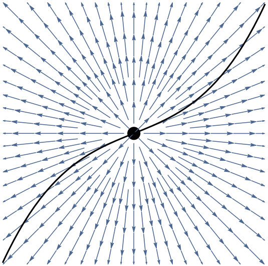
The structure of the nonlinear self-force

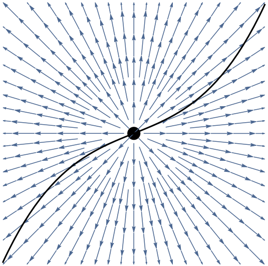
Abraham Harte

Centre for Relativity and Astrophysics
Dublin City University

Capra 2020



How do things move?



How do things move?

In GR, they move on geodesics...But geodesics of *which metric*?

An organizing principle

Test body laws of motion are preserved in the presence of self-interaction, but with *all fields replaced by effective counterparts*: $g \mapsto \hat{g}$, $F \mapsto \hat{F}$, ...

An organizing principle

Test body laws of motion are preserved in the presence of self-interaction, but with *all fields replaced by effective counterparts*: $g \mapsto \hat{g}$, $F \mapsto \hat{F}$, ...

1. Electromagnetism: $m\dot{u}^a = qF^a{}_b u^b \quad \mapsto \quad \hat{m}\dot{u}^a = q\hat{F}^a{}_b u^b$.

An organizing principle

Test body laws of motion are preserved in the presence of self-interaction, but with *all fields replaced by effective counterparts*: $g \mapsto \hat{g}$, $F \mapsto \hat{F}$, ….

1. Electromagnetism: $m\dot{u}^a = qF^a{}_b u^b \quad \mapsto \quad \hat{m}\dot{u}^a = q\hat{F}^a{}_b u^b$.
2. Gravity: $u^b \nabla_b u^a = 0 \quad \mapsto \quad u^b \hat{\nabla}_b u^a = 0$.

Central questions

1. How universal is this organizing principle?
2. *The* self-force question: What are $\hat{g}_{ab}$ or $\hat{F}_{ab}$ anyway?

Central questions

1. How universal is this organizing principle?

2. *The* self-force question: What are $\hat{g}_{ab}$ or $\hat{F}_{ab}$ anyway?

   In certain limits, $g_{ab} \mapsto \hat{g}_{ab}$ becomes a regularization procedure. But *limits and infinities are actually irrelevant.*

Linear theory is understood

An object moves as though the metric were $\hat{g}[g]$, not g.

$$\hat{g}(x) = g(x) - \int dx' \, G(x, x') \, \underbrace{\mathcal{D} g(x')}_{stress-energy}$$

$g \mapsto \hat{g}[g]$ is linear but *quasilocal*.

This is controlled by a particular 2-point function $G(x, x')$, namely the Detweiler–Whiting S-type Green function.

Nonlinear theories

Techniques developed for the linear case can be used to also understand objects coupled to nonlinear fields.

Nonlinear theories

Techniques developed for the linear case can be used to also understand objects coupled to nonlinear fields.

There's a "Volterra series"

$$\hat{g} = g_0 + \epsilon \left[g_1 - \int dx' \, G_1(x, x') \mathscr{D} g_1' \right]$$

Nonlinear theories

Techniques developed for the linear case can be used to also understand objects coupled to nonlinear fields.

There's a "Volterra series"

$$\hat{g} = g_0 + \epsilon \left[g_1 - \int dx' \, G_1(x,x') \mathcal{D} g_1' \right] + \epsilon^2 \Bigg[g_2 - \underbrace{\int dx' \, G_2(x,x') \mathcal{D} g_1'}_{\text{2-point}} - \underbrace{\int dx' \int dx'' \, H_2(x,x',x'') \mathcal{D} g_1' \mathcal{D} g_1''}_{\text{3-point}} \Bigg] + \dots$$

Towers of n–point functions

Order	2–point	3–point	4–point	5–point	$\cdots$
1	G_1	0	0	0	$\cdots$
2	G_2	H_2	0	0	$\cdots$
3	G_3	H_3	I_3	0	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
∞	$\mathscr{G}$	$\mathscr{H}$	$\mathscr{I}$	$\mathscr{J}$	$\cdots$

Everything is derivable from "resummed" $\mathscr{G}$, $\mathscr{H}$, $\mathscr{I}$, $\mathscr{J}$, etc.

What is the resumed 2-point function?

$\mathcal{G}(x, x'; \epsilon) = \sum_{n=1}^{\infty} \epsilon^n G_n(x, x')$ is the Detweiler-Whiting Green function obtained when the background is the *exact* $\hat{g}_{ab}(x; \epsilon)$.

What is the resumed 2-point function?

$\mathscr{G}(x, x'; \epsilon) = \sum_{n=1}^{\infty} \epsilon^n G_n(x, x')$ is the Detweiler–Whiting Green function obtained when the background is the *exact* $\hat{g}_{ab}(x; \epsilon)$.

1. **Reciprocity:** $\mathscr{G}(x, x'; \epsilon) = \mathscr{G}(x', x; \epsilon)$.

What is the resumed 2-point function?

$\mathcal{G}(x, x'; \epsilon) = \sum_{n=1}^{\infty} \epsilon^n G_n(x, x')$ is the Detweiler–Whiting Green function obtained when the background is the *exact* $\hat{g}_{ab}(x; \epsilon)$.

1. **Reciprocity:** $\mathcal{G}(x, x'; \epsilon) = \mathcal{G}(x', x; \epsilon)$.
2. **Locality:** $\mathcal{G}(x, x'; \epsilon) = 0$ when x, x' are timelike-separated.

What is the resummed 2-point function?

$\mathcal{G}(x, x'; \epsilon) = \sum_{n=1}^{\infty} \epsilon^n G_n(x, x')$ is the Detweiler–Whiting Green function obtained when the background is the *exact* $\hat{g}_{ab}(x; \epsilon)$.

1. **Reciprocity:** $\mathcal{G}(x, x'; \epsilon) = \mathcal{G}(x', x; \epsilon)$.
2. **Locality:** $\mathcal{G}(x, x'; \epsilon) = 0$ when x, x' are timelike-separated.
3. **Linearized Green:** $(\text{Lin Einst})_{\hat{g}} \mathcal{G} = \hat{\delta}$.

3-point functions

Sufficient conditions to *derive* laws of motion:

1. **Reciprocity:** $\mathscr{H}(x, x', x''; \epsilon) = \mathrm{Perm}_{x,x',x''}\left[\mathscr{H}(x, x', x''; \epsilon)\right]$.

3-point functions

Sufficient conditions to *derive* laws of motion:

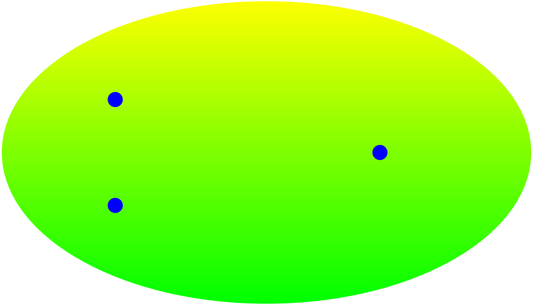
1. **Reciprocity:** $\mathscr{H}(x, x', x''; \epsilon) = \mathrm{Perm}_{x,x',x''}[\mathscr{H}(x, x', x''; \epsilon)]$.
2. **Locality:** $\mathscr{H}(x, x', x''; \epsilon) = 0$ if at least two (out of three) pairs of points are timelike-separated.

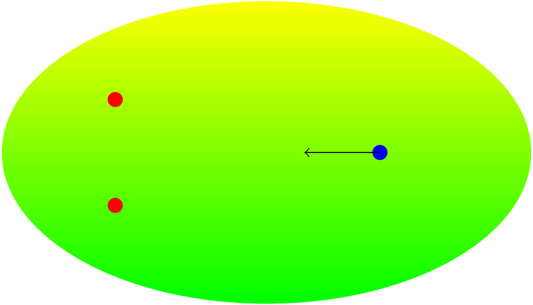
3-point functions

Sufficient conditions to *derive* laws of motion:

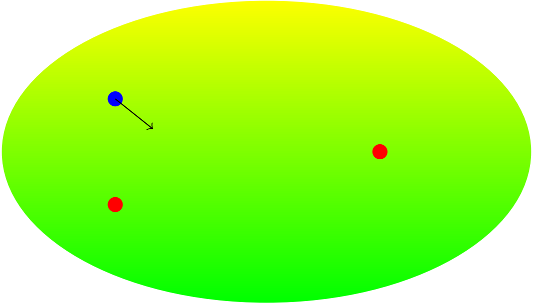
1. **Reciprocity:** $\mathscr{H}(x, x', x''; \epsilon) = \mathrm{Perm}_{x,x',x''}[\mathscr{H}(x, x', x''; \epsilon)]$.
2. **Locality:** $\mathscr{H}(x, x', x''; \epsilon) = 0$ if at least two (out of three) pairs of points are timelike-separated.
3. **2nd-order Green:** $(\text{Lin Einst})_{\hat{g}} \mathscr{H} \sim$ "$\mathscr{G}\mathscr{G}$".

n-point versions of Newton's 3rd law

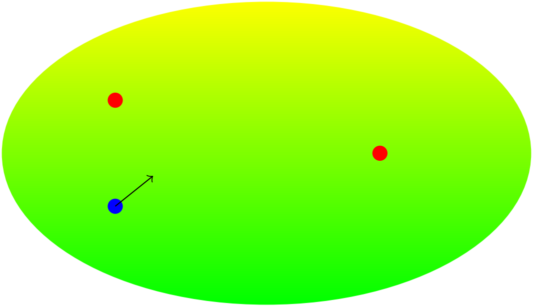




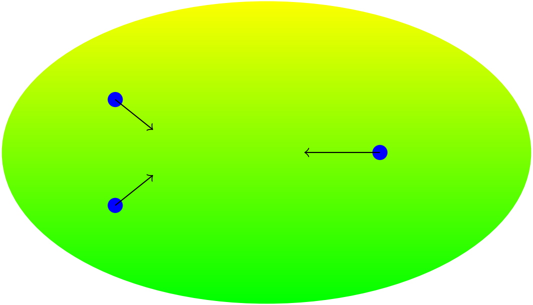
n–point versions of Newton's 3rd law



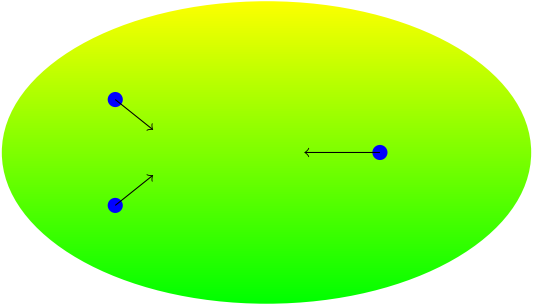
n-point versions of Newton's 3rd law



n-point versions of Newton's 3rd law



n-point versions of Newton's 3rd law



2-point: $\mathscr{L}_\xi \mathscr{G}(x, x'; \epsilon) \sim 0$
3-point: $\mathscr{L}_\xi \mathscr{H}(x, x', x''; \epsilon) \sim 0$

What do these things look like?

For the 2-point functions, we have the Hadamard form

$$\mathscr{G}(x, x'; \epsilon) = U\delta(\sigma) + V\Theta(\sigma).$$

Is there something similarly intuitive for $\mathscr{H}(x, x', x''; \epsilon)$?

A 3-point Hadamard form?

The following seems close to what's needed:

$$\mathscr{H}(x, x', x''; \epsilon) \sim \frac{\mathrm{Sym}_{x,x',x''}\left[\Theta(\sigma)\Theta(\sigma')\right]}{\underbrace{(\text{area of triangle with vertices } x,\, x',\, x'')}_{\frac{1}{2}\left[2\left(\sigma\sigma'+\sigma\sigma''+\sigma'\sigma''\right)-\left(\sigma^2+\sigma'^2+\sigma''^2\right)\right]^{1/2}}} + \ldots$$

A 3-point Hadamard form?

The following seems close to what's needed:

$$\mathcal{H}(x, x', x''; \epsilon) \sim \frac{\mathrm{Sym}_{x,x',x''}\left[\Theta(\sigma)\Theta(\sigma')\right]}{\underbrace{(\text{area of triangle with vertices } x,\, x',\, x'')}_{\frac{1}{2}\left[2\left(\sigma\sigma'+\sigma\sigma''+\sigma'\sigma''\right)-\left(\sigma^2+\sigma'^2+\sigma''^2\right)\right]^{1/2}}} + \ldots$$

Details to be worked out…

Conclusions

1. Motion of bodies with arbitrary size and structure can be understood using a nonlinearity expansion.
2. Nonlinear self-interaction involves n-point functions which generalize Detweiler–Whiting.

Conclusions

1. Motion of bodies with arbitrary size and structure can be understood using a nonlinearity expansion.
2. Nonlinear self-interaction involves n-point functions which generalize Detweiler–Whiting.

---

3. How can these functions be constructed?
4. Are they all just functional derivatives of one master/partition function?
5. Is there a fully non-perturbative "flow-type" description?