# Towards the Self-consistent Evolution of a Scalar Charge Around a Schwarzschild Black Hole (Yet Again, Again)

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in collaboration with
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# Geodesic vs. self-consistent evolution.

We wish to determine the self-forced motion and field (e.g. energy and angular momentum fluxes as well as full self-force) of a particle with scalar charge

$$\Box\psi^{\rm ret} = -4\pi q \int \delta^{(4)}(x - z(\tau))\, d\tau.$$

Two general approaches:

- Compute enough "geodesic"-based self-forces and then use these to drive the motion of the particle. (Post-processing, fast, accurate self-forces, relies on slow orbit evolution)
- Compute the "true" self-force while simultaneously driving the motion. (Potentially slow and expensive, potentially less accurate self-forces)

Not expected to perform the full evolution, but hopefully long enough evolutions to check errors in geodesic evolutions.

# Effective source approach.

... is a general approach to self-force and self-consistent orbital evolution that doesn't use any delta functions.

Key ideas

- Compute a regular field, $\psi^{\mathsf{R}}$, such that the self-force is

$$F_\alpha = \nabla_\alpha \psi^{\mathsf{R}}|_{x=z},$$

  where $\psi^{\mathsf{R}} = \psi^{\mathsf{ret}} - \psi^{\mathsf{S}}$, and the Detweiler-Whiting singular field $\psi^{\mathsf{S}}$ can be approximated via local expansions: $\psi^{\mathsf{S}} = \tilde{\psi}^{\mathsf{S}}(x|z,u,a) + O(\epsilon^n)$.

- The effective source, $S$, for the field equation for $\psi^{\mathsf{R}}$ is regular at the particle location

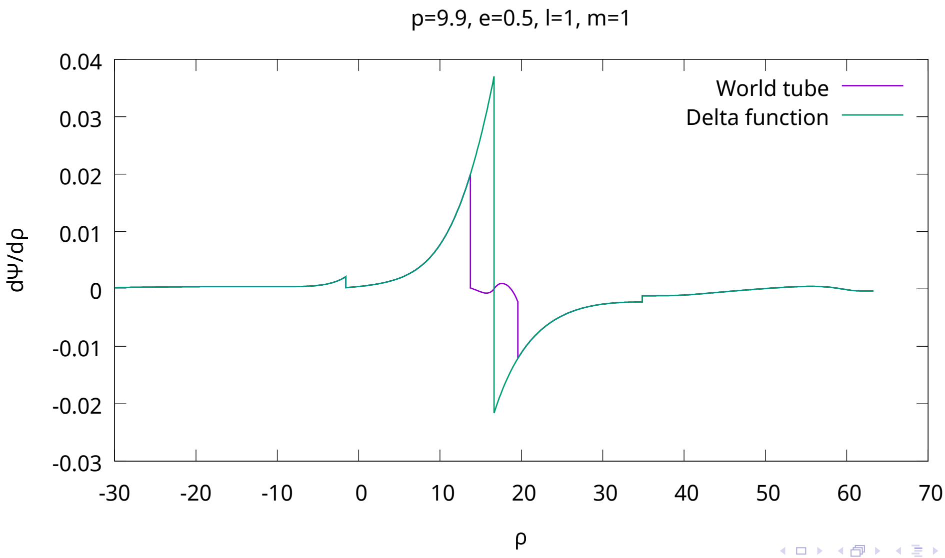
$$\Box\psi^{\mathsf{R}} = \Box\psi^{\mathsf{ret}} - \Box\tilde{\psi}^{\mathsf{S}} = S(x|z,u,a,\dot{a},\ddot{a}),$$

  where $\Box\tilde{\psi}^{\mathsf{S}} = -4\pi q \int \delta^{(4)}(x - z(\tau))\, d\tau - S$.

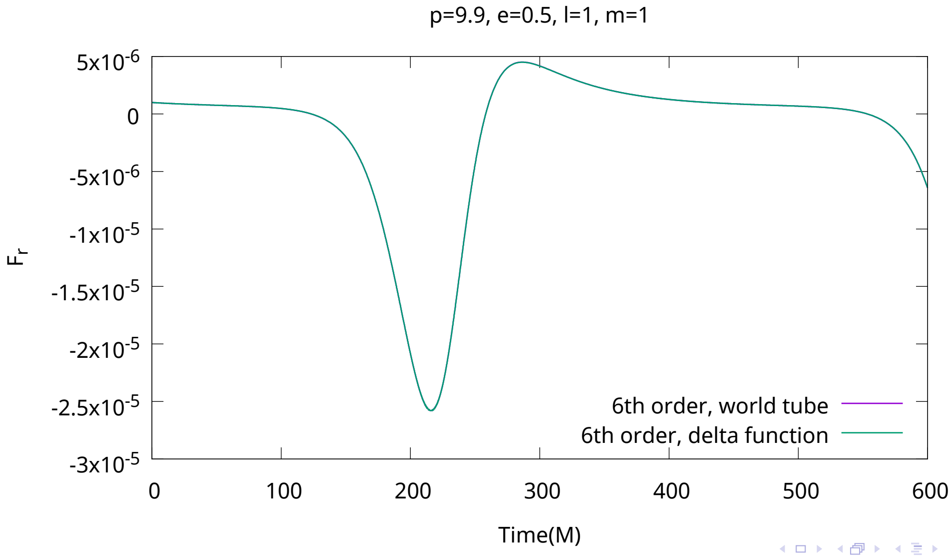
# Status at last years Capra.

- 1+1D Discontinuous Galerkin code (with time dependent coordinates as well as hyperboloidal slices) had been rewritten and released as open source software and included in the Einstein Toolkit.
- Still plagued by instabilities under most circumstances when back reaction was turned on.
- Only stable simulations when $\dot{a}$ and $\ddot{a}$ was left out.
- Note: $\dot{a}$ and $\ddot{a}$ only needed when evaluating the effective source away from the particle. They do not contribute at the particle itself.
- We have to use finite differences to calculate $\dot{a}$ and $\ddot{a}$. This adds numerical noise that seem to cause instabilities.
- At last Capra, Barry suggested to try to shrink the world tube down to zero size and only use the effective source to calculate jumps at the particle itself.
- Essentially switching from an effective source scheme to a real delta function point particle description, where the retarded field is evolved everywhere.
- This only seemed to require minimal changes to the code.

# Moving from effective source to delta-function

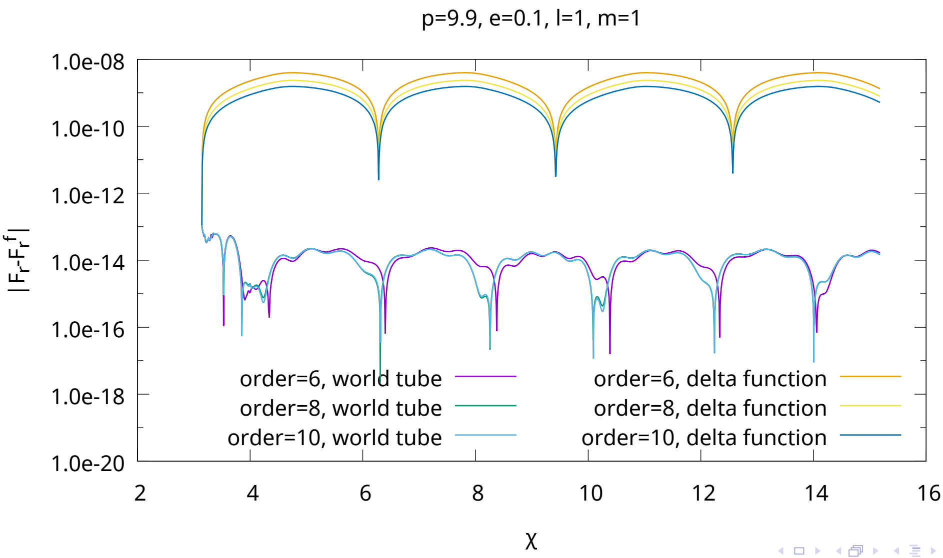


# First look



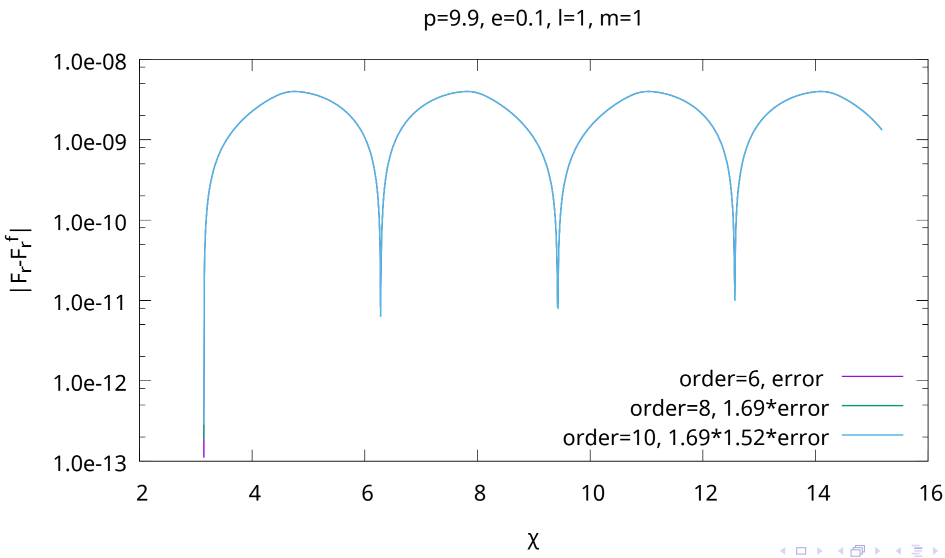
p=9.9, e=0.5, l=1, m=1
5x10-6
0
-5x10-6
-1x10-5
-1.5x10-5
-2x10-5
-2.5x10-5
-3x10-5
0
100
200
300
400
500
600
Fr
Time(M)
6th order, world tube
6th order, delta function

# Errors



p=9.9, e=0.1, l=1, m=1
1.0e-08
1.0e-10
1.0e-12
1.0e-14
1.0e-16
1.0e-18
1.0e-20
$|F_r - F_r^f|$
2
4
6
8
10
12
14
16
χ
order=6, world tube
order=8, world tube
order=10, world tube
order=6, delta function
order=8, delta function
order=10, delta function

# Scaled errors



# Outlook.

- I apparently do something really stupid when shrinking the world-tube size to zero that makes convergence only linear in the smallest node separation.
- Hopefully, long before next Capra, I will fix this and test whether shrinking the world-tube helps on the stability of self-consistent evolutions.
- The code is now part of the Black Hole Perturbation Toolkit as well.
- Teukolsky code in Kerr is in progress: Skinner (last summer), Gibbs (this summer).
- Checkpoint/restart capability in progress: Ogborn (this summer).
- Samuel Cupp is working on the Lorenz gauge and the Regge-Wheeler-Zerilli metric perturbation codes.