1 1st order phase transition in Maxwell-Chern-Simon QED

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Outline

1995 Kondo & Maris

3-dimensional QED with Chern-Simon term with 4-component fermion

$\theta$ : topological mass

small $\theta$ Parity and Chiral symmetry broken

$\theta_{cr} \rightarrow$ only Parity broken phase and chiral symmetric Phase
Method: Schwinger-Dyson eq in non-local gauge with massless loop correction

2011 Raya tested by Ladder Schwinger-Dyson eq and found $\theta_{cr} \sim 8 \times 10^{-3} e^2$

○ Our case covariant gauge with Ball-Chiu vertex which satisfy Ward-Takahashi-identity

○ Recently Mizutani showed the $\theta_{cr}$? in quenched case removing rolling error in low energy region.

○ Advantage of our case

1 order parameter is gauge invariant & low-energy mass is gauge invariant

( on-shell limit only transverse degree of freedom contribute)

2 $Z$ factor is gauge dependent
My results

Schwinger-Dyson eq with massless loop without vertex correction is solved in Landau gauge

and find the consistent solution.

infrared problem does not arise: massless loop soften the photon propagator as $1/p$.

with vertex correction: now in progress

Ward-Takahashi-identity by gauge invariance

S-D(equation of motion)

$$S_F^{-1}(p) = S_F^{(0)-1} - ie^2 \int \frac{d^3k}{(2\pi)^3} \Gamma_\mu(p, k) S_F(k) \gamma_\nu D_F^{\mu\nu}(p - k)$$

$$= A(p) p \cdot \gamma - B(p)$$

$$(p - q) \nu S_F'(p) \Gamma_\nu(p, q) S_F'(q) = S_F'(p) - S_F'(q)$$
2 Ball-Chiu Ansatz (1980)

Bashir, Pennington (1994)

\[ S_F^{-1}(p) = A(p) \gamma \cdot p - B(p) \]
\[ \Gamma^L_{\mu}(p, q) = \Gamma^L_{\mu}(p, q) + \Gamma^T_{\mu}(p, q) \]
\[ (p - q)_{\mu} \Gamma^L_{\mu}(p, q) = S_F^{-1}(q) - S_F^{-1}(p) \]
\[ (p - q)_{\mu} \Gamma^T_{\mu}(p, q) = 0 \]

Transverse vertex does not disturb the W-T identity.

Assume

\[ \Gamma^L_{\mu}(p, q) = a(p, q) \gamma_{\mu} + b(p, q)(p+q) \cdot \gamma(p+q)_{\mu} - c(p, q)(p+q)_{\mu} \]

solution
\[
\Gamma^L_{\mu}(p, q) = \frac{A(p) + A(q)}{2} \gamma_\mu + \frac{A(p) - A(q)}{2(p^2 - q^2)} (p + q) \cdot \gamma(p + q)_\mu \\
- \frac{B(p) - B(q)}{p^2 - q^2} (p + q)_\mu
\]

### 3 Chirality & Parity in (2+1)-dimension

\[\gamma_\mu = \{\gamma_0, \gamma_1, \gamma_2\}, \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \gamma_3, \gamma_5 \text{ Dirac representation}\]

\[\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},\]

(1)

\[\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\]

(2)

chiral transform \[\psi \mapsto \exp(i\alpha\gamma_3)\psi, \psi \mapsto \exp(i\beta\gamma_5)\psi\]
\[ \bar{\psi} \psi \rightarrow \cos(2\alpha) \bar{\psi} \psi (1) \psi + i \sin(2\alpha) \bar{\psi} (\gamma_3 \text{or} \gamma_5) \psi \]

\( m_e \bar{\psi} \psi \) is not singlet and violate chiral symmetry.

\[ \sigma = \bar{\psi} \psi, \pi = \bar{\psi} (\gamma_3 \text{or} \gamma_5) \psi, \]

\( \circ \) \( \sigma \) plays the role of Higgs and \( \pi \) restores symmetry as in the \( \sigma \) model.

Rotator or symmetric top.

Another mass \( m_o \bar{\psi} \pi \psi \) (spin density), \( \tau = [\gamma_3, \gamma_5]/2 \)

\( \bar{\psi} \psi \) is singlet under Parity \( \psi(x_0, x_1, x_2) = P \psi(x_0, -x_1, x_2) \),

\[ P = \gamma_1 \gamma_5 \]

\( \bar{\psi} \pi \psi \rightarrow -\bar{\psi} \pi \psi \) but singlet under chiral transform

\[ m = m_e + \tau m_o \]
Solving Dirac equation $\psi \mapsto \psi_+ + \psi_-$, chiral representation 2-spinor with $m_+, m_-$

$$m = m_e \begin{pmatrix} I_2 & 0 \\ 0 & I_2 \end{pmatrix} + m_o \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} = \begin{pmatrix} m_+ & 0 \\ 0 & m_- \end{pmatrix}$$

$$L = \bar{\psi}_+(x)(i\partial \cdot \gamma - m_+)\psi_+(x)$$
$$+ \bar{\psi}_-(x)(i\partial \cdot \gamma - m_-)\psi_-(x) \quad (3)$$

Propagator can be decomposed

$$S(p) = \frac{-1}{\gamma \cdot pA(p) - B(p)} = \frac{\gamma \cdot pA_+(p) + B_+(p)}{A_+(p)^2 p^2 + B_+(p)^2} \chi_+$$
$$+ \frac{\gamma \cdot pA_-(p) + B_-(p)}{A_-(p)^2 p^2 + B_-(p)^2} \chi_- \quad (4)$$

$$\chi_+ = (1 + \tau)/2 = \begin{pmatrix} I_2 & 0 \\ 0 & 0 \end{pmatrix}, \chi_- = (1 - \tau)/2 = \begin{pmatrix} 0 & 0 \\ 0 & I_2 \end{pmatrix}$$
S-D equation split into 2 spinor($S_+, S_-$) with full $\Gamma_\mu(p, q)$

2-kinds of mass:$m_e = (m_+ + m_-)/2, m_0 = (m_+ - m_-)/2$

4 Maxwell-Chern-Simon QED

$$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{\theta}{4}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}A_\rho + ...$$

$F \times A$ : Parity violating current

$$J_\nu = \partial_\mu F_{\mu\nu} + \frac{\theta}{2}\epsilon_{\alpha\beta\nu}F_{\alpha\beta}$$

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu} - p_\mu p_\nu/p^2 - i\theta\epsilon_{\mu\nu\rho\sigma}p_\rho/p^2}{p^2 - \theta^2 + i\epsilon} + \xi \frac{p_\mu p_\nu}{p^4}$$
\( \theta \) may be corrected by vacuum polarization

\[
\Pi_e = \frac{e^2}{8} \sqrt{k^2} (m = 0)
\]

massless loop \( c = e^2/8 \)

\[
D_e(k) = \frac{k^2 + c\sqrt{k^2}}{(k^2 + c\sqrt{k^2})^2 + \theta^2 k^2}, \quad (5)
\]

\[
D_O(k) = \frac{-\theta\sqrt{k^2}}{(k^2 + c\sqrt{k^2})^2 + \theta^2 k^2}. \quad (6)
\]

- use of complex number

\[
D_e(k) = \text{Re}\left( \frac{1}{k^2 + (c + i\theta)\sqrt{k^2}} \right),
\]

\[
D_O(k) = \text{Im}\left( \frac{1}{k^2 + (c + i\theta)\sqrt{k^2}} \right). \quad (7)
\]

is helpful to perform angular integration in S-D equation.
D Landau gauge with bare photon mass $\theta$

$$B_{\pm}(p) = \frac{e^2}{4\pi^2} \int_0^\infty dq q^2 \frac{dq}{2} \frac{dqq^2}{q^2 A_{\pm}(q)^2 + B_{\pm}(q)^2} \times [2(B_{\pm}(q)I_0(p, q) \mp \theta A_{\pm}(q)I_2(p, q_-)]].$$

$$p^2(A_{\pm}(p) - 1) = \frac{e^2}{4\pi^2} \int_0^\infty dq q^2 \frac{dq}{2} \frac{dqq^2}{q^2 A_{\pm}(q)^2 + B_{\pm}(q)^2} \times [2(A_{\pm}(q)I_3(p, q) \mp \theta B_{\pm}(q)I_2(p, q_+)]]$$

angular integral replace $\theta \to c + i\theta$

$$I_0(p, q) = \frac{-1}{2pq} \ln\left(\frac{(p - q)^2 + \theta)^2}{(p + q)^2 + \theta)^2}\right) \to \text{Re}(I_0(p, q, c, \theta))$$

$$I_2(p, q)_\pm = \frac{-1}{4pq} \ln\left(\frac{(p - q)^2 + \theta)^2}{(p + q)^2 + \theta)^2}\right)$$

$$\pm \frac{p^2 - q^2}{4\theta^2 pq} \ln\left(\frac{1 + \theta^2/(p - q)^2}{1 + \theta^2/(p + q)^2}\right) \to \text{Im}(I_2(p, q, c, \theta))$$

$$I_3(p, q) = \frac{(p^2 - q^2)^2}{8\theta^2 pq} \ln\left(\frac{1 + \theta^2/(p - q)^2}{1 + \theta^2/(p + q)^2}\right) - \frac{1}{2} - \frac{\theta^2}{8pq} \ln\left(\frac{(p - q)^2 + \theta^2}{(p + q)^2 + \theta^2}\right) \to \text{Re}(I_3(p, q, c, \theta)).$$
Add vertex correction terms.

5 Numerical results shown later

\[ 10^{-5} \leq p \leq 10^5, \, B_{\pm}(p), \, A_{\pm}(p), \, 50 \, \text{Digit} \]

We searched the value of \( \theta_{cr} \) where \( \langle \bar{\psi}\psi \rangle \simeq 0 \)

\[ \langle \bar{\psi}\psi \rangle_e \simeq 0 \leftrightarrow \langle \bar{\psi}\psi \rangle_e \leq 10^{-5}, \text{ at } \theta = 0 \, \langle \bar{\psi}\psi \rangle \simeq 3 \times 10^{-3} e^4, \]

at \( \theta \simeq 1 \times 10^{-2} e^2, \, \langle \bar{\psi}\psi \rangle_e \simeq 2 \times 10^{-5} , \text{ without vertex correction: Parity broken and Chiral symmetric} \)

\( \theta \) dependence is slow in case with massless loop.

quenched case it is clear to see

Phase transition is not discussed. We need more dynamics as effective potential.
6 Summary

- $\theta = 0$ QED3 gauge invariant & chiral symmetry breaking: Maris et al
- 4-comonent case: small $\theta \leftrightarrow$ chiral symmetry & Parity broken

$m_e \neq 0$, small $m_o \neq 0$

- $\theta_{cr} \leftrightarrow$ Parity broken & chiral symmetric $m_e = 0, m_+ = -m_-, m_o \neq 0$(Kondo & Maris, 1995) non-local gauge quenched Landau gauge $\theta_{cr} = 8 \times 10^{-3} e^2$ (Raya, 2011)

vertex correction Parity broken, chiral symmetric ($MIZUTANI$) $\theta_{cr} \sim 8 \times 10^{-3} e^2$

My case massless loop + Landau gauge $\theta_{cr} \sim 10^{-2} e^2$
with vertex correction of BC, $\langle \overline{\psi}\psi \rangle_e \neq 0$ for large $\theta$?

Chirality and Parity both broken

Now in progress

infrared behavior is soft by massless loop correction.

Our study has relations to finite density case.

with chemical potential which violates parity, C-S is induced by anomaly.