Three-body Dynamics in Extreme Mass Ratio Inspirals

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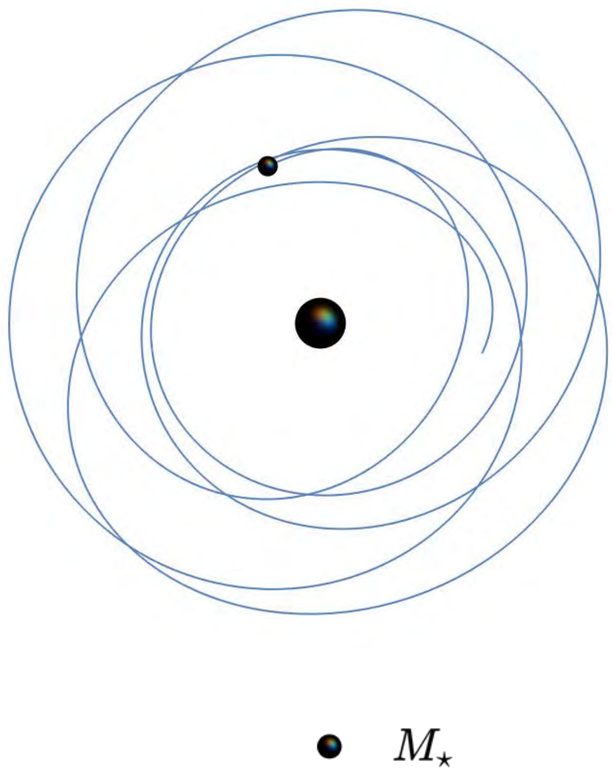
CAPRA Meeting 2020, June 22, 2020

Why EMRIs are interesting?

- Mass and spin of the supermassive black hole can be measured to high accuracy -useful for understanding spin growth in supermassive black holes.
- EMRI merger rate is related to stellar distribution near the supermassive black hole.
- Many orbital cycles: small deviation in Kerr metric may leave relative large footprint in the waveform.
- Environmental effects of EMRIs: AGN disks, close objects, etc.

Close stellar-mass objects to EMRIs



- Roughly $2{\times}10^4$ black holes are predicted to settle in the inner parsec of our galaxy centre [Miralda & Gould `00].

- A density cusp of X-ray binaries observed near Sgr A* [Hailey et al. `18].

How to produce close tidal perturbers?

- Mass segregation (+ dynamical friction) so that more massive black holes sink to the centre [Emami & Loeb `19]: 40 $M_\odot$ black holes at mean distance ~5AU from Sgr A*

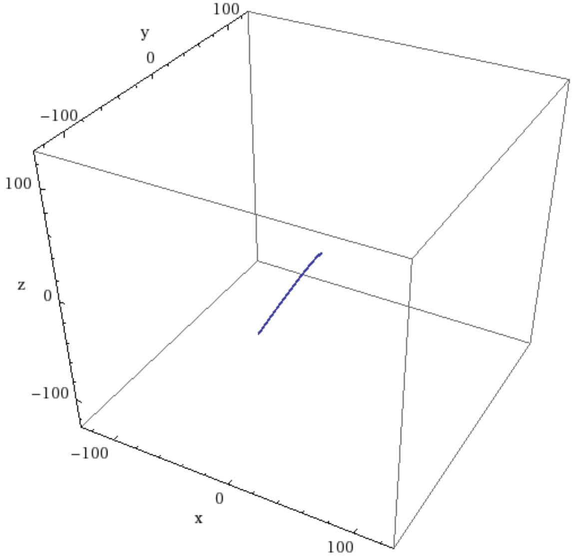
- Black holes trapped in AGN disks migrate towards the supermassive black hole (~O(10%) galaxies are active galaxies). Same scenario recently proposed to explain hierarchical mergers to produce ~100 $M_\odot$ black holes [Y. Yang et al. `19].

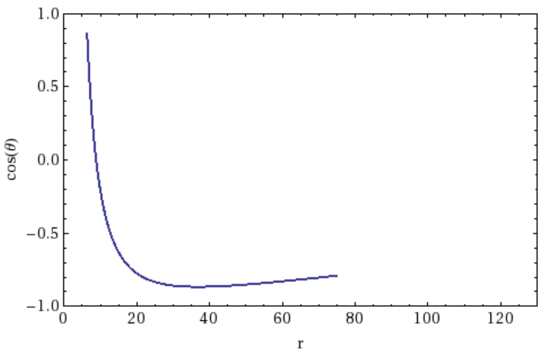
- Objects scattered to the vicinity of the supermassive black hole: usually with high eccentricity.

EMRI evolution

- Orbital timescale (minutes) << radiation reaction timescale (months-years)

- Basic strategy: two timescale expansion [Hinderer & Flanagan `08]. Zeroth order approximation: the trajectory is approximately a geodesic with separable motion.





$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J})$$
$$\frac{dJ_i}{d\tau} = 0$$
$$\mathbf{J} = \{\mu, E, Q, L_z\}$$

Credit: Rob Cole

EMRI evolution with radiation reaction

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \eta g^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathrm{J}) + \mathcal{O}(\eta^2)$$
$$\frac{dJ_i}{d\tau} = \eta G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2)$$

Adiabatic approximation:

$$\frac{dq_i}{d\tau} \approx \omega_i(\mathbf{J})$$
$$\frac{dJ_i}{d\tau} \approx \eta \langle G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J})\rangle$$

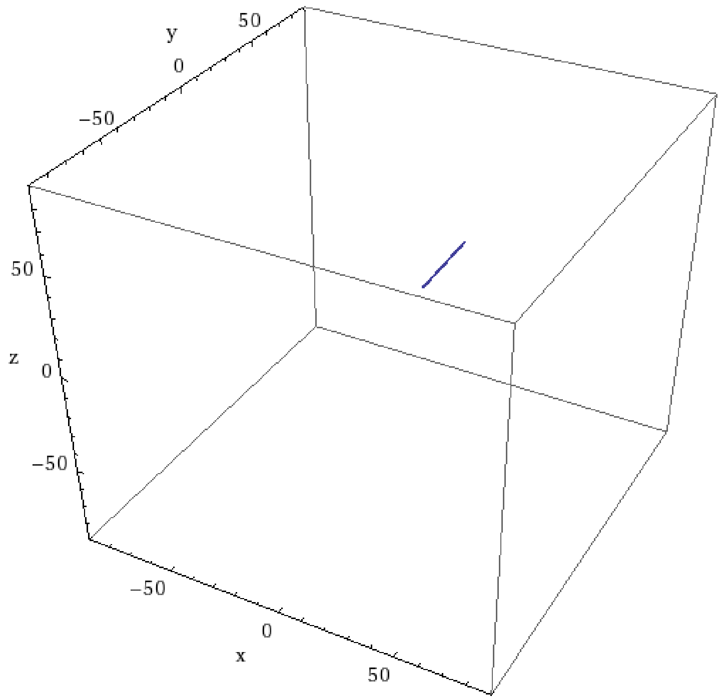
Averaging

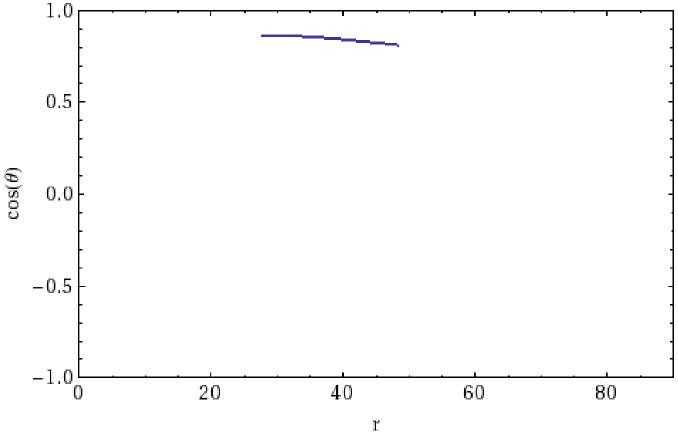
$$G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) = \sum_{k,n} G^{(1)}_{i,\mathrm{sf,kn}}(\mathbf{J}) e^{i(kq_\theta + nq_r)}$$

$$\Rightarrow \quad \langle G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) \rangle = G^{(1)}_{i,\mathrm{sf,00}}(\mathbf{J})$$

Not true if $k\omega_\theta + n\omega_r \approx 0$

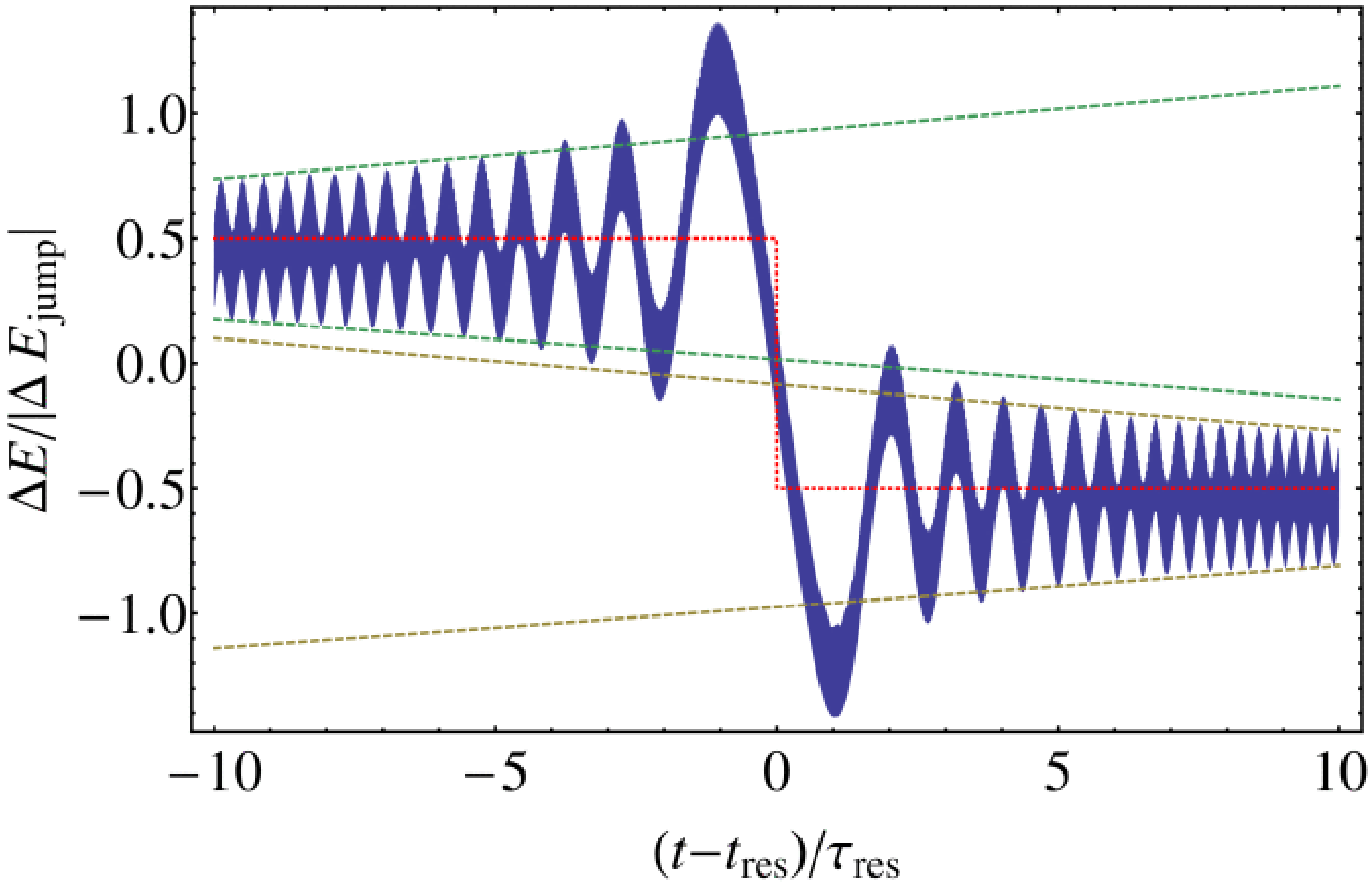
Resonant Orbit





Credit: Rob Cole

Evolution across Transient Resonance

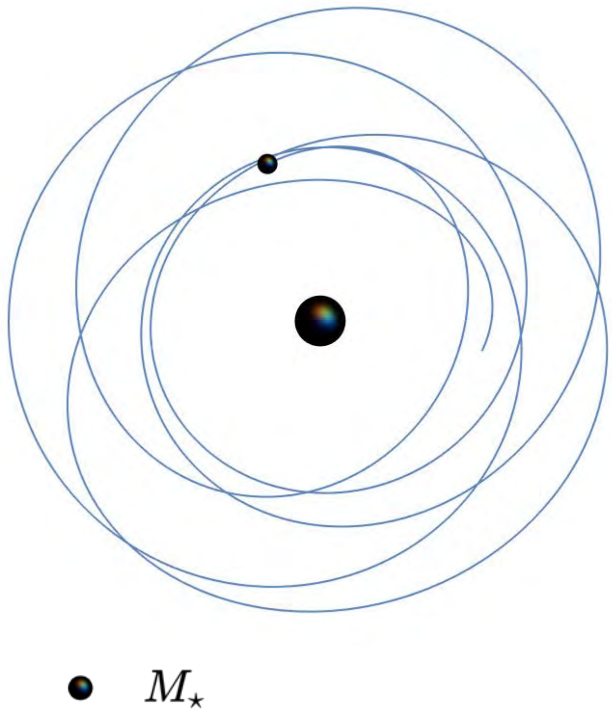


Are transient resonances astrophysically important?

- They happen generically for EMRIs in LISA band.
- Kick size is small but if early in the inspiral phase: significant dephasing
- Bigger kicks when (1) more eccentric orbit (2) lower order resonances (smack k and n)

Only lose a few percent for detection purpose [Berry et al. `16], but important for parameter estimation

With a tidal perturber…



$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \eta g^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathrm{J}) + \epsilon g^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathrm{J})$$

$$\frac{dJ_i}{d\tau} = \eta G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \epsilon G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J})$$

Adiabatic approximation:

$$\frac{dq_i}{d\tau} \approx \omega_i(\mathbf{J})$$

$$\frac{dJ_i}{d\tau} \approx \eta \langle G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J})\rangle + \epsilon \langle G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J})\rangle$$

Averaging

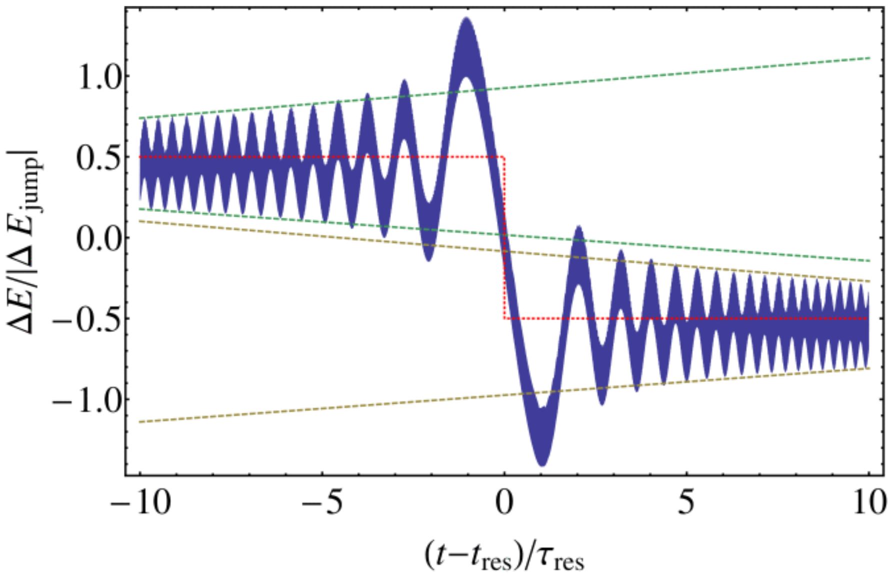
$$G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J}) = \sum_{k,n,m} G^{(1)}_{i,\mathrm{tide,knm}}(\mathbf{J}) e^{i(kq_\theta + nq_r + mq_\phi)}$$

$$\Rightarrow \quad \langle G^{(1)}_{i,\mathrm{tide}}(q_\theta, q_r, q_\phi, \mathbf{J})\rangle = G^{(1)}_{i,\mathrm{tide,000}}(\mathbf{J})$$

Not true if $k\omega_\theta + n\omega_r + m\omega_\phi \approx 0$

Tidal Resonance

- A kick in Q, Lz across the resonance
- Perturbed conserved quantities lead to perturbed orbital frequencies.
- Sample resonance (n:k:m=2:1:-2).

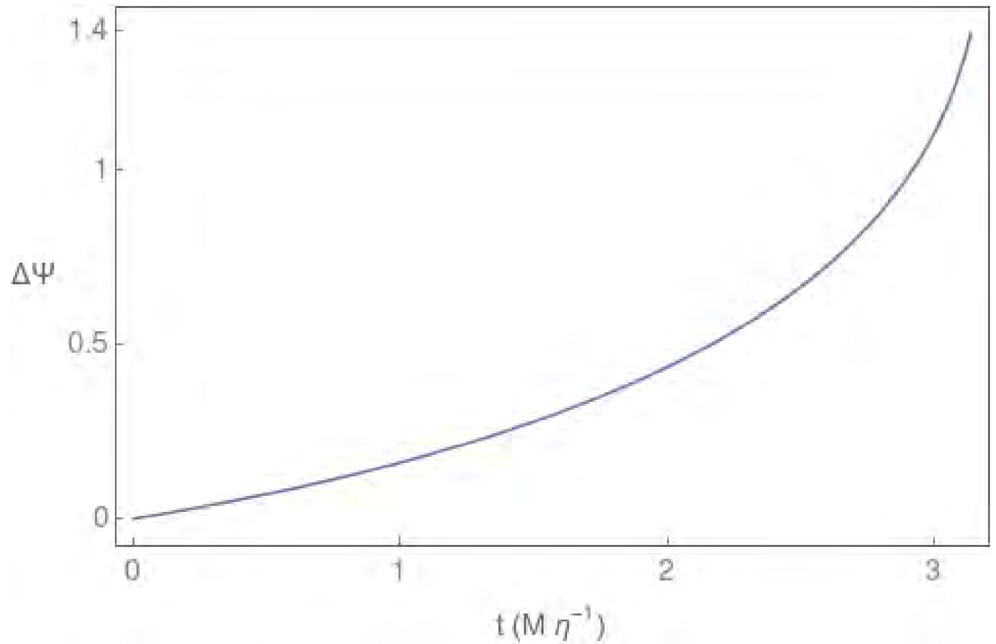


a^a	$r_{\rm min}$	$r_{\rm max}$	$\theta_{\rm min}{}^b$	$\dot{Q}_{-2,2,1}$	$\dot{L}_{z\,-2,2,1}$
0.7	3.5	5.1628033	$\pi/3$	$1.66 + 2.27i$	$-0.35 - 0.47i$
0.9	3	6.6159726	$\pi/4$	$6.60 + 7.70i$	$-1.72 - 2.01i$
0.99	3	5.3718120	$\pi/4$	$4.46 + 3.43i$	$-1.23 - 0.95i$

Dephasing

$$\Delta\Psi := \int_0^{T_{\rm plunge}} 2\Delta\omega_\phi dt$$

$$= 1.4\left(\frac{\mu}{10M_\odot}\right)^{-\frac{1}{2}}\left(\frac{M}{M_{\rm SgrA^*}}\right)^{\frac{7}{2}}\left(\frac{M_*}{10\,M_\odot}\right)\left(\frac{R}{4.3\,\rm AU}\right)^{-3}$$



Including the dynamics of the outer object

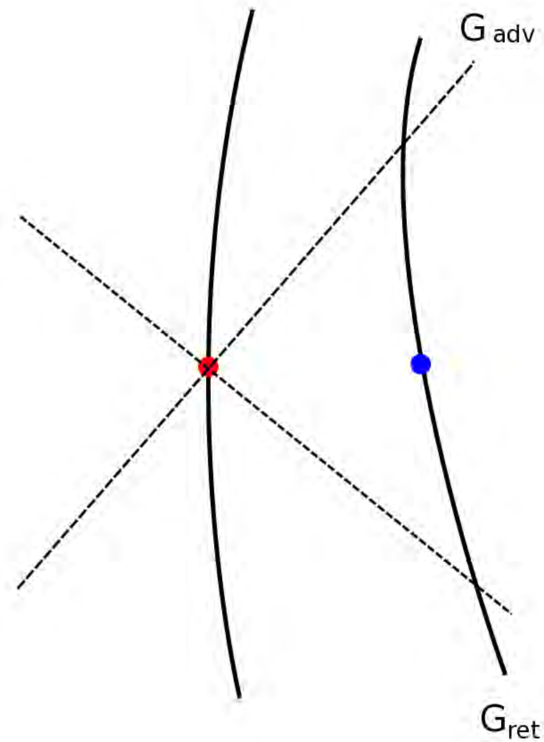
- Not previously explored in the fully relativistic setting.

- Conservative dynamics can be casted in Hamiltonian language.

$$\mathcal{H} = \frac{\mu}{2} g^{\rm Kerr}_{\alpha\beta}(x) u^\alpha u^\beta + \frac{\mu}{2} g^{\rm Kerr}_{\alpha\beta}(\underline{x}) \underline{u}^\alpha \underline{u}^\beta + \frac{\mu}{2} \underline{h}_{\alpha\beta}(x) u^\alpha u^\beta + \frac{\mu}{2} h_{\alpha\beta}(\underline{x}) \underline{u}^\alpha \underline{u}^\beta$$

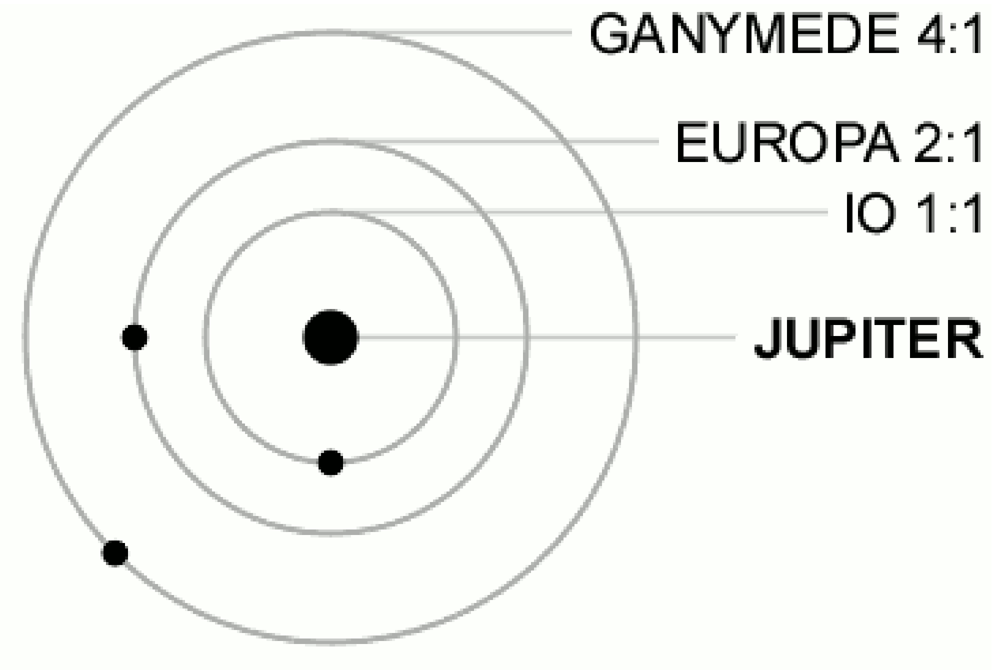
- With metric perturbation :

$$h_{\alpha\beta}(x) = \mu \int d\tau' \, G_{\alpha\beta\rho\sigma}(x; x') u'^\rho u'^\sigma$$



$$\boxed{G = \frac{1}{2}(G_{\rm ret} + G_{\rm adv})}$$

Newtonian limit: mean motion resonance



Mean motion resonance

- Newtonian mean motion resonance requires approximately commensurate orbital periods :

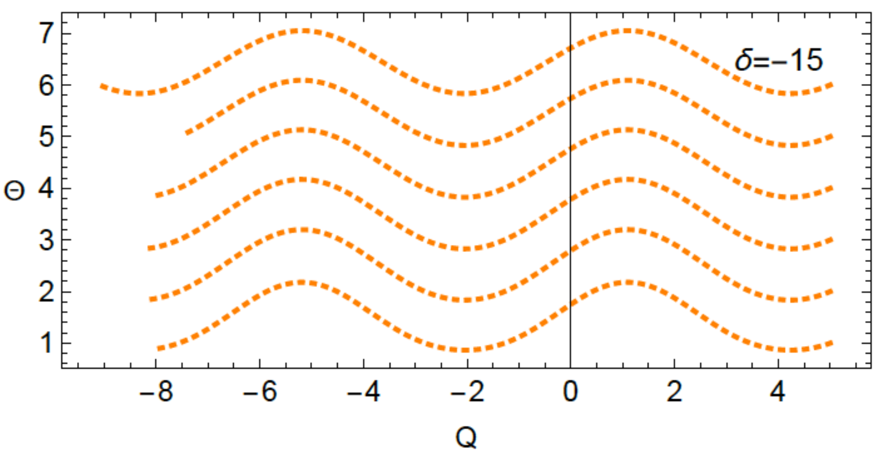
$$j\omega - (j-k)\omega' \approx 0$$

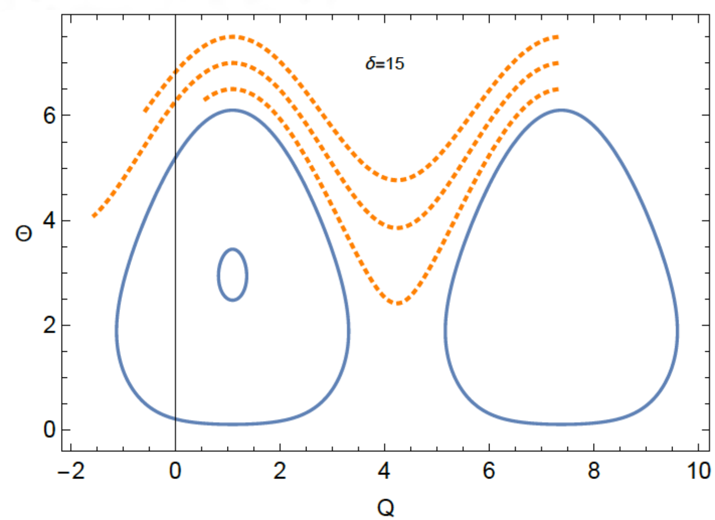
- Relativistic mean motion resonance requires:

$$m\omega_\phi + k\omega_\theta + n\omega_r + m'\omega'_\phi + k'\omega'_\theta + n'\omega'_r \approx 0$$

- In both cases, the resonant Hamiltonian can be reduced to

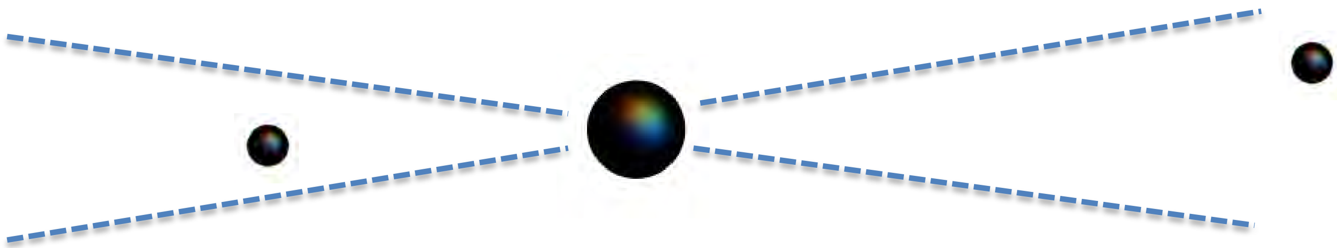
$$\mathcal{H} = \delta\Theta + \beta\Theta^2 + \sqrt{\Theta}\cos Q$$



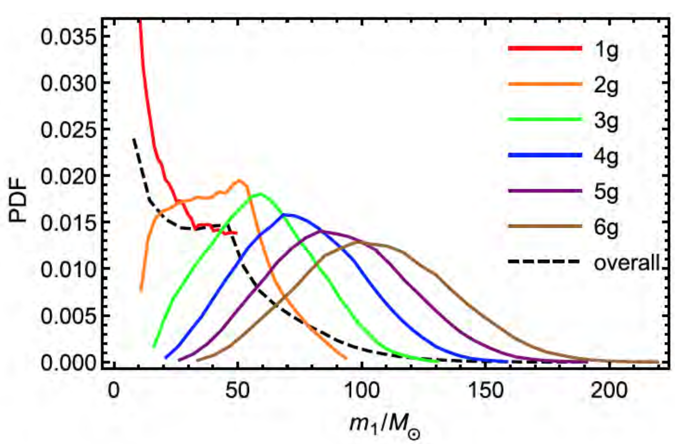


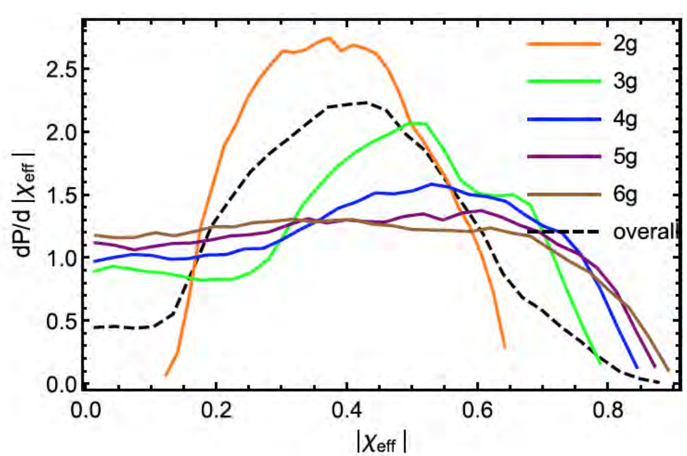
Possible formation mechanism

- Supermassive black hole +AGN disk. Sample evolution with REBOUND: $10^6\,M_\odot$+ $10\,M_\odot$ + $10\,M_\odot$,+ thin disk + PN equation of motion.

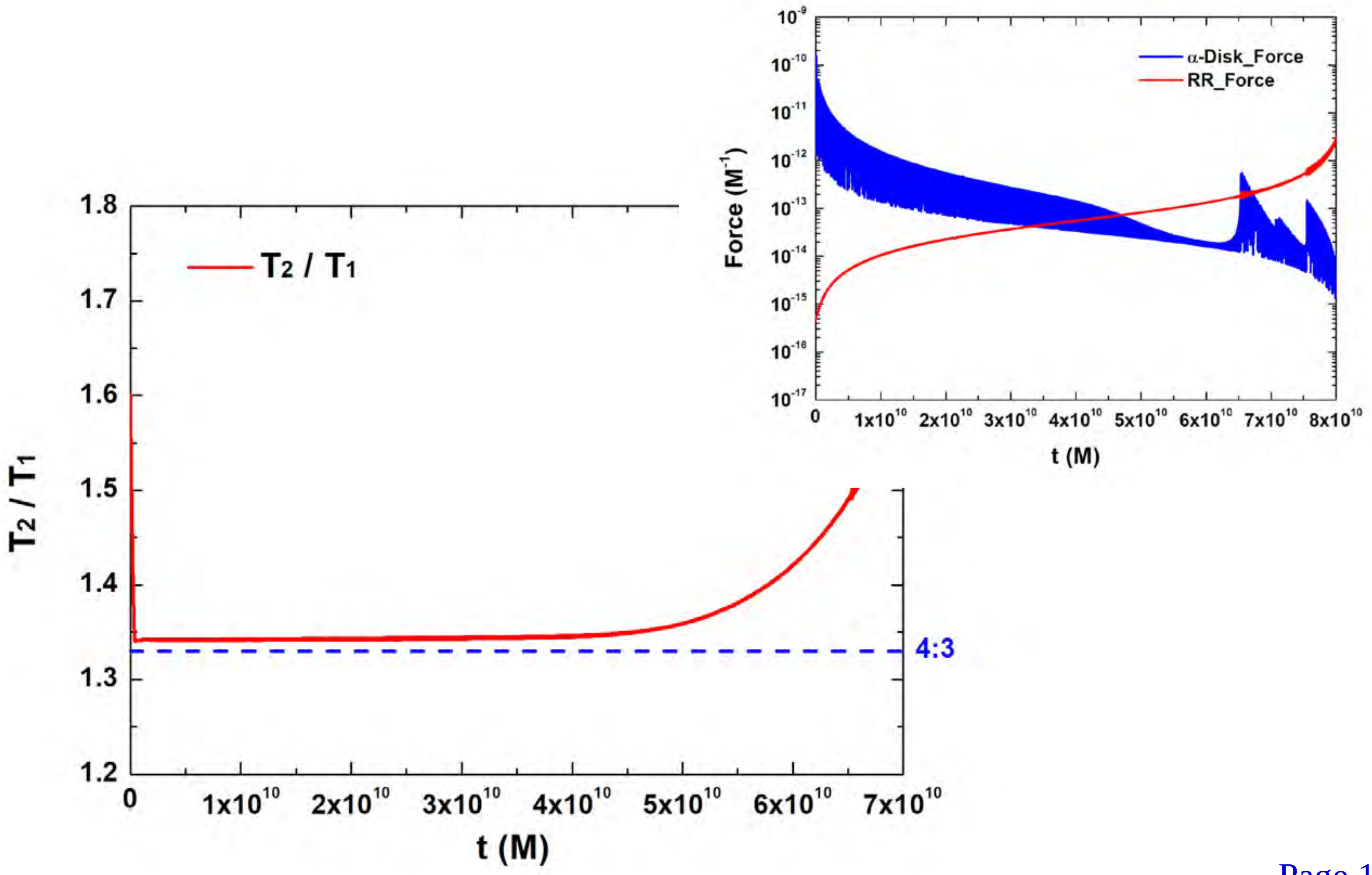


- Similar scenario recently proposed to explain hierarchical mergers to produce ~100 $M_\odot$ black holes [Y. Yang et al. `19].

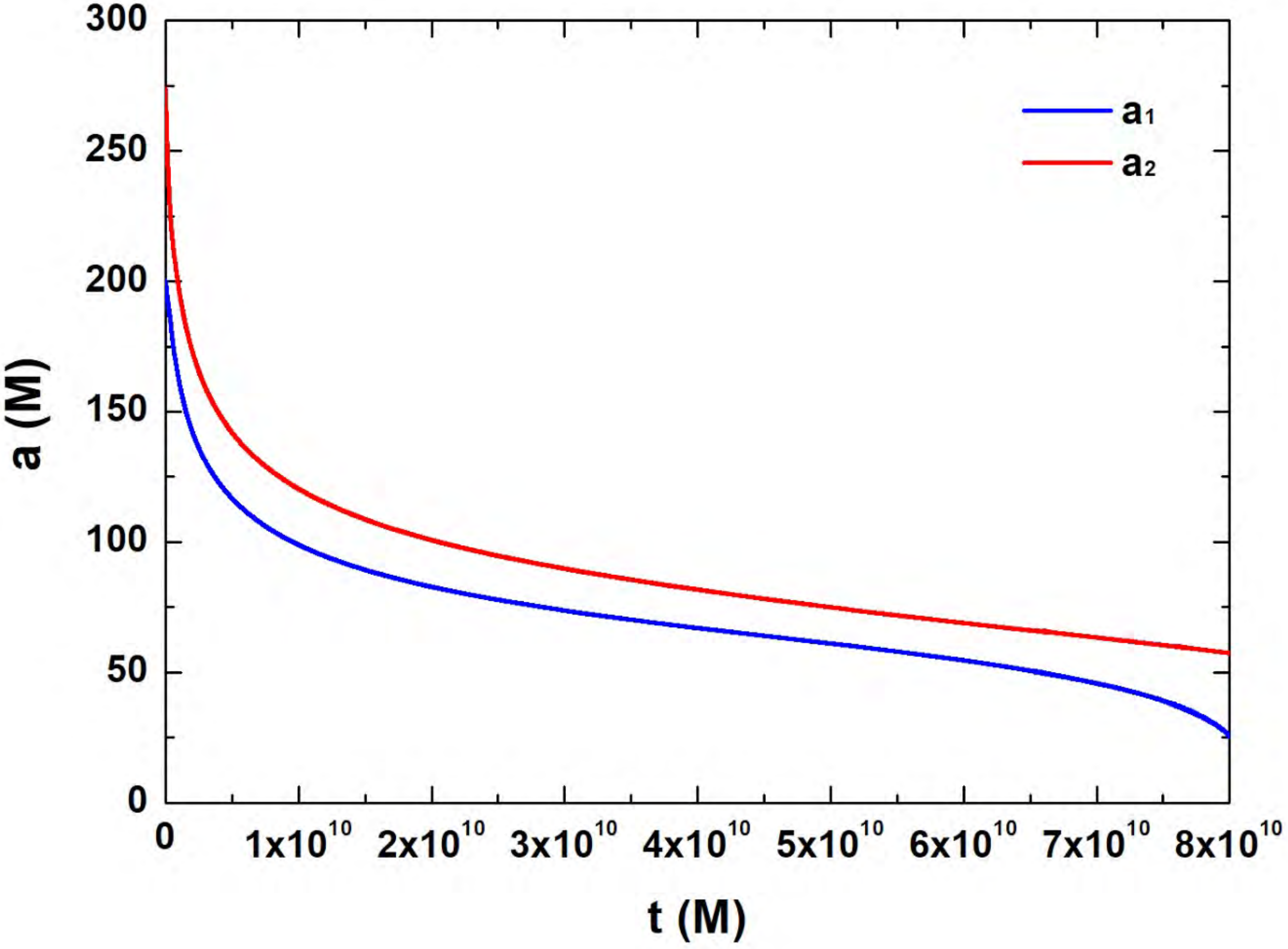




Resonance breaks when radiation reaction dominates



Outer object as a tidal perturber



Conclusion

- Tidal resonance can change EMRI waveform appreciably depending on the distance and mass of the tidal perturber.

- Opportunity to learn about light black holes that are close to massive black holes.

- In the AGN disk, pairs of stellar-mass black holes can be locked into mean motion resonance in the relativistic regime.

- Such pairs may be observed with waveform including tidal resonance effects. Unbroken pairs may be observed for immediate black holes?