

## Exploring the small mass ratio binary black hole merger with Numerical Relativity

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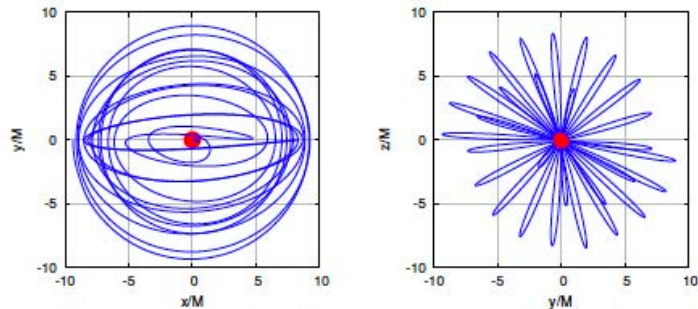
# Introduction

- In Capra 14 (2011) We presented first full numerical simulation 100:1 for two orbits before merger:

C. O. Lousto and Y. Zlochower, Phys. Rev. Lett. **106**, 041101 (2011), arXiv:1009.0292 [gr-qc].

- More recently, we studied the GW beaconing with precessing  $q=1/7$ ,  $q=1/15$  binaries and found excellent results with updated techniques and AMR grid:

C. O. Lousto and J. Healy, Phys. Rev. **D99**, 064023 (2019), arXiv:1805.08127 [gr-qc].



- We will revisit the scenario of the nonspinning small mass ratio binaries as we did for up to  $q=1/10$  in:

J. Healy, C. O. Lousto, and Y. Zlochower, Phys. Rev. **D96**, 024031 (2017), arXiv:1705.07034 [gr-qc].

[Presented in Capra 20 (2017)]

-> But we now push it to  $q=1/15$ ,  $1/32$ ,  $1/64$ , and  $1/128$

# Numerical Simulations

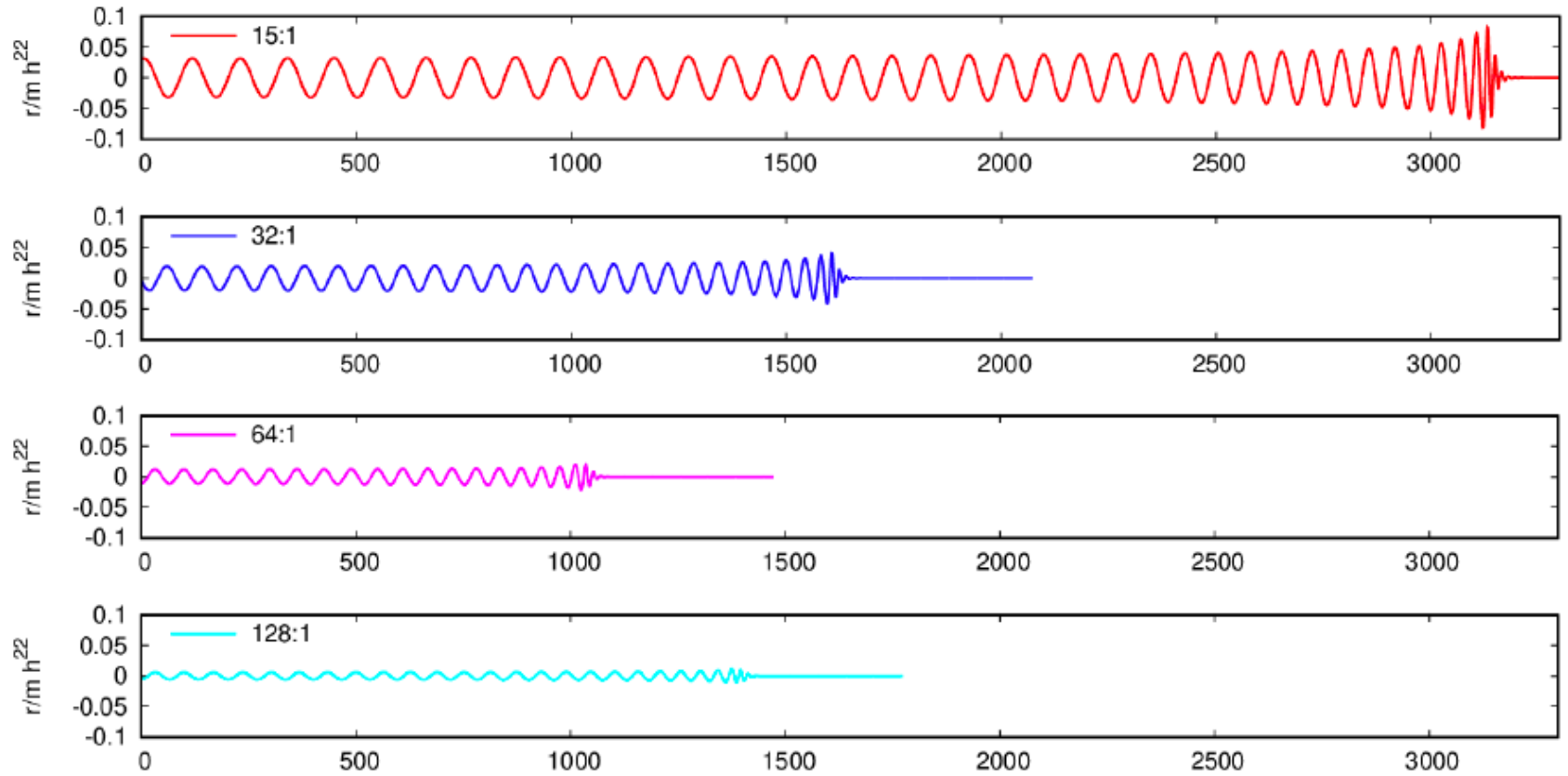
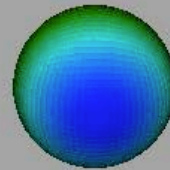


FIG. 1. (2,2) modes (real part) of the strain waveforms versus time ( $t/m$ ), for the  $q = 1/15, 1/32, 1/64, 1/128$  simulations.

# Numerical Simulations



128:1 merger orbit and horizons  
Viz: Nicole Rosato.

# Numerical Simulations

Convergence:

TABLE I. The final black hole mass  $M_{\text{rem}}/m$ , spin  $\alpha_{\text{rem}}$ , and its recoil velocity  $v_m$ , and the Luminosity  $\mathcal{L}$ , waveform frequency  $\omega_{22}$  at the maximum amplitude  $h_{\text{peak}}$ , for each resolution of the  $q = 1/15$  simulations. Extrapolation to infinite resolution and order of convergence is derived.

resolution	$M_{\text{rem}}/m$	$\alpha_{\text{rem}}$	$v_m$ [km/s]	$\mathcal{L}_{\text{peak}}$ [ergs/s]	$m\omega_{22}^{\text{peak}}$	$h_{\text{peak}}$
n100	0.994837	0.188442	33.45	1.659e+55	0.2902	0.08526
n120	0.994876	0.188874	34.67	1.678e+55	0.2860	0.08473
n140	0.994891	0.188987	35.24	1.683e+55	0.2866	0.08466
$n \rightarrow \infty$	0.994905	0.189047	36.07	1.687e+55	0.2868	0.08464
order	4.63	6.87	3.40	6.01	10.33	10.82

TABLE II. The final black hole mass  $M_{\text{rem}}/m$ , spin  $\alpha_{\text{rem}}$ , and its recoil velocity  $v_m$ , and the Luminosity  $\mathcal{L}$ , waveform frequency  $\omega_{22}$  at the maximum amplitude  $h_{\text{peak}}$ , for the sequence of the  $q = 1/32, 1/64, 1/128$  simulations. Also given are the initial simple proper distance, SPD, and number of orbits to merger  $N$  for these simulations.

$q$	$M_{\text{rem}}/m$	$\alpha_{\text{rem}}$	$v_m$ [km/s]	$\mathcal{L}_{\text{peak}}$ [ergs/s]	$m\omega_{22}^{\text{peak}}$	$h_{\text{peak}}$	SPD	N
1/32	0.9979	0.1006	9.14	4.260e+54	0.2820	0.0424	9.51	13.02
1/64	0.9990	0.0520	2.34	1.113e+54	0.2812	0.0220	8.22	9.98
1/128	0.9996	0.0239	0.96	3.313e+53	0.2746	0.0116	8.19	12.90

Speeds: 2.7M/h ( $q=1/15$  with 6<sup>th</sup> order) on 8 nodes (448 cores) in Frontera. CFL=1/4.  
 1.1 M/h ( $q=1/32$  with 8<sup>th</sup> order), 0.6M/h ( $q=1/64$ ), 0.32M/h ( $q=1/128$ )

# Results

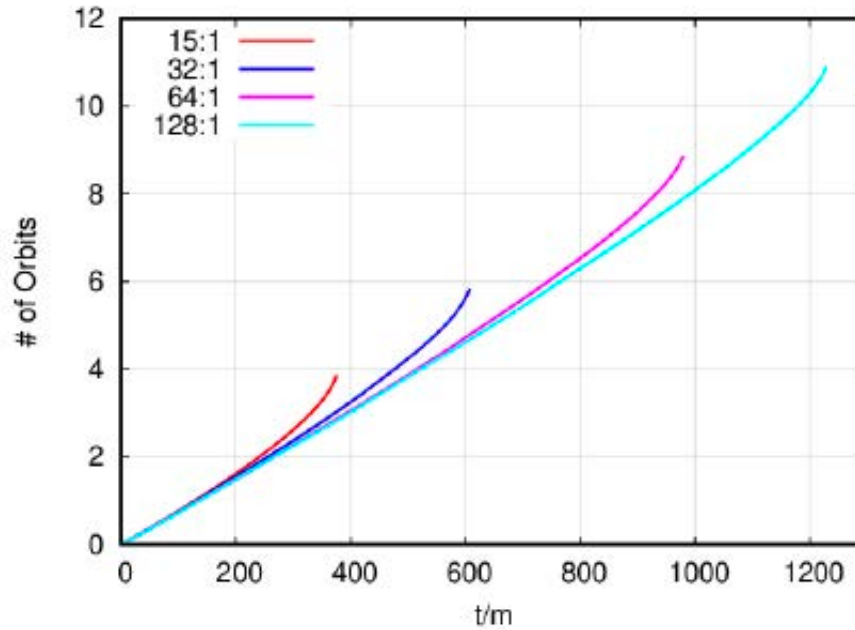
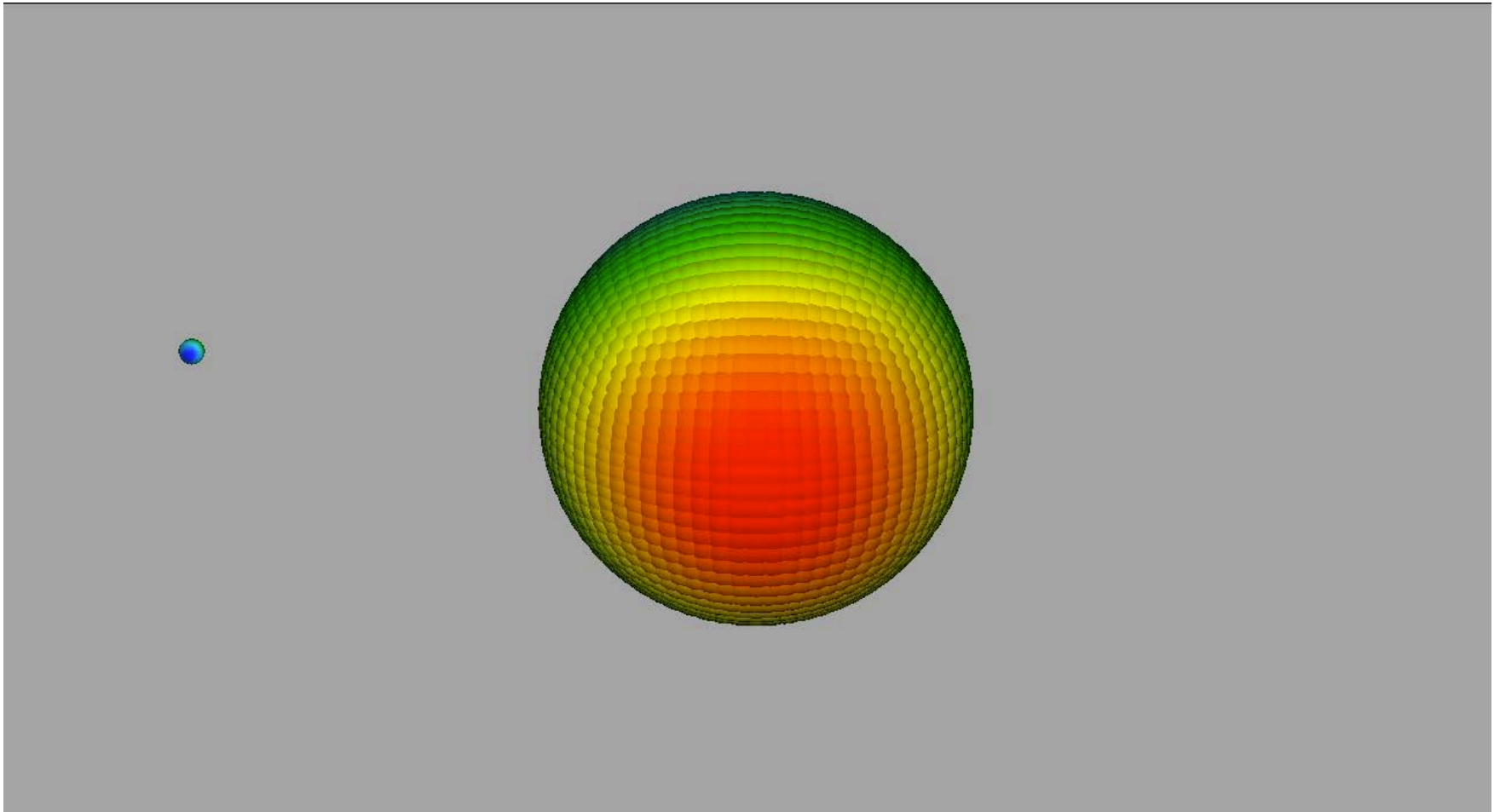


FIG. 2. Comparative number of orbits and time to merger, from a fiducial orbital frequency  $m\Omega_i = 0.0465$  for the  $q = 1/15, 1/32, 1/64, 1/128$  simulations.

$$T_{\text{merger}} \sim (83.2M) \text{eta}^{-0.56}, \text{eta} = m_1 m_2 / m^2$$

# Results



128:1 merger horizons (rescaled) Curvature K  
Viz: Nicole Rosato.

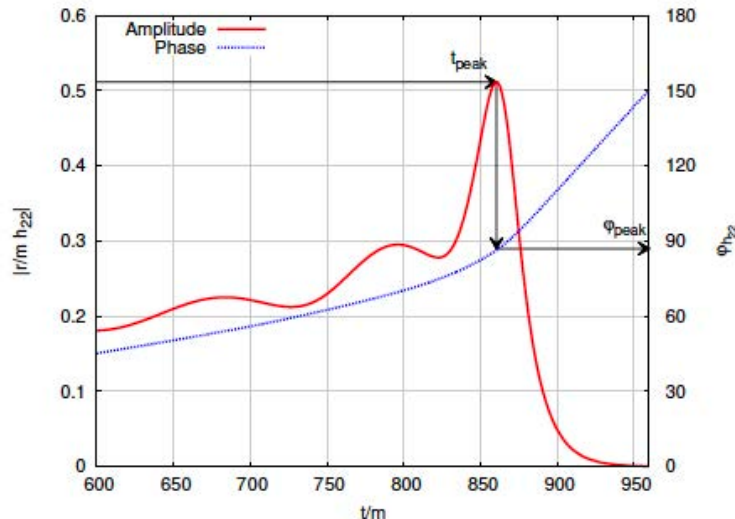
# Analysis

$$\frac{M_{\text{rem}}}{m} = (4\eta)^2 \left\{ M_0 + K_{2d} \delta m^2 + K_{4f} \delta m^4 \right\} + \left[ 1 + \eta(\tilde{E}_{\text{ISCO}} + 11) \right] \delta m^6, \quad (1)$$

where  $\delta m = (m_1 - m_2)/m$  and  $m = (m_1 + m_2)$  and  $4\eta = 1 - \delta m^2$ .

$$\alpha_{\text{rem}} = \frac{S_{\text{rem}}}{M_{\text{rem}}^2} = (4\eta)^2 \left\{ L_0 + L_{2d} \delta m^2 + L_{4f} \delta m^4 \right\} + \eta \tilde{J}_{\text{ISCO}} \delta m^6. \quad (2)$$

$$v_m = \eta^2 \delta m (A + B \delta m^2 + C \delta m^4). \quad (3)$$



$$h_{\text{peak}} = (4\eta)^2 \left\{ H_0 + H_{2d} \delta m^2 + H_{4f} \delta m^4 \right\} + \eta \tilde{H}_p \delta m^6, \quad (4)$$

where  $\tilde{H}_p(\alpha_{\text{rem}})$  is the particle limit, taking the value  $H_p(0) = 1.4552857$  in the nonspinning limit [18].

$$\mathcal{L}_{\text{peak}} = (4\eta)^2 \left\{ N_0 + N_{2d} \delta m^2 + N_{4f} \delta m^4 \right\}. \quad (5)$$

$$m\omega_{22}^{\text{peak}} = (4\eta) \left\{ W_0 + W_{2d} \delta m^2 + W_{4f} \delta m^4 \right\} + \tilde{\Omega}_p \delta m^6, \quad (6)$$

where  $\tilde{\Omega}_p(\alpha_{\text{rem}})$  is the particle limit, taking the value  $\tilde{\Omega}_p(0) = 0.279525$  in the nonspinning limit [18].

[18] A. Bohé *et al.*, Phys. Rev. D **95**, 044028 (2017), arXiv:1611.03703 [gr-qc].



# Analysis

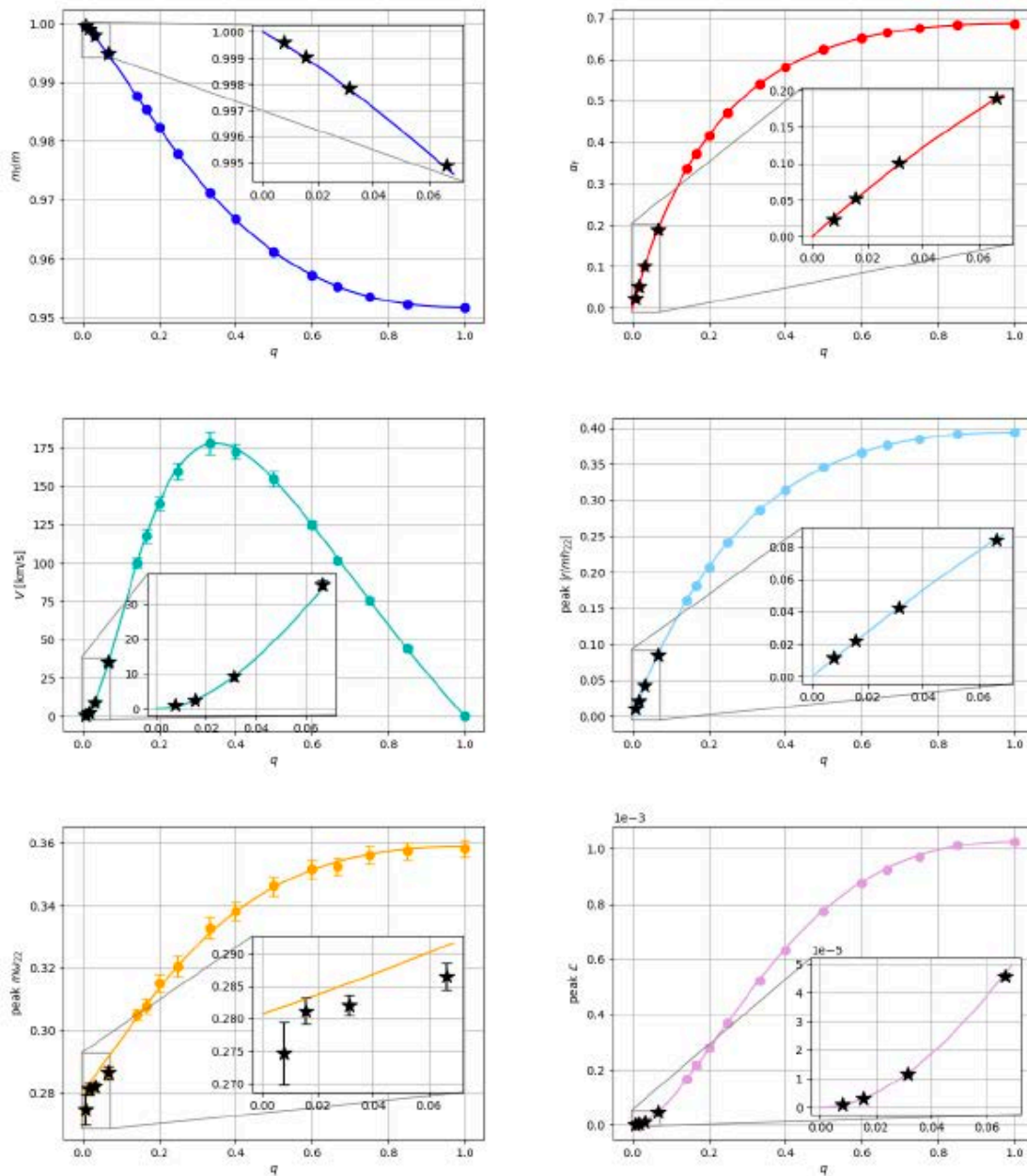


FIG. 3. Final mass, spin, recoil velocity, peak amplitude, frequency, and luminosity. Predicted vs. current results for the  $q = 1/15, 1/32, 1/64, 1/128$  simulations. Each panel contains the prediction from the original fits in Ref. [13] (solid line), data used to determine the original fits (filled circles), and the data for the current results (stars). An inset in each panel zooms in on the new simulations. Again, we stress no fitting to the new data is performed in this plot.

[13] J. Healy, C. O. Lousto, and Y. Zlochower, Phys. Rev. D **96**, 024031 (2017), arXiv:1705.07034 [gr-qc].

# Conclusions

- We have passed all first accuracy tests up to  $q=1/128$
- Assessed errors  $\sim 2\%$
- Adding spin to large black hole with same grid is straightforward
- Can still use speed ups for massive productions for applications to
  - 3G GW detectors
  - LISA for calibration of perturbative techniques

# Appendix: Refitted coefficients

TABLE III. Fitting coefficients of the phenomenological formulas (1)-(6)

$M_0$	$K_{2d}$	$K_{4f}$
$0.95165 \pm 0.00002$	$1.99604 \pm 0.00029$	$2.97993 \pm 0.00066$
$L_0$	$L_{2d}$	$L_{4f}$
$0.68692 \pm 0.00065$	$0.79638 \pm 0.01086$	$0.96823 \pm 0.02473$
$A$	$B$	$C$
$-8803.17 \pm 104.60$	$-5045.58 \pm 816.10$	$1752.17 \pm 1329.00$
$N_0 \times 10^3$	$N_{2d} \times 10^4$	$N_{4f} \times 10^4$
$(1.0213 \pm 0.0004)$	$(-4.1368 \pm 0.0652)$	$(2.46408 \pm 0.1485)$
$W_0$	$W_{2d}$	$W_{4f}$
$0.35737 \pm 0.00097$	$0.26529 \pm 0.01096$	$0.22752 \pm 0.01914$
$H_0$	$H_{2d}$	$H_{4f}$
$0.39357 \pm 0.00015$	$0.34439 \pm 0.00256$	$0.33782 \pm 0.00584$