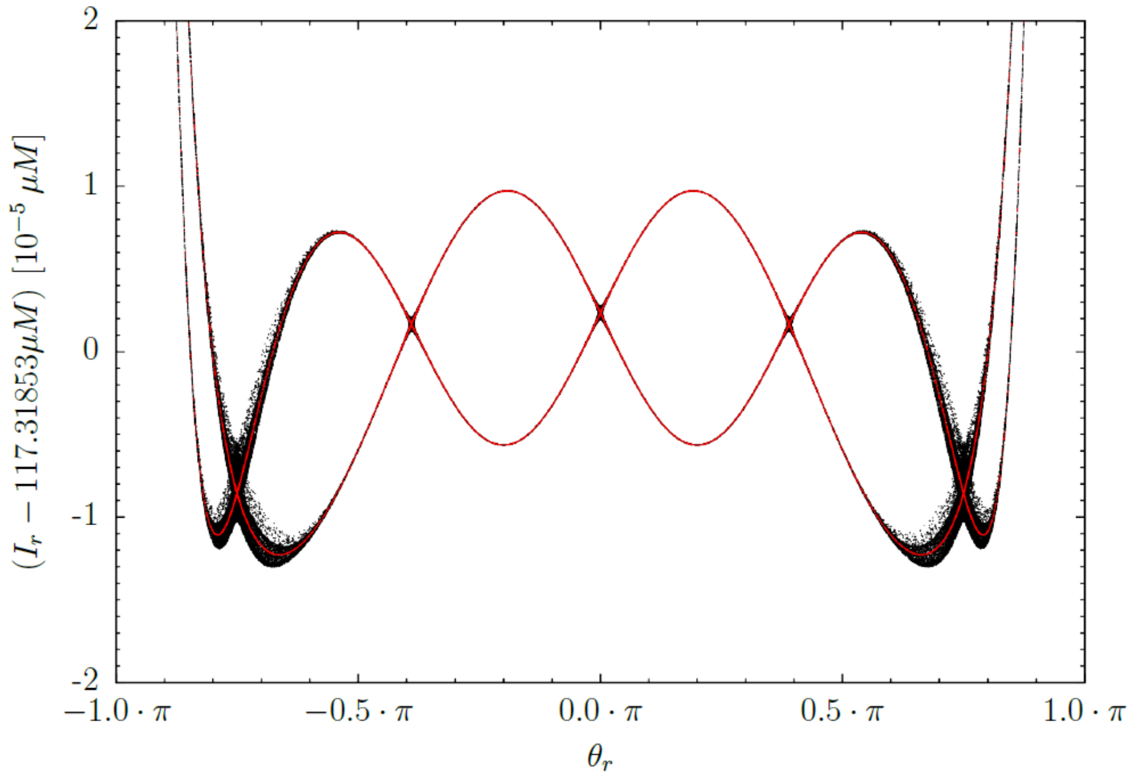
Hence, thickness of resonant layer scales only as $\sqrt{S^2} = S$

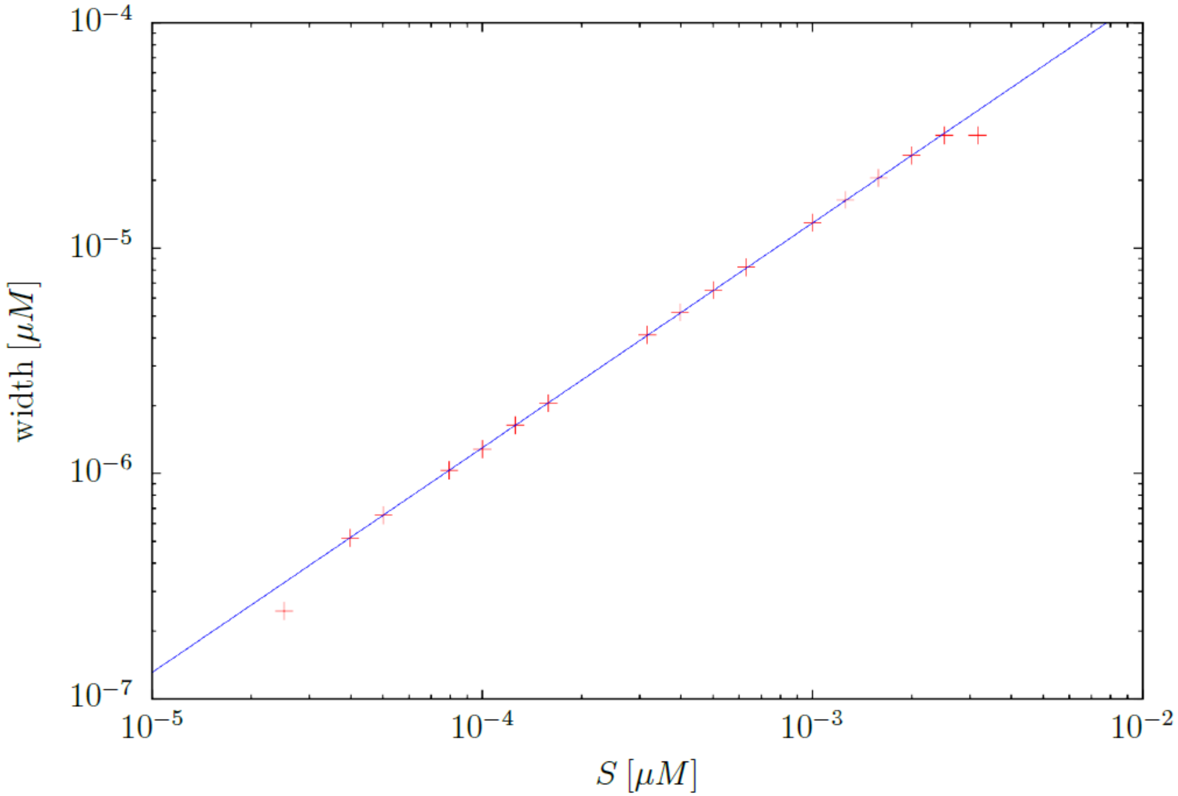
(For gory details see my notes from the plane here)

Numerical evidence

Ondřej Zelenka

Georgios Lukes-Gerakopoulos



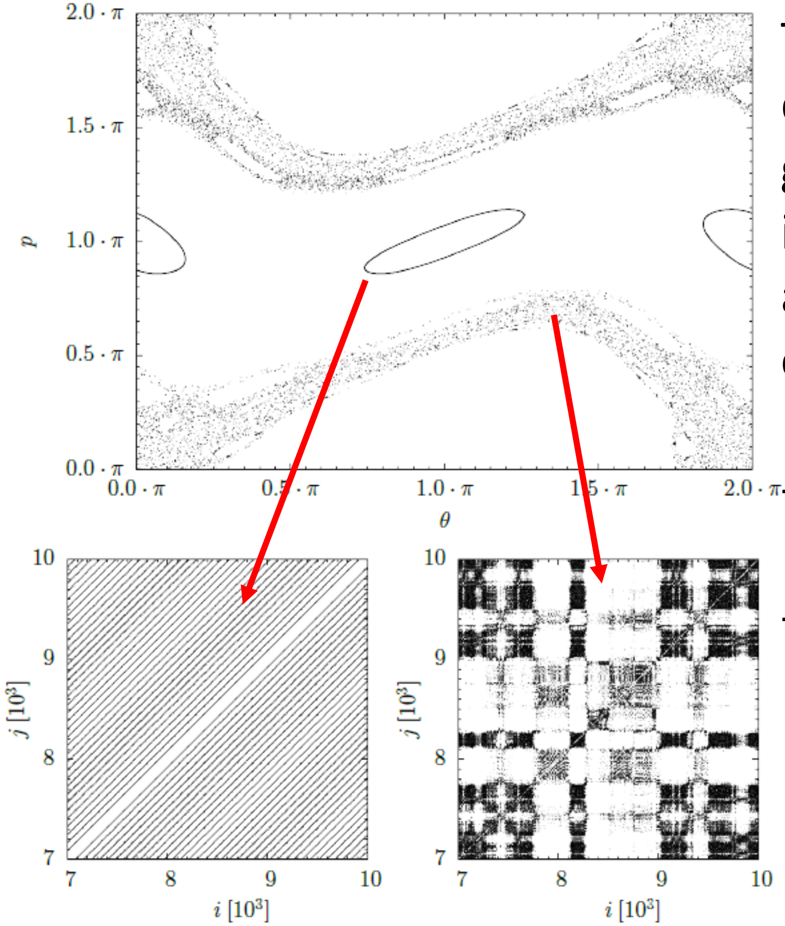


(To be published soon, ask me for pdf of Ondřej's Master thesis)

Hunting for chaos

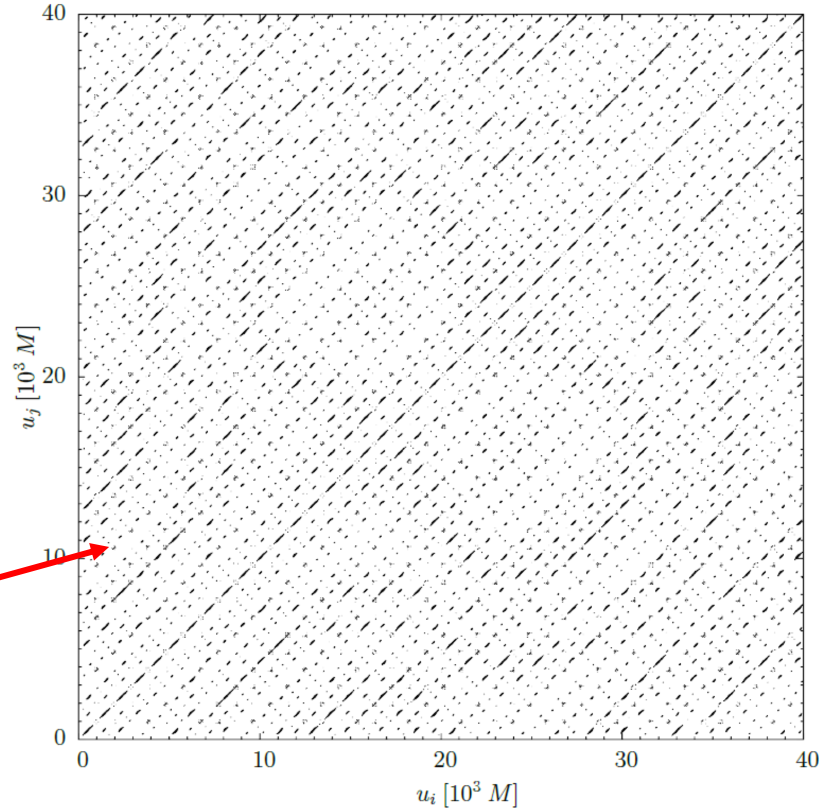
Ondřej Zelenka

Georgios Lukes-Gerakopoulos



Take a time-series of any dynamical variable from a given system and observe its recurrences – you are able to discern regular from chaotic.

- ***Can you do this for GW strain?***
- For a weakly chaotic orbit for mass ratio 10^{-4} this is now limited by the noise in the Teukolsky solver



[Lukes-Gerakopoulos & Kopáček 17, Zelenka+ 19]

Conclusions

- You need only spin-curvature from finite-size effects in compact-object EMRIs
- Evolution of spin is analytically solvable at the accuracy we need, so is the average influence on the orbit
- Spin-orbit resonances are not strong enough, chaos as well
- You need to compute more for post-adiabatic EMRIs, specifically immediate perturbations of orbit (→fluxes), and $\langle \dot{S}_{\parallel} \rangle_{\mathrm{gsf1}}$

This talk was supported by Grant No. GACR-17-06962Y of the Czech Science Foundation