D-brane States and Annulus Amplitudes in OSp Invariant Closed String Field Theory

Koichi Murakami (RIKEN)

This talk is based on

Yutaka Baba (RIKEN), Nobuyuki Ishibashi (Tsukuba) and K.M.,

- JHEP **07** (2008) 046 [arXiv:0805.3744]

cf • JHEP **0710**, 008 (2007) [arXiv:0706.1635]
- JHEP **0705**, 020 (2007) [arXiv:hep-th/0703216]
- JHEP **0605**, 029 (2006) [arXiv:hep-th/0603152]

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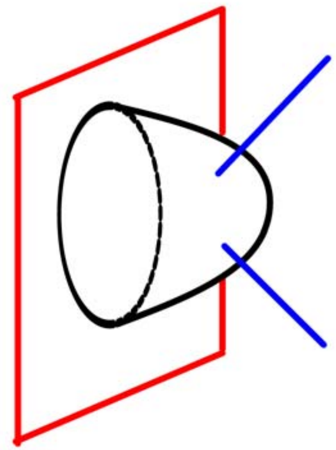
▶ What we would like to study

D-branes in closed SFT

(cf. Hashimoto-Hata)

▶ D-brane states in OSp invariant SFT (Baba-Ishibashi-K.M

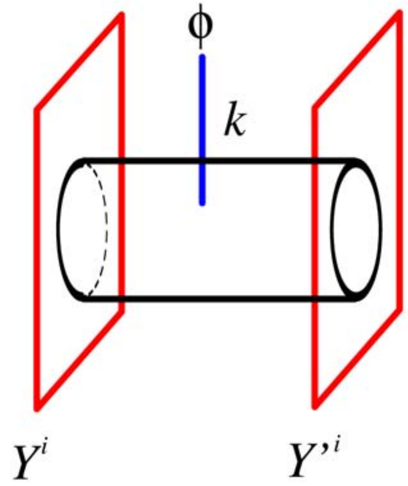


⇒
- coincident N D-branes
- including ghost D-branes
- states for one D-brane in constant B background

D-brane tension; disk amplitudes are correctly reproduced

▶ What we did

- states for N (ghost) D-branes separately located
- annulus amplitudes



→ reproduce usual amplitudes with correct normalizations

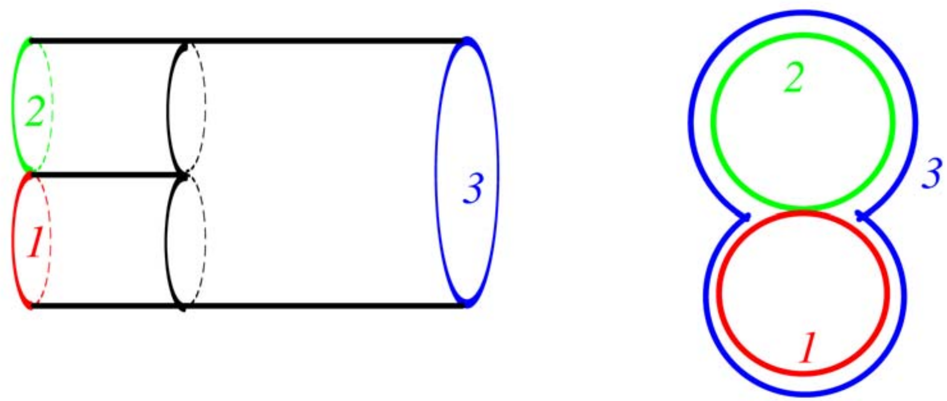
In particular, the moduli space of amplitude is covered completely and only once.

▶ OSp invariant SFT (Siegel, Uehara, Neveu, West, Zwiebach, Kugo, Kawano)

can be regarded as a light-cone gauge SFT with "transverse" coordinates $X^M = (X^\mu; C, \bar{C}) \in \mathrm{OSp}(25,1|2)$ $\mu = 0, \ldots, 25$ $C, \bar{C}$: ghosts (spin 0)

$$S = \int dt \left[\frac{1}{2} \Phi \cdot \left(\alpha i \frac{\partial}{\partial t} - \left(L_0 + \tilde{L}_0 - 2 \right) \right) \Phi + \frac{2g}{3} \Phi \cdot (\Phi * \Phi) \right]$$

- "light-cone variables" $t(= x^+)$ (proper time), $\alpha(= 2p^+)$ (string length)
  $\rightarrow$ unphysical ones canceled with ghosts via Parisi-Sourlas mechanism
- $*$: joining-splitting type interaction



- BRST invariance $\leftarrow$ (non-linear) J^{-C} of $\mathrm{OSp}(25,2|2)$
  similar to J^{-i} of $\mathrm{SO}(25,1)$ light-cone gauge SFT

$\blacktriangleright$ State for one flat Dp-brane sitting at $X^i = Y^i$ $(i = p+1, \ldots, 25)$

(Baba-Ishibashi-K.M.)

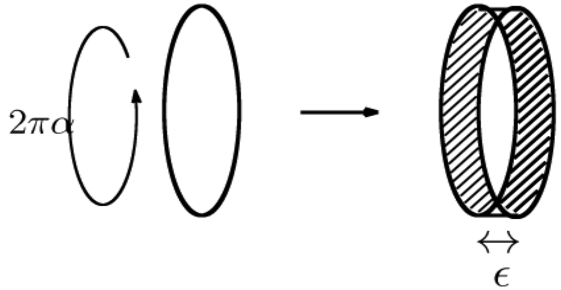
$$|D(Y)\rangle\rangle = \lambda \int d\zeta\, \bar{\mathcal{O}}_D(\zeta, Y)\ |0\rangle\rangle\ ,$$

$$\bar{\mathcal{O}}_D(\zeta, Y) = \exp\left[a \int_{-\infty}^{0} d\alpha\, e^{\zeta\alpha}\ {}^{\epsilon}\langle B(Y)|\bar{\psi}\rangle + b\zeta^2 \right],$$

$$a = \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}$$

$$b = \frac{(2\pi)^{13}\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}}\sqrt{\pi}g}$$

$$|B(Y)\rangle^{T} = e^{-\frac{T}{|\alpha|}(L_0+\tilde{L}_0-2)}|B(Y)\rangle\ ,$$

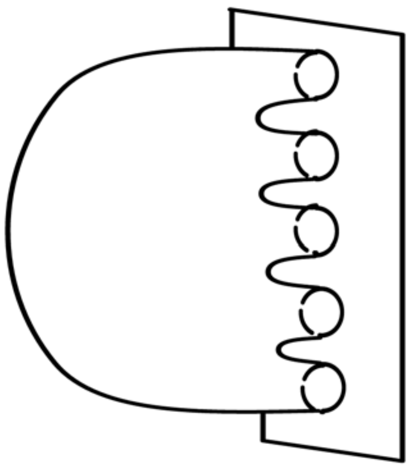


$|B(Y)\rangle$: boundary state for Dp-brane at $X^i = Y^i$

- a and b are determined by BRST invariance
- $|D\rangle\rangle \sim e^{aB\cdot\Phi}|0\rangle\rangle$

  $\to$ insert boundaries in the closed string worldsheet

  $\to$ a determines the D-brane tension



▶ States for separately located N flat D-branes (for $(Y^i_{(I)} - Y^i_{(J)})^2 \neq 0 \ (I \neq J)$)

simply given as a direct product of $\int d\zeta \, \bar{\mathcal{O}}_D$

$$|D_N(Y_{(I)})\rangle\rangle = \lambda_{N+} \prod_{I=1}^{N} \int d\zeta_I \, \bar{\mathcal{O}}_D(\zeta_I, Y_{(I)})|0\rangle\rangle$$

$\Leftarrow$ BRST invariant (in the leading order in ϵ)

$\Leftarrow$ $|B(Y)\rangle^\epsilon * |B(Y')\rangle^\epsilon$ is suppressed by $\epsilon^{\frac{(\Delta Y)^2}{4\pi^2}}$ with $\Delta Y^i = Y^i - Y'^i$

(cf. Kishimoto-Matsuo-Watanabe)

cf. Coincident case: (Baba-Ishibashi-K.M.)

$$|D_N\rangle\rangle = \lambda_{N+} \int d^N\zeta \, \underline{\triangle_N^2(\zeta_1, \ldots, \zeta_N)} \prod_{i=1}^{N} \bar{\mathcal{O}}_D(\zeta_i)|0\rangle\rangle$$

$\triangle_N$: Vandermonde det $\Rightarrow$ ζ_i as eigenvalues of a matrix T

i.e. $|D_N\rangle\rangle = \int dT \, \exp\left[a \int d\alpha \, \mathrm{Tr} e^{T\alpha\epsilon} \langle B|\bar{\psi}\rangle + b \mathrm{Tr} T^2 \right] |0\rangle\rangle$

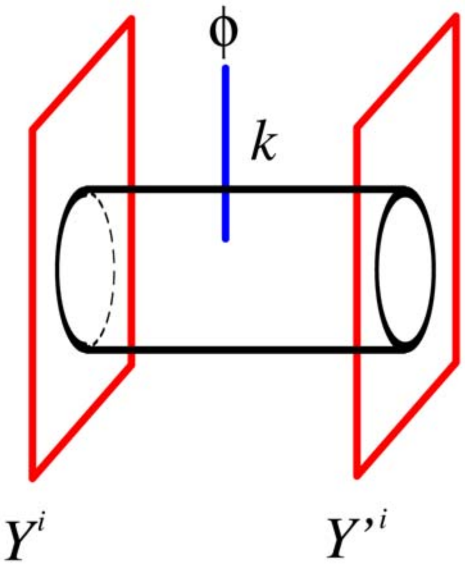
$\rightarrow$ non-abelian nature of coincident D-branes(??): $U(1)^N \rightarrow U(N)$

▶ Annulus Amplitudes

To verify our construction of D-brane states further

⇒ Evaluate annulus amplitudes

- with one closed string external line
- the annulus is suspended between two parallel Dp-branes sitting at $X^i = Y^i$ and Y'^i



▶ How to read the S-matrix elements (proposal)

(i) evaluate the correlation function at $\mathcal{O}(g^1)$

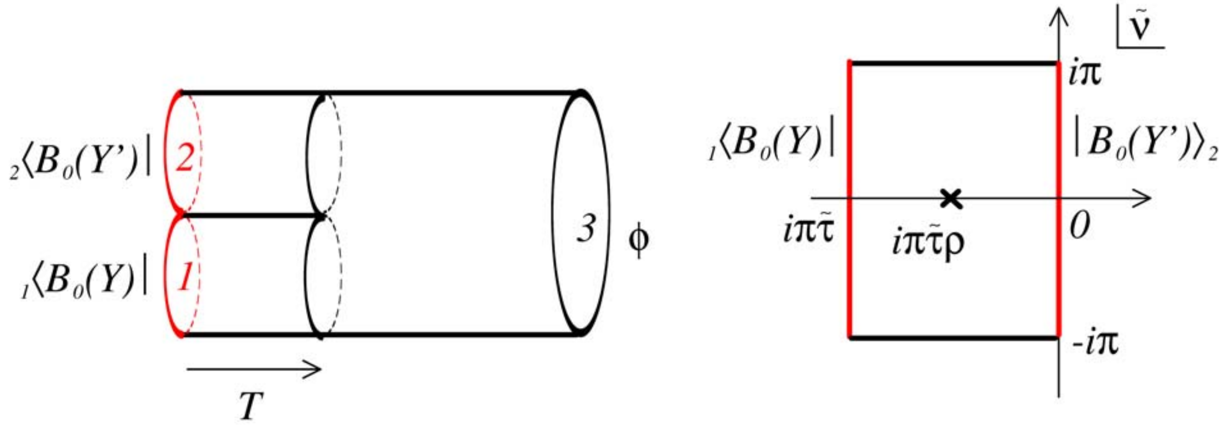
$$\langle \mathcal{O}_\phi(t,k) \rangle\rangle_{D_2(Y;Y')} \equiv \frac{\langle\langle 0|\mathcal{O}_\phi(t,k)|D_2;Y,Y'\rangle\rangle}{\langle\langle 0|D_2;Y,Y'\rangle\rangle}$$

$$\simeq -3!a^2 \int_{\alpha_1,\alpha_2>0,\alpha_3<0} d\alpha_1 d\alpha_2 d\alpha_3 \int_0^t dT \frac{2g}{3} \langle V_3^0(1,2,3)|B(Y)\rangle_1^T |B(Y')\rangle_2^T e^{-\frac{T-t}{\alpha_3}H^{(3)}} |\mathrm{DDF}_\phi;k\rangle_3$$

with $\mathcal{O}_\phi(t,k) = \int d\alpha \left({}_{\mathrm{ghost}}\langle 0| \otimes {}_X\langle \mathrm{DDF}_\phi;k| \right) |\Phi(t)\rangle$: observable

■ changing the integration variables $(\alpha_3, \alpha_1, T) \to (\alpha, \varrho, \tilde{\tau})$

mapping to a cylinder of circumference 2π and length $-i\pi\tilde{\tau}$ ($\tilde{\nu}$-plane)



$$\rho(\nu) = \alpha \ln \frac{\vartheta_1\left(\frac{i}{2\pi}(\tilde{\nu} + i\pi\tilde{\tau}\varrho)\,|\tilde{\tau}\right)}{\vartheta_1\left(\frac{i}{2\pi}(\tilde{\nu} - i\pi\tilde{\tau}\varrho)\,|\tau\right)} + \alpha\varrho\tilde{\nu}\ , \quad \varrho = \frac{\alpha_1}{\alpha}\ , \quad \alpha = -\alpha_3$$

(ii) residue of the one-shell pole at $k^2 + M^2 = 0$ for the closed string external state $\to$ S-matrix element

$$S_{\phi DD} = -4\pi^2 i g a^2 \int_0^1 d\varrho \int_0^{i\infty} d\tilde{\tau}\tilde{\tau} \prod_{n=1}^{\infty} (1 - e^{2\pi i n\tilde{\tau}})^2$$

$$\times_X \left\langle B(Y') \left| e^{i\pi\tilde{\tau}\varrho(L_0^X + \tilde{L}_0^X - 2)}\ V_\phi\ e^{i\pi\tilde{\tau}(1-\varrho)(L_0^X + \tilde{L}_0^X - 2)} \right| B(Y) \right\rangle_X$$

This coincides with the S-matrix in the 1st quantization

- $\prod_{n=1}^{\infty}(1-e^{2\pi i n\tilde{\tau}})^2$: coincides with the ghost contribution
- as an integral over the moduli space of the one-punctured cylinder

$$\begin{cases} \underline{\text{correct measure}} \\ \text{moduli space is covered } \underline{\text{completely and only once}} \end{cases}$$

▶ Summary

Using the OSp invariant SFT for bosonic strings,

- construct 2nd quantized states corresponding to N parallel D-branes separately located ⇐ BRST invariant (in the leading order of ϵ)
- show that above states reproduce usual results with the correct integral region and measure over the moduli space

■ Here we just present the annulus amplitude involving one external closed string state.

The extension with more general case is possible: with N external legs, annulus ending on the same D-brane....