Endpoint of the precessional instability in aligned-spin binary black holes

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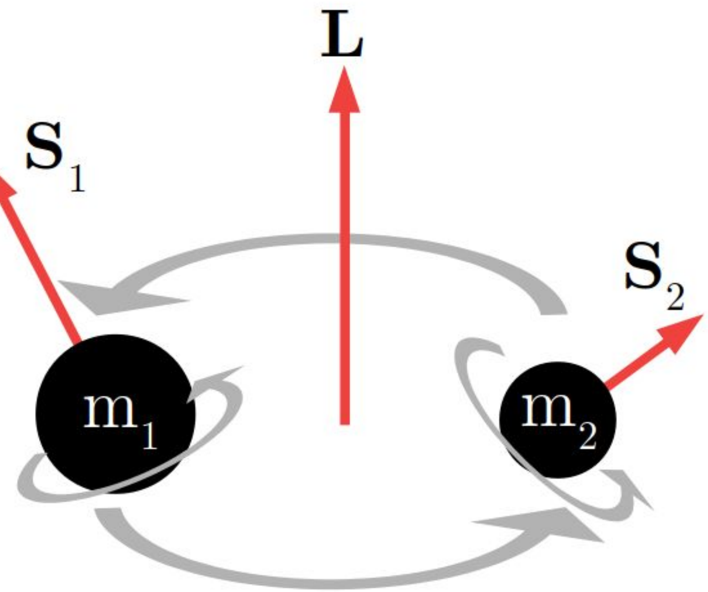
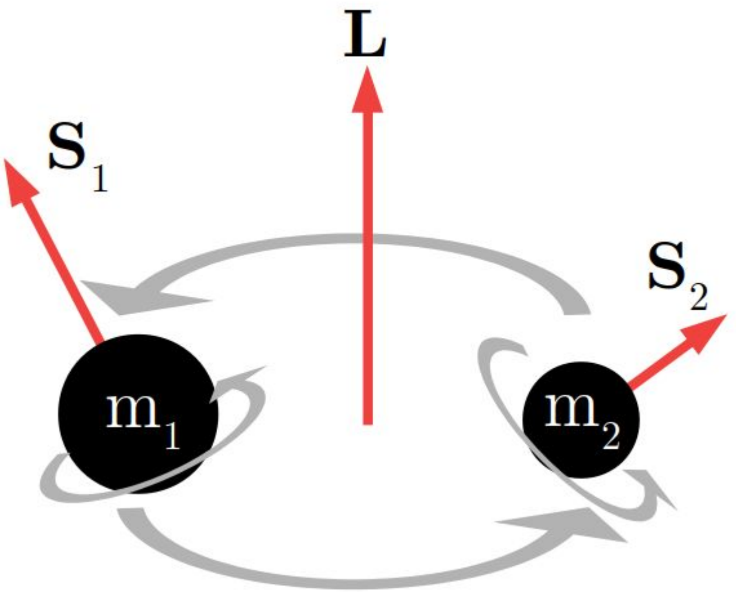
Generic binary black holes

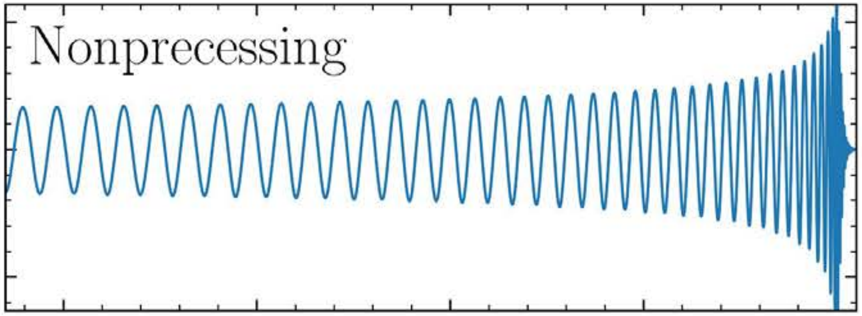
- Geometric units: $G = c = 1$
- Mass ratio: $q = \dfrac{m_2}{m_1} < 1$
- Total mass: $M = m_1 + m_2$
- Orbital separation: r
- Dimensionless spins: $0 \leq \chi_1 \, , \, \chi_2 \leq 1$

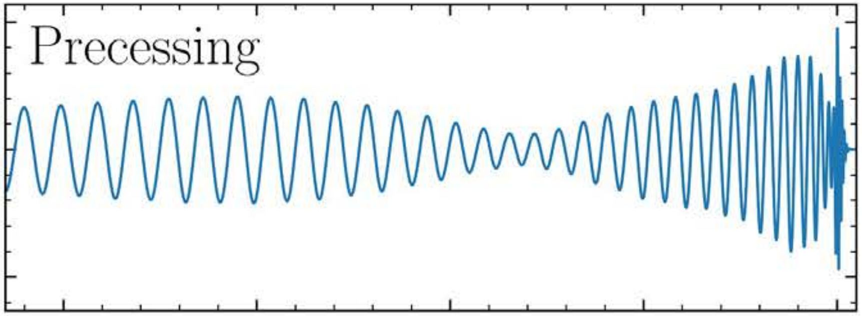


Spin precession

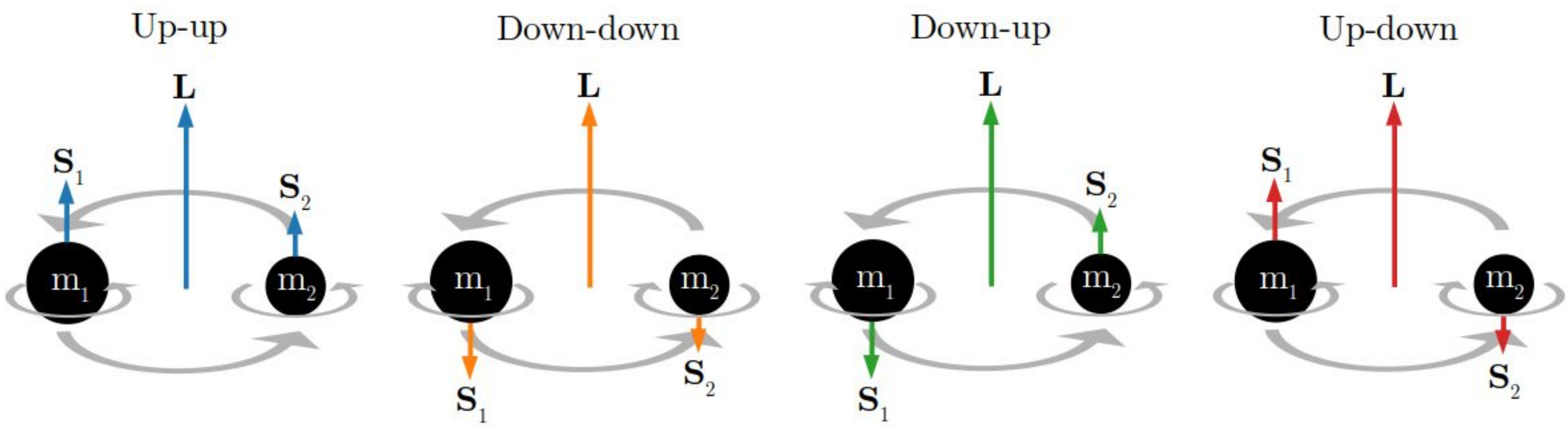
- In generic binaries the angular momenta **precess**
- Aligned-spin configurations are **non-precessing**







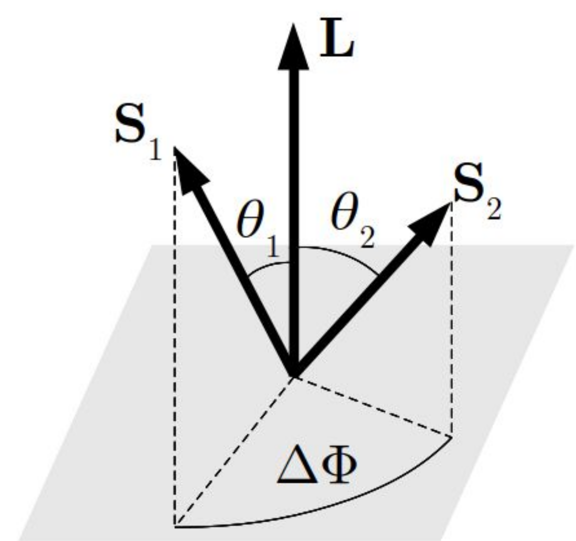
Do spin-aligned binaries stay aligned?



- Test configurations

- Astrophysics:
  - Field formation
  - AGN

[Graham+ 20](#) - BBH merger counterpart?!



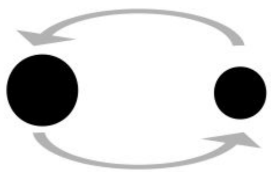
Multi-timescale analysis

- Orbit timescale $t_{\rm orb} \sim r^{3/2}$
- Precession timescale $t_{\rm pre} \sim r^{5/2}$
- Radiation reaction timescale $t_{\rm RR} \sim r^4$

In the post-Newtonian (large separation) regime:

$$r \gg M \implies \boxed{t_{\rm orb} \ll t_{\rm pre} \ll t_{\rm RR}}$$

Orbit << precession



$$t_{\rm orb} \ll t_{\rm pre}$$

- 2PN orbit-averaged evolution equations:

$$\frac{d\mathbf{S}_i}{dt} = \mathbf{\Omega}_i \times \mathbf{S}_i\,, \qquad \frac{d\mathbf{L}}{dt} = \mathbf{\Omega}_L \times \mathbf{L} + \frac{dL}{dt}\hat{\mathbf{L}}$$

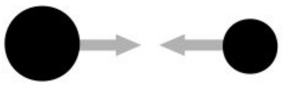
- Conserved quantity - effective spin:

$$\xi = \frac{1}{M^2}\left[(1+q)\mathbf{S}_1 + \left(1+\frac{1}{q}\right)\mathbf{S}_2\right]\cdot\hat{\mathbf{L}}$$

Darmour 01

Racine 08

Precession << inspiral

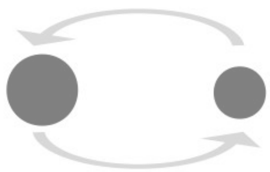


$$t_{\mathrm{pre}} \ll t_{\mathrm{RR}}$$

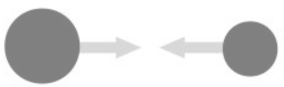
$$L = |\mathbf{L}| \qquad J = |\mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2|$$

$$M\,,q\,,\chi_1\,,\chi_2\,,\xi\,,J\,,L$$

Motion on the precession timescale



Orbit << **Precession** << Inspiral



Kesden+ 15, Gerosa+ 15

- Motion parametrized by a single time-varying quantity:

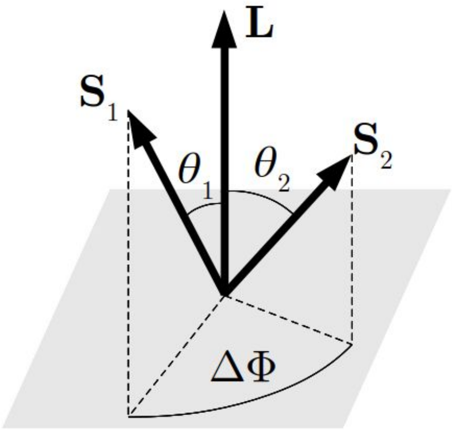
$$S = |\mathbf{S}_1 + \mathbf{S}_2|$$

$$\frac{dS^2}{dt} = \sqrt{-A^2(S^2 - S_+^2)(S^2 - S_-^2)(S^2 - S_3^2)}$$

$$S^2 = S_+^2 - (S_+^2 - S_-^2)\,\mathrm{sn}^2(\psi, m)$$

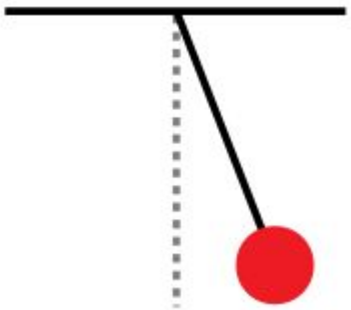
Chatziioannou+ 17

Perturbing the aligned configurations



$$\alpha_i = \cos\theta_{i*} = \pm 1$$

$$S_* = |\alpha_1 S_1 + \alpha_2 S_2|$$



- Perturbations to aligned-spin configurations evolve as a **harmonic oscillator**:

Lousto+ 16, Mould+ 20

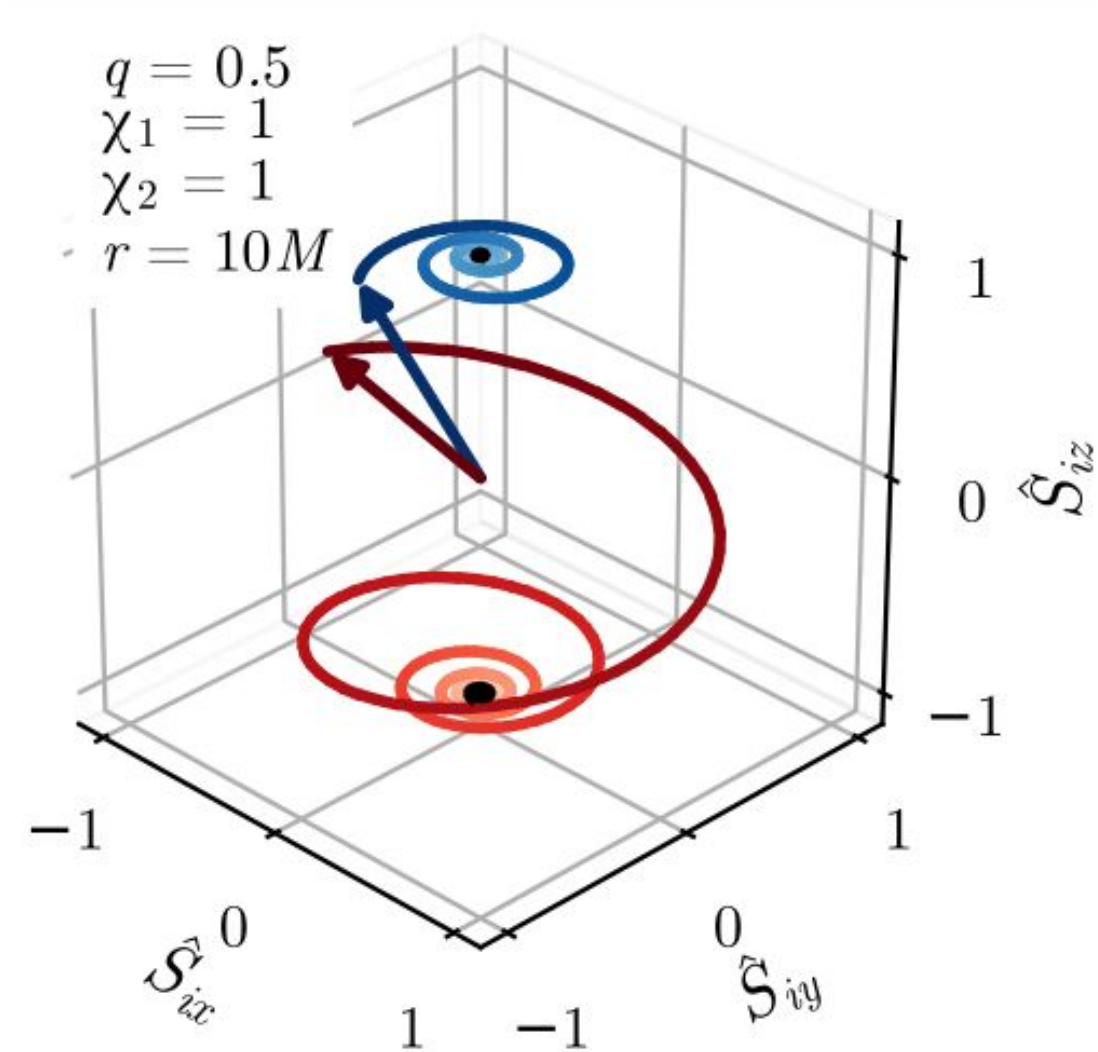
$$\boxed{\frac{d^2}{dt^2}(S^2 - S_*^2) + \omega^2 (S^2 - S_*^2) \simeq 0}$$

Is the frequency complex?

$$\omega^2(L) = \left[L^2 - 2\frac{q\alpha_1 S_1 - \alpha_2 S_2}{1-q}L + \left(\frac{q\alpha_1 S_1 + \alpha_2 S_2}{1-q}\right)^2\right]$$
$$\times \left(L - \frac{q\alpha_1 S_1 + \alpha_2 S_2}{1+q}\right)^2 \left[\frac{3M^9 q^5 (1-q)}{2(1+q)^{11} L^7}\right]^2$$

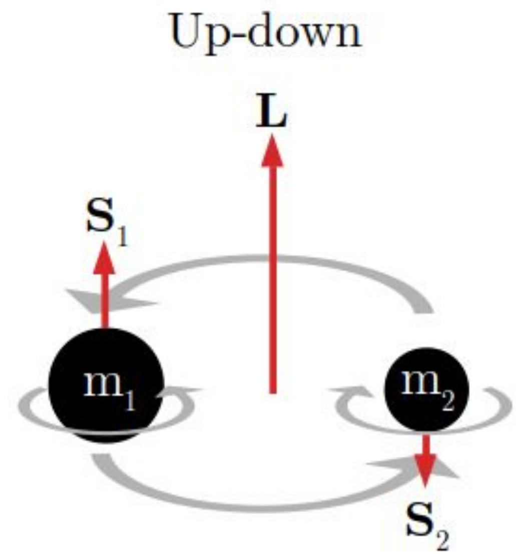
- **Stability?**
  - real frequency ⇒ small amplitude oscillations
  - imaginary frequency ⇒ **dynamical instability**

Up-down precessional instability



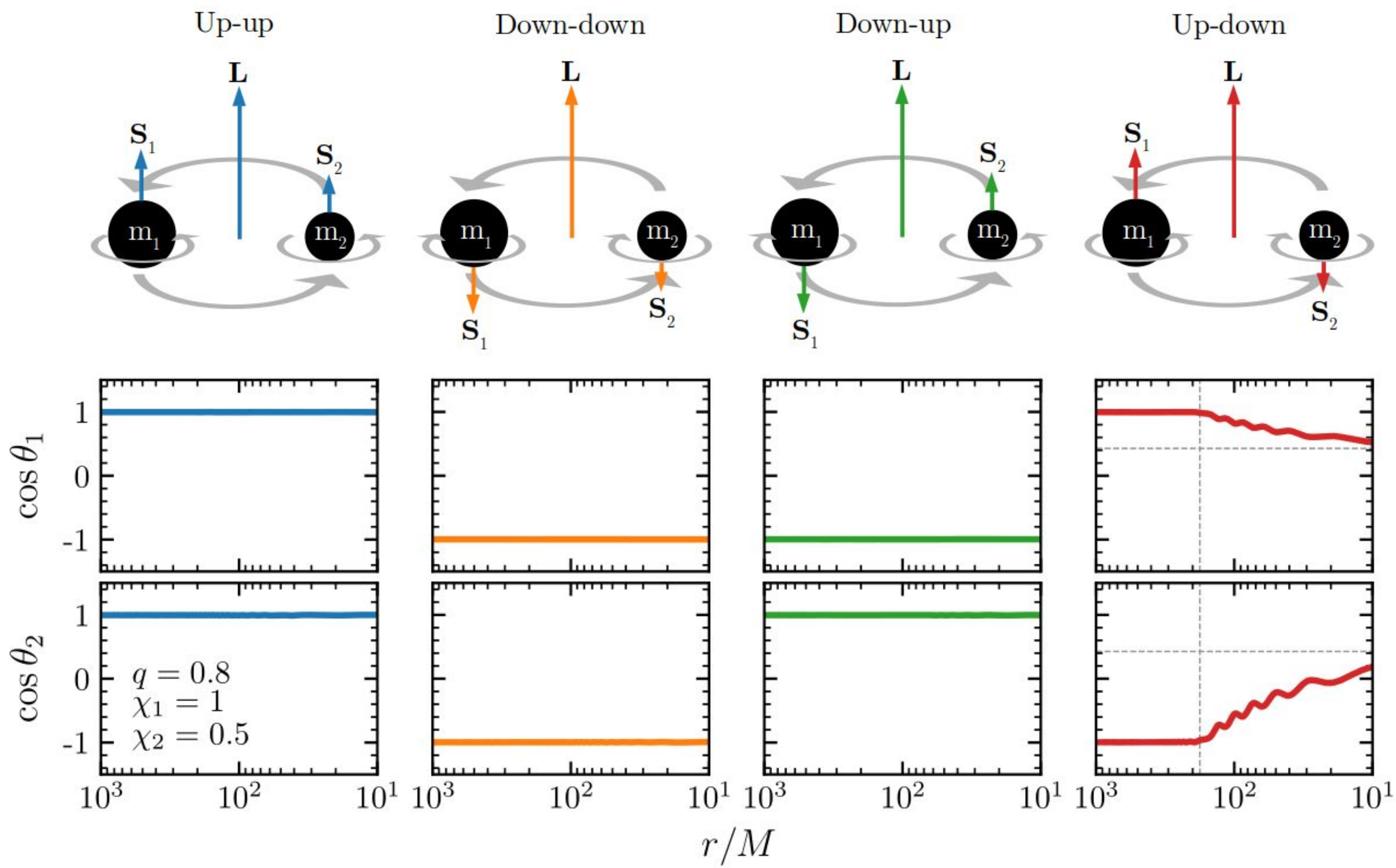
Only **up-down** is unstable

Gerosa+ 15, Lousto+ 16

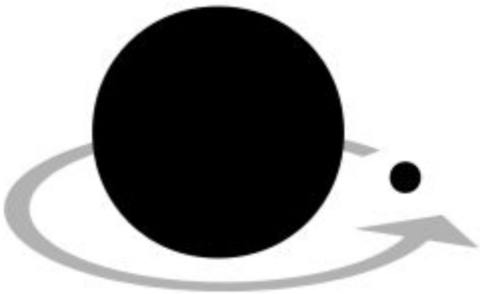


$$\frac{r_{\mathrm{ud}+}}{M} = \frac{\left(\sqrt{\chi_1} + \sqrt{q\chi_2}\right)^4}{(1-q)^2}$$

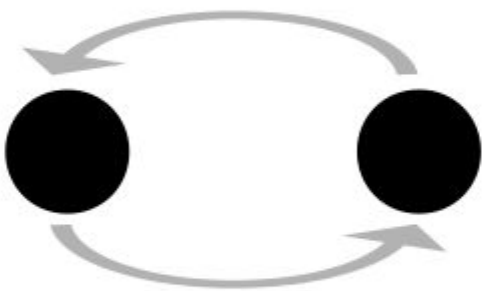
Only the up-down configuration is unstable



Mass ratio limits



$q \to 0$

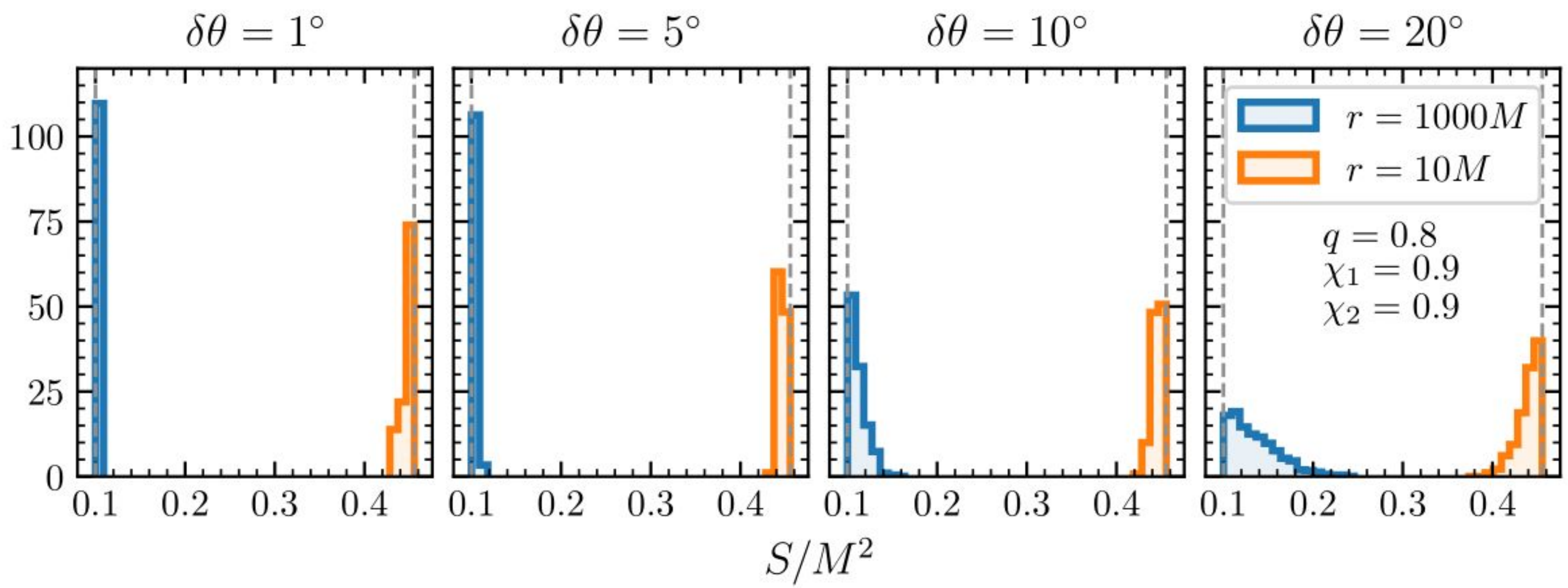


$q \to 1$

$$\frac{dS}{dt} = 0$$

$$\omega = 0$$

How does the instability evolve?



- Unstable up-down binaries do not disperse in the available parameter space, but **cluster to defined endpoints**

Spin-orbit resonances

- The three angular momenta remain coplanar:
  $\Delta\Phi = 0, \pi$ [Schnittman 04]

- Spin precession is a quasi-periodic motion:
  $S_- \leq S \leq S_+, \;\; \xi_-(S) \leq \xi \leq \xi_+(S)$

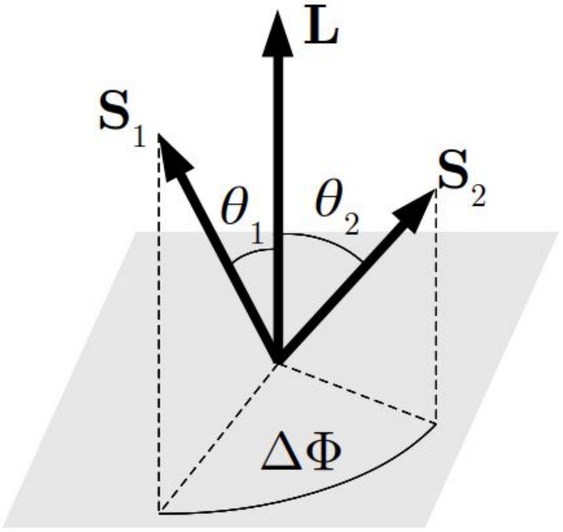
- Resonances are the limiting case: $S_- = S_+$

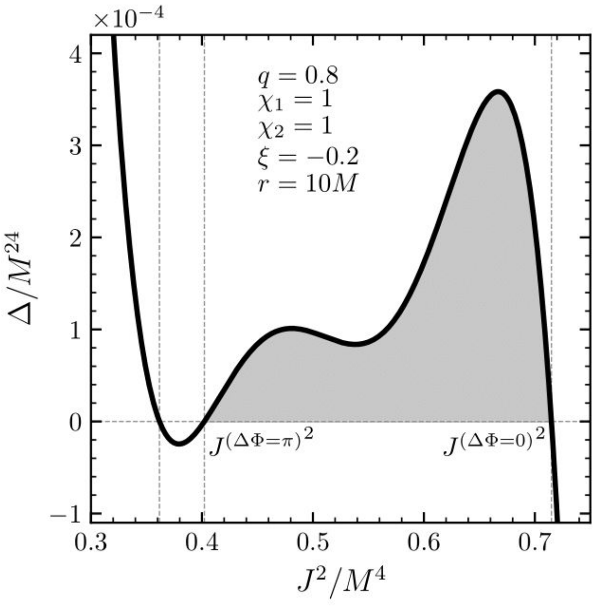
$$\left(\frac{dS^2}{dt}\right)^2 = -A^2(S^2 - S_+^2)(S^2 - S_-^2)(S^2 - S_3^2) = 0$$

- Physical when there are at least 2 real roots:

$$\Delta(J^2) = \delta_{10}J^{10} + \delta_8 J^8 + \delta_6 J^6 + \delta_4 J^4 + \delta_2 J^2 + \delta_0$$

- **Up-down is a resonance pre-instability**





Asymptotic resonances

$$r \to 0 \implies J \to S$$

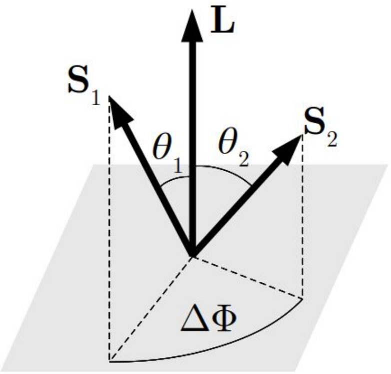
$$\lim_{L/M^2 \to 0} \frac{\Delta}{\delta_{10}} = \prod_{j=1}^{5} (S^2 - \lambda_j)$$

$$\max(\lambda_3, \lambda_4) \leq S^2 \leq \lambda_5$$

$$\cos\theta_i$$

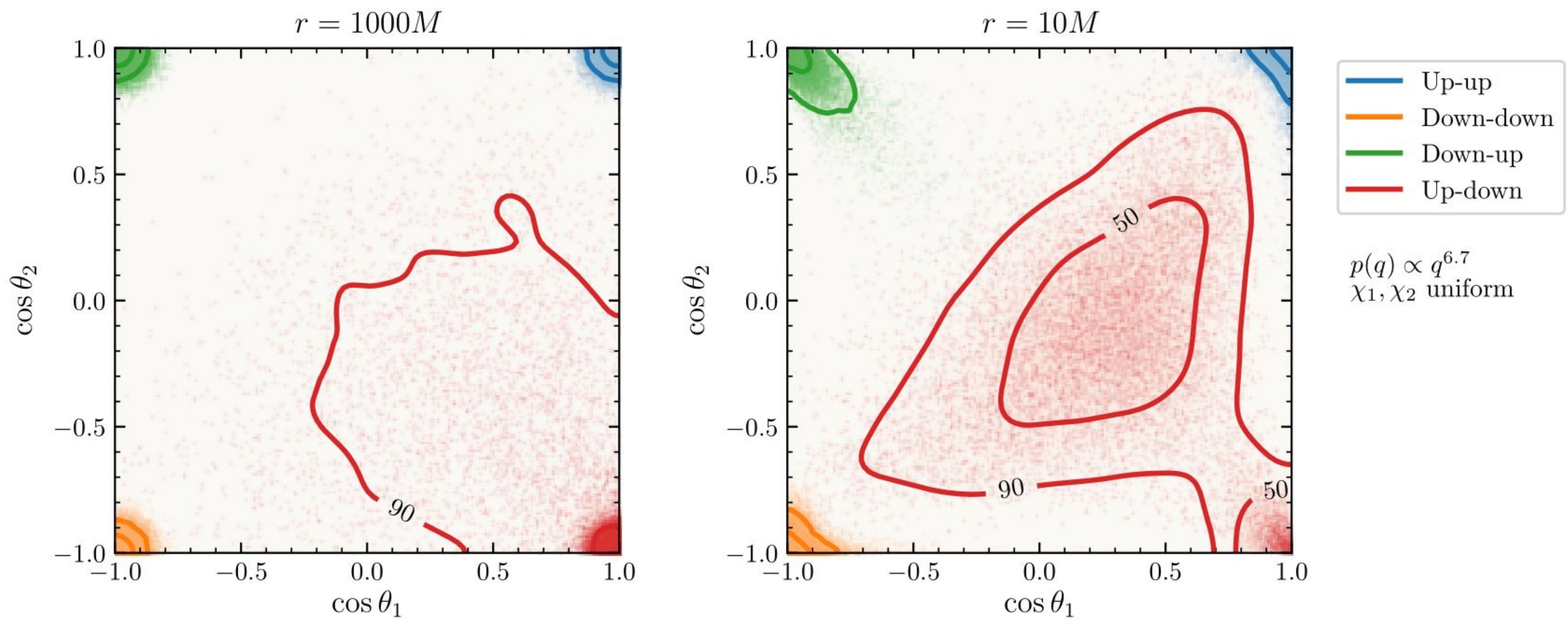
$$\Delta\Phi = \pi \qquad\qquad \Delta\Phi = 0$$

Endpoint of the up-down instability



$$\cos\theta_1 = \cos\theta_2 = \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2} \quad \text{and} \quad \Delta\Phi = 0 \quad (S = S_1 + S_2)$$

Mould+ 20



Summary

- Up-down binaries are **unstable to precession**
- Instability leads to **predictable endpoints**

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Also discuss prospects for detection

Ongoing work

- Is the precessional instability present in the **numerical relativity** regime?
- Do **all** resonances act as harmonic oscillators when perturbed?
- New implementation of the Python package *precession* (Gerosa+ 16)

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