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Superstring in the plane-wave background with RR-flux as a conformal field theory

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1. Introduction : PP-wave as a Limit of $\mathbf{AdS_5 \times S^5}$
2. Canonical Analysis and Conformal Symmetry
3. BRS Quantization and Physical Spectrum
4. Summary and Future Problems

Based on the Collaboration with Yoichi Kazama, **JHEP 0803 (2008) 057** (arXiv:0801.1561).

1 Introduction : PP-Wave as a Limit of $\mathbf{AdS_5} \times \mathbf{S_5}$

● AdS/CFT Correspondence : One of the Profound Structures in String Theory

Type IIB Superstring Theory on $\mathbf{AdS_5} \times \mathbf{S^5}$
$\Updownarrow$ Duality
4-Dim. $\mathcal{N} = 4$ $SU(N)$ Super Yang-Mills Theory

◇ Parameter Correspondence

$$g^2_{\text{YM}} N = 4\pi g_s N = R^4/\alpha'^2$$

★ Balance Between RR-Flux N and Curvature R $\sim$ Large RR-Flux is Crucial.

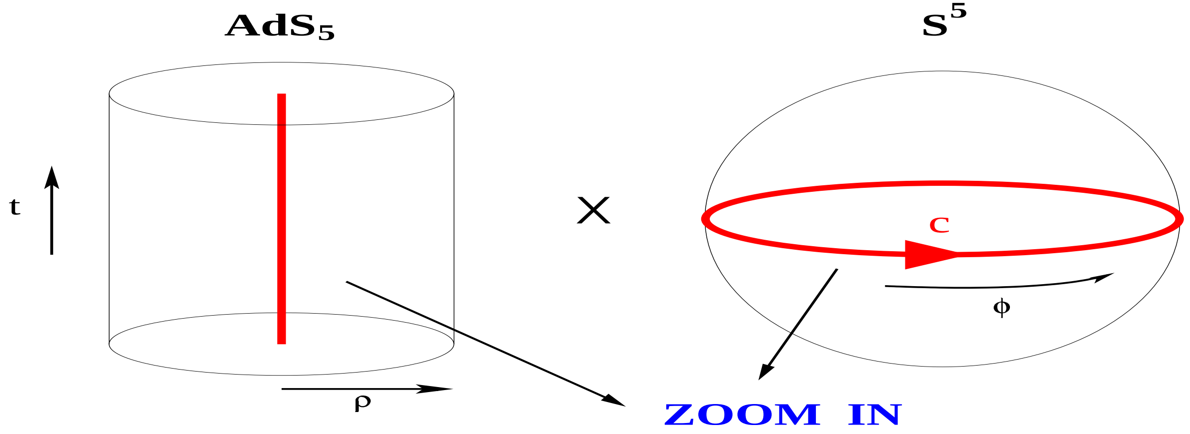
RNS Formulation Has (Severe) Problem with RR-Flux $\Longrightarrow$ Green-Schwarz Formalism

● Green-Schwarz Action on $\mathbf{AdS_5} \times \mathbf{S^5}$ with RR-Flux Has Been Constructed.

E.g. Bosonic Part is a Non-Linear Sigma Model on $\frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}$.

- Interacting and "Massive" Theory $\Longrightarrow$ Left and Right Moving Sectors Couple.

● PP-wave as a Limit of $\mathbf{AdS_5} \times \mathbf{S^5}$ $\Longleftarrow$ Zoom-In of a Null Geodesics



PP-wave Geometry : $ds^2 = 2\,dx^+dx^- - \mu^2 x_I^2\,dx^{+2} + dx_I^2 \quad (I = 1 \sim 8)$,

RR-flux : $F_{+1234} = F_{+5678} = \frac{\mu}{2}$

★ Non-Trivial Curvature and RR-flux is Still There !

The Green-Schwarz Action in Light-Cone Gauge $\Longrightarrow$ Massive Free Field Theory. (Metsaev)

But, ANY String Theory Has Also Massless CFT Description. (Cf. Friedan-Martinec-Shenker)

How Can We Reconcile the “Massive” Picture with Powerful CFT Description ?

$\Longrightarrow$ We Try to Formulate and Quantize the String Theory as EXACT CFT.

2 Canonical Analysis and Conformal Symmetry

- GS Action in a Conformally Inv. Gauge $g_{ij} = \eta_{ij}, \ \gamma^+\theta^A = 0 \ (A = 1, 2)$[1] :

$$\mathcal{L}_{\text{GS}} = \mathcal{L}_{\text{Kin}} + \mathcal{L}_{\text{WZ}},$$

$$\mathcal{L}_{\text{Kin}} = -\frac{T}{2}\eta^{ij}\Big(2\partial_i X^+ \partial_j X^- + \partial_i X^I \partial_j X^I \underbrace{-\mu^2 X^{I2}\partial_i X^+ \partial_j X^+}_{\text{Coupling to Curvature}}\Big)$$

$$+ i T \eta^{ij}\Big(\partial_i X^+ \left(\theta^1 \partial_j \theta^1 + \theta^2 \partial_j \theta^2\right) \underbrace{+2\mu \partial_i X^+ \partial_j X^+ \theta^1 \theta^2}_{\text{Coupling to RR-Flux}}\Big),$$

$$\mathcal{L}_{\text{WZ}} = -i\sqrt{2}T\epsilon^{ij}\partial_i X^+ \left(\theta^1 \partial_j \theta^1 - \theta^2 \partial_j \theta^2\right),$$

- Soln. of EoM $: \partial_+ \partial_- X^+ = 0 \Longrightarrow X^+ = \mathcal{X}_L^+(\sigma_+) + \mathcal{X}_R^+(\sigma_-)$,

$$X^I = \Sigma_n \left(a_n^I u_n + \tilde{a}_n^I \tilde{u}_n\right), \quad \theta^A = \Sigma_n \left(b_n^A u_n + \tilde{b}_n^A \tilde{u}_n\right).$$

$$u_n \ (\tilde{u}_n) = e^{-i\left(\lambda_n^{\pm}\mathcal{X}_R^+ + \lambda_n^{\mp}\mathcal{X}_L^+\right)}, \quad \lambda_n^{\pm} = \frac{\omega_n \pm n}{\alpha' p^+}, \quad \omega_n = \sqrt{M^2 + n^2}.$$

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[1] To Fix κ-Symmetry. This Becomes Free Massive Theory in the LC Gauge $\partial_0 X^+ \propto p^+$.

★ The Solutions are Inseparable Functions of $\boldsymbol{\sigma}^+$ and $\boldsymbol{\sigma}^-$.

Can we Construct PURELY Left (or Right) Moving Virasoro Generator $\mathcal{T}^+(\sigma_+)$?

Virasoro Generators $\mathcal{T}_\pm$ in terms of the Soln.[2] $\Longrightarrow$ $\mathcal{T}_+ = \frac{T}{2}\left(\partial_+ \mathcal{X}_L^+\right)^2 f_+\left(\sigma_+\right)$.

★ Depends on ONLY σ_+ $\Longrightarrow$ However, Completely Different from Flat ($\mu = 0$) Case.

● Hamiltonian Analysis : Brackets for Modes (a_n, b_n) from the ETC for Fields.

$\Longrightarrow$ We Do NOT Know the Completeness for u_n and Can NOT Obtain the Brackets.

◇ Soln. of EoM + $\underbrace{\text{Brackets for } t\text{-Indep. Modes}}_{\text{Not Obtained Here}}$ $\Longrightarrow$ Correlators at Unequal-Times.

However, String Theory Has Conformal Symmetry including the HAMILTONIAN.

★ Virasoro Alg. Based on Fields at $t = 0$ Gives Dynamical Information !

Physical Spectrum $\Longleftarrow$ (Gauge) Constraints

Dynamics $\Longleftarrow$ Construction of Physical Primary Fields

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[2] For Free Boson, $\mathcal{T}_+ \sim \frac{1}{2}\left(\partial_+ \phi_L\right)^2$. f_+ is an Arbitrary Fn. of σ_+

With Dimensionless Fields $(A^\mu(\sigma) \sim X^\mu, B^\mu(\sigma) \sim P^\mu, S^A(\sigma) \sim \theta^A)$ at $t=0$,

$$2\pi\,\mathcal{T}_+ = \tilde{\Pi}^+\tilde{\Pi}^- + \frac{1}{2}\tilde{\Pi}_I^2 + \frac{i}{2}S^2\partial_\sigma S^2 + \frac{\hat{\mu}^2}{2}\tilde{\Pi}^+\Pi^+A_I^2 - \frac{i\hat{\mu}}{\sqrt{2}}\sqrt{\tilde{\Pi}^+\Pi^+}S^1S^2,$$

where $\tilde{\Pi} = \frac{1}{\sqrt{2}}(B + \partial_\sigma A) \sim \partial X$, $\Pi = \frac{1}{\sqrt{2}}(B + \partial_\sigma A) \sim \bar{\partial} X$.

Quantum Op. Requires Ordering $\Longrightarrow$ Phase-Space Normal Ordering for Fourier Modes
★ $A_n\ (n \geq 1), B_n\ (n \geq 0), S_n^A\ (n \geq 1)$ as “Annihilation Operators”.

Except for Central Charge Terms, Quantum Operator Anomalies Appear from

$$C_B = \frac{1}{(2\pi)^2}\left(\left[\frac{1}{2}\tilde{\Pi}_I^2(\sigma),\ \frac{\hat{\mu}^2}{2}\tilde{\Pi}^+\Pi^+A_I^2(\sigma')\right] - (\sigma \leftrightarrow \sigma')\right),$$

$$C_F = \frac{1}{(2\pi)^2}\left[\frac{i\hat{\mu}}{\sqrt{2}}\sqrt{\tilde{\Pi}^+\Pi^+}S^1S^2(\sigma),\ \frac{i\hat{\mu}}{\sqrt{2}}\sqrt{\tilde{\Pi}^+\Pi^+}S^1S^2(\sigma')\right],$$

$$C_B = -C_F = -\frac{i\hat{\mu}^2}{\pi}\left(2\tilde{\Pi}^+\Pi^+\delta'(\sigma-\sigma') + \partial_\sigma(\tilde{\Pi}^+\Pi^+)\delta(\sigma-\sigma')\right).$$

★ These Two Operator Anomalies Exactly Cancel Out !

3 BRS Quantization and Physical Spectrum

BRS Quantization Requires NILPOTENT BRS Charge Q_B

★ Virasoro Generator with Central Charge 26 is Needed.

$\Longrightarrow$ Quantum Correction Term $\Delta\mathcal{T}_+ = -\frac{1}{2\pi}\partial_\sigma^2 \ln \tilde{\Pi}^+$ should be Introduced.

From the Virasoro Generator, $Q_B = \sum_n \left(\tilde{c}_{-n} L_n^+ - \frac{1}{2}\sum_m (m-n)\tilde{c}_{-m}\tilde{c}_{-n}\tilde{b}_{m+n}\right)$.

- Physical States as Q_B-Cohomology

The Decomposition $Q_B = Q_{-1} + Q_0 + Q_{n\geq 1}$ by Light-Cone No.: $\tilde{\Pi}_n^\pm \Rightarrow \pm 1$.

Isomorphism : Q_B-Cohomology $\simeq$ Q_{-1}-Cohomology $\simeq$ $\mathcal{H}_T$ with $L_0^+|\Psi\rangle = 0$

$L_0^+ = L_0^- = 0$ in $\mathcal{H}_T$ $\Longrightarrow$ On-Shell Condition and Level-Matching.

★ On-Shell Condition $H = 0$ Gives Light-Cone Hamiltonian with "Massive Oscillators" :

$$\tilde{\alpha}_n^I = \frac{1}{\sqrt{2}}(B_n^I - \omega_n A_n^I), \quad [\tilde{\alpha}_n^I, \tilde{\alpha}_m^J] = \omega_n \delta^{IJ}\delta_{m+n,0}.$$

◇ Correctly Reproduce the Light-Cone Gauge Spectrum as BRS-Cohomology !

4 Summary and Future Problems

● Summary

- We Have Investigated Both the Classical and Quantum Aspects of Superstring Theory in the PP-Wave Background with a Conformally Invariant Gauge as an Exact CFT.
- Quantum Virasoro Generators[3] are Constructed and the Light-Cone Gauge Spectrum is Correctly Reproduced as the BRS-Cohomology.

I Hope (Interesting) Details will Be Reported Somewhere ! (Here ?)

● Future Problems (Now in Progress)

- Analysis of Global Symmetries : Realization of the PP-Wave Superalgebra.
- $(1, 1)$ Primary Fields and Correlation Fn. $\Longleftarrow$ Flat GS-String as a CFT.
- Application to the BMN and AdS/CFT Correspondence, $\cdots\cdots$

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[3] Purely Bosonic Part and Other Orderings Suffer from Operator Anomalies.