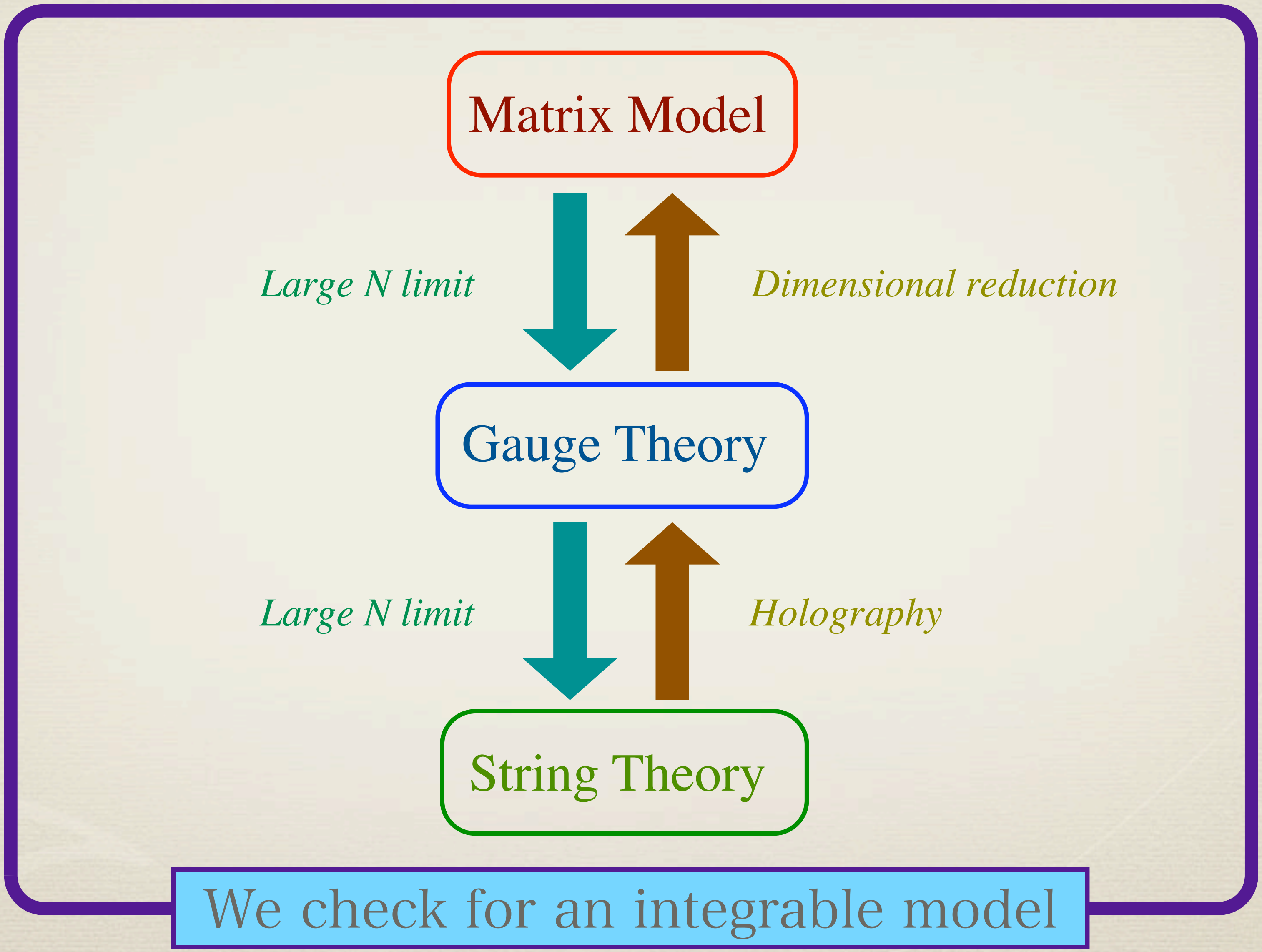
TWO-DIMENSIONAL GAUGE THEORY AND MATRIX MODEL

Kazutoshi Ohta
Department of Physics, Tohoku University

Collaboration with G. Ishiki, S. Shimasaki, and A. Tsuchiya
[arXiv:0811.3569]

Introduction



We consider 2-dimensional Yang-Mills theory [Witten (1992)]

Witten shows that the non-Abelian localization works in the 2-dim $U(N)$ Yang-Mills theory on S^2 and the partition function is given by a weighted sum over critical points

$$
\begin{aligned}
Z &= \int \mathcal{D}A \, e^{-\frac{i}{4g_{\mathrm{YM}}^2}\int d^2x\sqrt{g}\,\mathrm{Tr}F^2} \\
&= \sum_{\vec{n}\in\mathbb{Z}^N} w(\vec{n}) e^{-\frac{2\pi^2}{g_{\mathrm{YM}}^2 A}\vec{n}^2}
\end{aligned}
$$

Non-Abelian localization theorem

where A is an area of S^2 and $\vec{n}$ is instanton (monopole) charges of classical solutions to the equation of motion

$$D^\mu F_{\mu\nu} = 0$$

Minahan and Polychronakos determined the weight function $w(\vec{n})$ using the Poisson resummation formula from Migdal's partition function

$$\mathcal{Z}_{\text{2dYM}} = \sum_R (\dim R)^2 e^{-g_{\text{YM}}^2 A C_2(R)}$$

$$= C \sum_{\vec{p}\in\mathbb{Z}^N} \prod_{i<j} (p_i - p_j)^2 e^{-\frac{1}{2} g_{\text{YM}}^2 A \sum_{i=1}^N p_i^2}$$

discretized (Gaussian) matrix model

Poisson resummation

$$\mathcal{Z}_{\text{2dYM}} = \sum_{\vec{n}\in\mathbb{Z}^N} w(\vec{n}) e^{-\frac{2\pi^2}{g_{\text{YM}}^2 A} \sum_{i=1}^N n_i^2}$$

where

$$w(\vec{n}) = C \int \prod_{i=1}^N dy_i \prod_{i<j} ((y_i - y_j)^2 - (n_i - n_j)^2) e^{-\frac{1}{2 g_{\text{YM}}^2 A} \sum_i y_i^2}$$

2d YM (=BF theory + mass term) on S^2

$$S_{\rm 2dYM} = -\frac{1}{4g_{\rm YM}^2}\int_{S^2} d^2x\sqrt{g}\,{\rm Tr}F^2$$

$$\to \int_{S^2} d^2x\sqrt{g}\,{\rm Tr}\left\{i\Phi F + \frac{\mu}{2}\Phi^2\right\}$$

dim reduction

$$S_{\rm MM} = -\frac{1}{g^2}{\rm Tr}\left\{\frac{i}{3}\epsilon^{ijk}X_i[X_j, X_k] + X_i^2\right\}$$

This matrix model is called the $\mathcal{N}$=1* matrix model in the context of Dijkgraaf-Vafa theory.

⇔ mass deformed superpotential of $\mathcal{N}$=4 theory

Classical solutions ⇔ $SU(2)$ representations

For later convenience, we redefine the $M \times M$ matrices as

$$Z = X_1 + iX_2, \quad Z^\dagger = X_1 - iX_2, \quad \Phi = X_3$$

then the action becomes

$$S_{\mathrm{MM}}(\Phi, Z, Z^\dagger) = -\frac{1}{g^2} \mathrm{Tr} \left\{ Z[\Phi, Z^\dagger] + (1 - i\epsilon) Z Z^\dagger + \Phi^2 \right\}$$

Integrating Z and $Z^\dagger$ first

$$\mathcal{Z}_{\mathrm{MM}} = \lim_{\epsilon \to 0} \frac{1}{M!} \int \prod_{i=1}^{M} d\phi_i \prod_{i \neq j} \frac{\phi_i - \phi_j}{\phi_i - \phi_j + 1 - i\epsilon} e^{-\frac{i}{g^2} \sum_{i=1}^{M} \phi_i^2}$$

Using $\lim_{\epsilon \to 0} \frac{1}{x - i\epsilon} = \mathcal{P} \frac{1}{x} + i\pi\delta(x)$, we find

$$\mathcal{Z}_{\mathrm{MM}} = \sum_{|Y|=M} \mathcal{N}_Y \mathcal{P} \int \prod_{l=1}^{l(Y)} dy_l \prod_{l<m} \frac{(y_l - y_m)^2 - (d_l - d_m)^2}{(y_l - y_m)^2 - (d_l + d_m)^2} e^{-\frac{i}{g^2} \sum_{l=1}^{l(Y)} \left(\frac{1}{4} d_l y_l^2 + \frac{1}{12} d_l^3 - \frac{1}{12} d_l \right)}$$

The partition function is given by integrals over zero modes y_l and summation over partitions Y of M

N blocks sector of $\mathcal{N}=1^*$ matrix model partition function

$$\mathcal{Z}_N = \sum_{\vec{d}\in\mathbb{Z}^N} \mathcal{P}\int \prod_{l=1}^{N} dy_l \prod_{l<m} \frac{(y_l-y_m)^2-(d_l-d_m)^2}{(y_l-y_m)^2-(d_l+d_m)^2} e^{-\frac{i}{g^2}\sum_{l=1}^{N}\left(\frac{1}{4}d_l y_l^2+\frac{1}{12}d_l^3-\frac{1}{12}d_l\right)}$$

Large matrix size limit ($M\to\infty$)

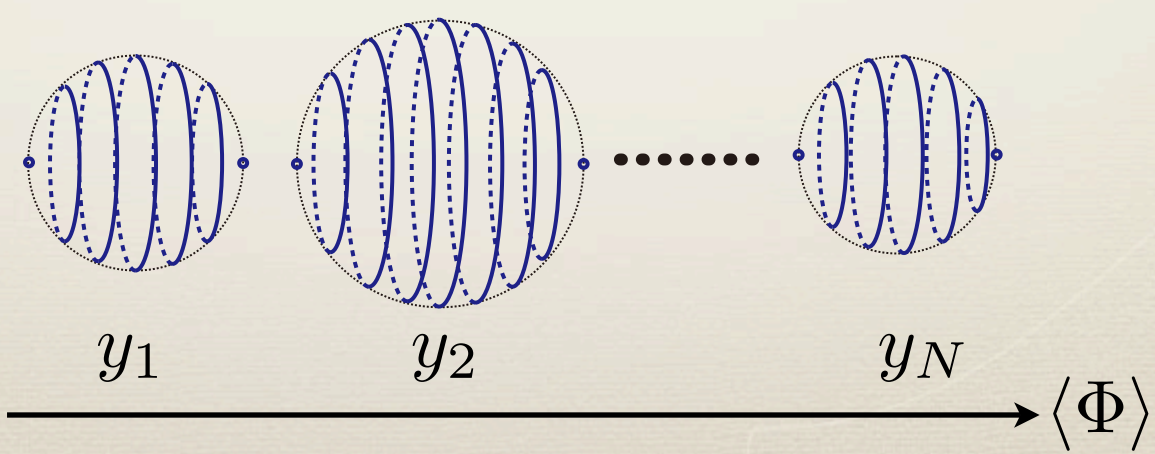
N:fix

$d_l \equiv N_0 + n_l$

$N_0 \to \infty$

$$\mathcal{Z}_{\text{"2d YM"}} \simeq \mathcal{C} \sum_{\vec{n}\in\mathbb{Z}^N} \int \prod_{l=1}^{N} dy_l \prod_{l<m} \left\{(y_l-y_m)^2-(n_l-n_m)^2\right\} e^{-\frac{2\pi^2}{g_{\text{YM}}^2 A}\sum_{l=1}^{N}\left(y_l^2+n_l^2+N_0 n_l\right)}$$

Fluctuations of the size of blocks ⇒ instanton (monopole) charges



Conclusion

* We obtain the 2d YM partition function from the reduced matrix model.

* The localization theorem also works in the matrix model.

* Using the relation between the 2d YM and non-critical strings, we can obtain the string from the matrix model in the large N (rank) limit.

$\mathcal{N}=1^*$ Matrix Model → 2d Yang-Mills → Gross-Taylor String