2d $\mathcal{N} = (2, 2)$ SYM on computer

Hiroshi Suzuki

Theoretical Physics Laboratory, RIKEN

Dec. 20, 2008 @ RIKEN

- I. Kanamori, H.S., arXiv:0809.2856, Nucl. Phys. B in press
- I. Kanamori, H.S., arXiv:0811.2851

- Seiberg-Witten \Leftrightarrow IIA NS5 wrapping Σ in $CY_3 \Leftrightarrow$ Nekrasov formula
- ISS ⇔ NS5-D6-D4 ⇔ Seiberg duality

- Seiberg-Witten \Leftrightarrow IIA NS5 wrapping Σ in $CY_3 \Leftrightarrow$ Nekrasov formula
- ISS ⇔ NS5-D6-D4 ⇔ Seiberg duality

• (Highly) consistent solution? or Real establishment?

- Seiberg-Witten \Leftrightarrow IIA NS5 wrapping Σ in $CY_3 \Leftrightarrow$ Nekrasov formula
- ISS ⇔ NS5-D6-D4 ⇔ Seiberg duality
- (Highly) consistent solution? or Real establishment?
- Quest for complementary approach?

- Seiberg-Witten \Leftrightarrow IIA NS5 wrapping Σ in $CY_3 \Leftrightarrow$ Nekrasov formula
- ISS ⇔ NS5-D6-D4 ⇔ Seiberg duality
- (Highly) consistent solution? or Real establishment?
- Quest for complementary approach? Lattice?

- Nonperturbative dynamics of SUSY gauge theories
 - Seiberg-Witten \Leftrightarrow IIA NS5 wrapping Σ in $CY_3 \Leftrightarrow$ Nekrasov formula
 - ISS ⇔ NS5-D6-D4 ⇔ Seiberg duality
- (Highly) consistent solution? or Real establishment?
- Quest for complementary approach? Lattice?
- Gauge/gravity correspondence

Manifest SUSY would be impossible, because

$$\{Q^{A}_{\alpha}, (Q^{B}_{\beta})^{\dagger}\} = 2\delta^{AB}\sigma^{m}_{\alpha\dot{\beta}}P_{m}$$

• However, at least a linear combination ${\it Q}$ of ${\it Q}^{\it A}_{\alpha}$ and $({\it Q}^{\it B}_{\beta})^{\dagger}$ such that

$$\{Q,Q\}=2Q^2=0$$

could be realized even on the lattice

• Moreover, if the continuum action S can be written as

$$S = QX$$

Q-invariance of *S* could be promoted to lattice symmetry!

- The above lattice formulation possesses a manifest lattice symmetry Q
 - Cohen, Kaplan, Katz, Ünsal, Endres
 - Sugino
 - Catterall
 - D'Adda, Kanamori, Kawamoto, Nagata
 - Damgaard, Matsuura
 - Kikukawa, Sugino

< 4 →

- The above lattice formulation possesses a manifest lattice symmetry Q
 - Cohen, Kaplan, Katz, Ünsal, Endres
 - Sugino
 - Catterall
 - D'Adda, Kanamori, Kawamoto, Nagata
 - Damgaard, Matsuura
 - Kikukawa, Sugino
- But how about other SUSY generators?
- The best thing we may hope is that these are restored in the continuum limit *a* → 0
- It can be argued, owing to *Q*, this really occurs for lower dimensional SUSY gauge theories...

• Using Sugino's lattice formulation for 2d $\mathcal{N} = (2, 2)$ SYM,

(I) < ((()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) < (()) <

- Using Sugino's lattice formulation for 2d $\mathcal{N} = (2, 2)$ SYM,
- Numerically confirmed restoration of full SUSY

(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))

- Using Sugino's lattice formulation for 2d $\mathcal{N} = (2, 2)$ SYM,
- Numerically confirmed restoration of full SUSY
 - Partially Conserved SuperCurrent relation

$$\partial_\mu \left< (oldsymbol{s}_\mu)_i(oldsymbol{x})(f_
u)_i(oldsymbol{0})
ight> = rac{\mu^2}{g^2} \left< (f)_i(oldsymbol{x})(f_
u)_i(oldsymbol{0})
ight>$$

(I) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1)) < ((1))

- Using Sugino's lattice formulation for 2d $\mathcal{N} = (2, 2)$ SYM,
- Numerically confirmed restoration of full SUSY
 - Partially Conserved SuperCurrent relation

$$\partial_\mu \left< (oldsymbol{s}_\mu)_i(oldsymbol{x})(f_
u)_i(oldsymbol{0})
ight> = rac{\mu^2}{g^2} \left< (f)_i(oldsymbol{x})(f_
u)_i(oldsymbol{0})
ight>$$

• This strongly indicates 2d $\mathcal{N}=(2,2)$ SYM (with SUSY breaking scalar mass) on computer

- Using Sugino's lattice formulation for 2d $\mathcal{N} = (2, 2)$ SYM,
- Numerically confirmed restoration of full SUSY
 - Partially Conserved SuperCurrent relation

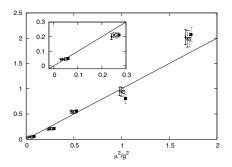
$$\partial_\mu \left< (oldsymbol{s}_\mu)_i(oldsymbol{x})(f_
u)_i(oldsymbol{0})
ight> = rac{\mu^2}{g^2} \left< (f)_i(oldsymbol{x})(f_
u)_i(oldsymbol{0})
ight>$$

- This strongly indicates 2d $\mathcal{N}=(2,2)$ SYM (with SUSY breaking scalar mass) on computer
- Numerically studied some physical questions
 - Gapless mass spectrum
 - Static potential between charges in the fundamental reps.



• The continuum limit of the ratio

$$\frac{\partial_{\mu} \langle (\boldsymbol{s}_{\mu})_{i}(\boldsymbol{x})(f_{0})_{i}(\boldsymbol{0}) \rangle}{\langle (f)_{i}(\boldsymbol{x})(f_{0})_{i}(\boldsymbol{0}) \rangle} \left(\Rightarrow \frac{\mu^{2}}{g^{2}} \right)$$



Hiroshi Suzuki (RIKEN)

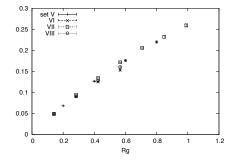
2

900

Static Potential between Charges in Fund. Reps.

• Static potential between charges in the fundamental representation V(R)/g

$$-\ln \{W(T,R)\} = V(R)T + c(R)$$



• This confining behavior appears distinct with a conjecture in the '90s by Armoni, Frishman and Sonnenschein

Hiroshi Suzuki (RIKEN)

2d $\mathcal{N} = (2, 2)$ SYM on computer