Neutrino Spectral Density at Electroweak Scale Temperature


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References

Leptogenesis, Particularly in Low Energy Scale

- The standard model (SM) seems to fail to explain the observed baryon asymmetry of the universe (BAU): $\eta_B = (n_B - \bar{n}_B)/n_\gamma \simeq 6.1 \times 10^{-10}$.

- An extension of the SM by adding right-handed Majorana neutrinos ($N_R$) may have a chance to account for the BAU (Fukugita et al. ('86)): A decay of $N_R$ (e.g. Fig.) generates a net lepton number, which are partially converted into the baryon number via sphaleron process in the electroweak (EW) phase trans. (Kuzmin et al. ('85), Klinkhamer et al. ('84), Arnold et al. ('87)).

- If the mass difference between two $N_R$s is in the order of their CP-violating decay width, the CP asymmetry is dynamically enhanced (Pilaftsis ('97)), and the leptogenesis in the EW scale can be relevant (Pilaftsis et al. ('05)).
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If the standard-model leptons have non-trivial spectral properties in EW scale plasma, the lepton number creation via the $N_R$ decay may be significantly modified (c.f. Kiessig et.al., PRD. 2010).
If the standard-model leptons have non-trivial spectral properties in EW scale plasma, the CP asymmetry would be modified.
There is a growing interest in the collective nature of the fermions in the scenario of thermal leptogenesis (Drewes, arXiv:1303.6912).

In QED and QCD at extremely high $T$, the Hard Thermal-Loop (HTL) approx. indicates that a probe fermion interacting with thermally excited gauge bosons and anti-fermions admits a collective excitation mode (See, The text book by LeBellac).

In the neutrino dispersion relation in the electroweak scale plasma, the existence of a novel branch in the ultrasoft-energy region has been indicated by using the HTL and the unitary gauge (Boyanovsky, PRD. 2005).
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From QGP to Particle Cosmology

**Goal:** We investigate Neutrino Spectral Density at $T \gtrsim M_W, Z$.

1. Without restricting ourselves to the dispersion,
2. In $R_\xi$ Gauge (Fujikawa et.al. PRD. 1972),
3. And discuss a possible implication to the leptogenesis.

**Hints in QGP Physics**

- The Spectral Density of massless fermion coupled with the massive mesonic mode in plasma (an effective description of QGP) has been investigated and shown to have a Three-Peak Structure with a Ultrasoft Mode. (Kitazawa et.al. ('05-'06), Harada et.al. ('08), w.o. HTL).
- In particular, when the fermion is coupled with the massive *vectorial* meson, the Gauge Independent Nature of the three-peak structure has been confirmed (Satow et.al. ('10)) by using the Stueckelberg formalism.
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   • Neutrino Spectral Density: Overview
   • Three Peak Structure: In Details
   • Gauge Parameter $\xi$ Dependence of Spectral Property
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Setups

- Massless Lepton Sector:

\[
\mathcal{L}_L = \sum_{i=e, \mu, \tau} \left[ (\bar{\nu}^i, \bar{l}^i_L) i \not{\partial} \left( \nu^i, l^i_L \right) + \bar{l}^i_R i \not{\partial} l^i_R \right] + \left[ W^\dagger \mu J^\mu_W + J^\mu_W W^\mu + Z^\mu J^\mu_Z + A^\mu_{\text{EM}} J^\mu_{\text{EM}} \right],
\]

(2)

- Weak Bosons in \( R_\xi \) Gauge:

\[
G_{\mu\nu}(q, T) = -\frac{(g_{\mu\nu} - q_\mu q_\nu/M_{W,Z}^2(T))}{q^2 - M_{W,Z}^2(T)} + \frac{q_\mu q_\nu/M_{W,Z}^2(T)}{q^2 - \xi M_{W,Z}^2(T)}.
\]

(3)

- Weak Boson Masses \( T \ll \nu(T) \) (c.f. Manuel, PRD. 1998):

\[
M_W(T) = \frac{g \nu(T)}{2} + \mathcal{O}(gT), \quad M_Z(T) = \frac{\sqrt{g^2 + g'^2}}{2} \nu(T) + \mathcal{O}(gT).
\]

(4)
Higgs Effective Potential ($R_\xi$ Gauge)

$$V_{\text{eff}} = -\frac{\mu_0^2}{2} \left[ 1 - \frac{T^2(2\lambda + 3g^2/4 + g'^2/4)}{4\mu_0^2} \right] v^2(T) + \frac{\lambda}{4} v^4(T), \quad (5)$$

- The $\xi$ dependences cancel out between the Nambu-Goldstone modes and the ghost contributions (Text Book by Kapusta).
- The effective potential leads to the second-order phase transition. Note that in reality the possibility of the strong first-order transition has been ruled out within the standard model (Kajantie et.al.('96), Y.Aoki et.al.(‘99), Csikor et.al.(‘99, ’00)).
- The temperature region satisfying $M_{W,Z}(T) \lesssim T \ll v(T)$ should exist, and a non-trivial spectral property is anticipated there.
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For the massless left-handed neutrinos, the finite-\(T\) effects are solely encoded in the coefficients in the decomposition

\[
\Sigma_{\text{ret}}^{(\nu)}(p, \omega; T) = \sum_{s=\pm} \left[ \mathcal{P}_R \Lambda_{s,p} \gamma^0 \mathcal{P}_L \right] \Sigma_s^{(\nu)}(|p|, \omega; T),
\]

\[(6)\]

\[
\mathcal{P}_{L/R} = \frac{(1 \mp \gamma_5)}{2}, \quad \Lambda_{\pm,p} = \frac{(1 \pm \gamma^0 \gamma \cdot p/|p|)}{2}.
\]

\[(7)\]

For the spectral density, similarly,

\[
\rho^{(\nu)}(p, \omega; T) = \sum_{s=\pm} \left[ \mathcal{P}_R \Lambda_{s,p} \gamma^0 \mathcal{P}_L \right] \rho_s^{(\nu)}(|p|, \omega; T)
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\[
\rho_{\pm}^{(\nu)}(|p|, \omega; T) = \frac{-\text{Im} \Sigma_{\pm}^{(\nu)}(|p|, \omega; T)/\pi}{\{\omega - |p| \mp \text{Re} \Sigma_{\pm}^{(\nu)}(|p|, \omega; T)\}^2 + \{\text{Im} \Sigma_{\pm}^{(\nu)}(|p|, \omega; T)\}^2}.
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\[
T/v_0 = 0.2, \quad T/M_W(T) \simeq 0.63, \quad (8)
\]

\[
\omega - |p| - \text{Re} \Sigma_+(\omega, |p|, T) = 0
\]
Intermediate Temperature Region I

\[ T/v_0 = 0.42, \quad T/M_W(T) \simeq 1.45, \quad (9) \]

\[ \omega - |p| - \text{Re} \sum_+ (\omega, |p|, T) = 0 \]
Intermediate Temperature Region II

\[ T/v_0 = 0.5, \quad T/M_W(T) \simeq 1.83, \quad (10) \]

\[ \omega - |p| - \Re \Sigma_+(\omega, |p|, T) = 0 \]
\[ \frac{T}{v_0} = 0.8, \quad \frac{T}{M_W(T)} \simeq 4.9. \]

- The spectral property becomes closer to the HTL result.
- \( T/v(T) \simeq 1.59 > 1 \): The additional thermal-loop corrections may modify the spectral property (Hidaka-Satow-Kunihiro, Ncul.Phys.A, 2012).
The spectral property becomes closer to the HTL result.

- $T/v_0 \approx 1.59 > 1$: The additional thermal-loop corrections may modify the spectral property (Hidaka-Satow-Kunihiro, Nucl.Phys.A, 2012).
Spectral Density at $(|p|/v_0, T/v_0) = (0.02, 0.5)$
Landau Damping (Fig.) makes the imaginary part being finite in the spacelike region:

\[
\text{Im} \Sigma_+^{(\nu)} \supset \int_k \delta \left[ \omega + |k| - \sqrt{|p - k|^2 + M_{W,Z}^2} \right] \\
\times \left[ N_F(1 + N_B) + N_B(1 - N_F) \right] \cdot \left[ \cdots \right].
\]  

For a small external momentum \((\omega, p)\) and a not small \(M_{W,Z}\), the phase space in \(\int_k\) admitting the Landau Damping will be restricted.
Landau Damping (Fig.) makes the imaginary part being finite in the spacelike region:

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(11)

For a small external momentum \((\omega, p)\) and a not small \(M_{W,Z}\), the phase space in \(\int_k\) admitting the Landau Damping will be restricted.
Landau Damping Suppression

\[ G(x_0) = \int_{x_0}^{\infty} dx \ xN_B(x) = \sum_{n=1}^{\infty} \frac{e^{-nx_0}}{n^2} \left[ 1 + nx_0 \right], \quad x_0 = \frac{\omega^2 - |p|^2 - M_{W,Z}^2}{2T(\omega - |p|)} > 0. \]

c.f. HTL Limit \( T \gg M_{W,Z}, \omega, |p| \): \( x_0 \to 0 \) and \( G(x_0) \to \zeta(2) \gg 0. \)

The condition \( T \sim M_{W,Z} \) and the resultant finite \( x_0 \) leads to \( G(x) \ll \zeta(2) \) \( \rightarrow \) the suppression of the Landau damping.
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\[ x_0 \rightarrow 0 \ and \ G(x_0) \rightarrow \zeta(2) \rightarrow 0. \]

\[ c.f. \ HTL \ Limit \ T \gg M_{W,Z}, \omega, \ |p|: \ x_0 \rightarrow 0 \ and \ G(x_0) \rightarrow \zeta(2) \rightarrow 0. \]
Self-Energy at Three-Peak Region

\[ \frac{T}{v_0} = 0.5 \, , \, \frac{|p|}{v_0} = 0.02 \, . \]  \hspace{1cm} (12)

In the right panel, the crossing points corresponds to the solutions of

\[ \omega - |p| - \text{Re} \Sigma_+(\omega, |p|, T) = 0 \]

Spectral Density at \((|\mathbf{p}|/v_0, T/v_0) = (0.02, 0.5)\)
\( \xi \) Dependence of Three-Peak Spectral Density

\[ (p/v_0, T/v_0) = (0.02, 0.5) \]

\[ \xi = 0.1 \quad \xi = 1 \quad \xi = 10 \]

\text{U-Gauge}  

\( \rho_+ v_0 \) vs. \( \omega/v_0 \)
The net baryon number $N_b$ is produced in the sphaleron process when the changing rate of $N_b$ is larger than the expanding rate of the universe,

$$\left| \frac{1}{N_b} \frac{dN_b}{dt} \right| \geq H(T), \quad (13)$$

where,

$$H(T) = 1.66 \sqrt{N_{dof}} \frac{T^2}{M_{PL}} \simeq T^2 \times 1.41 \times 10^{-18} \text{ (GeV)}, \quad (14)$$

$$\frac{1}{N_b} \frac{dN_b}{dt} = -1023 \cdot g^7 \nu(T) \exp \left[ -1.89 \frac{4\pi \nu(T)}{gT} \right], \quad (15)$$

and we obtain

$$T \geq T_\ast \simeq 160 \text{ GeV}, \quad T_\ast / \nu_0 \simeq 0.65. \quad (16)$$
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and we obtain

$$T \geq T^* \simeq 160 \text{ GeV} , \quad T^*/\nu_0 \simeq 0.65 .$$  \hspace{1cm} (16)
Neutrino Spectral Density around $T = T_*$

\[ \rho_+ v_0 \text{ at } p/v_0 = 0.007 \]

\[
\frac{T_*}{v_0} \approx 0.65, \quad \frac{T_*}{v(T)} \sim 1.
\] (17)
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4. Summary
We have investigated the spectral properties of standard-model left-handed neutrinos at finite $T$ around the electroweak scale in a way where the gauge invariance is manifestly checked ($R_ξ$ gauge).

The spectral density of SM neutrino has the three-peak structure with the ultrasoft mode with a physical significance when $T/M_{W,Z} \gtrsim 1$.

The collective excitation which involves the ultrasoft mode appears at temperature comparable to $T_*$ within the present approximation. The three-peak collective modes could affect the leptogenesis at $T \gtrsim T_*$. 

Future Work: It is desirable to estimate how large the effects of two-loop or higher-order diagrams are on the neutrino spectral density.

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5 Buckups
Spectral Density Example: Superconductivity

\[ \frac{dl}{dV} \sim \frac{d}{dV} \left[ \int_{\epsilon_f}^{\epsilon_f + eV} d\omega \text{ Dos}(\omega) \right] \sim \text{Dos}(\omega) \sim \int_p \rho(\omega, p), \quad (18) \]

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Neutrino Spectral Density at Electroweak Scale Temperature

Slightly Underdoped Bi2212 STM Superconductor Scan

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