Topological Phases of Eternal Inflation

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The problem addressed in this work:

What happens when gravity is coupled to a theory with metastable vacuum?

- e.g. scalar field which has a false vacuum and a true vacuum
  \[ V(\Phi_F) > 0, \quad V(\Phi_T) = 0 \]

- If we ignore gravity, first order phase transition:
  - Nucleation of bubbles of true vacuum (Callan, Coleman, ...)
  - The whole space eventually turns into true vacuum.
Bubble of true vacuum

- Described by Coleman-De Luccia instanton (Euclidean “bounce” solution).
  Nucleation rate: \( \Gamma \sim e^{-(S_{\text{cl}} - S_{\text{deSitter}})} \)

- Open FRW universe inside a bubble:
  Spatial slice: 3D hyperboloid
  \[ ds^2 \sim -dt^2 + t^2 (dR^2 + \sinh^2 R d\Omega^2) \]

- If \( \Gamma \ll H^4 \), bubble nucleation cannot catch up the expansion of space, and false vacuum exists forever (“Eternal Inflation”)

\[ \]
View from the future infinity

• Consider conformal future infinity of de Sitter.

\[ ds^2 \sim \frac{-d\eta^2 + d\vec{x}^2}{H^2 \eta^2} \quad (-\infty < \eta < 0) \]

• A bubble: represented as a sphere cut out from de Sitter.

• “Scale invariant” distribution of bubbles

Bubbles nucleated earlier:
appear larger: radius \( \sim H^{-3} |\eta|^3 \)
rarer: volume of nucleation sites \( \sim |\eta|^{-3} \)
Model for eternal inflation

- Mandelbrot model (Fractal percolation)
  - Start from a white cell. (One horizon volume of inflating region)
  - Divide the cell into cells with half its linear size. (The space grows by a factor of 2.)
  - Paint each cell in black with probability P. (Bubble is nucleated and takes up a horizon volume. $P \sim \Gamma$)
  - Subdivide the surviving (white) cells, and paint cells in black w/ probability P. Repeat this infinite times.
Three phases of eternal inflation

From the result on the 3D Mandelbrot model


In order of increasing $P$ (or $\Gamma$), there are

(white = inflating, black = non-inflating)

• **Black island phase**: Black regions form isolated clusters; $\exists$ percolating white sheets.

• **Tubular phase**: Both regions form tubular network; $\exists$ percolating black and white lines.

• **White island phase**: White regions are isolated; $\exists$ percolating black sheets.
Spacetime inside the (cluster of) bubbles
Black island phase (isolated cluster of bubbles)

Small deformations of open FRW universe.

- Basic fact: A collision of two bubbles (of the same vacuum) does not destroy the bubble [c.f. Bousso, Freivogel, Yang, ‘07]
  - Spatial geometry approaches smooth $H^3$ at late time.
  - Residual symmetry $SO(2,1)$: spatial slice has $H^2$ factor
  - Negative curvature makes the space expand.

- Local geometry near collision will be similar to the two bubble case even when many bubbles collide.
Tubular phase (tube-like structure of bubbles)

In the late time limit: spatial slice is a negatively curved space whose boundary has infinite genus.

Late time geometry: \[ ds^2 = -dt^2 + t^2 ds^2_{H/\Gamma} \]

– Spatial geometry: \( H^3 \) modded out by discrete elements of isometry
– Boundary genus = # of elements
– The whole space is accessible to a single observer.

Genus 1 case
• Simpler example: true vacuum with toroidal boundary
  [Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, ‘08]

Consider ring-like initial configuration of bubbles
Late time geometry: negatively curved space with toroidal boundary.

\[ ds^2 = -f(t) dt^2 + f^{-1}(t) dz^2 + t^2 dH_2^2 \]
\[ f(t) = 1 + t^2 / \ell^2 \quad \text{(de Sitter)} \]
\[ f(t) = 1 - t_n / t \quad \text{(in region } n; \ t_n: \text{ const.)} \]

\[ f_{n+2}(t_{*,n+2}) f_n(t_{*,n+2}) = (f_{n+1}(t_{*,n+2}))^2 \]
\[ t_{*,n}: \text{ time of the } n\text{-th collision} \]
White island phase (isolated inflating region)

An observer in the black region is “surrounded” by the white region (contrary to the intuition from Mandelbrot model).

- Simple case: two white islands (with $S^2$ symmetry)
  [Kodama et al ’82, BFSSSYY ’08]

- An observer can see only one boundary; the other boundary is behind the black hole horizon. [c.f. “non-traversability of a wormhole”, “topological censorship”]
Summary

Three phases of eternal inflation and their cosmology:

• Black island phase:
  Small deformation of an open FRW

• Tubular phase:
  Negatively curved space with an infinite genus boundary

• White island:
  Observer sees one boundary and one or more black hole horizons (behind which there are other boundaries).