Multi-timescale post-Newtonian dynamics of spinning black-hole binaries: a status update

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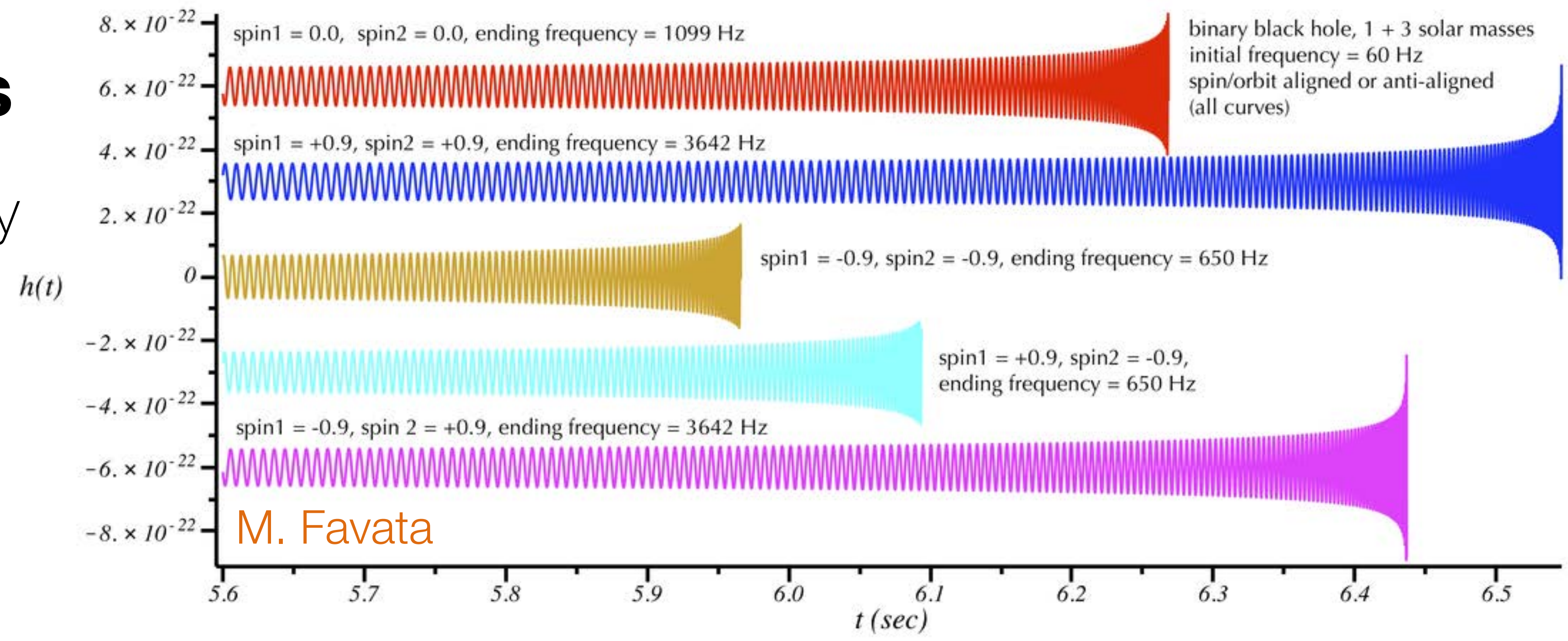
Outline

1. Spinning BH binaries
2. PN timescale separation
3. Spin morphologies
4. Predictions

Spin in the waveform

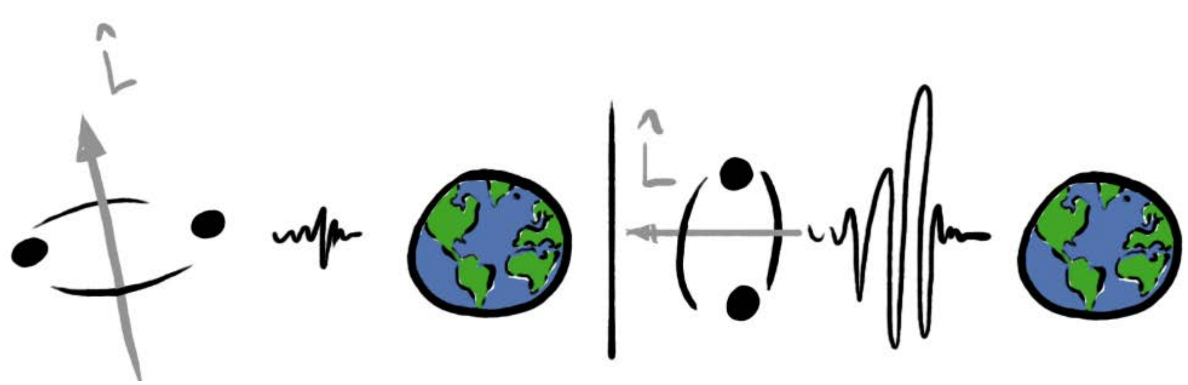
Aligned components

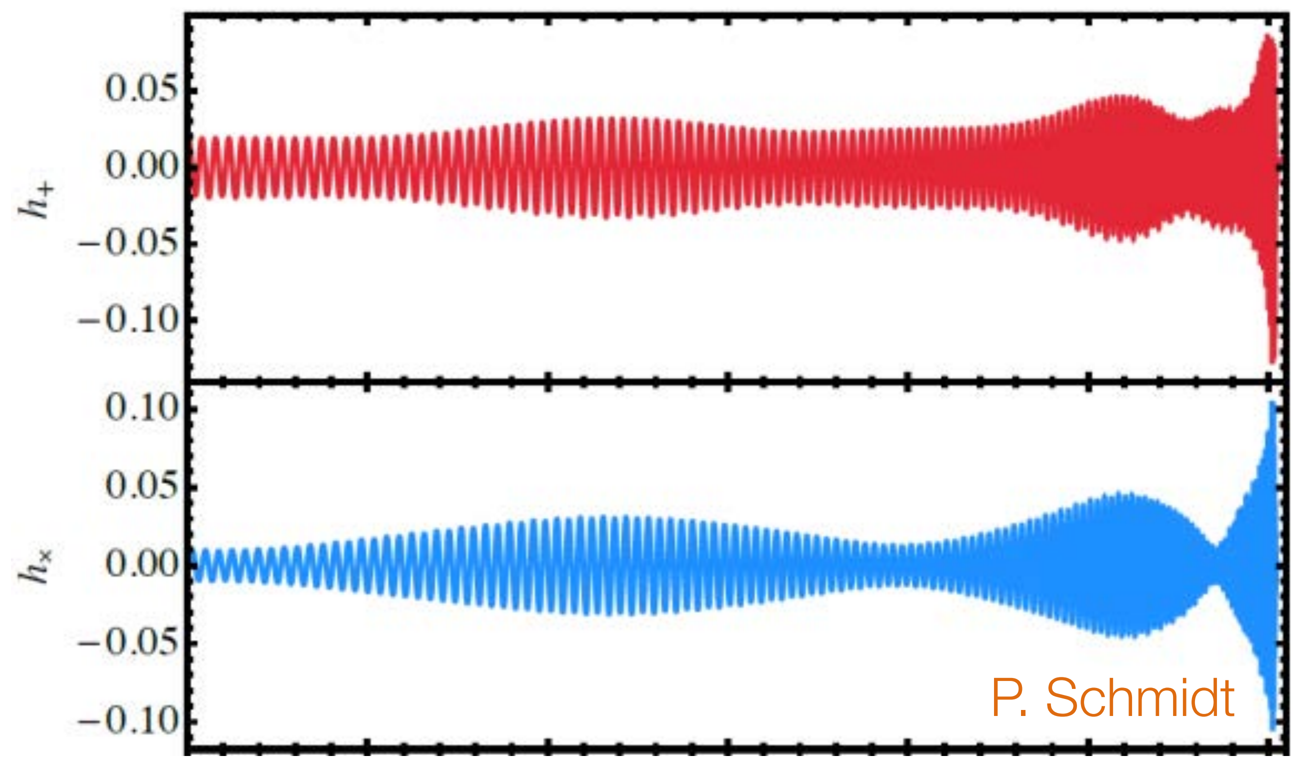
- Different merger frequency (like the ISCO)
- Aligned spins take longer to merge



In-plane components

- spin precession; orbital plane precession
- Peculiar waveform modulations



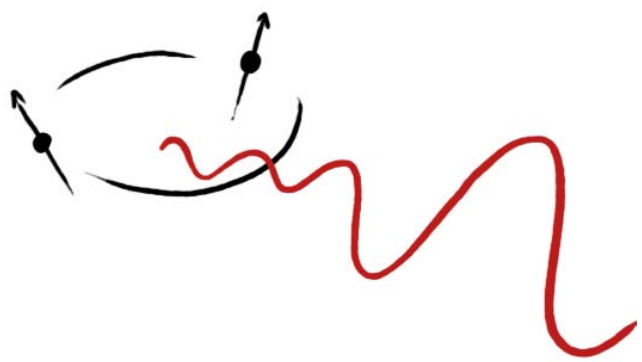


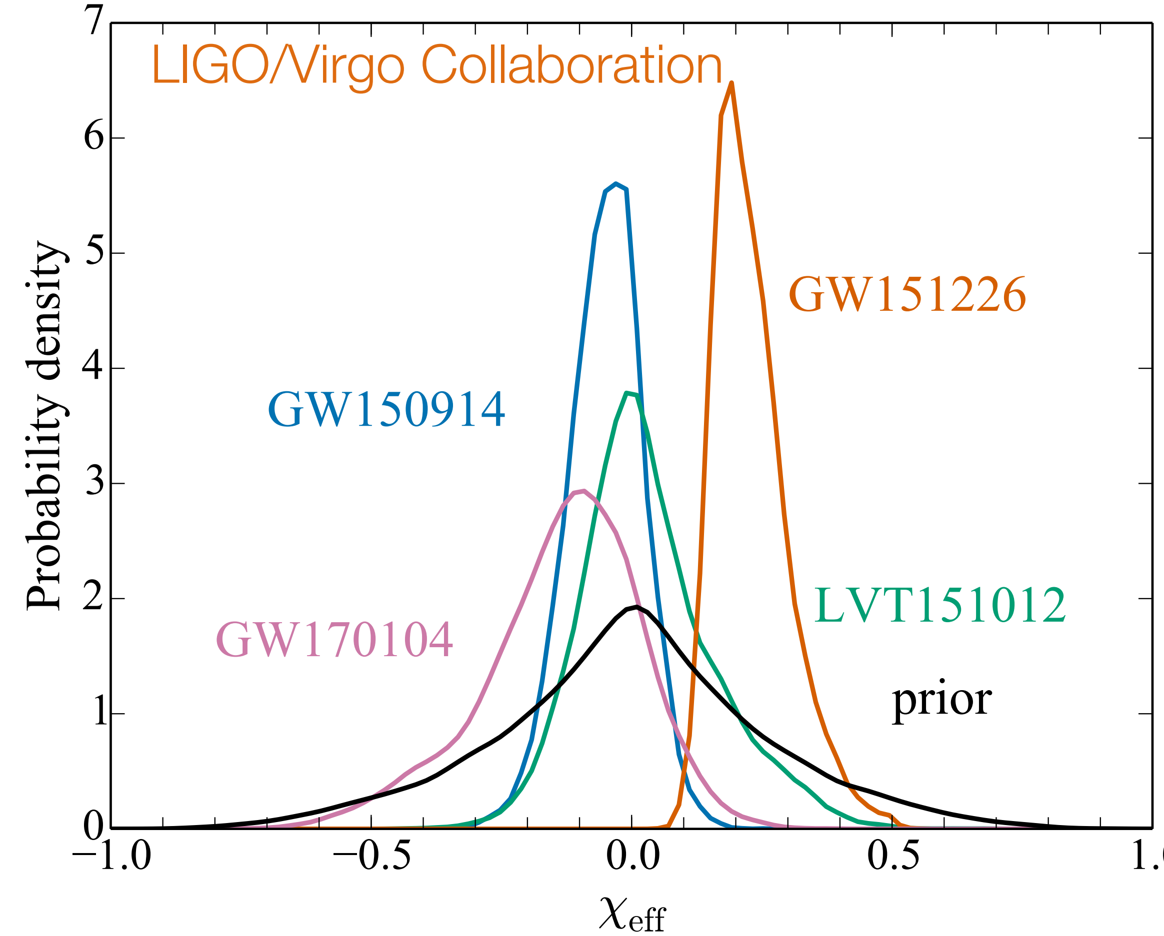
Spin measurements

- Best measured quantity: effective spin

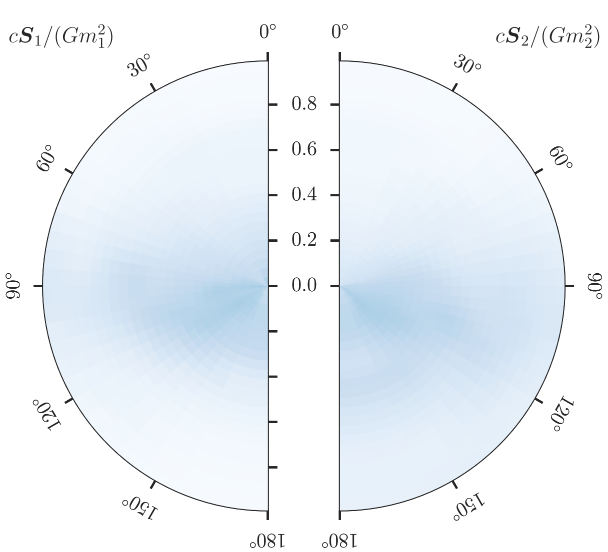
$$\chi_{\rm eff} = \left(\frac{\mathbf{S_1}}{m_1} + \frac{\mathbf{S_2}}{m_2} \right) \frac{\hat{\mathbf{L}}}{M}$$

- Constant of motion at 2PN Racine 2008; **DG**+ 2015a

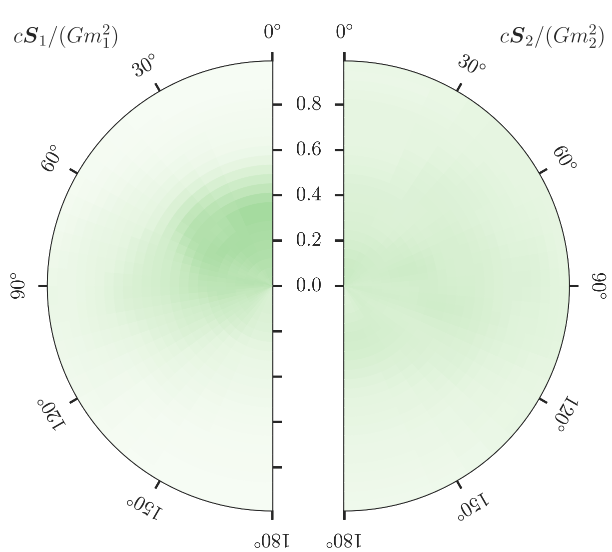




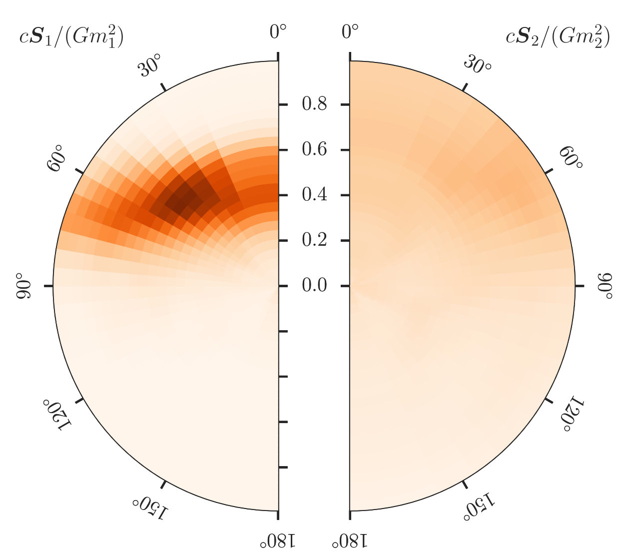
GW150914



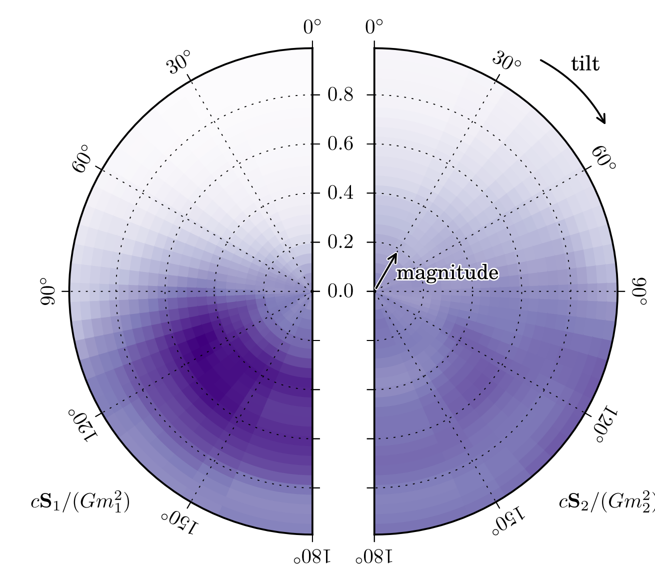
GW151012



GW151226



GW170104



LIGO/Virgo Collaboration

A tale of three timescales

DG+ 2015a; Kesden, DG+, 2014

1. **Orbital** motion $t_{\rm orb} \propto (r/r_g)^{3/2}$ Kepler's third law
2. Spin & orbital-plane **precession** $t_{\rm pre} \propto (r/r_g)^{5/2}$ Apostolatos+ 1994
3. GW emission and **inspiral** $t_{\rm RR} \propto (r/r_g)^{4}$ Quadrupole formula Peters & Matthews 1963

Post-Newtonian: $r \gg r_g = GM/c^2$ timescale separation

Orbit $\ll$ Precession $\ll$ Inspiral

BH binary **multi-timescale** analysis:

1. Solve the dynamics (hopefully analytically) on the shorter time
2. Quasi-adiabatic evolution ("average") on the longer time

Common practice in binary dynamics

- periastron precession
- osculating orbital elements
- variation of constants

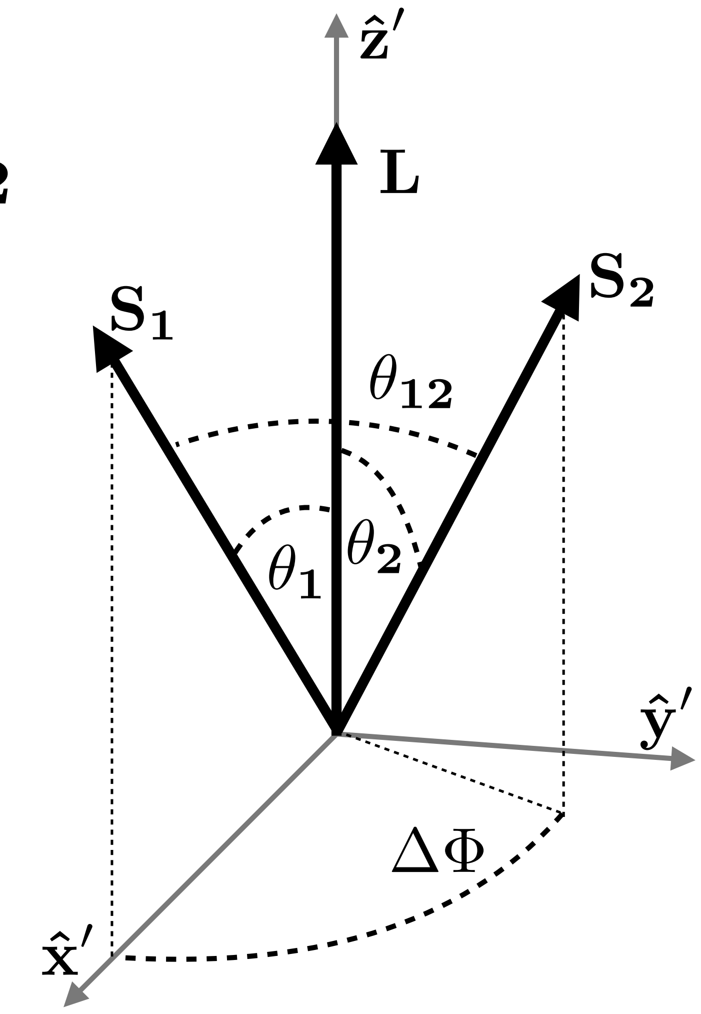
Two-spin PN dynamics

Orbit $\ll$ **Precession** $\ll$ Inspiral

Variables • Three momenta, **9** components $\mathbf{L}, \mathbf{S_1}, \mathbf{S_2}$

Constraints • Spin magnitudes are constants

• Reference frame (3 constraints)



Spin precession is a **4D** problem!

$$r,\ \theta_1,\ \theta_2,\ \Delta\Phi$$

Timescale separation: freeze GW emission

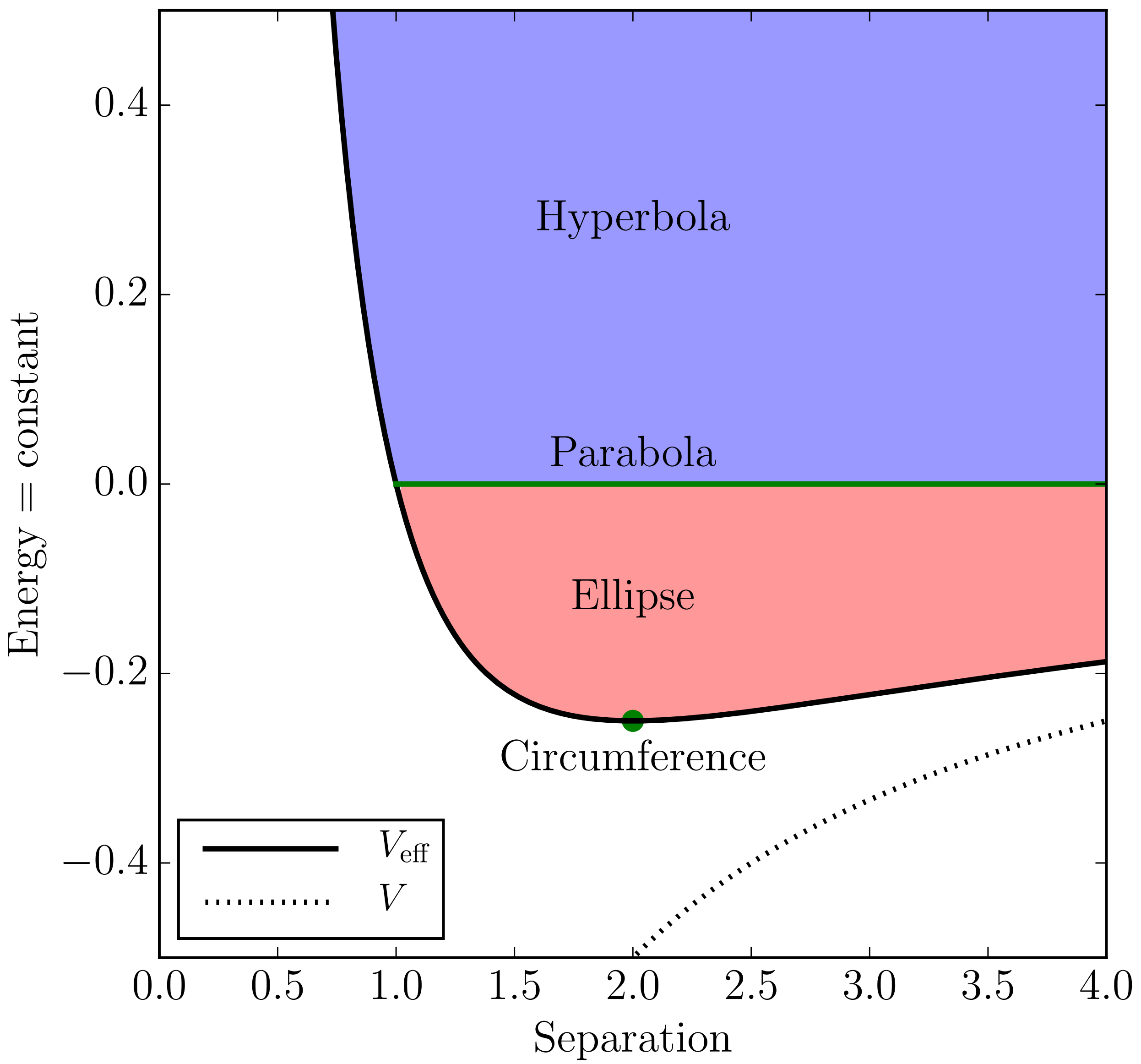
- r and $J = |\mathbf{L} + \mathbf{S_1} + \mathbf{S_2}|$ vary on t_{RR}
- Effective spin is constant at 2PN $(\xi = \chi_{\mathrm{eff}})$ Damour 2001; Racine 2008

Spin precession is (actually) a **1D** problem!

Chosen parameter is $S = |\mathbf{S_1} + \mathbf{S_2}|$

Analytical solution!

Kepler's two-body problem



What you do:

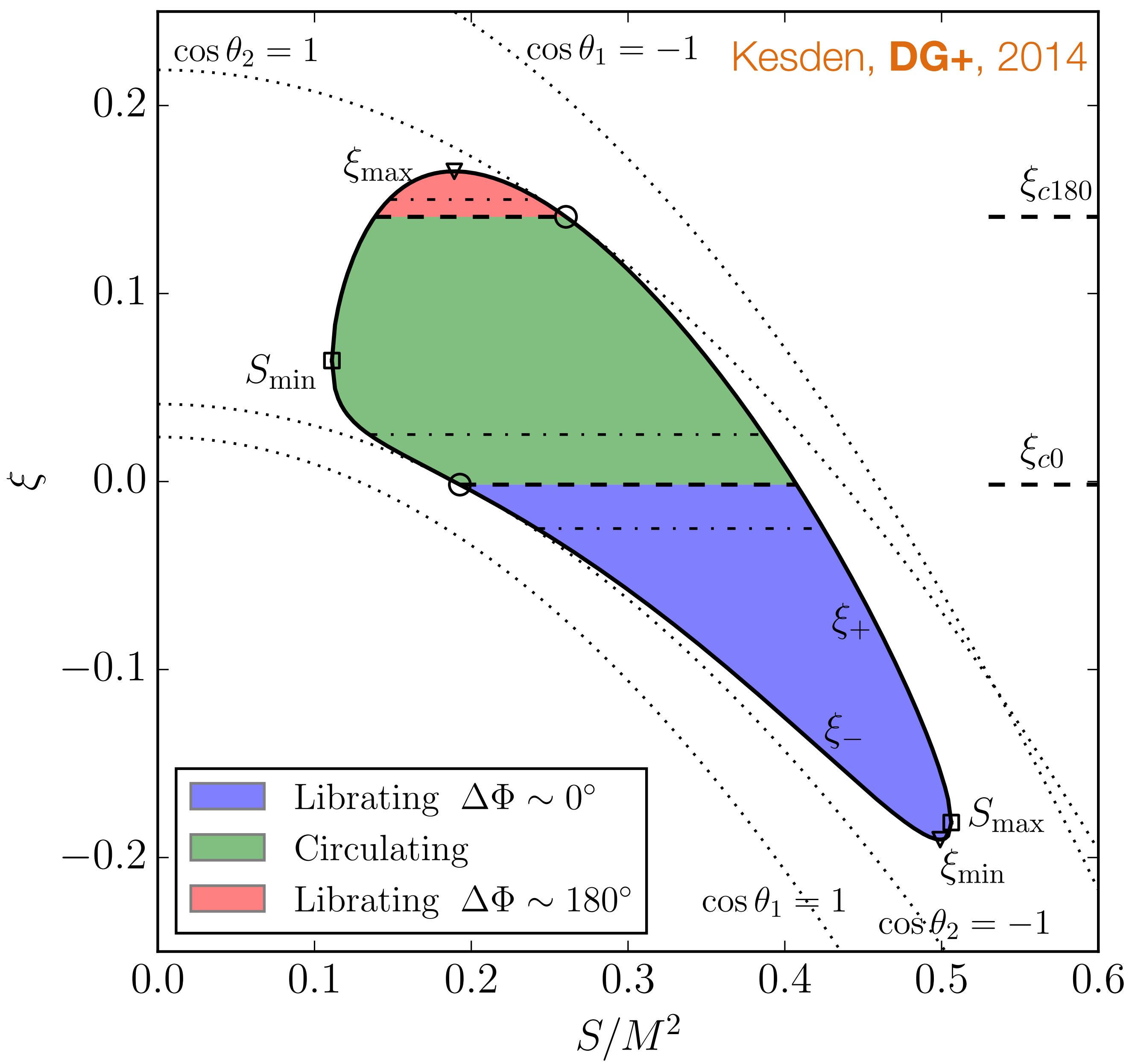
- One effective particle: 3D
- 3D to 2D problem:
  L is a constant of motion!
- Energy is constant: 2D to 1D?
- Effective potential

What you get:

- A lot of understanding
- Solutions are Kepler's orbits
- **Phases**: bound, unbound

Integrate GMm/r^2 to get a bunch of points along an orbit or…
knowing that that curve is an ellipse!

Effective potentials for spin precession



What you do:

- Start from 4D problem
- 4D to 2D problem: GW are frozen, r and J are constant,
- Further constant of motion, effective spin: 2D to 1D
- Effective potentials for BH binary spin precession

What you get:

- Analytical solutions
- Phases, resonances
- A lot of understanding

Integrating the PN eq. to get a bunch of points on a precession cone or…
knowing the shape of that cone!

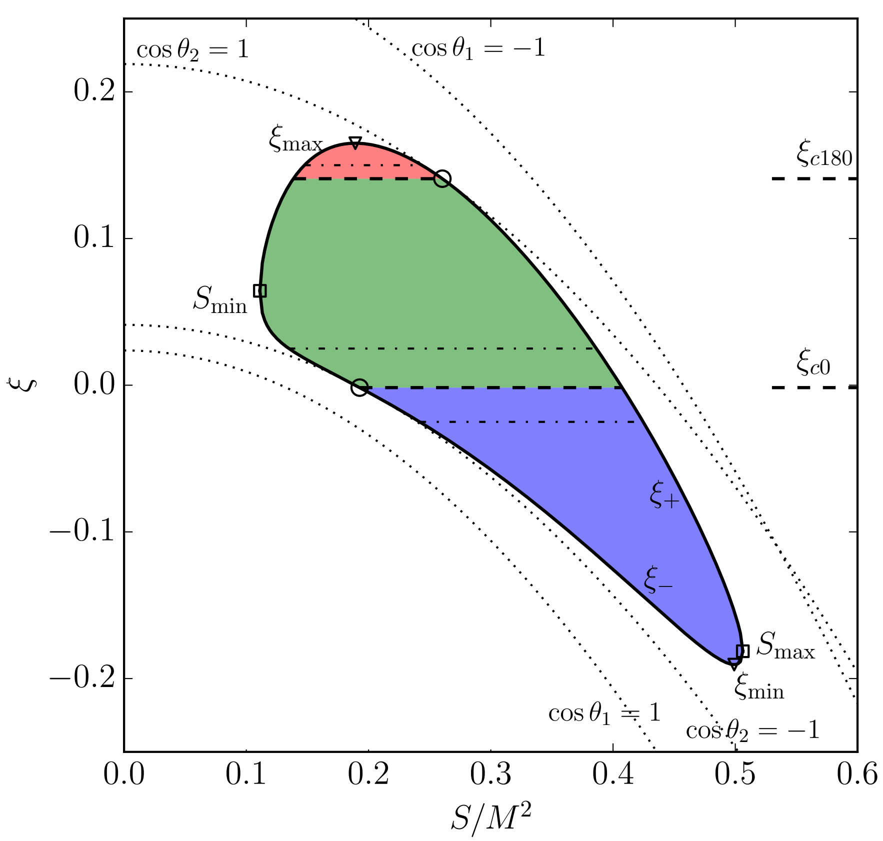
Spin morphologies

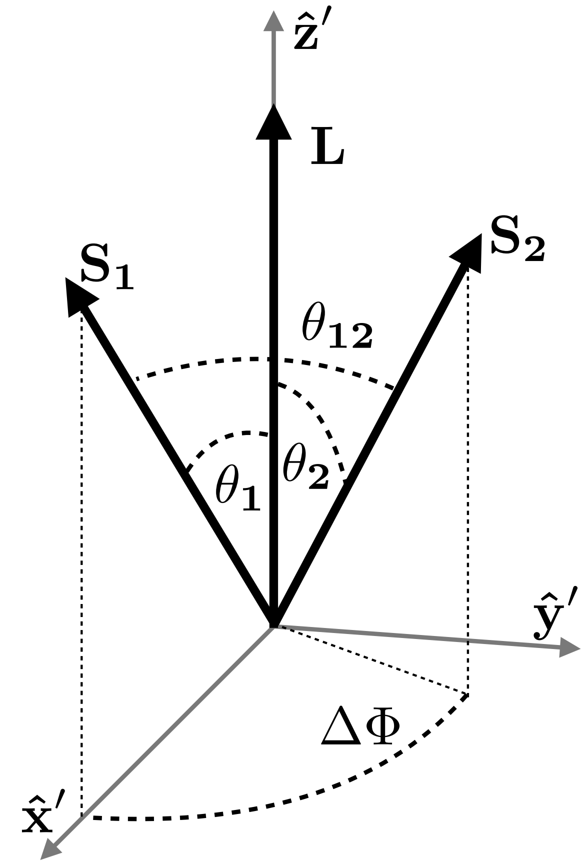
Spin tilts $\theta_1, \ \theta_2$

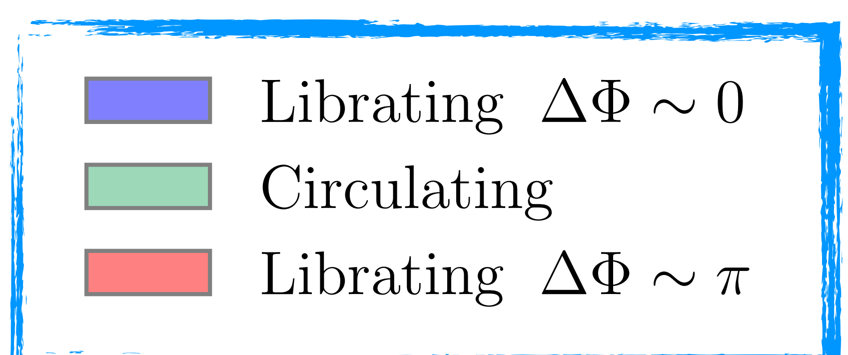
- Monotonic oscillations: **nutations**
- Bounded by the effective potentials

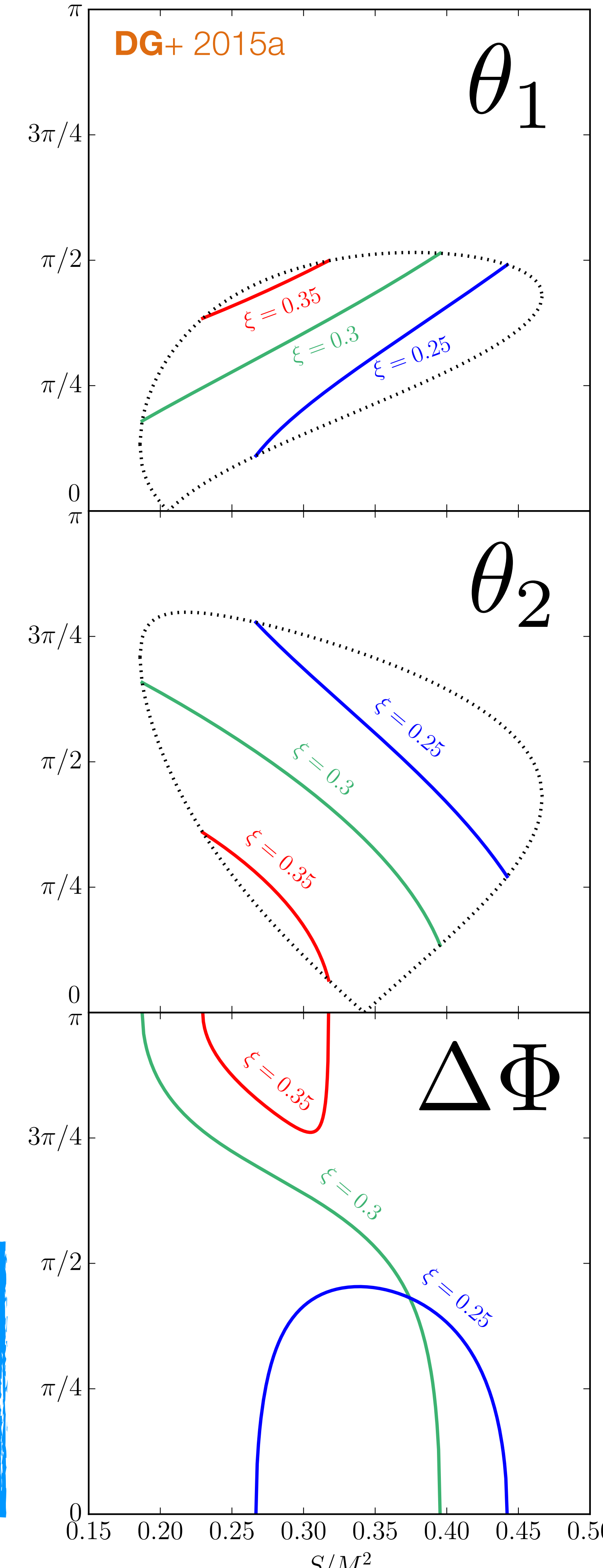
Azimuthal projection $\Delta\Phi$

- Three different **morphologies**
- Boundaries if aligned



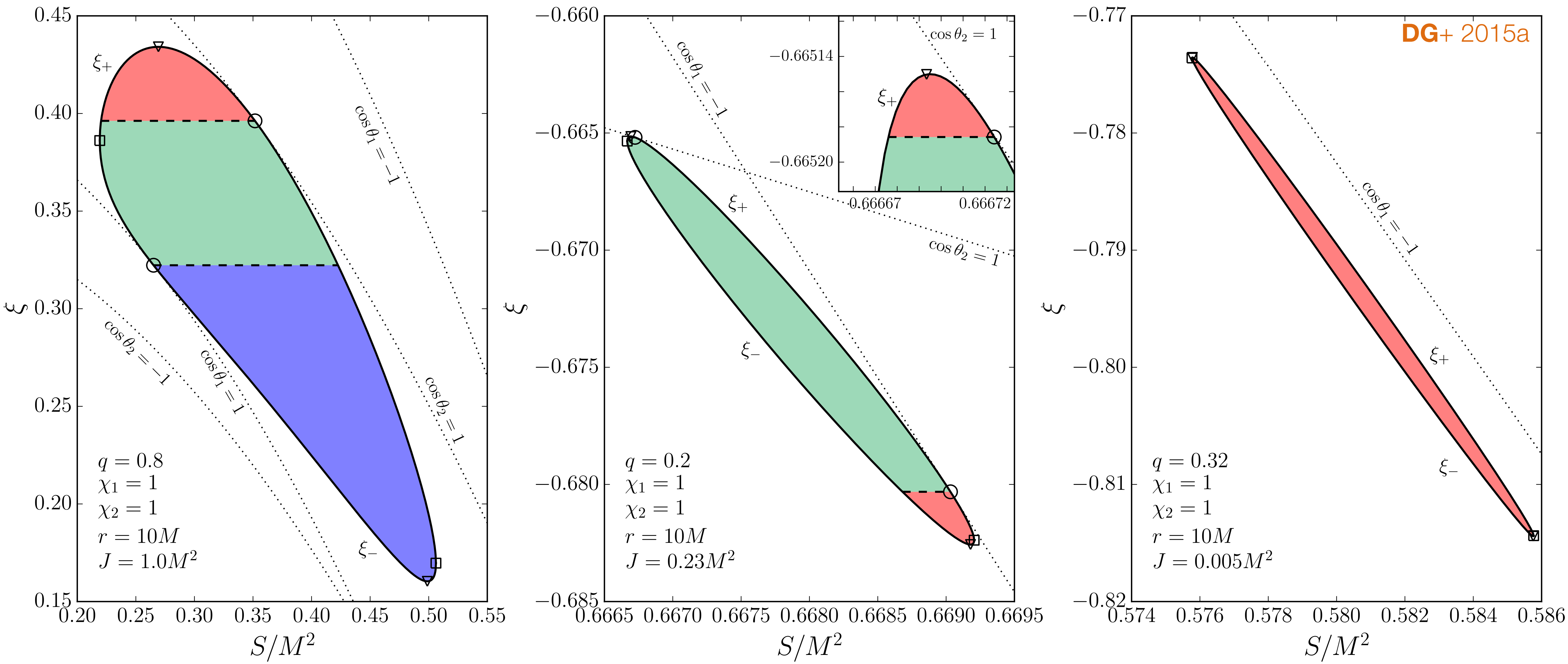






A complete classification

- Effective potentials allow a full classification of the parameter space
- Using only geometry and constant of motions!
- Extremely rich phenomenology!

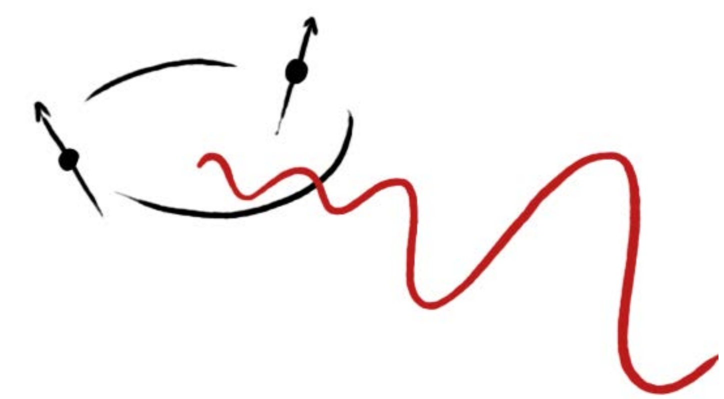


Averaging the average

Orbit $\ll$ Precession $\ll$ **Inspiral**

Let's **turn on** GW emission

- Quasi-adiabatic approach
- Only r and J vary on $t_{\rm RR}$. One single ODE.



Usual **orbit** average

$$\langle X \rangle_{\rm orb} = \frac{\int d\psi \; X \; dt/d\psi}{\int d\psi \; dt/d\psi}$$

Some parameters for the dynamics (here ψ is Kepler's true anomaly)

Orbital period

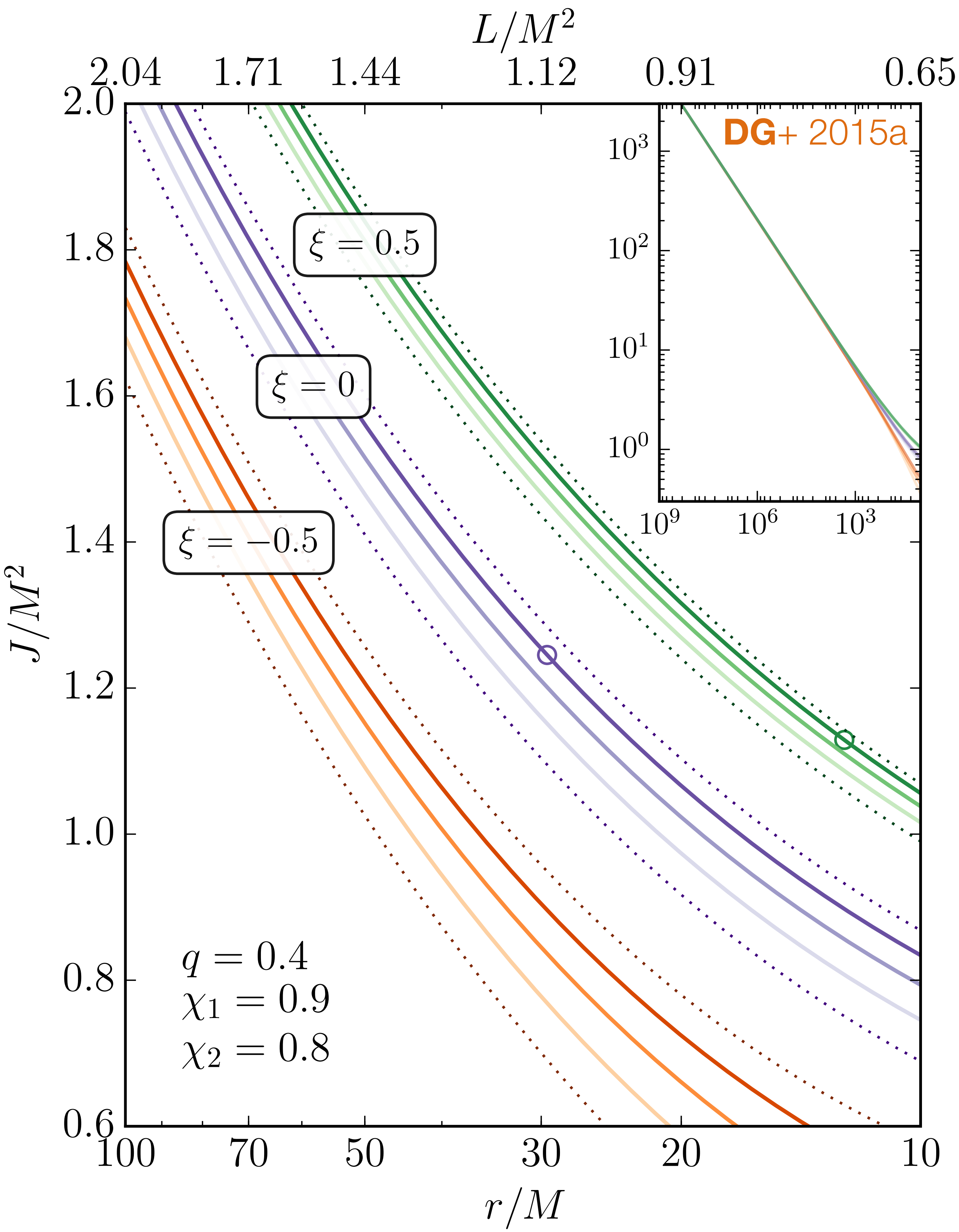
New **precession** average

$$\langle X \rangle_{\rm pre} = \frac{\int dS \; \langle X \rangle_{\rm orb} \; dt/dS}{\int dS \; dt/dS}$$

Dynamics is now parametrized by S

Precession period

Precession-averaged inspiral

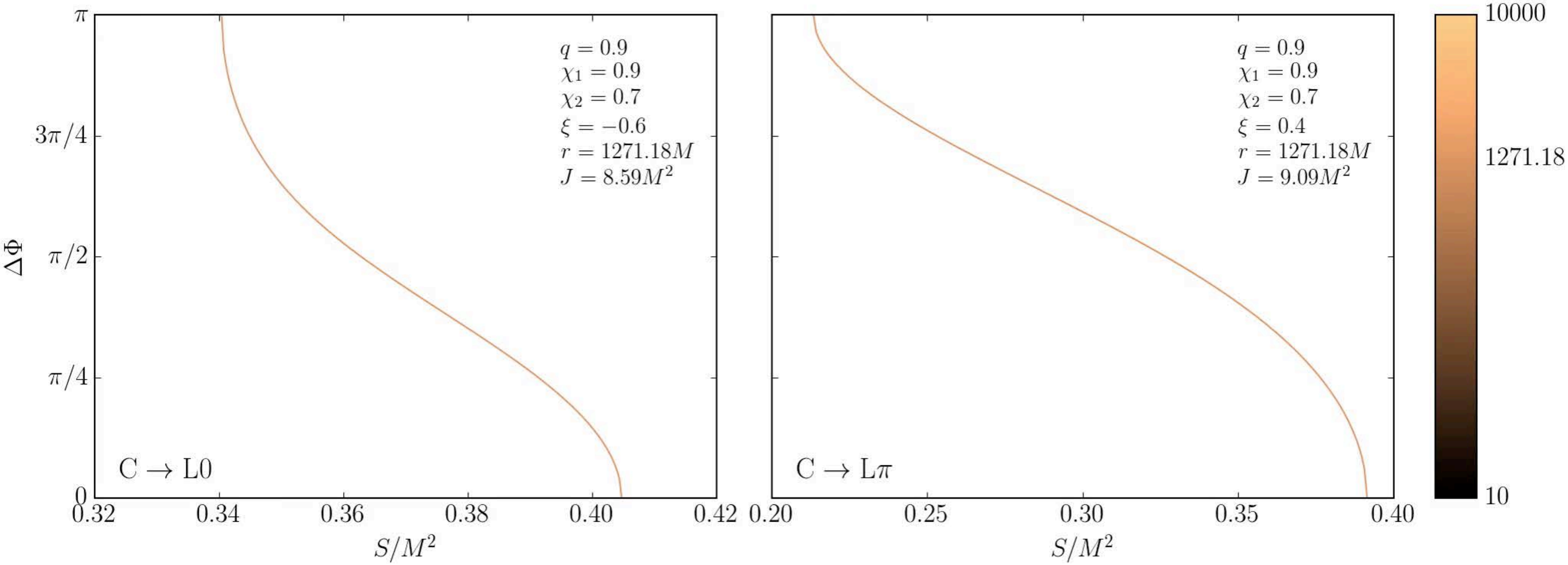


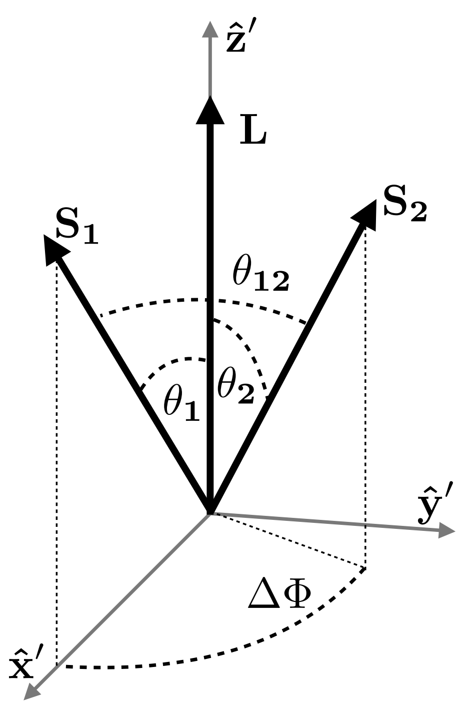
Result is very simple!

$$\left\langle \frac{dJ}{dL} \right\rangle_{\rm pre} = \frac{1}{2LJ}\left(J^2 + L^2 - \langle S^2 \rangle_{\rm pre}\right)$$

- PN evolution is reduced to solving **one single ODE**
- Computationally, very easy
- Domain can be compactified to integrate from $r/M = \infty$
- Precession-timescale quantities are then resampled

Morphological phase transitions



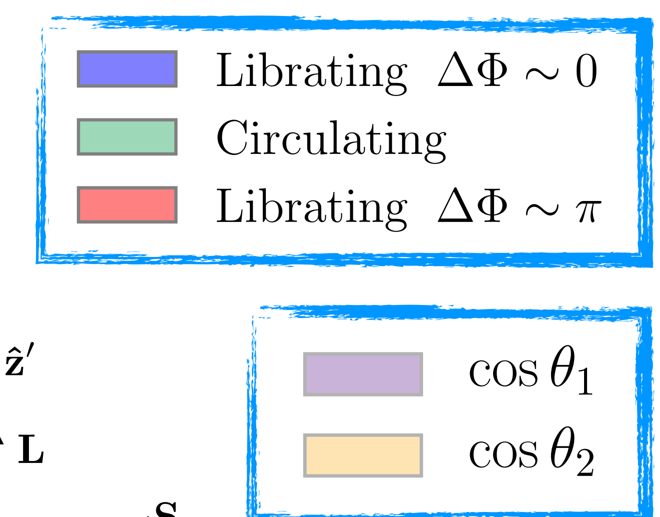


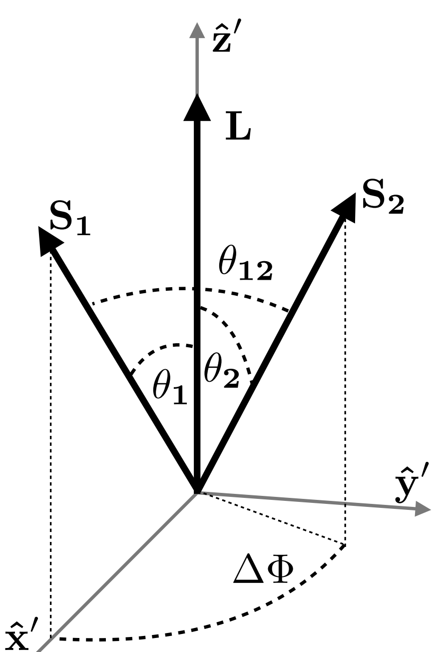
Phase transitions:

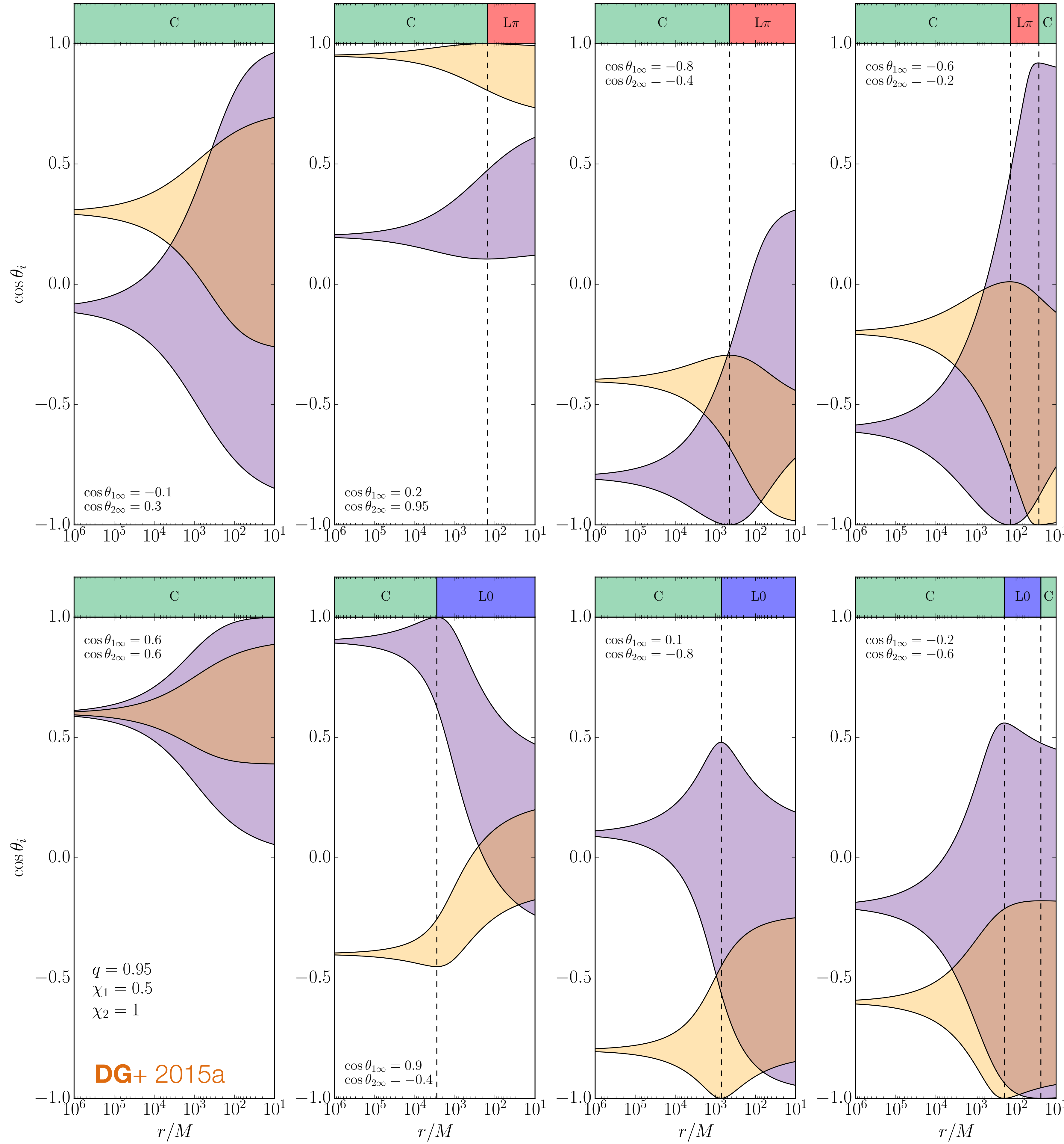
- Discontinuities in the evolution of $\Delta\Phi$
- At large separations, circulation $\Delta\Phi \in [-\pi, \pi]$
- At small separations: libration $\Delta\Phi \sim 0$ or $\Delta\Phi \sim \pi$

The role of alignment

- The range grows fatter
- "Bounce" at the alignment configuration…
- … and sharp transition





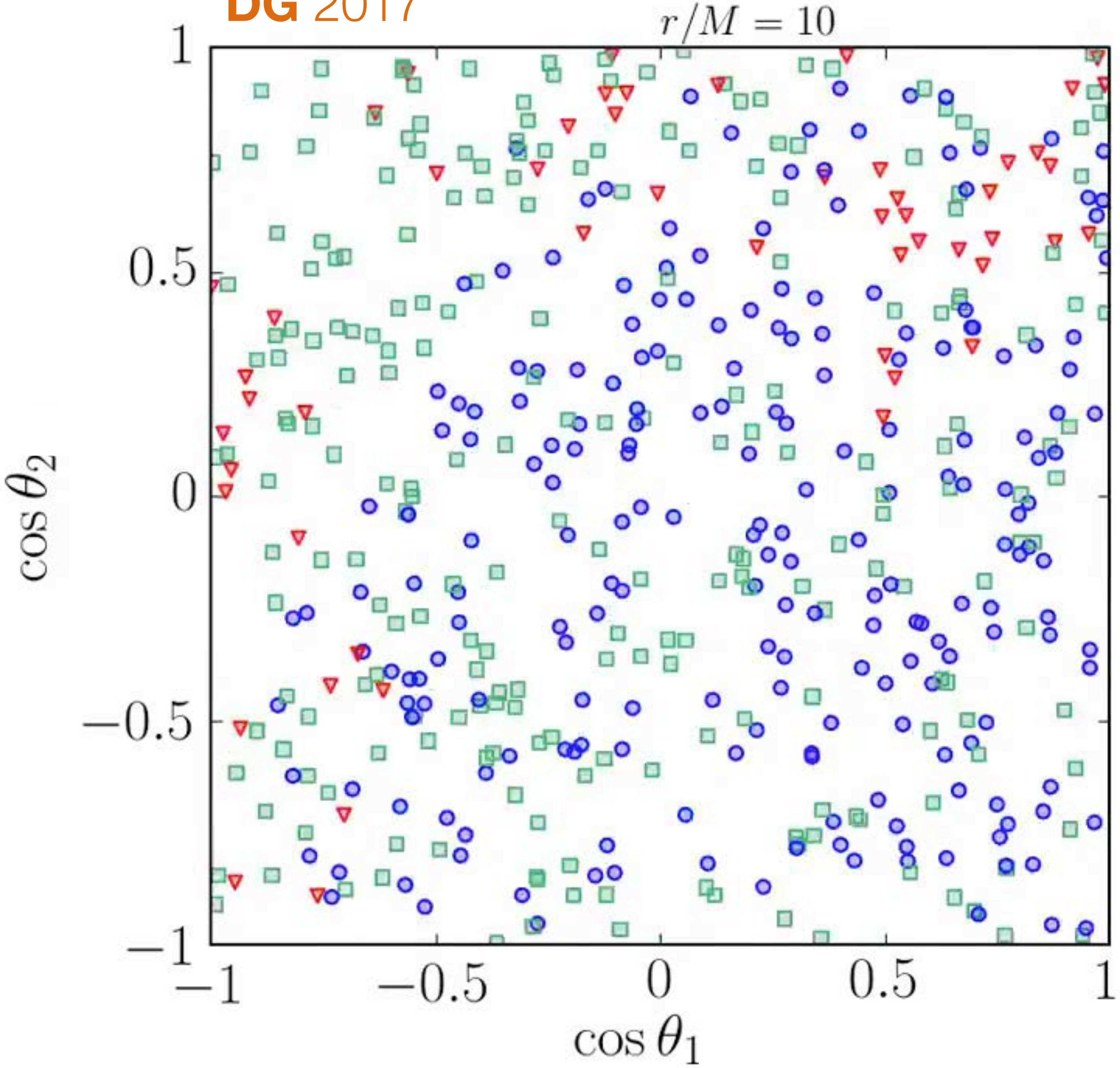


A predictive statement

- **Morphology**: feature of spin precession that does <u>not</u> vary on the precessional time!
- The final spin orientations are scattered around, **but back in the days…**

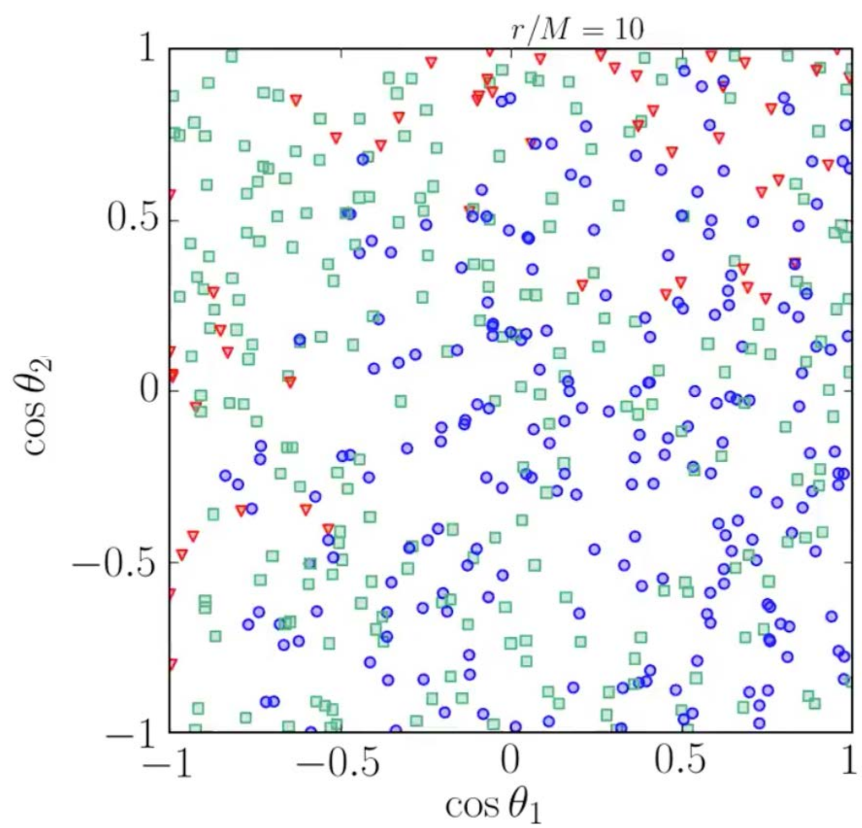
The final morphology encodes the initial spin orientation:
how BHs form!

DG 2017



At formation

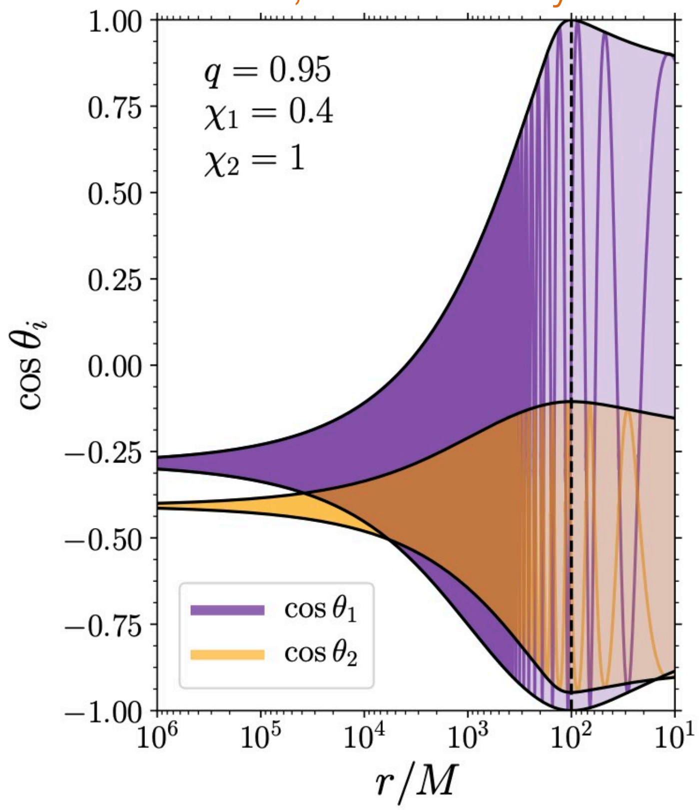
At detection



New predictions

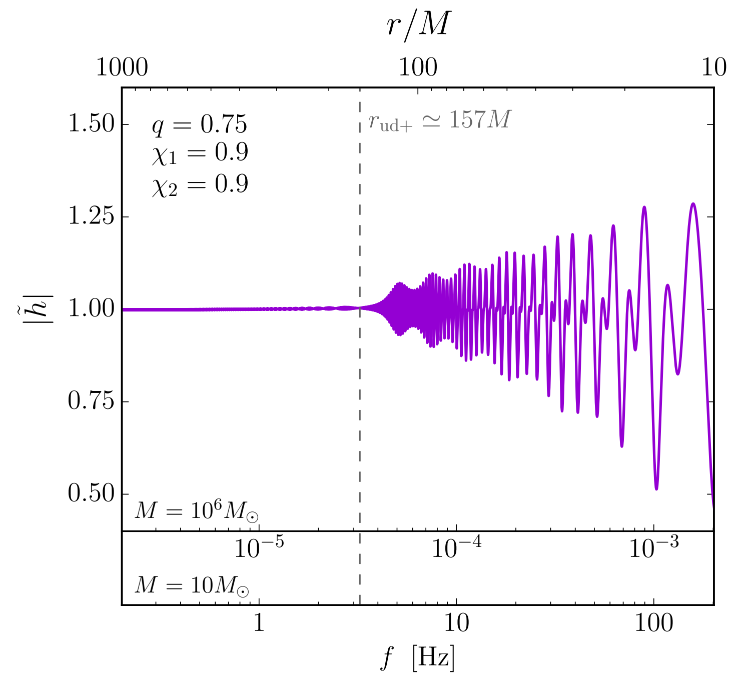
Wide nutation

DG+ 2019, Lousto Healy 2014



Updown instability

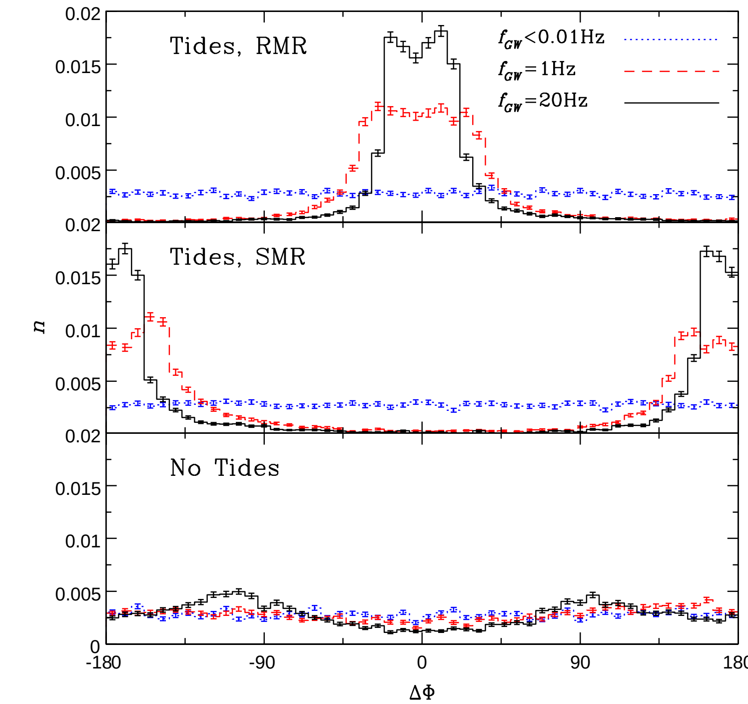
DG+ 2015b, Mould DG 2020



Matt's talks right now!

Binary-star astrophysics

DG+ 2013, 2018, 2019



Discontinuous limits

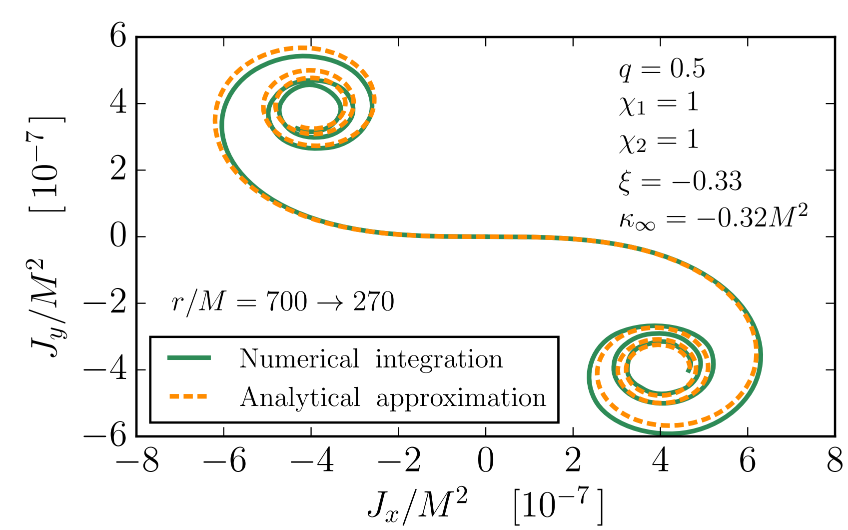
DG+ 2017

Morphologies can be distinguished

DG+ 2014; Trifiro',DG+ 2016, Afle+2018

Nutational resonances

Zhao, Kesden, DG 2017



Asymptotic mapping

Reali, Mould, DG+ 2020

Luca's talk yesterday

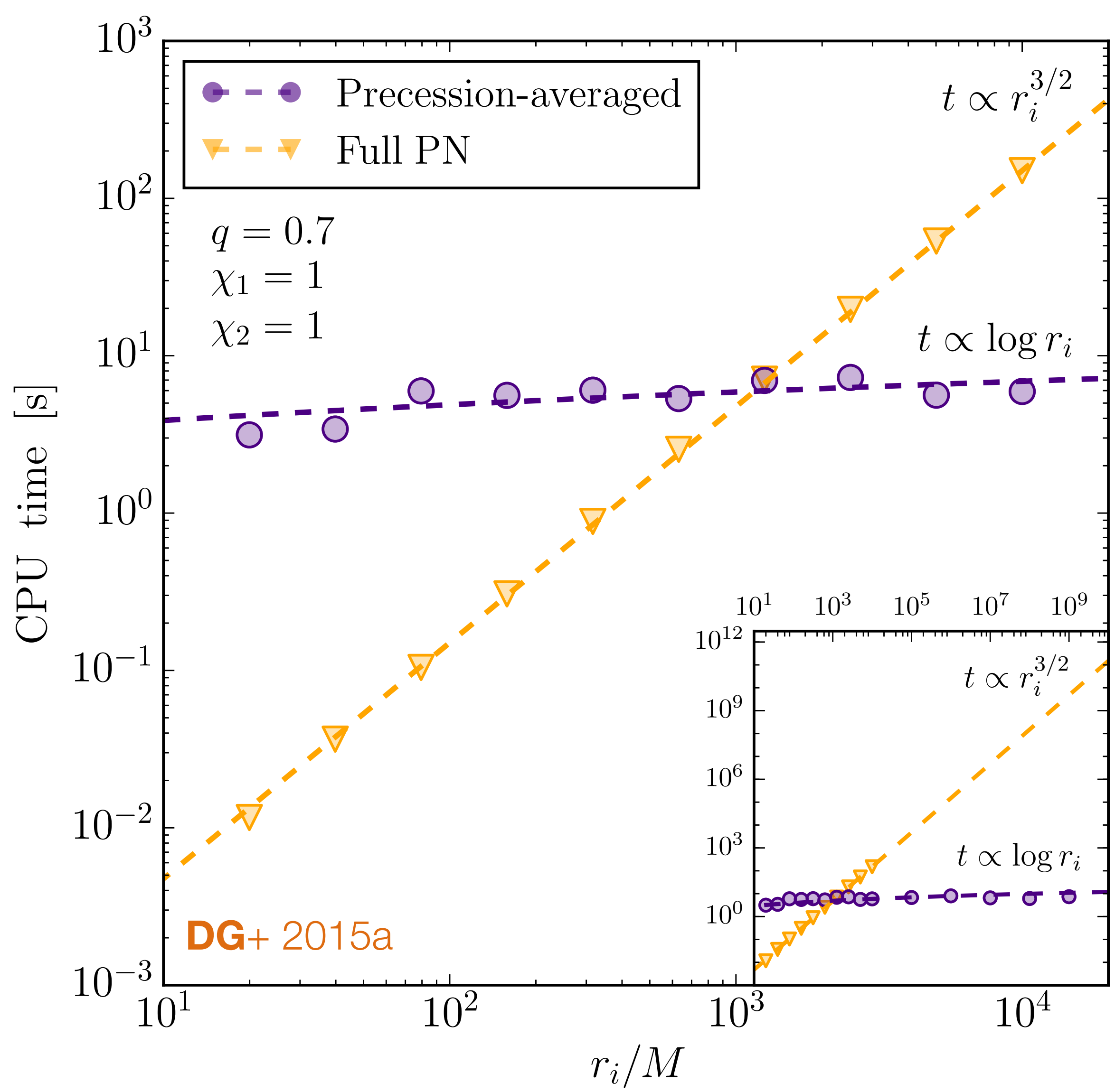
Fast LIGO waveforms

Chatziioannou+ 2016

“precession” python module

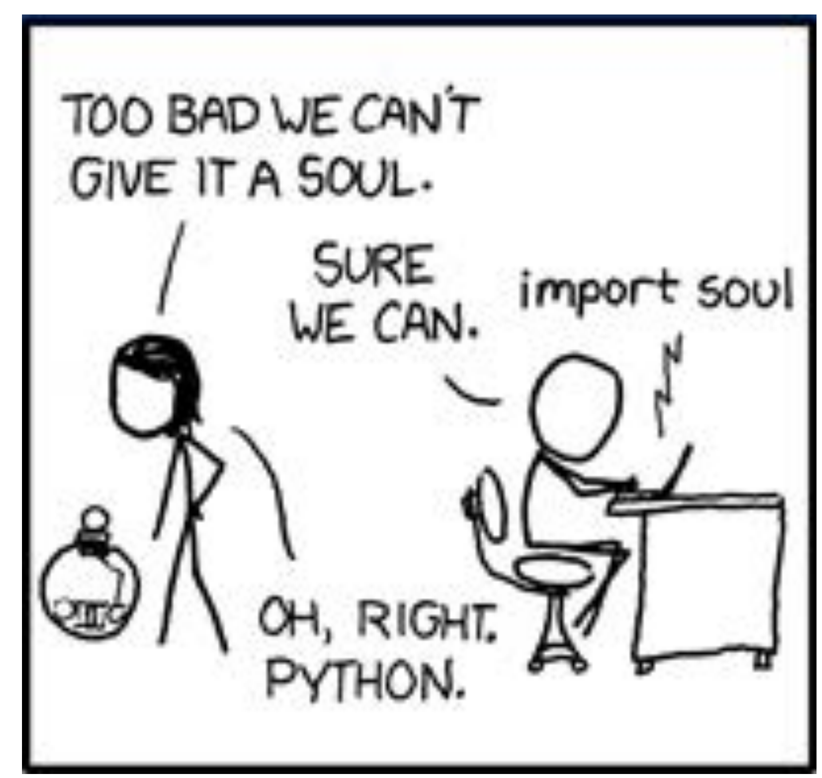
DG, Kesden 2016

Used in 25+ papers to date




```
pip install precession
>>> import precession
```

- Precessional dynamics
- Orbit-averaged inspirals
- Precession-averaged inspirals
- Remnant predictions



Stay tuned! New awesome version coming

DG, Mould 2020 (in prep)

Outline

Thanks! M. Kesden, E. Berti, R. O’Shaughnessy, U. Sperhake, A. Klein, M. Mould, L. Reali, A. Lima, J. Vosmera, X. Zhao, D. Trifiro'

1. Spinning BH binaries
2. A new approach to BH spin precession
3. Astrophysics with BH spins