Black Holes and the Fluctuation Theorem

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+ work in progress
Backgrounds

Gravity

◆ Black hole thermodynamics

\[ \begin{align*}
\text{1st law} : & \quad \frac{k}{8\pi G} \Delta A_{BH} = \Delta M \\
\text{2nd law} : & \quad \Delta A_{BH} \geq 0
\end{align*} \]

◆ Hawking radiation

– classical gravity + quantum field
– Planck distribution with temperature

\[ T_H = \frac{1}{8\pi GM} \]

\[ \begin{align*}
\text{1st law} : & \quad T_H \Delta S_{BH} = \Delta M \\
\text{generalized 2nd law} : & \quad \Delta S_{BH} + \Delta S_{\text{matter}} \geq 0
\end{align*} \]

◆ What about non-equilibrium?

– Asymptotically flat Schwarzschild BHs are unstable system, “negative specific heat”
Backgrounds

Non-equilibrium physics

◆ The Fluctuation Theorem [Evans, Cohen & Morris ’93]
  – non-equilibrium fluctuations satisfy
    \[
    \frac{\text{Prob(entropy difference } = \Delta S')}{\text{Prob(entropy difference } = -\Delta S')} = e^{\Delta S}
    \]
  – violation of the second law of thermodynamics

The fluctuation theorem
\[
\frac{\rho(\Delta S)}{\rho(-\Delta S)} = e^{\Delta S}
\]
Microscopic, Equality

The second law of thermodynamics
\[\Delta S \geq 0\]
Macroscopic, Inequality

– A bridge between microscopic theory with **time reversal symmetry**
and the **second law of thermodynamics**
Motivations

- We want to investigate **non-equilibrium** nature of black holes
  - cf. Einstein’s theory of Brownian motion, **fluctuations are important**
  - Analyzing an asymptotically flat BH as a thermodynamically **unstable** system

- We apply the fluctuation theorem to BH with matter
  - We expect to see **entropy decreasing probability**
  - We expect to get **the generalized second law as a corollary**

Ambitions

- Information loss problem
- Connection between Jacobson’s idea “the Einstein equation of states”
- Application for gauge/gravity duality
Outline

- We will consider a spherically symmetric BH with scalar field

- Construct an effective EOM of scalar field near the horizon

\[ \langle (\partial_t - \partial_{r*})\phi \rangle |_{r_H+\epsilon} = 0 \]

**Ingoing boundary condition (friction)\]**

\[ \langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle \sim 2\frac{T_H}{\eta}\delta(t-t') \]

**Hawking radiation (noise)**

\[ (\partial_t - \partial_{r*})\phi_{i,m}|_{r=r_H+\epsilon} = \xi \]

Langevin equation

\[ r = r_H \quad (r_* \to -\infty) \]

\[ (\partial_t^2 - \partial_{r*}^2 + V_i(r)) \phi_{l,m} \]
Outline

- By using 1st law \[ \frac{1}{T_H} \int_0^T dt A_{BH} T_t^r(r_\epsilon) = \Delta S_{BH} \]

\[ \frac{\rho(\Delta S_{BH} + \Delta S_{M})}{\rho(-(-\Delta S_{BH} + \Delta S_{M}))} = e^{\Delta S_{BH} + \Delta S_{M}} \]

The fluctuation theorem for BH and matter

\[ \langle (\Delta S_{BH} + \Delta S_{M}) \rangle \geq 0 \]

The generalized second law

- Green-Kubo relation \[ \bar{J} = L^{(1)}(\beta_H - \beta) + L^{(2)}(\beta_H - \beta)^2 + \cdots \]

\[ L^{(1)} = \frac{1}{2} \int_0^\infty dt A_{BH}^2 \langle T_t^r(0, r_\epsilon) T_t^r(t, r_\epsilon) \rangle |_{T=T_H} \]

Response correlation

Extension to non-linear response