Black Holes and the Fluctuation Theorem

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with Satoshi Iso and Sen Zhang, arXiv:1008.1184

+ work in progress

Backgrounds

Gravity

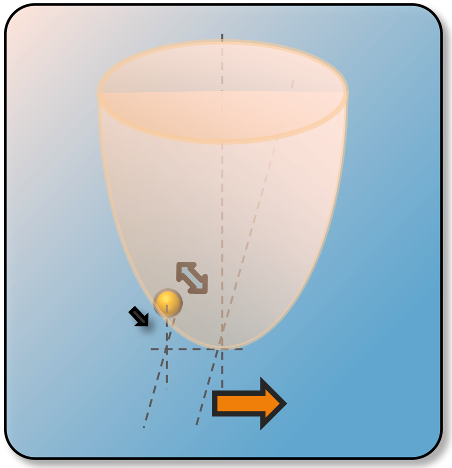
- Black hole thermodynamics $\begin{cases} \text{1st law}: \frac{\kappa}{8\pi G}\Delta A_{BH} = \Delta M \\ \text{2nd law}: \Delta A_{BH} \geq 0 \end{cases}$

- Hawking radiation
  - classical gravity + quantum field
  - Planck distribution with temperature $T_H = \dfrac{1}{8\pi GM}$

$$\Rightarrow \begin{cases} \text{1st law}: T_H \Delta S_{BH} = \Delta M \\ \text{generalized 2nd law}: \Delta S_{BH} + \Delta S_{\text{matter}} \geq 0 \end{cases}$$

- What about non-equilibrium?
  - Asymptotically flat Schwarzschild BHs are unstable system, “negative specific heat”

Backgrounds



Non-equilibrium physics

- The Fluctuation Theorem [Evans, Cohen & Morris '93]
  - non-equilibrium fluctuations satisfy

$$\frac{\text{Prob}(\text{entropy difference} = \Delta S)}{\text{Prob}(\text{entropy difference} = -\Delta S)} = e^{\Delta S}$$

  - violation of the second law of thermodynamics

The fluctuation theorem		The second law of thermodynamics
$\frac{\rho(\Delta S)}{\rho(-\Delta S)} = e^{\Delta S}$	$\supset$	$\Delta S \geq 0$
Microscopic, Equality		**Macroscopic, Inequality**

  - A bridge between microscopic theory with **time reversal symmetry** and **the second law of thermodynamics**

Motivations

- We want to investigate non-equilibrium nature of black holes
  - cf. Einstein's theory of Brownian motion, fluctuations are important
  - Analyzing an asymptotically flat BH as a thermodynamically unstable system

- We apply the fluctuation theorem to BH with matter
  - We expect to see entropy decreasing probability
  - We expect to get the generalized second law as a corollary

- Ambitions
  - Information loss problem
  - Connection between Jacobson's idea "the Einstein equation of states"
  - Application for gauge/gravity duality

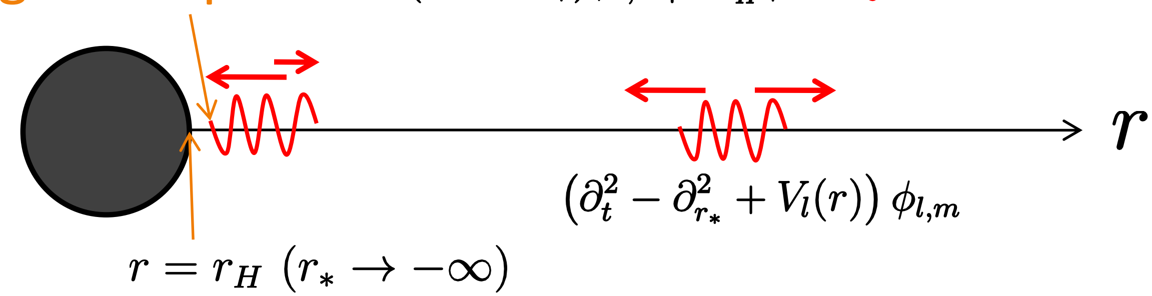
Outline

- We will consider a spherically symmetric BH with scalar field
- Construct an effective EOM of scalar field near the horizon

cf. real -time AdS/CFT [Herzog, Son '03 , Son, Teaney '09]

$\langle (\partial_t - \partial_{r_*})\phi \rangle \vert_{r_H+\epsilon} = 0$	Ingoing boundary condition **(friction)**
$\langle \xi(t) \rangle = 0\ ,\ \langle \xi(t)\xi(t') \rangle \simeq 2\dfrac{T_H}{\eta}\delta(t-t')$	Hawking radiation **(noise)**

Langevin equation $(\partial_t - \partial_{r_*})\phi_{l,m}|_{r=r_H+\epsilon} = \xi$



Outline

- By using 1st law $\frac{1}{T_H}\int_0^{\tau} dt A_{BH} T_t^r(r_\epsilon) = \Delta S_{BH}$

$$\frac{\rho(\Delta S_{BH}+\Delta S_M)}{\rho(-(\Delta S_{BH}+\Delta S_M))} = e^{\Delta S_{BH}+\Delta S_M}$$

- Microscopic
- Equality

The fluctuation theorem for BH and matter

$$\cup$$

$$\langle(\Delta S_{BH} + \Delta S_M)\rangle \geq 0$$

- Macroscopic
- Inequality

The generalized second law

- Green-Kubo relation $\bar{J} = L^{(1)}(\beta_H - \beta) + L^{(2)}(\beta_H - \beta)^2 + \cdots$

$$L^{(1)} = \frac{1}{2}\int_0^{\infty} dt A_{BH}^2 \langle T_t^r(0, r_\epsilon) T_t^r(t, r_\epsilon)\rangle|_{T=T_H}$$

Response — correlation

⇒ Extension to non-linear response