Extreme mass ratio inspirals with spinning secondary*

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Why should we consider a spinning secondary in EMRIs?

- Secondary spin enters at 1-post-adiabatic order in the GW phase.

$$\Phi_{\rm GW} = \underbrace{q^{-1}\mathcal{C}^{(0)}}_{\text{adiabatic}} + \underbrace{q^{-1/2}\mathcal{C}^{(1/2)}}_{\text{resonances}} + \underbrace{q^{0}\mathcal{C}^{(1)}}_{\text{post-1-adiabatic}} + \mathcal{O}(q)$$

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- Measuring S might provide insights on the small object nature
  In units μM

$$\sigma := \frac{S}{\mu M} = \frac{S}{\mu^2} q = \chi q \qquad \text{with } q \ll \sigma \ll 1 \text{ in EMRIs}$$

  Some examples:
  $|\chi|_{\text{Earth}} \approx 140$, $|\chi|_{\text{fastest white dwarf}} \approx 10$ and $|\chi|_{\text{fastest pulsar}} \approx 0.3$
  For Kerr BHs $|\chi| \leq 1$ (Kerr bound)

Mathisson-Papapetrou-Dixon (MPD) equations of motion

- For a small enough secondary, its $T^{\mu\nu}$ can be expanded in multipoles

$$T^{\mu\nu} = \int \frac{\mathrm{d}\lambda}{\sqrt{-g}} \left\{ \delta\big(\vec{x} - \vec{z}(\lambda)\big) p^{(\mu} v^{\nu)} - \mu \nabla_\sigma \left[S^{\sigma(\mu} v^{\nu)} \delta\big(\vec{x} - \vec{z}(\lambda)\big) \right] \right\}$$

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- ${T^{\mu\nu}}_{;\mu} = 0$ gives the MPD equations for a spinning body

$$\nabla_{\vec{v}} p^\mu = -\frac{1}{2} {R^\mu}_{\nu\alpha\beta} v^\nu S^{\alpha\beta} \qquad \nabla_{\vec{v}} S^{\mu\nu} = 2 p^{[\mu} v^{\nu]} \qquad v^\mu = \frac{\mathrm{d}z^\mu}{\mathrm{d}\lambda}$$

where $S^{\mu\nu} = -S^{\nu\mu} \qquad \mu^2 := -p^\nu p_\nu \qquad 2S := S^{\mu\nu} S_{\mu\nu}$

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- For a Kerr space-time, the first integrals E and J_z are:

$$E = -p_t + \frac{1}{2} g_{t\mu,\nu} S^{\mu\nu} \qquad J_z = p_\phi - \frac{1}{2} g_{\phi\mu,\nu} S^{\mu\nu}$$

Circular equatorial orbits (CEO) with spin (anti)aligned

- Primary spin a aligned to positive z-axis
- Given $\epsilon_{\mu\nu\alpha\beta}$ the Levi-Civita tensor, the *spin vector* is defined as

$$S_\mu \equiv \frac{1}{2\mu}\epsilon_{\mu\nu\alpha\beta}p^\nu S^{\alpha\beta}$$

  Spin vector parallel to a when $S^\mu = \delta^\mu_\theta S^\theta$

  S^μ is aligned ($S > 0$) or anti-aligned ($S < 0$) to the primary spin.
- For the ISCO we used the effective potential [1]

$$V_\sigma(r) = \frac{1}{r^4}(\alpha_\sigma E^2 - 2\beta_\sigma E + \gamma_\sigma)$$

  Only *stable* $\left(\frac{\mathrm{d}^2 V_\sigma}{\mathrm{d}r^2}\big|_{r=r_0} < 0\right)$, *prograde orbits* were considered .

[1] Jefremov et al, 2015

Radiation reaction effects and balance laws

- At first order in the secondary spin [2]

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{GW}} = -\frac{\mathrm{d}E}{\mathrm{d}t} \qquad \left(\frac{\mathrm{d}J_z}{\mathrm{d}t}\right)_{\mathrm{GW}} = -\frac{\mathrm{d}J_z}{\mathrm{d}t}$$

- There are no equations for $S^{\mu\nu}$ radiation reaction evolution based on asymptotic fluxes
- We neglected radiation reaction on $S^{\mu\nu}$ and assumed $S = cost$
- Therefore, for a spin (anti)aligned CEO

$$\Omega = \frac{\partial E}{\partial r}\left(\frac{\partial J_z}{\partial r}\right)^{-1} \quad \text{or} \quad \frac{\mathrm{d}E}{\mathrm{d}t} = \Omega \frac{\mathrm{d}J_z}{\mathrm{d}t}$$

- Still to prove that "circular remains circular" for a spinning body [3]

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[2] Akcay et al, 2019

[3] Kennefick, 1998

A few words about the numerical routines

To calculate the Teukolsky fluxes, we employed the Mathematica packages of the Black Hole Perturbation Toolkit (BHPToolkit)
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- Spin-weighted spheroidal harmonics calculated with the package "SWSH"
- Homogeneous radial solutions calculated with two routines:
  1) MST method of the package "Teukolsky"
     Faster and more accurate at low frequency
  2) Sasaki-Nakamura (SN) method, using new boundary conditions
     Faster and more accurate at high frequency

Source term for the radial Teukoulsky equation

- Teukolsky source term for radial equation reads [4]

$$\mathcal{T}_{\ell m\omega} = 4\int \mathrm{d}t\mathrm{d}\theta \sin\theta \mathrm{d}\phi \frac{\left(B_2'[T^{\mu\nu}] + B_2'^{*}\left[\,T^{\mu\nu}\right]\right)}{\bar{\rho}\rho^5} {}_{-2}S^{a\omega}_{\ell m}(\theta) e^{-i(m\phi+\omega t)}$$

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- The amplitudes $Z^{H,\infty}_{\ell m\omega}$ become

$$Z^{H,\infty}_{\ell m\hat{\omega}} = C^{H,\infty}_{\ell m\omega} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i(\omega t - m\phi(t))} \sum_{i=0}^{3} A_i \frac{\mathrm{d}^i}{\mathrm{d}r^i} R^{\mathrm{in,up}}_{\ell m\omega}$$

with $A_i = A_i(\theta(t), r(t), S^{\mu\nu}, p^{\mu}, v^{\mu})$ and $A_3 = 0$ for $\sigma = 0$

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- For a spin aligned CEO

$$Z^{H,\infty}_{\ell m\omega} = \delta(\omega - m\Omega) 2\pi C^{H,\infty}_{\ell m\omega} \sum_{i=0}^{3} A_i(r_0, \pi/2, S) \frac{\mathrm{d}^i}{\mathrm{d}r^i} R^{\mathrm{in,up}}_{\ell m\omega}$$

[4] Tanaka et al, 1996

Orbital evolution and GW phase

- Prototype binary: $q = 3 \times 10^{-5}$ (with $\mu = 30\mathrm{M}_\odot, M = 10^6\mathrm{M}_\odot$). All fluxes were calculated up to $l = 20$.
- Orbital evolution given by

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -q^2 \mathcal{F}(r) \left(\frac{\mathrm{d}E}{\mathrm{d}r} \right)^{-1} \qquad \frac{\mathrm{d}\Phi}{\mathrm{d}t} = \Omega(r(t))$$

At $t = 0$, $\Phi(0) = 0$ while $r(0)$ such that $\Omega(0)$ is the same for all σ.

Crosscheck analysis:

- for $q = 1$, fluxes $\mathcal{F}$ agrees with Harms et al, 2016 (time domain)
- fluxes for $\sigma = 0$ agree with BHPToolkit data
- linear spin corrections of the fluxes in Schwarzschild agree with Akcay et al, 2019

Linear spin corrections to the fluxes

Linear spin corrections $\delta\mathcal{F}^\sigma$ were obtained by interpolating

$$\mathcal{F}(r,\Omega) - \mathcal{F}^0(r,\Omega) = \sigma\delta\mathcal{F}^\sigma(r,\Omega) + \mathcal{O}(\sigma^2)$$

$\mathcal{F}^0$ are the fluxes for $\sigma = 0$.

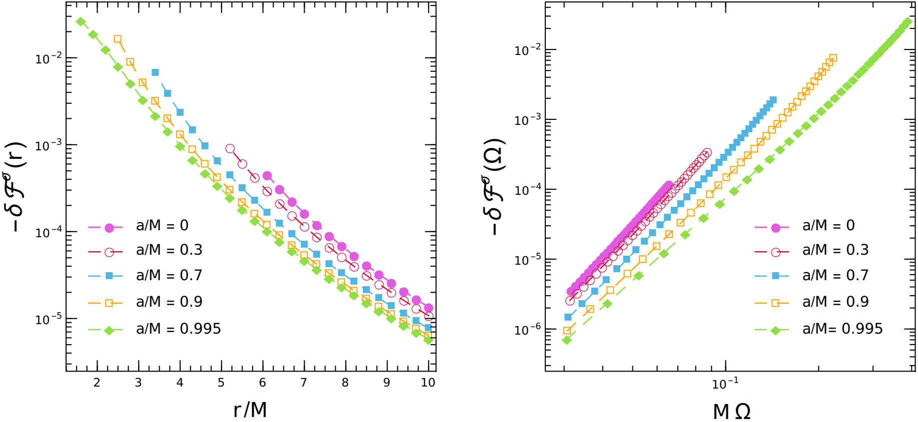


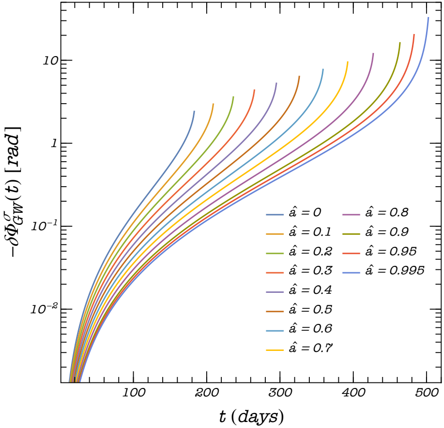
Figure: fixed r   **Figure:** fixed Ω

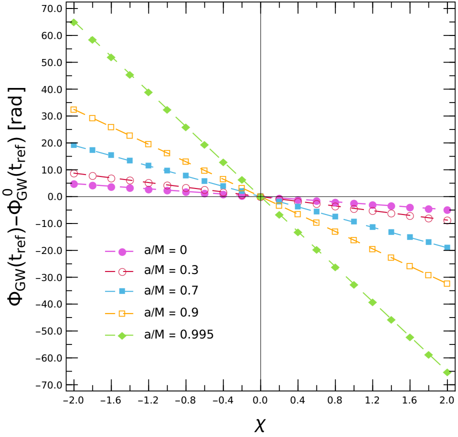
Linear spin corrections to the GW phase

GW phase is $\Phi_{\mathrm{GW}} = 2\Phi$ for dominant mode.

$$\Phi_{\mathrm{GW}}(t) - \Phi^0_{\mathrm{GW}}(t) = (\sigma/q)\delta\Phi^\sigma_{\mathrm{GW}}(t) + \mathcal{O}(\sigma^2/q)$$

Φ^0_{GW} is the GW phase for $\sigma = 0$.

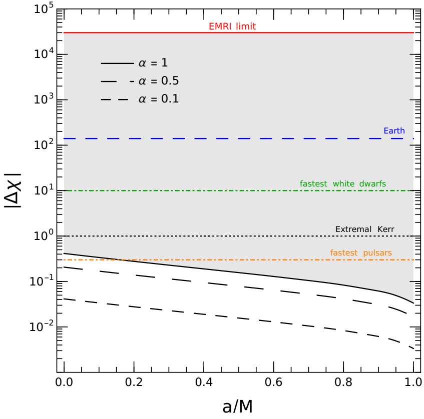




Spin resolution and Kerr bound test

Minimum $\Delta\chi = \chi_B - \chi_A$ leading to a phase difference of $\alpha\,\mathrm{rad}$ is:

$$|\Delta\chi| > \frac{\alpha}{|\delta\Phi^{\sigma}_{\mathrm{GW}}(a, t_{\mathrm{ref}})|}$$



- With $\alpha = 1\,\mathrm{rad}$, for

$$a/M = 0.7 \qquad |\Delta\chi| > 0.1$$
$$a/M = 0.9 \qquad |\Delta\chi| > 0.05$$

- For $\chi \approx a/M \approx 0.7$, $|\Delta\chi/\chi| \sim 15\%$
- Spin χ of a secondary BH might be measured with great accuracy

Conclusions and future work

Summary:

- Calculated the GW fluxes for a spinning secondary in spin-aligned CEO orbits in Kerr
- Computed GW phase for spinning objects during adiabatic inspiral
- LISA might discriminate between a fast spinning BH or a slowly rotating NS secondary

Future work:

- More sophisticated statistical analysis (see Huerta and Gair, 2011)
- Include conservative first order in q self force and spin evolution into the dynamics
- Consider noncircular orbits and spin precession
- Include the secondary quadrupole moment?

Final notes and acknowledgments

- code and data are available at the GitHub repository https://web.uniroma1.it/gmunu/resources
- this work was made possible thanks to the Mathematica packages of the BHPToolkit https://bhptoolkit.org/
- all tensors computation were performed with the Mathematica package "xAct" www.xact.es
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Thank you for you attention!