Modelling black-hole binaries
in the
intermediate-mass-ratio regime

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Why IMRIs?

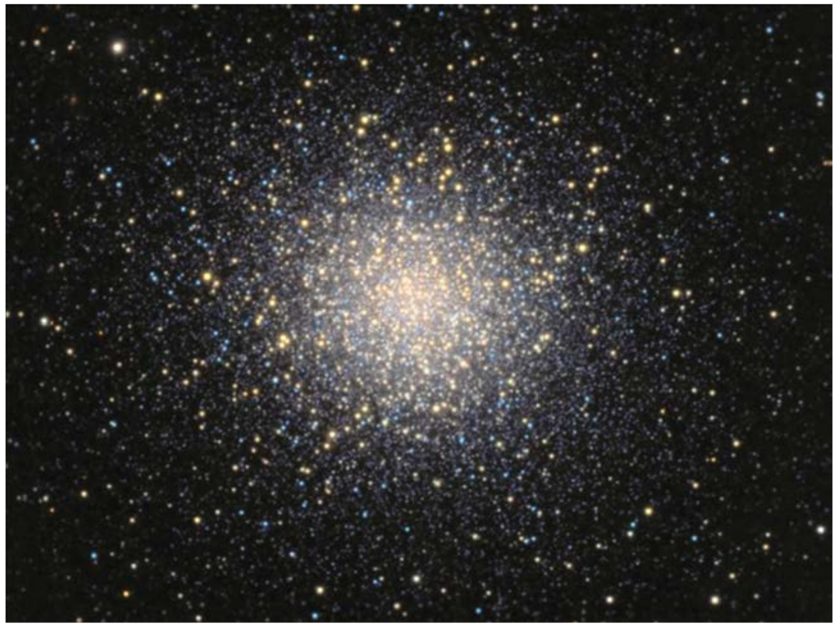
Formation Methods

#1: Stellar-mass CO + IMBH = IMRI with Advanced LIGO

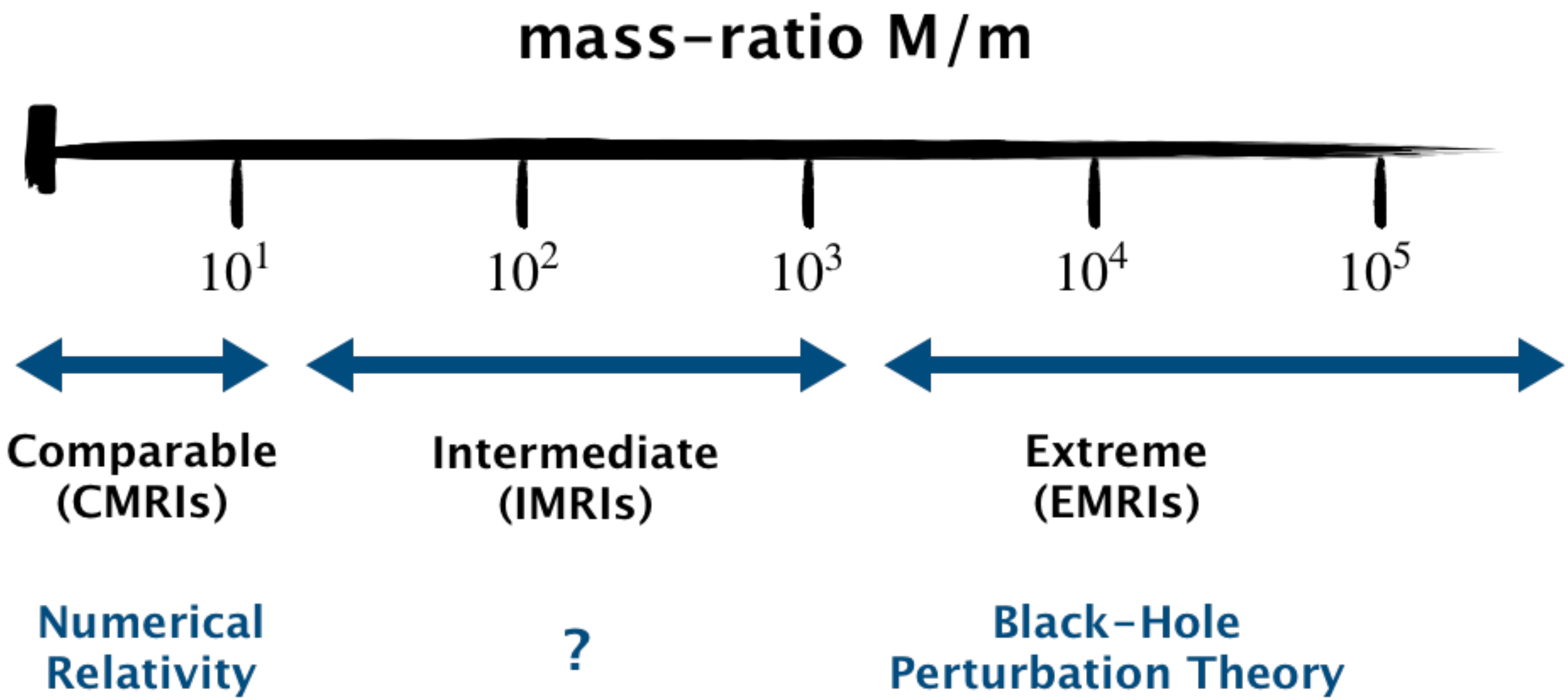
➡ Explore the dynamics of globular clusters

#2: IMBH + SMBH = IMRI with LISA

➡ Explore the dynamics of galactic nuclei



BBH Mergers


mass-ratio M/m
10^1
10^2
10^3
10^4
10^5
Comparable (CMRIs)
Intermediate (IMRIs)
Extreme (EMRIs)
Numerical Relativity
?
Black-Hole Perturbation Theory

NR runtime with M/m

$\left(\frac{M}{m}\right)$ ➡ **Number of orbits (physics)**

$\left(\frac{M}{m}\right)$ ➡ **Time steps per orbit (numerics)**

$$\text{NR runtime} \propto \left(\frac{M}{m}\right)^2$$

If 1:10 ~ 100 days ~ 3 months
Then 1:100 ~ 10,000 days ~ 27 years!

IMRIs: 1:100–1,000

Existing Work

Mesh Refinement Techniques

Lousto and Healy

1:128 binary, 13 orbits before merger

arXiv:2006.04818v1

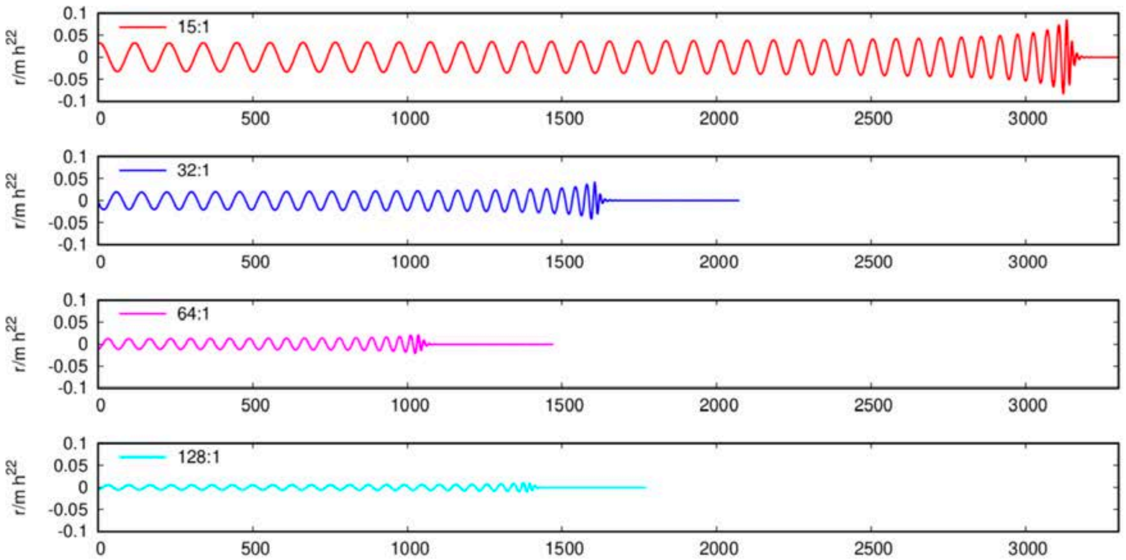


FIG. 1. (2,2) modes (real part) of the strain waveforms versus time (t/m),for the $q = 1/15, 1/32, 1/64, 1/128$ simulations.

Fernando, Neilsen, Lim, Hirschmann and Sundar

1:100 binary

arXiv:1807.06128v2

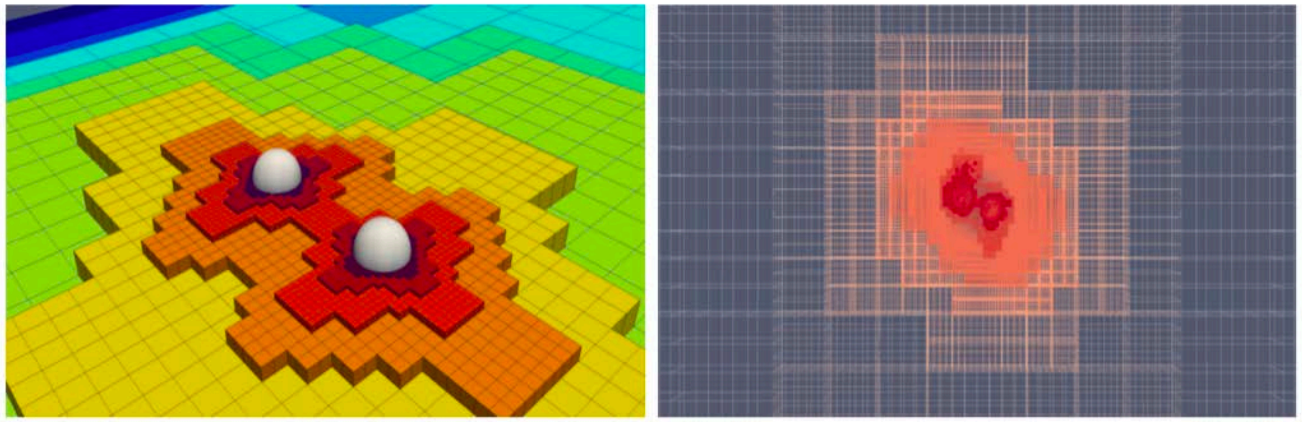
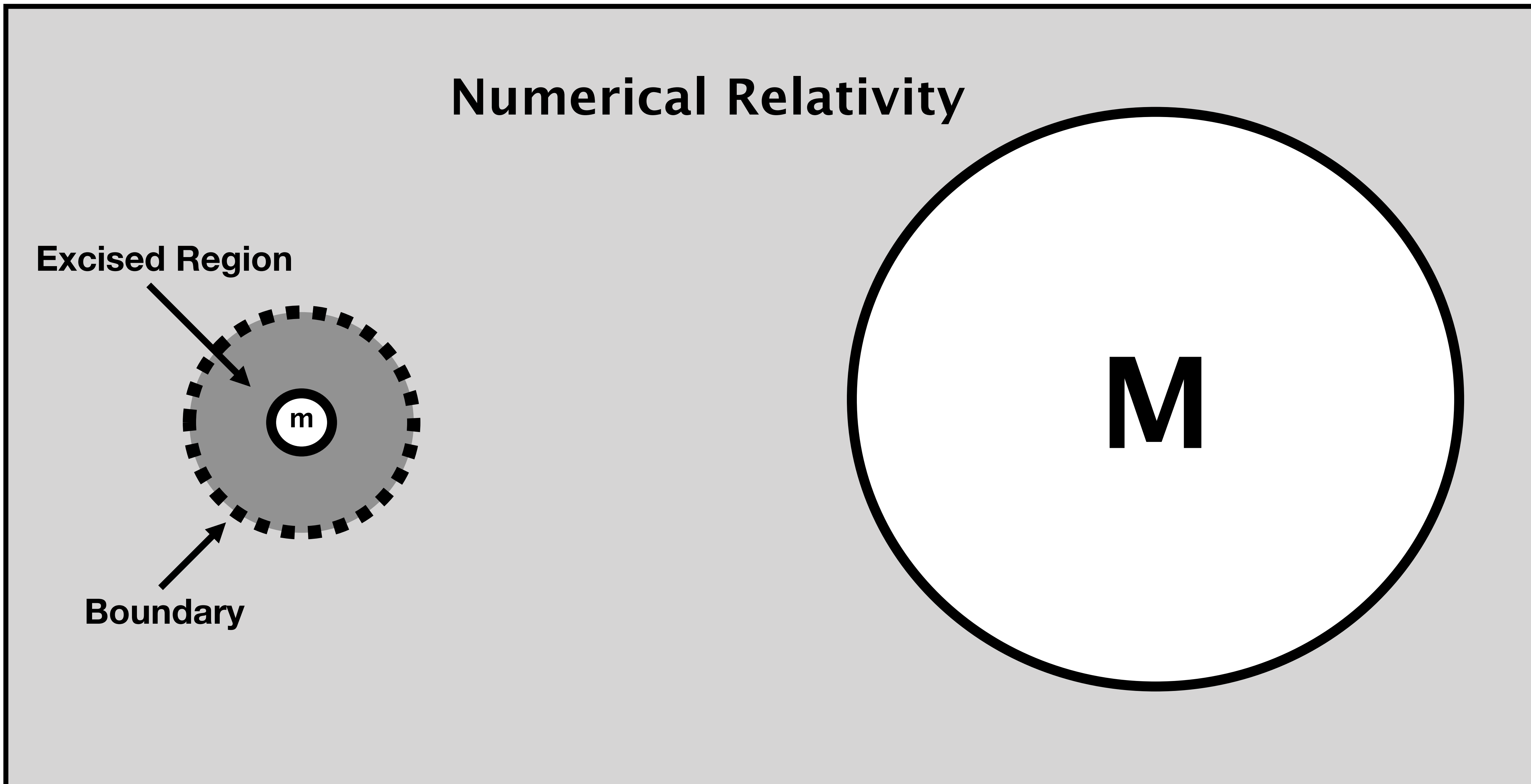


Fig. 2: (**left**) A example of the adaptive mesh created by DENDRO for the binary black-hole system. (**right**) the hierarchical wavelet grids generated for the binary black hole system.

Proposed Method (3+1)

Matching approximate analytical solution to NR elsewhere in the spacetime

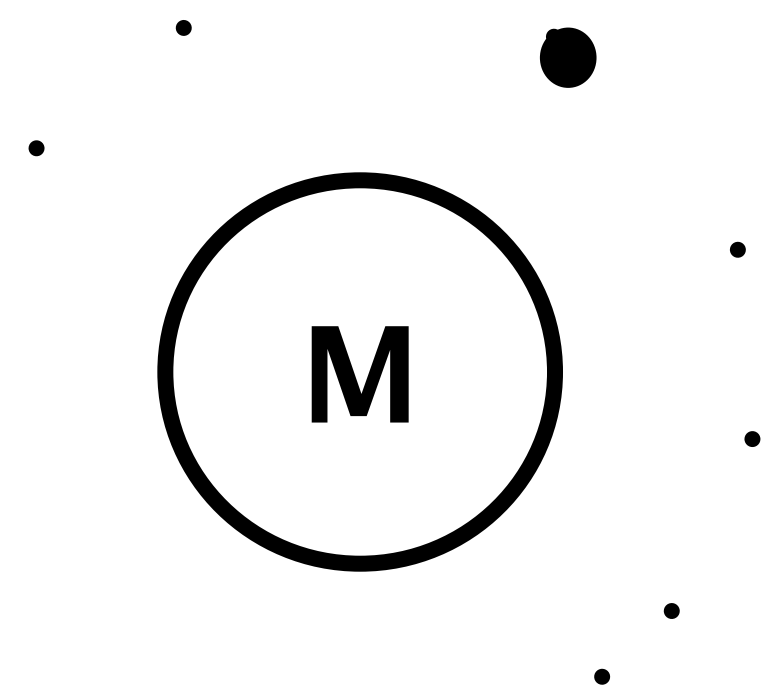


Toy Model (1+1)
Scalar Field on Schwarzschild

$$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi(x) = -4\pi\rho(x)$$

Spherical Harmonic Mode Decomposition

$$\Phi = \frac{1}{r}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\Psi_{lm}(r,t)Y_{lm}(\theta,\phi)$$

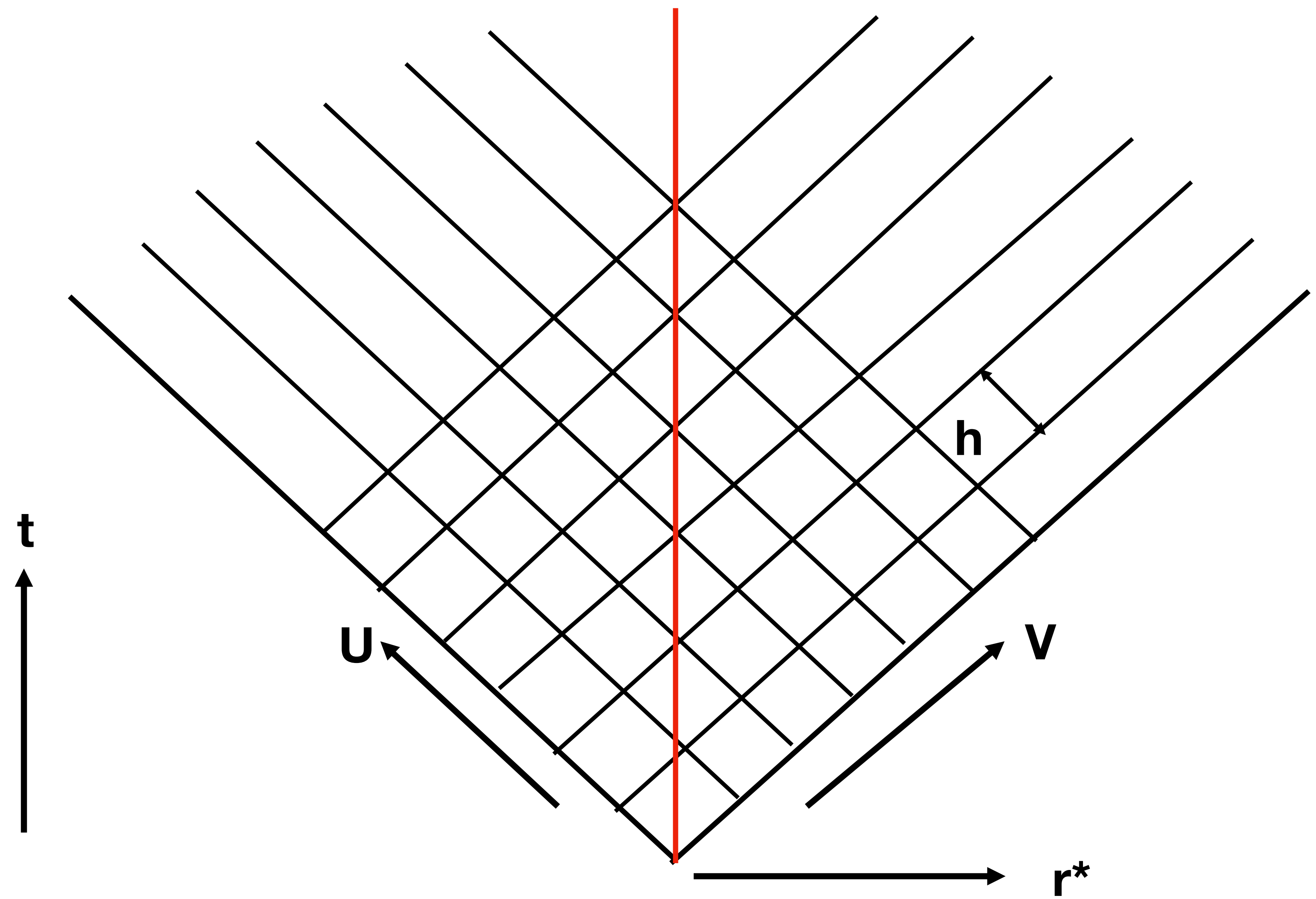


$$\rho = \frac{Q}{r^2(t)}\frac{F(r(t))}{E(r_p)}\sum_{lm}Y_{lm}(\theta,\phi)Y^*_{lm}\left(\frac{\pi}{2},\phi_p(t)\right)\delta(r-r_p)$$

Characteristic Grid

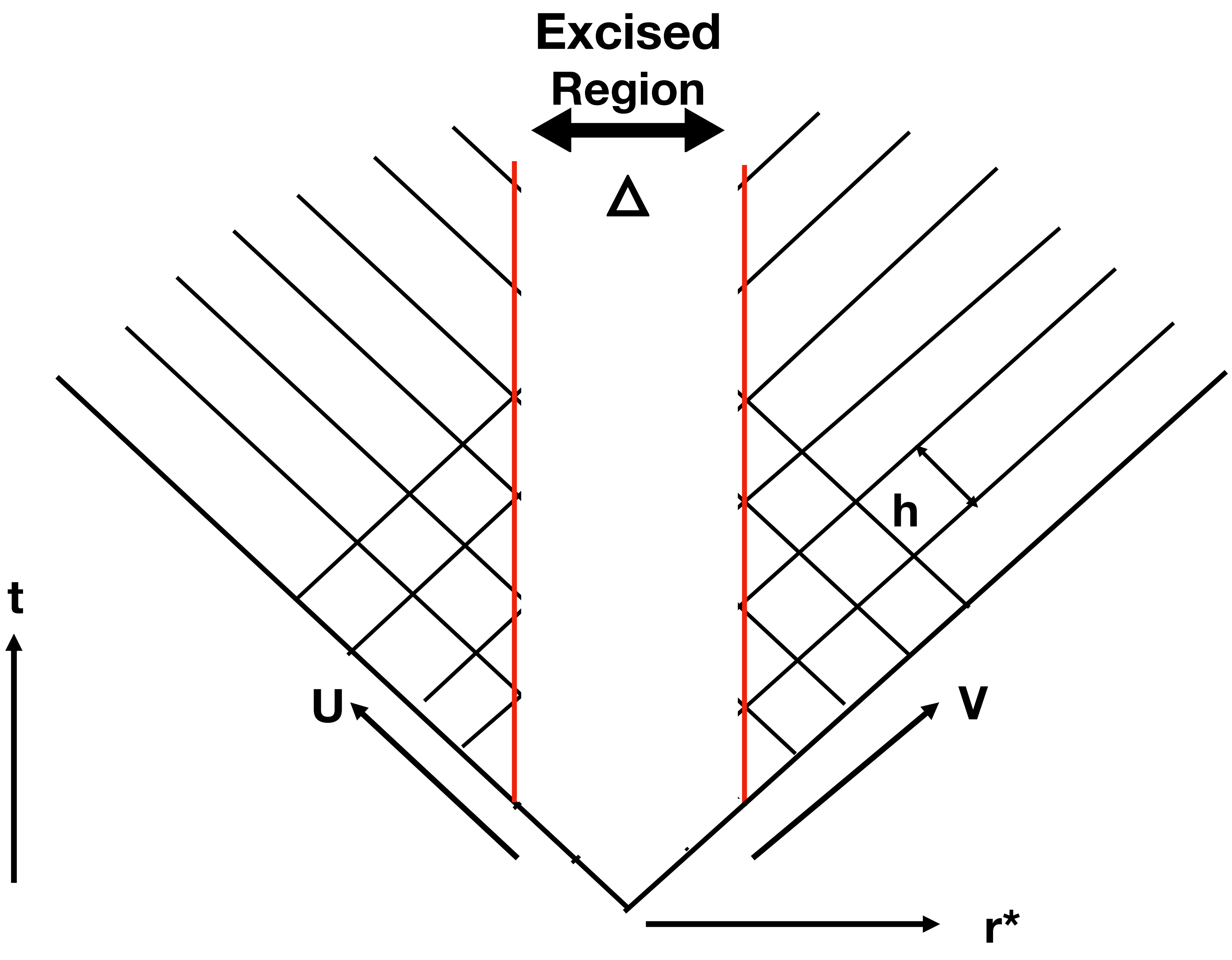
Transformation to Double-Null Coordinates

$$\partial_u \partial_v \Psi + V(r)\Psi = S(r(t))\delta(r - r_p)$$



Lousto and Price arXiv:gr-qc/9705071

Toy Model
Worldtube Excision on Schwarzschild



Approximate Analytical Solution inside Worldtube

Matching the retarded field across the worldtube boundary

$$\Phi^{ret}(x) \approx \Phi^P + \Phi^R$$

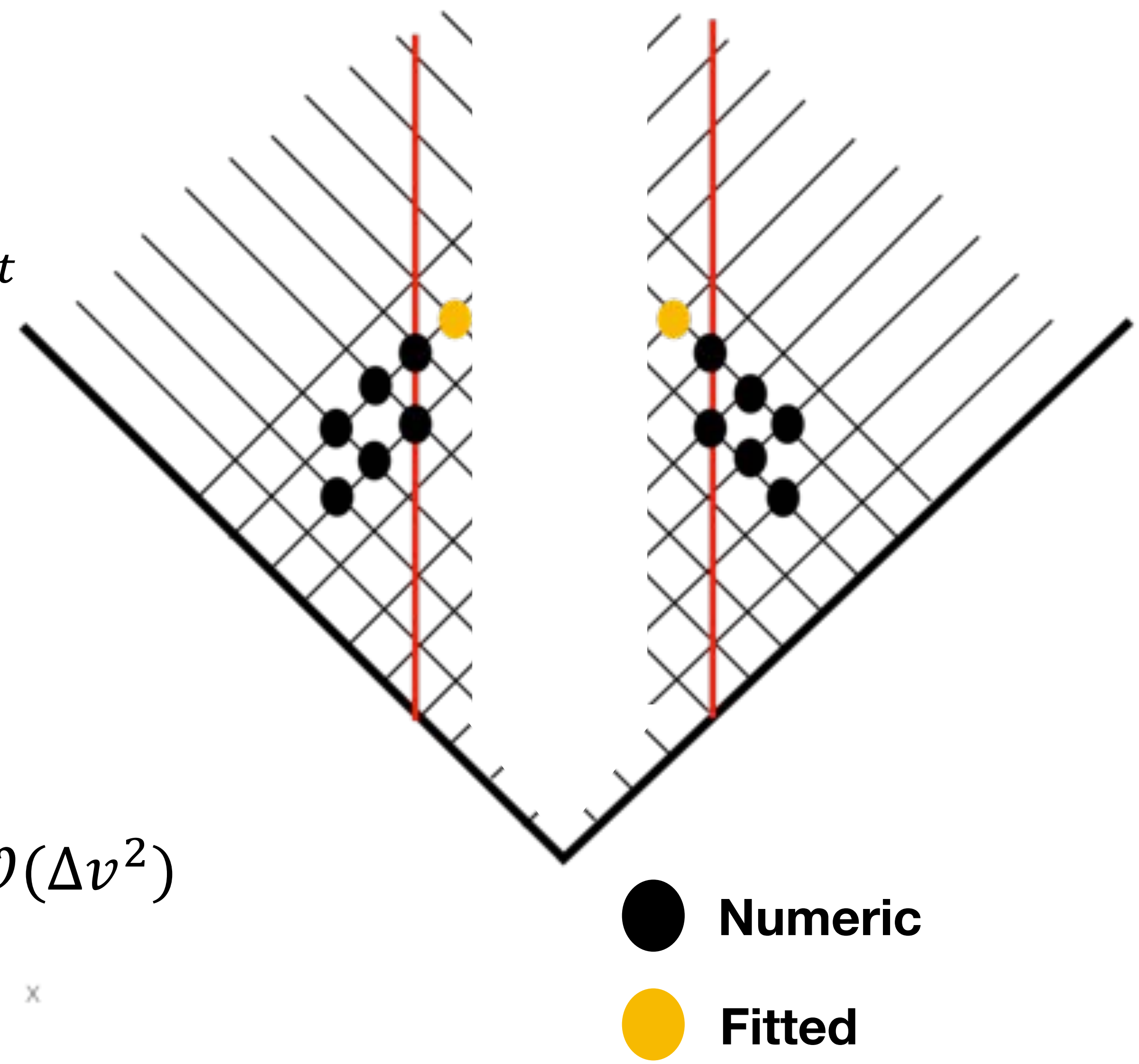
$$\Phi^P_{l,m}(t,r) = [a_0 + a_1\Delta r + b_0|\Delta r| + \mathcal{O}(\Delta r^2)]e^{im\Omega t}$$

a_0, a_1, b_0 = known coefficients

$$\Phi^R_{l,m} = \underline{\Phi^{R0}_{l,m}} + \underline{\Phi^{R1}_{l,m}}\Delta r + \mathcal{O}(\Delta r^2)$$

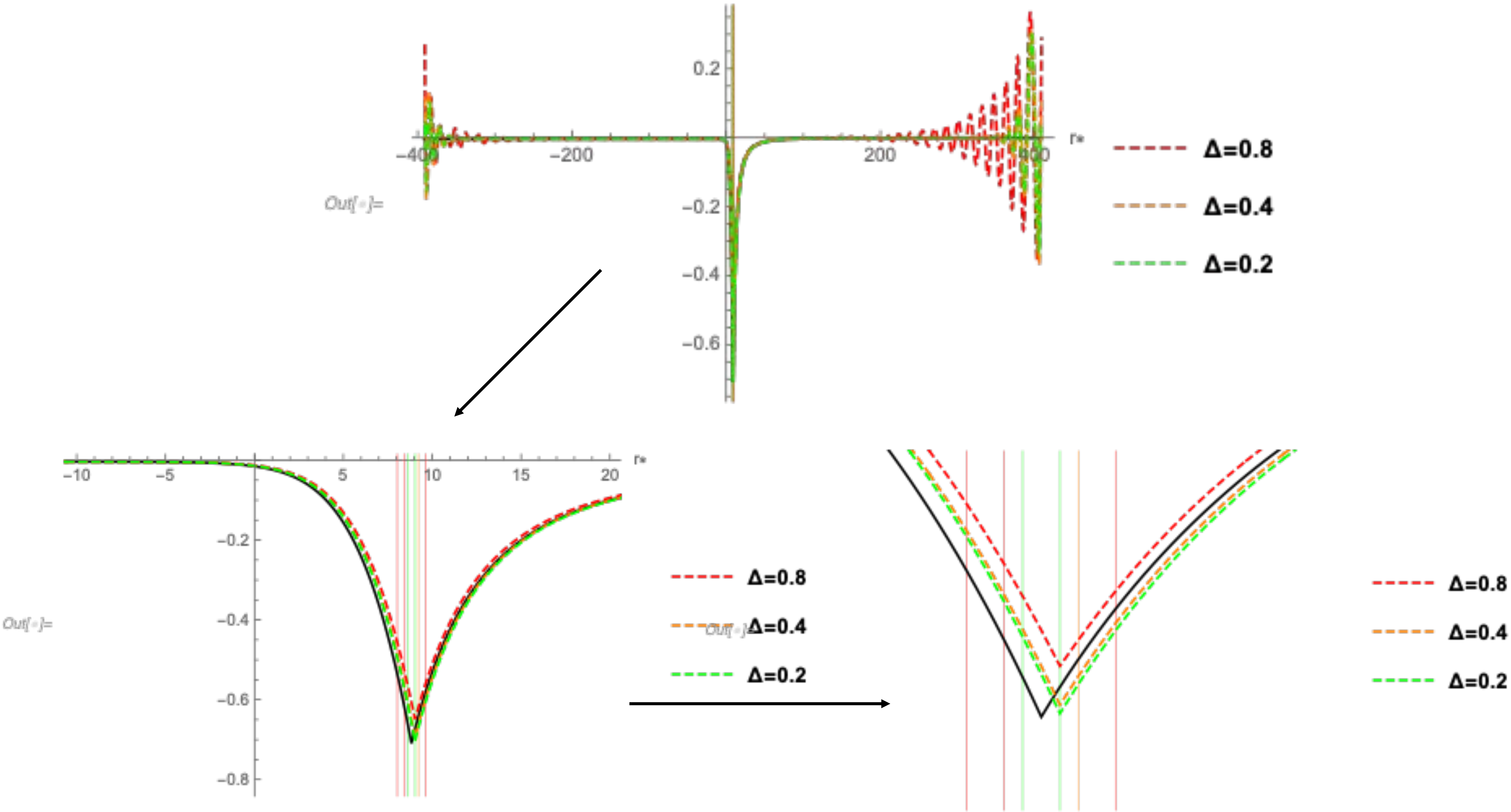
$$\downarrow$$

$$\Phi^R_{l,m} = \underline{\Phi^0_{l,m}} + \underline{\Phi^u_{l,m}}\Delta u + \underline{\Phi^v_{l,m}}\Delta v + \mathcal{O}(\Delta u^2) + \mathcal{O}(\Delta v^2)$$



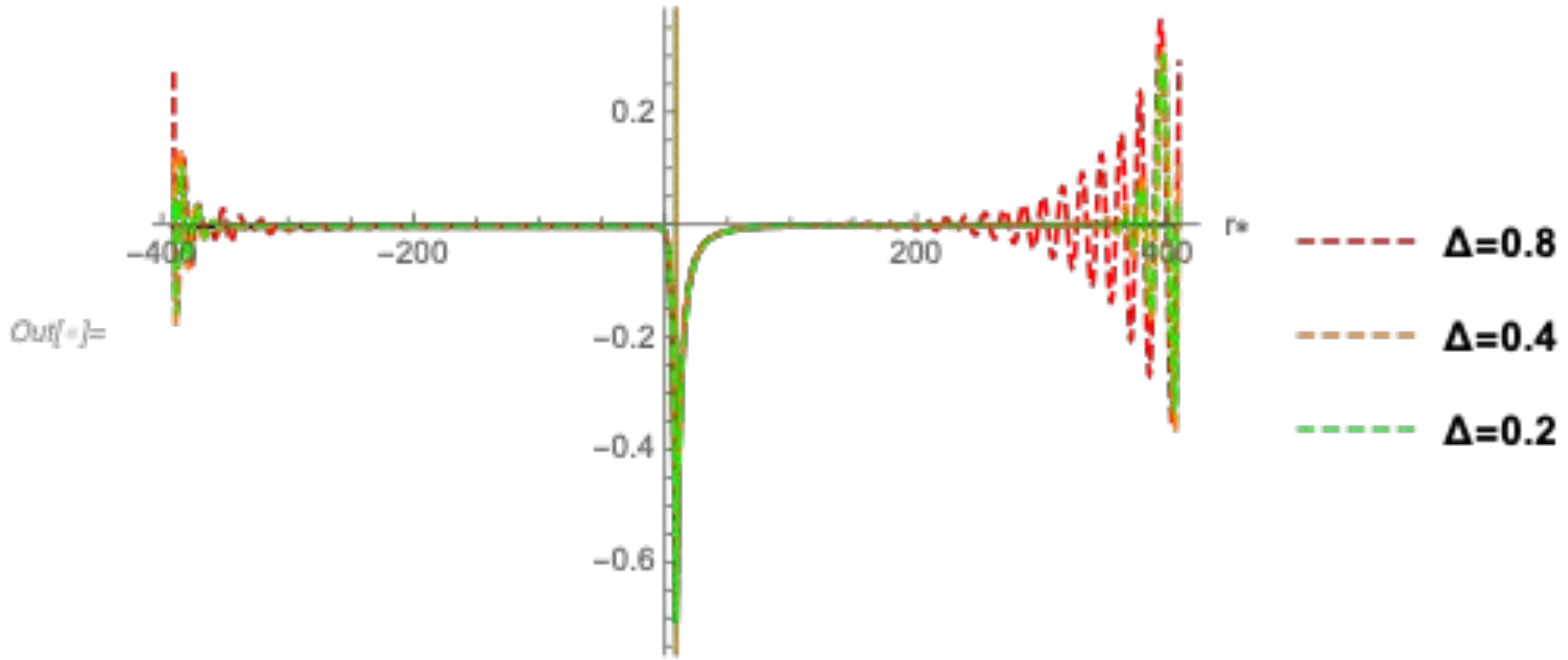
Sample Results

(l=2, m=0, h=0.1)

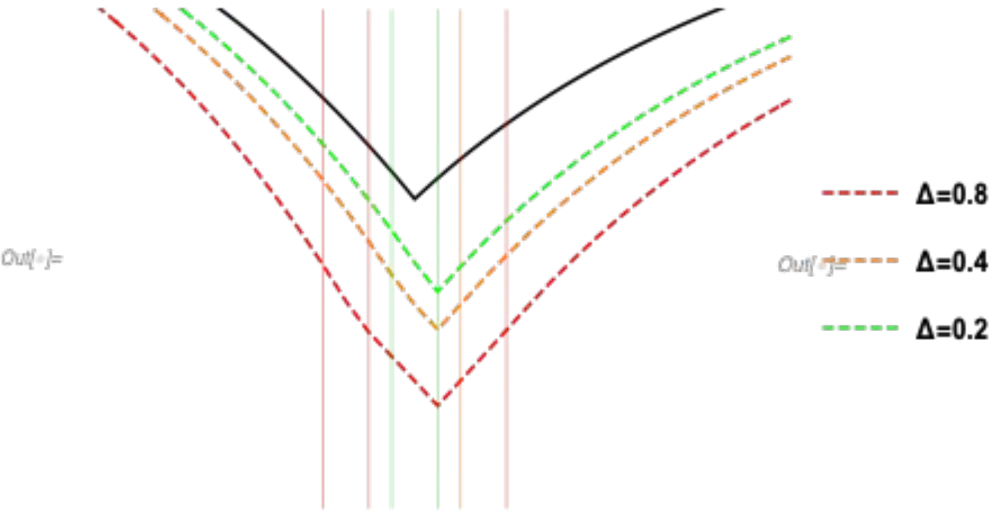


Model Order Comparison: Δ

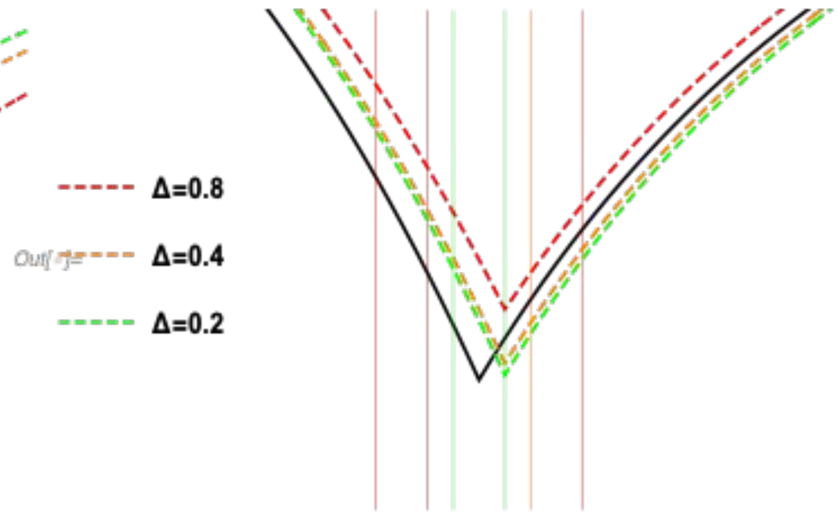
(l=2, m=0, h=0.1)



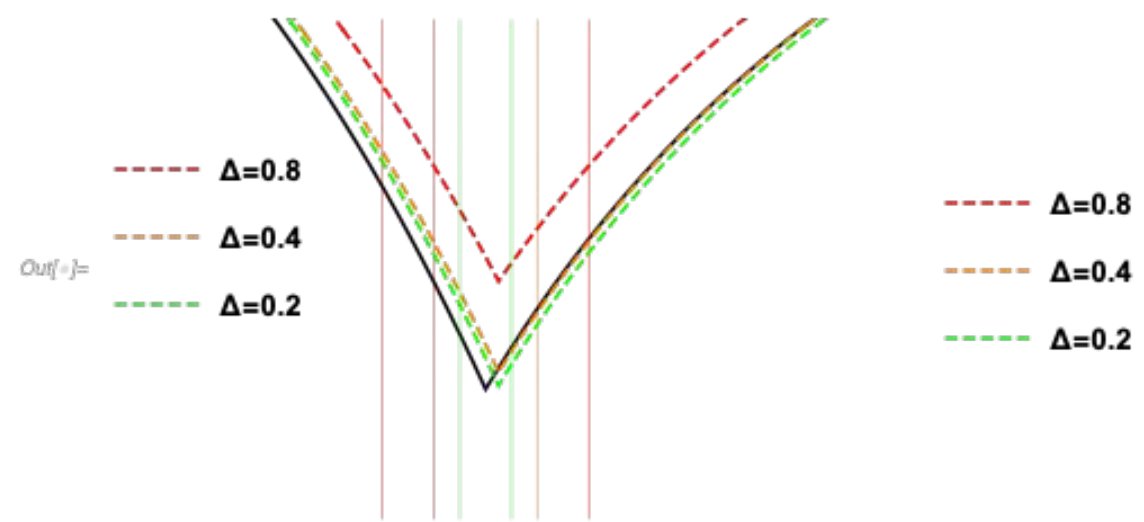
Leading



Quadratic



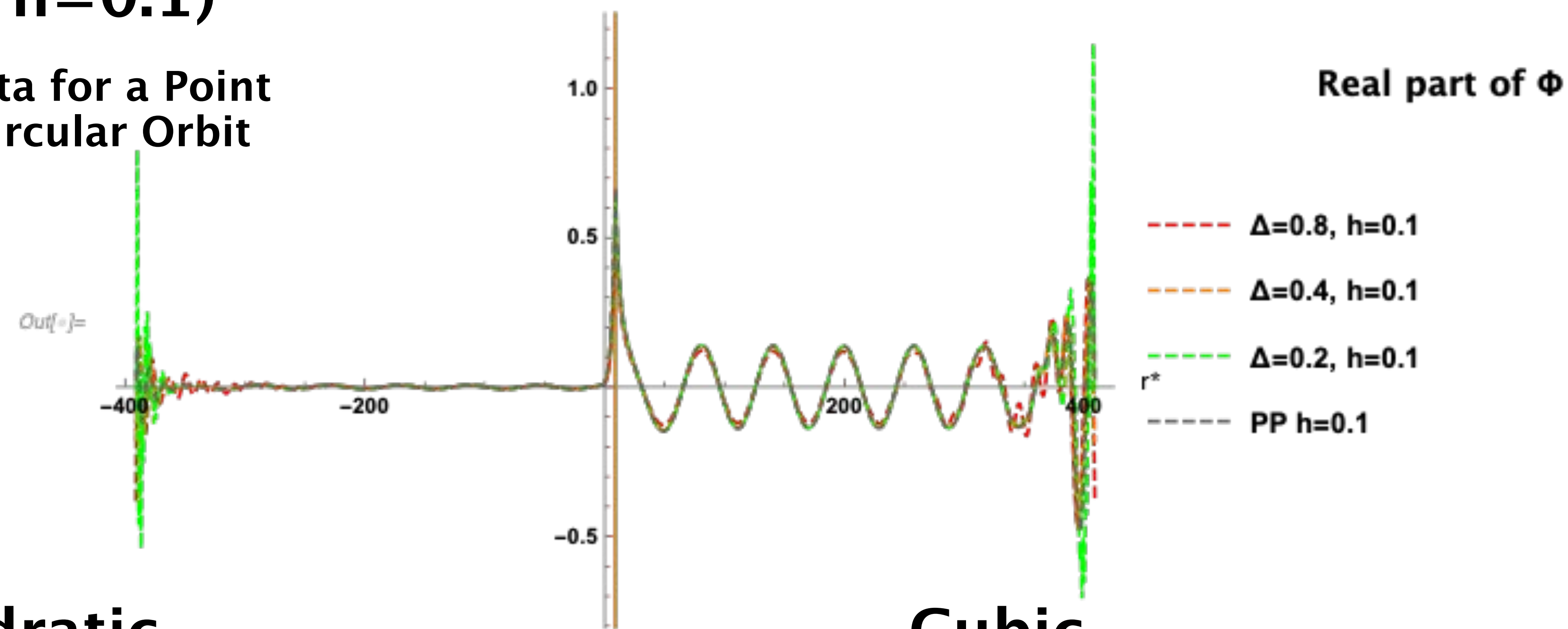
Cubic



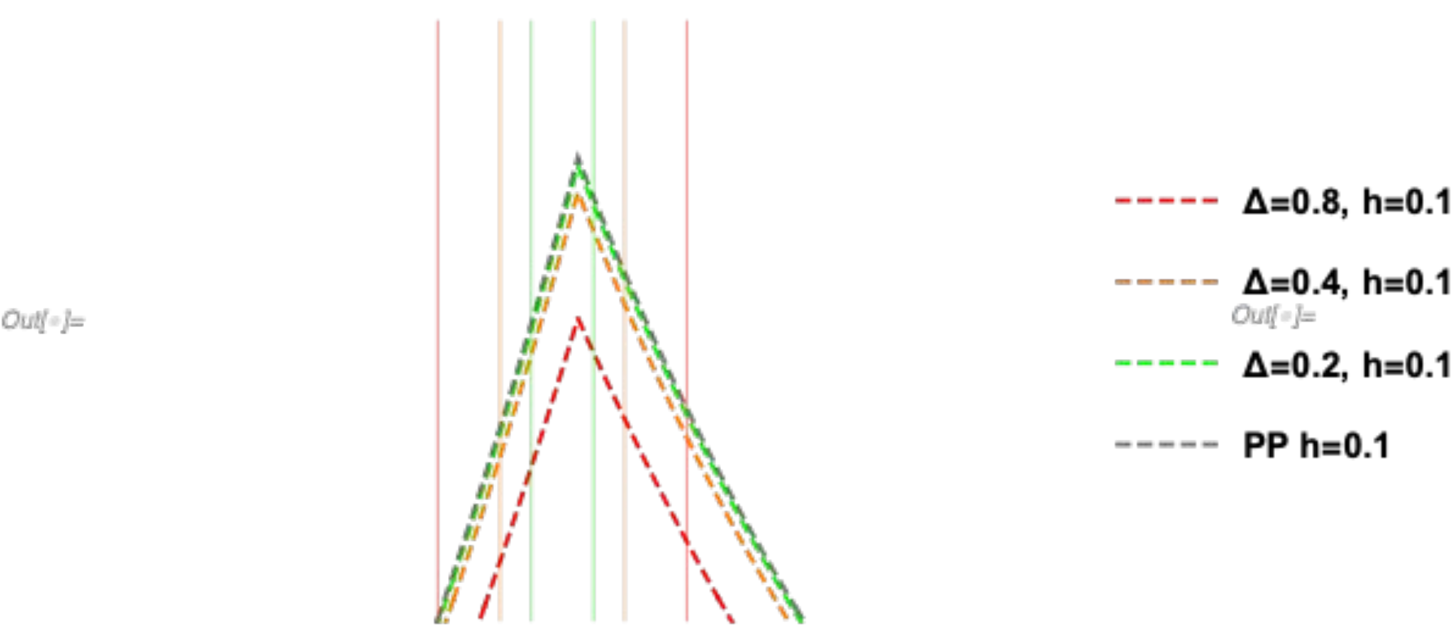
Non-Static Modes

(l=2, m=2, h=0.1)

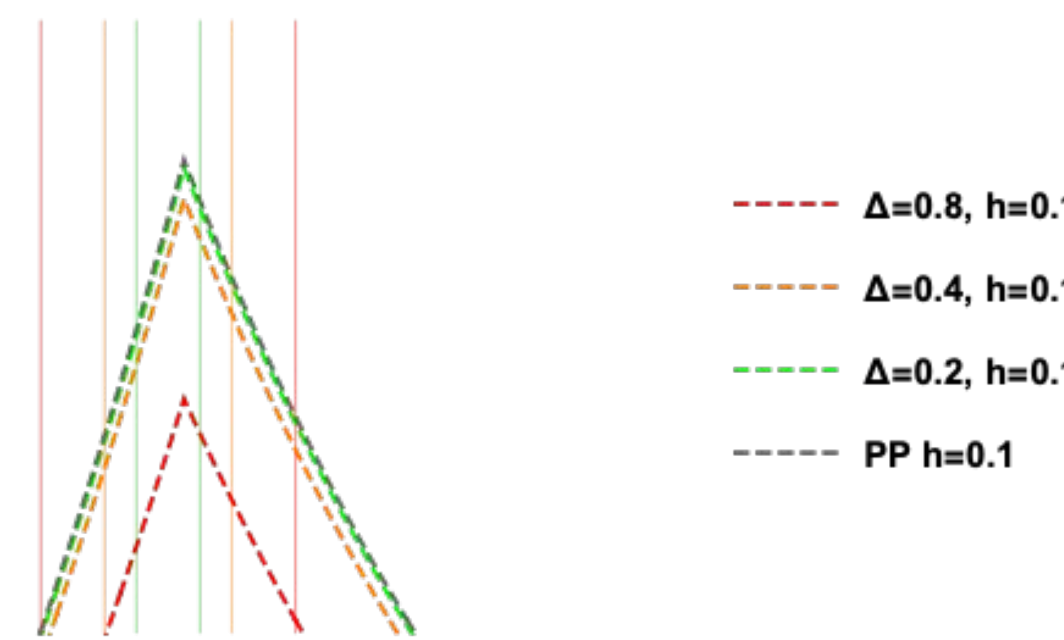
PP=Numeric Data for a Point Particle on a Circular Orbit



Quadratic



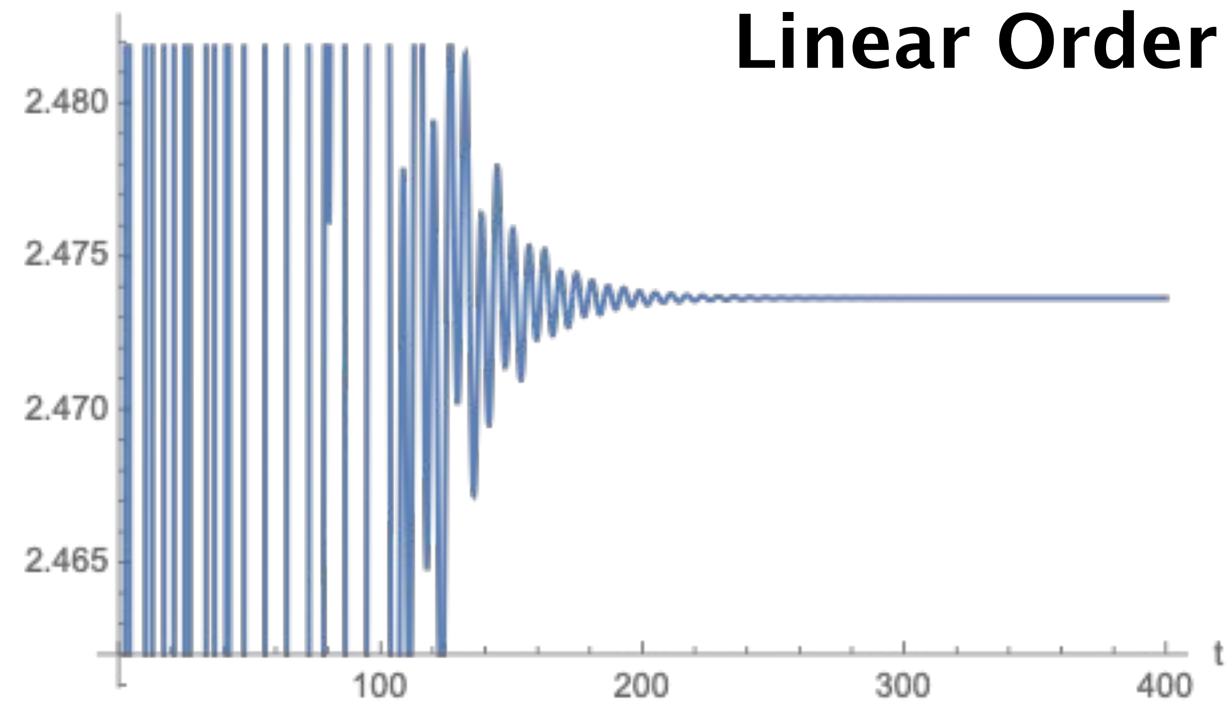
Cubic



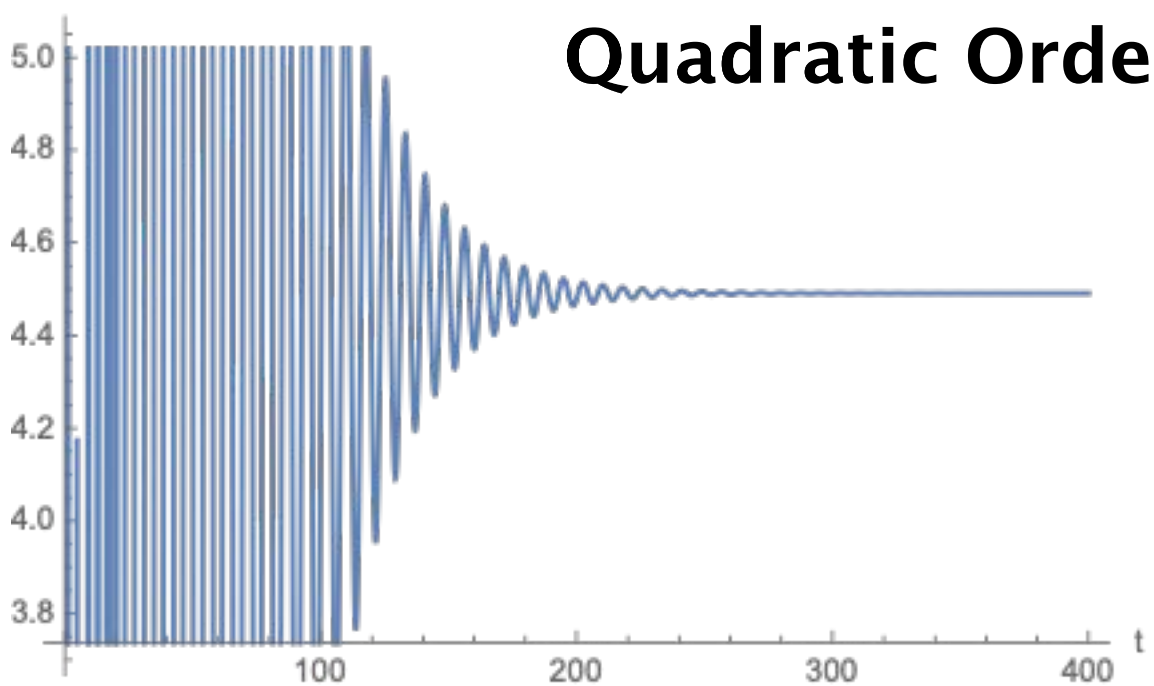
Convergence in Δ

$$r = \frac{\Phi(4\Delta) - \Phi(2\Delta)}{\Phi(2\Delta) - \Phi(\Delta)}$$

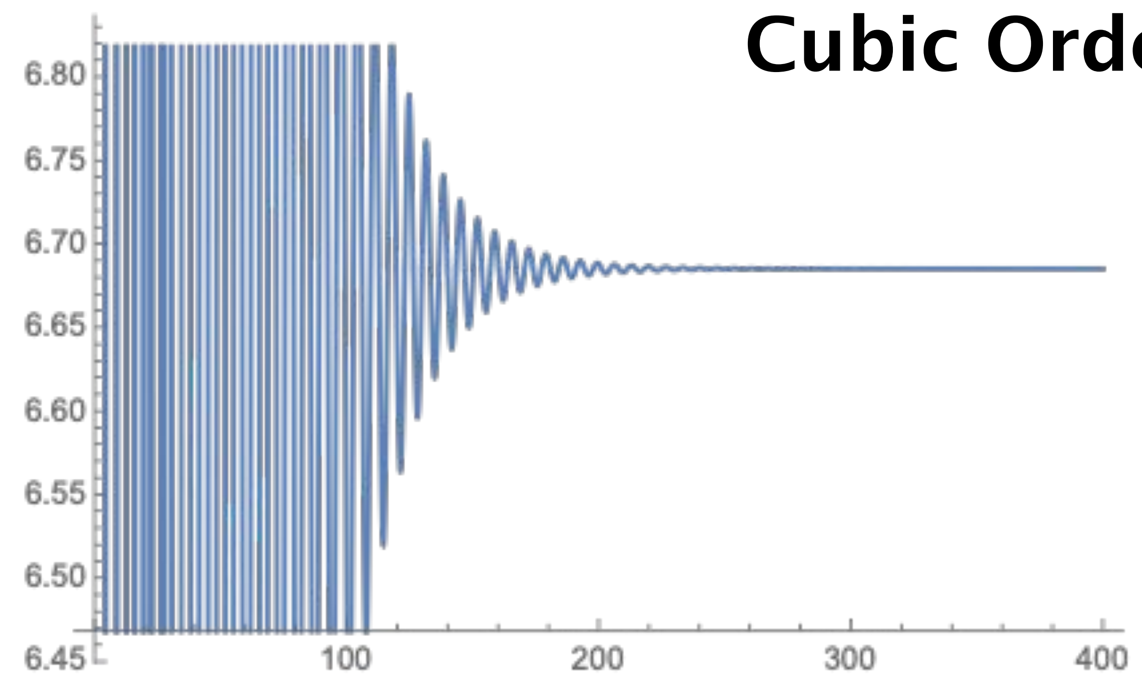
Linear Order



Quadratic Order



Cubic Order

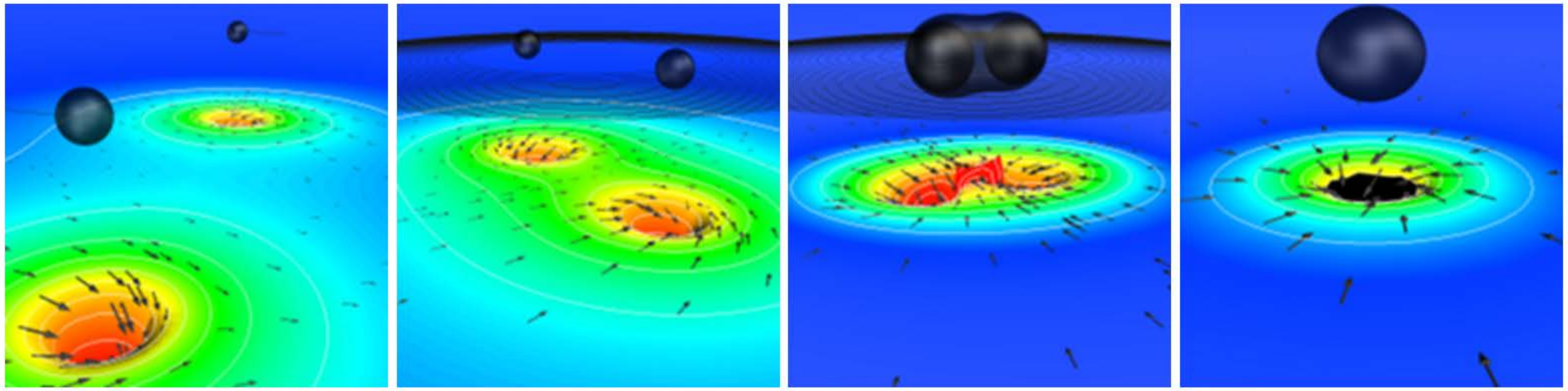


STAG RESEARCH CENTRE

UNIVERSITY OF Southampton

Next Steps

- Spectral Implementation of 1+1 Scalar Model
- Spectral Implementation of 3+1 Scalar Model
- Inclusion of Orbital Evolution
- 3+1 Gravity Model in SpEC



SpEC simulation of inspiral and merger of two black holes