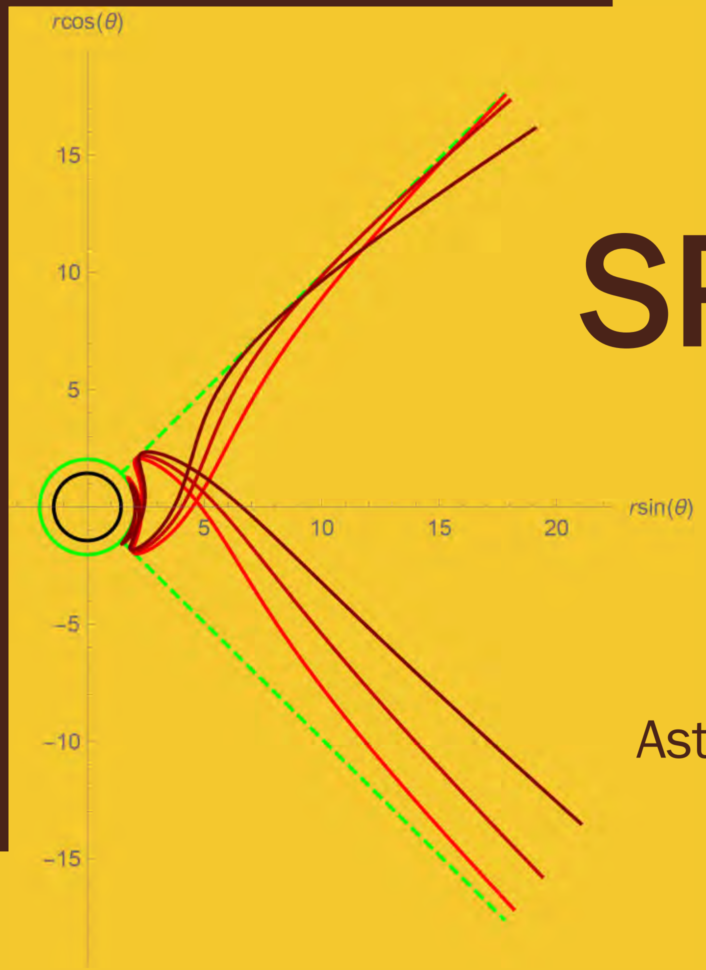
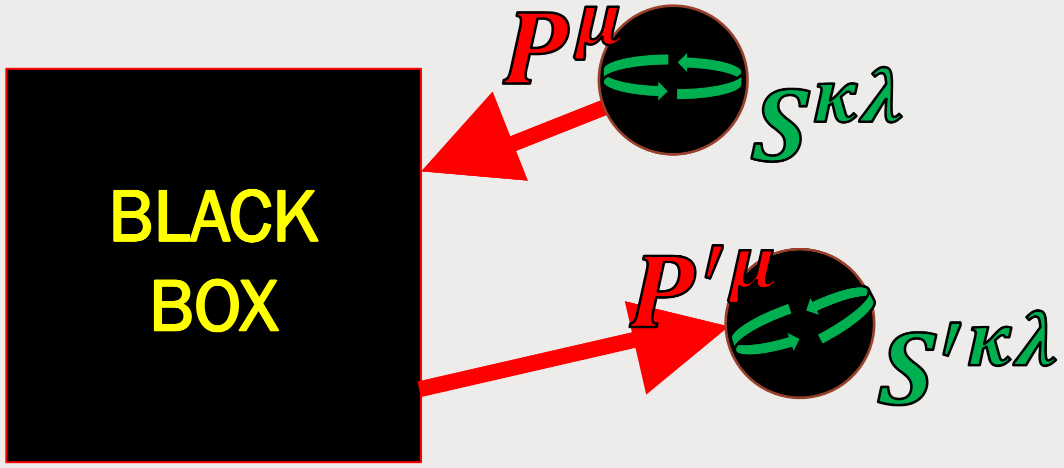


SCATTERING OF SPINNING PARTICLES BY BLACK HOLES

Vojtěch.Witzany @ ASU.CAS.CZ
Astronomical Institute of the Czech Academy of Sciences, Prague,

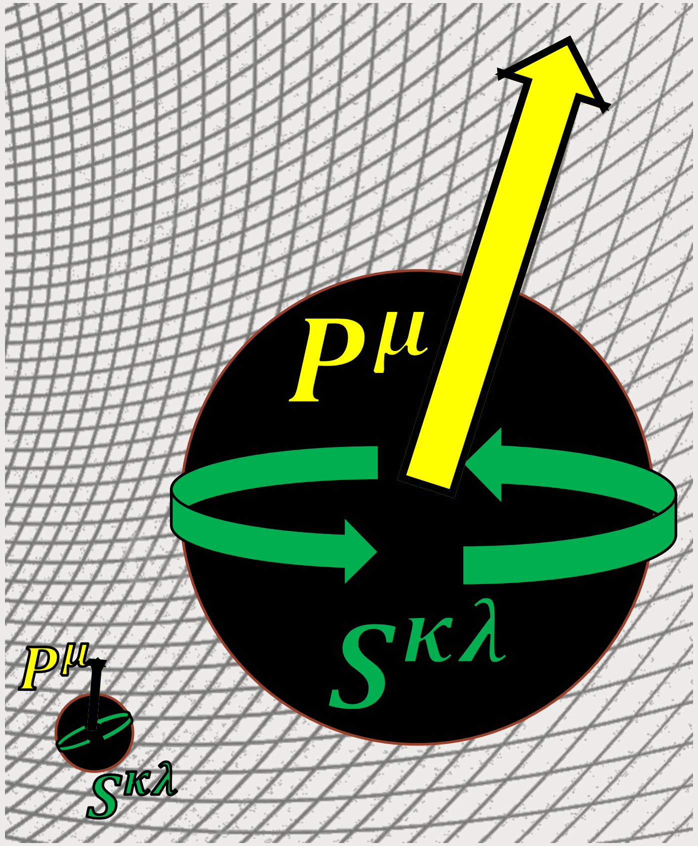
23rd Capra, online, 26.6. 2020

Outline of the talk



- Demonstrate the Maupertuis principle on geodesics
- Apply it to spin-perturbed motion
- Some magic with contours, Partie Finie regularization
- Profit

Motivation



- Secondary spin shows up at first post-adiabatic order of the large-mass-ratio expansion (for rapidly spinning compact bodies)
- In the post-Newtonian expansion it shows up at 1 PN or 1.5PN order, depending on whether body is compact (able to rapidly spin without falling apart) or not
- Spin brings about one new degree of freedom, coupled to the orbit via the „spin-curvature coupling“
- The 1PA effects can be computed considering only spinning test particles
- People seem to care about post-Minkowski scattering lately, they should have a strong-field reference

Previous work

- Ham.-Jac. Geodesics in Kerr by Carter (1968), scattering e.g. Chandrasekhar (1985), see also bound orbits by Fujita & Hikida (2009)
- The equations of motion (Mathisson 1937, Papapetrou 1951, Dixon 1964,…)
- Hamiltonian formalism for spin (Barausse+ 2009, Vines+ 2016, Witzany+2019)
- Hamilton-Jacobi for spin in Kerr, fundamental frequencies (Witzany 2019)
- Aligned spin equatorial Kerr by Bini+ (2017)
- *Here: generic spin, generic inclination solution!*

GEODESIC SCATTERING

Scattering from Maupertuis

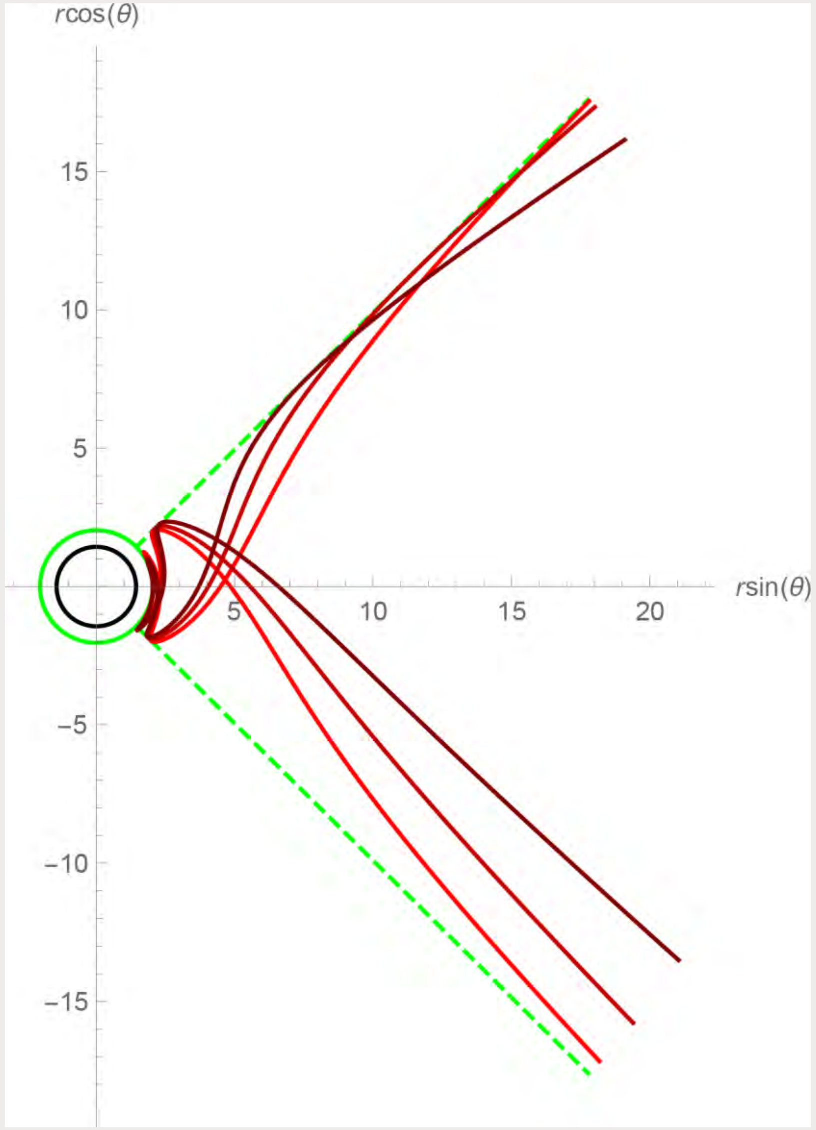
- Take the action integrated between the in and out points

$$W(r,\vartheta,\varphi,t;Q,L,E) = -E(t_o - t_i) + L(\varphi_o - \varphi_i) + \int_{\vartheta_i}^{\vartheta_o} \pm\sqrt{\Theta(\vartheta;Q,L,E)}\mathrm{d}\vartheta + \int_{\infty}^{\infty} \pm\sqrt{R(r;Q,L,E)}\mathrm{d}r$$

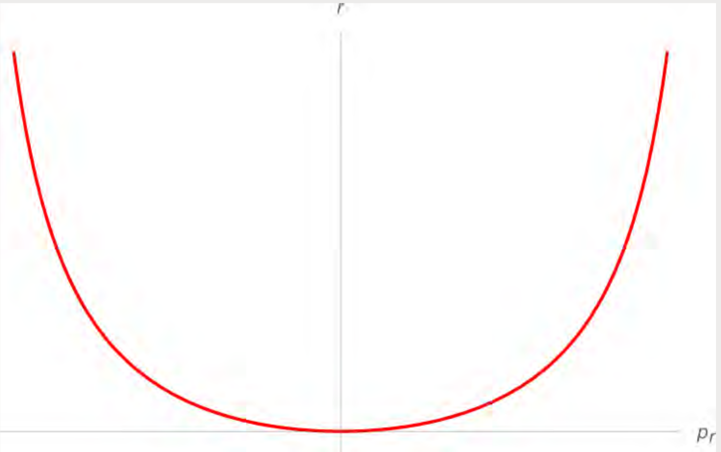
- Maupertuis principle: variation of on-shell action wrt separation constants is zero → determines endpoints!
- Variation wrt Q: „Carter-Mino time 'spent' on r should match that 'spent' on ϑ"

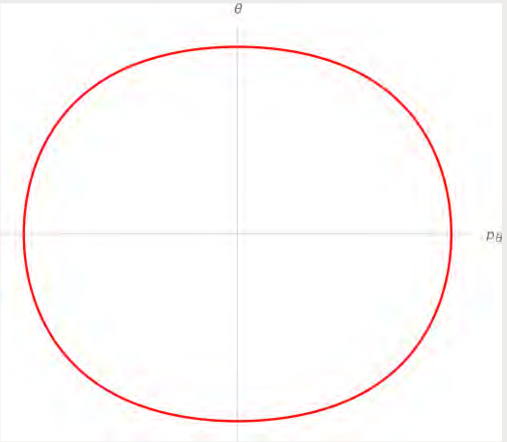
$$\int_{\vartheta_i}^{\vartheta^o} \frac{\pm\mathrm{d}\vartheta}{\sqrt{\Theta(\vartheta;Q,L,E)}} = \int_{\infty}^{\infty} \frac{\pm\mathrm{d}r}{\sqrt{R(r;Q,L,E)}}$$

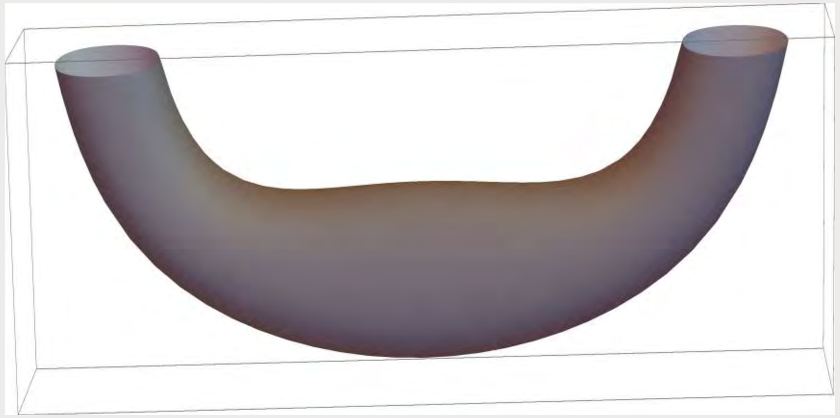
- Determines $\vartheta_o(\vartheta_i,Q,E,L)$ in closed form. Put in finite bounds, determines $\vartheta(r)$!

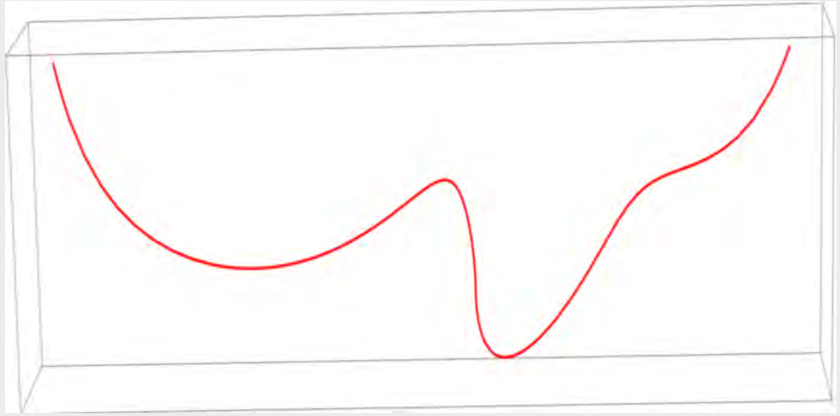


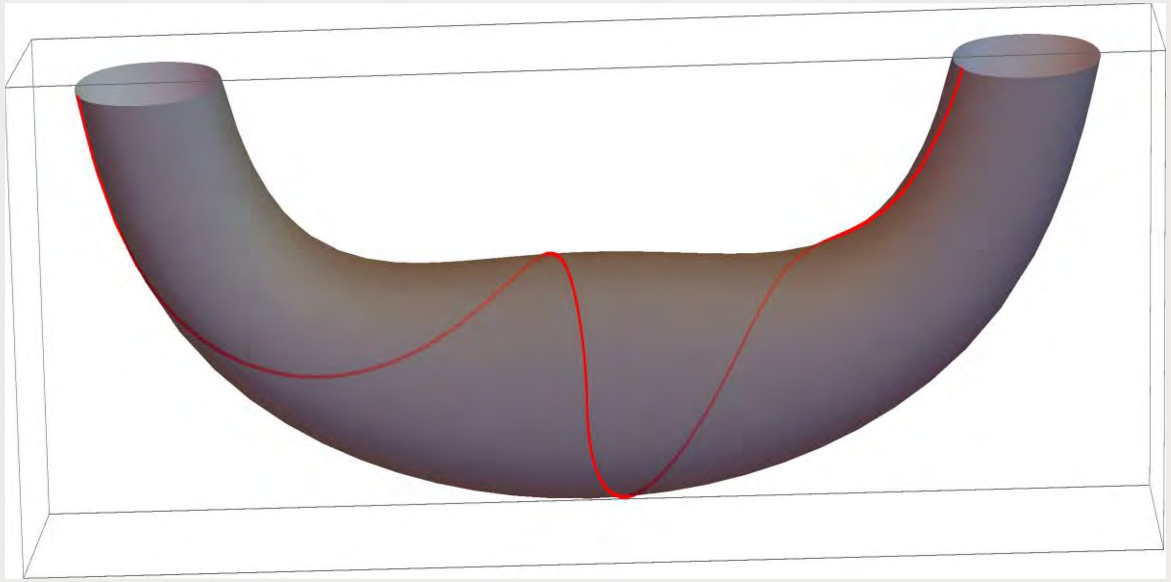
Contour magic











But any contour on torus with fixed ends and same winding is fine!

SPIN IT

Same, but different: Maupertuis

$$W(r,\vartheta,\varphi,t,\phi_s;K_{so},L_{so},E_{so},s_\parallel) = -E_{so}t + L_{so}\varphi + s_\parallel\phi_s$$
$$+\int_{\vartheta_i}^{\vartheta_o} \pm(\sqrt{w_\vartheta'^2(\vartheta;K_{so},\dots) - e_{0\vartheta}e^\kappa_{C;\vartheta}e_{D\kappa}\tilde{s}^{CD}} + e^\kappa_{C;\vartheta}e_{D\kappa}\tilde{s}^{CD}/2)\mathrm{d}\vartheta$$
$$+\int_\infty^\infty \pm(\sqrt{w_r'^2(r;K_{so},\dots) - e_{0r}e^\kappa_{C;r}e_{D\kappa}\tilde{s}^{CD}} + e^\kappa_{C;r}e_{D\kappa}\tilde{s}^{CD}/2)\mathrm{d}r$$

- Reminder: $s_\parallel = s^\mu l_\mu/|l| = s^\mu Y_{\mu\nu}u^\nu/\sqrt{K}$
- Same game, $\vartheta_o(\vartheta_i,K_{so},E_{so},L_{so})$ determined by $\frac{\partial W}{\partial K_{so}} = 0$, the rest also by their respective variations
- Connection terms not separable, but we can deform contour to split computation into 1D integrals
- Even $\vartheta(r)$ parametrization possible! (Phase error $O(\lambda s^2)$)

Intermezzo: *Partie finite*

- Expand integrals with perturbatively changing bounds (Damour+ 1980s)

$$\int_{\text{Roots of } f+\epsilon g} \frac{\mathrm{d}x}{\sqrt{f(x)+\epsilon g(x)}}$$
$$= \int_{\text{Roots of } f} \frac{\mathrm{d}x}{\sqrt{f(x)+\epsilon g(x)}} + \mathrm{Pf}\int_{\text{Roots of } f} \frac{\epsilon g(x)}{2f(x)^{3/2}}\mathrm{d}x + O(\epsilon^2)$$

- Pf „automatically selects the right root“. In practice change to a phase variable χ, singularity $\sim F_1/\chi^2$ and integral ends up as

$$\mathrm{Pf}\int_0^{\cdots} F(\chi)\mathrm{d}\chi = \int_\kappa^{\cdots} F(\chi)\mathrm{d}\chi + \frac{1}{\kappa} F_1 + O(\kappa^2)$$

- Numerically $\kappa \sim \epsilon \sim s$ yields results valid up to $O(s)$ so that is fine

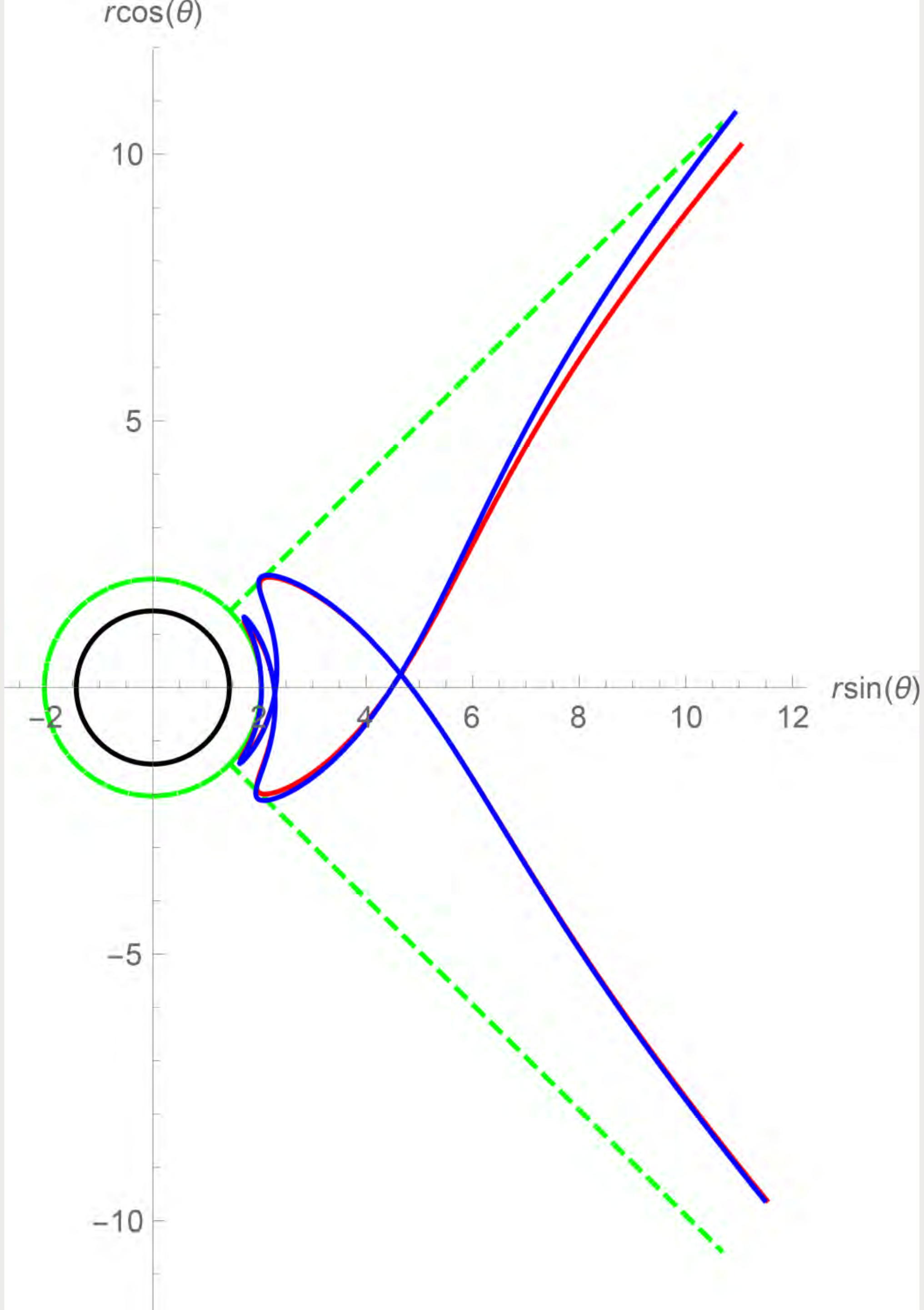
Sample result expression

$$\vartheta_{\mathrm{o}}^{\mathrm{sp}}\left(\vartheta_i, K_{so}, E_{so}, L_{so}, s_\parallel\right) = \vartheta_o^g(\vartheta_i, K_{so}, E_{so}, L_{so}) + \delta\vartheta_o^{sp}(\vartheta_i, K_{so}, E_{so}, L_{so}, s_\parallel)$$

$$\delta\vartheta_o^{sp} = \sqrt{\Theta\left(\vartheta_o^g\right)}\,\mathrm{Pf}\int_{\infty}^{\infty} \frac{-s_\parallel\sqrt{K_{so}}\,\frac{E_{so}\left(r^2+a^2\right)-aL_{so}}{r^2+K_{so}} - e_{0r}e_{C;r}^{\kappa}e_{D\kappa}\tilde{s}^{CD}/2}{\Delta R^{\frac{3}{2}}}\,\mathrm{d}r$$

$$+\sqrt{\Theta\left(\vartheta_o^g\right)}\,\mathrm{Pf}\int_{\vartheta_i}^{\vartheta_o^g} \frac{as_\parallel\sqrt{K_{so}}\,\frac{L_{so}-aE_{so}\sin^2\vartheta}{K_{so}-a^2\cos^2\vartheta} + e_{0\vartheta}e_{C;\vartheta}^{\kappa}e_{D\kappa}\tilde{s}^{CD}/2}{\Theta^{3/2}}\,\mathrm{d}\vartheta$$

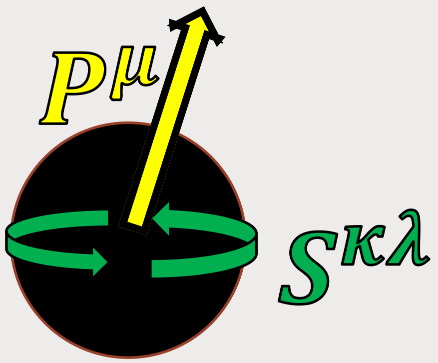
- Contour chosen so that integrals simple



Tentative Mathematica implementation

- Energy 2.26, azimuthal angular momentum 5.5 M, BH spin $0.9M$, „vertical angular momentum squared“ (Q) 28 M^2
- Secondary spin „aligned“ $s_{\|} = s$, magnitude 0.02 M
- Remember: finite Mino time, so trajectory globally faithful

Outlooks



- Quantitative validation needed, reexpressing in observables (asymptotic momenta, impact parameter,...)
- Better implementation
- PN/PM expansions to obtain simpler order-by-order formulas for scattering angle
- You tell me what this is good for
- Look out for preprint (soon?)