Generation, Quark/Lepton Mass Hierarchy and Flavor Mixing from Point Interactions in an Extra Dimension

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soon with 高橋 亮（阪大-->北海道大）

Mysteries of the Standard Model

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  Why so different the structure of flavor mixing is between quark and lepton !?

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in the context of 5d gauge theories on a circle.

Ideas & Features

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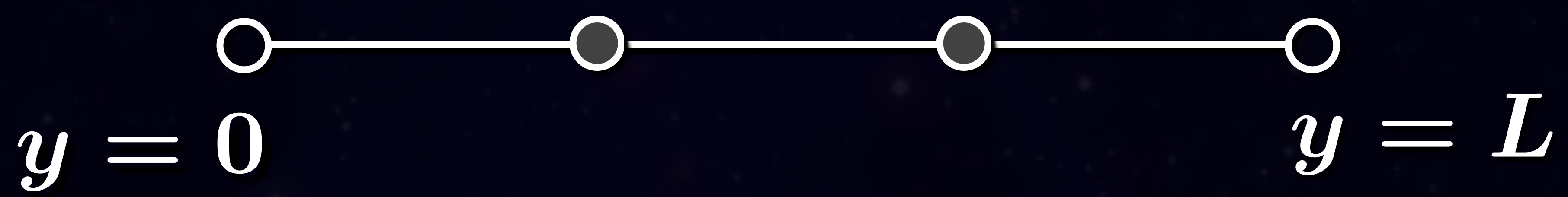

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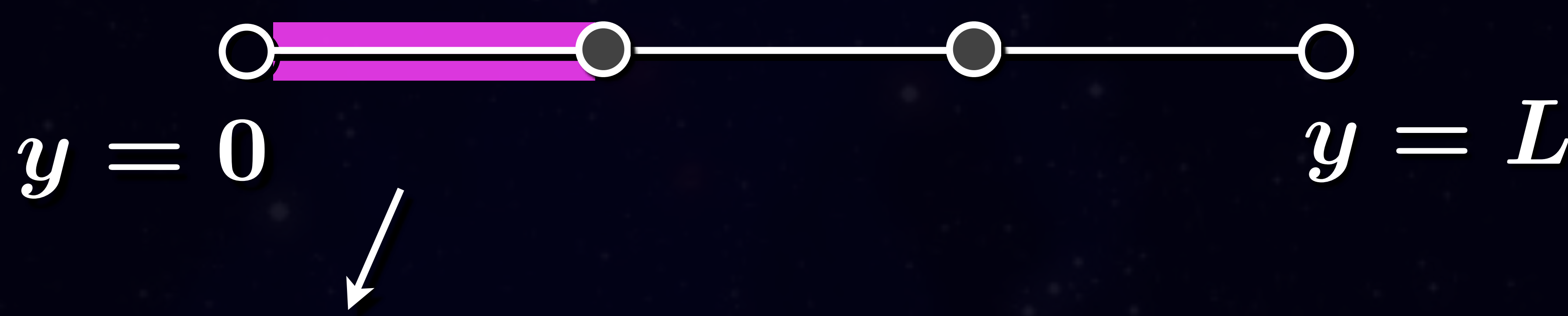

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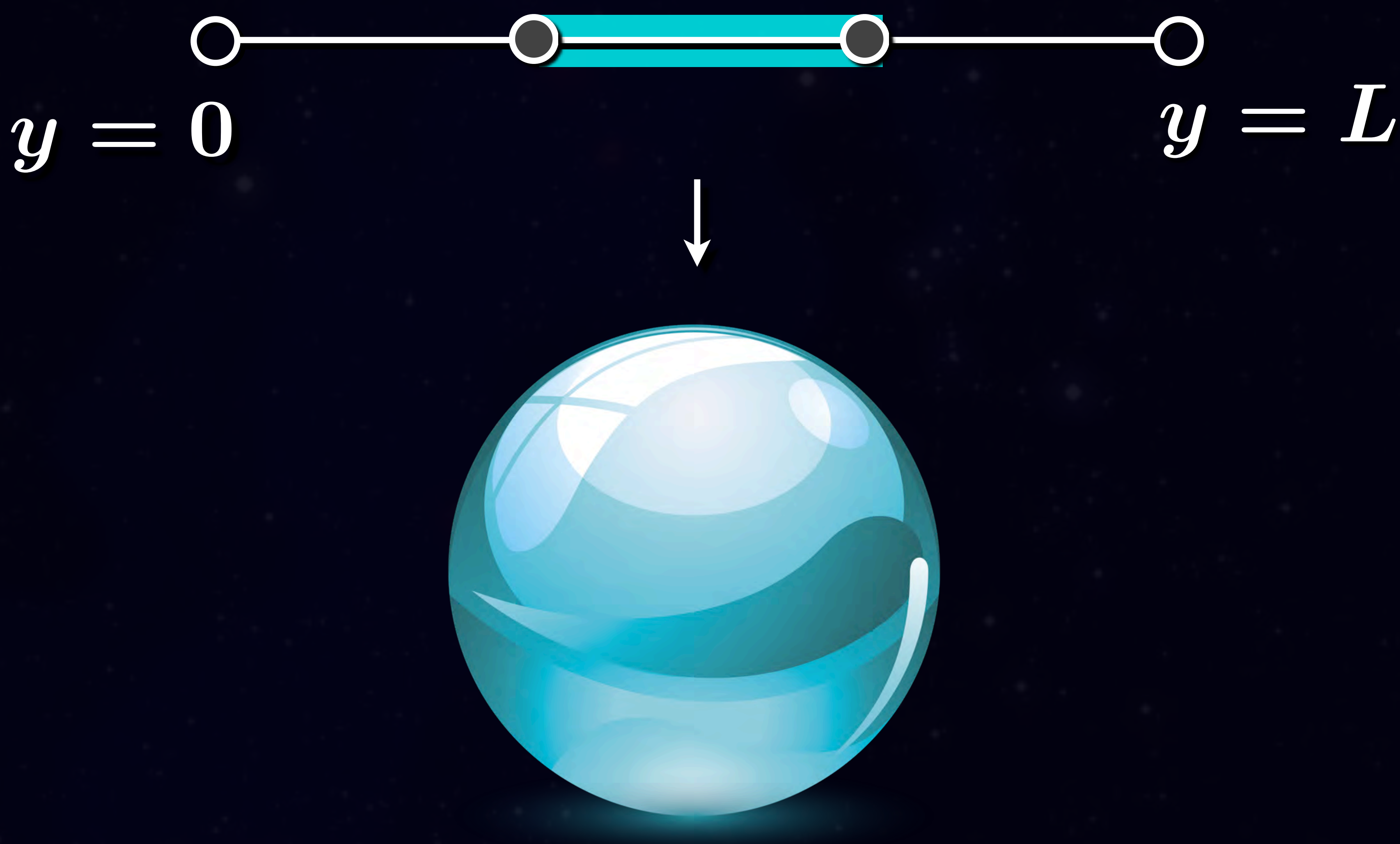

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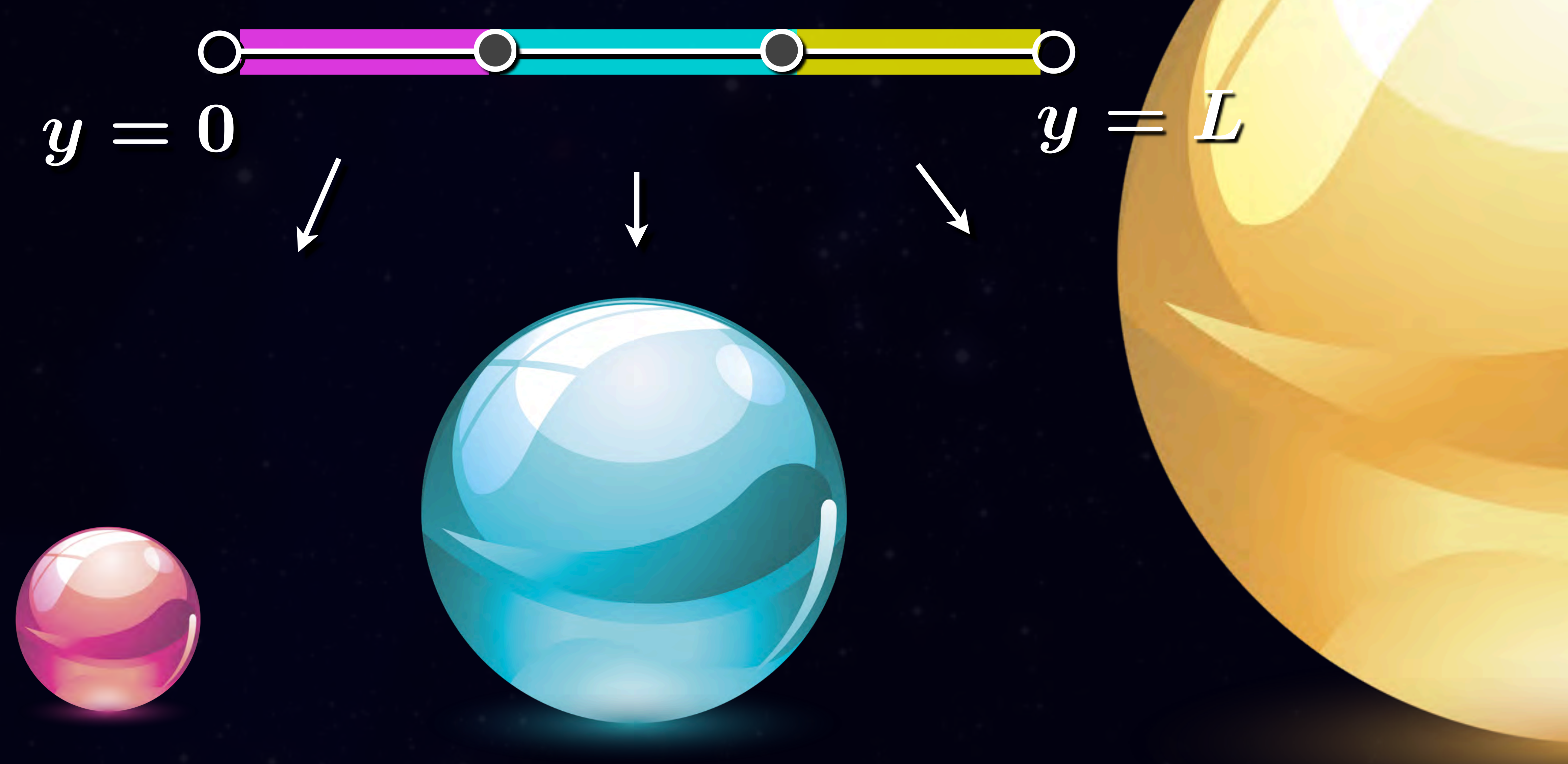

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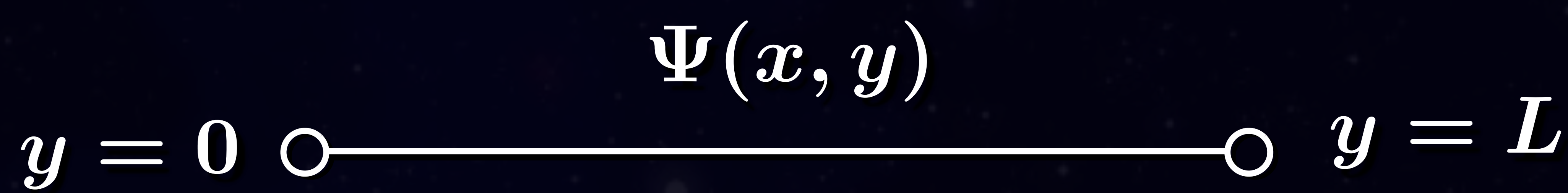

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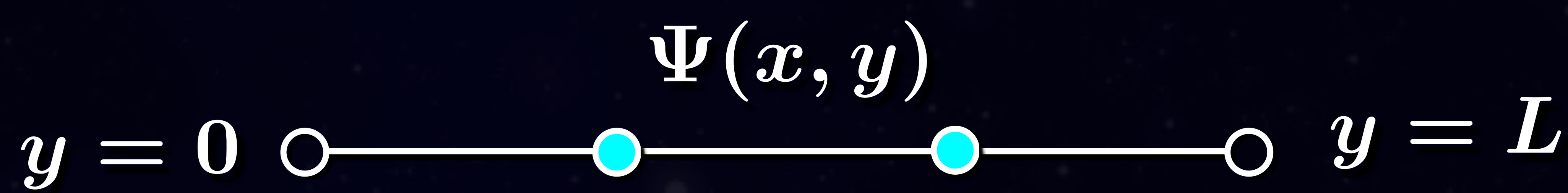

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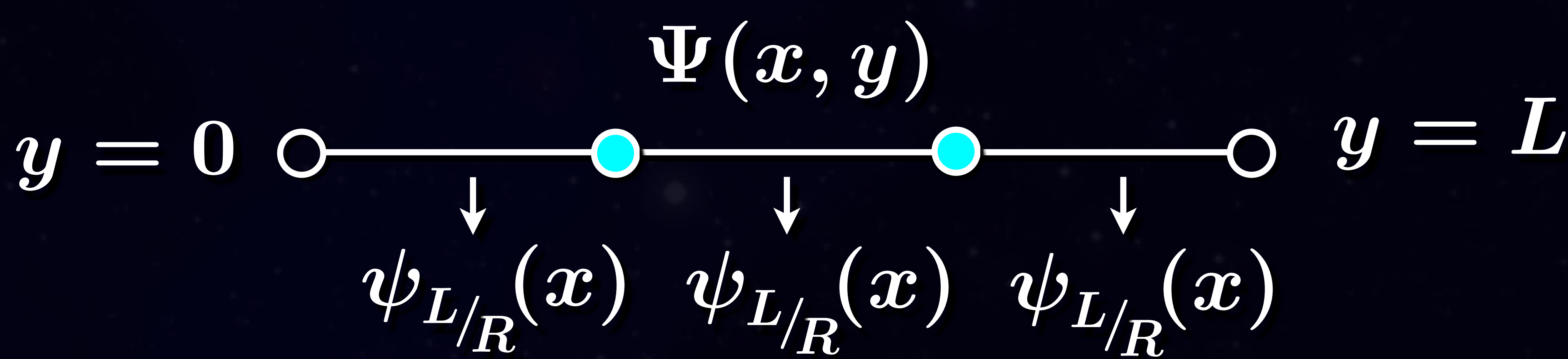

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- ★ Flavor mixing is determined by a configuration of extra dimension.

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- **5d gauge theory on a circle**

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with $\left\{ \begin{array}{l} \textbf{5d fermions (one generation)} \\ \textbf{5d Higgs field \& singlet scalar} \end{array} \right.$

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◆ 5d gauge theory on a circle

with { 5d fermions (one generation)
5d Higgs field & singlet scalar

Gauge fields

$$W_M^a(x,y)$$
$$B_M(x,y)$$

Higgs field
Singlet scalar

$$H(x,y)$$
$$\Phi(x,y)$$

Fermions

$$\begin{pmatrix} u(x,y) \\ d(x,y) \end{pmatrix} \quad \begin{pmatrix} e(x,y) \\ \nu(x,y) \end{pmatrix}$$

$$u'(x,y) \qquad e'(x,y)$$
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Higgs field
Singlet scalar

$H(x,y)$ ← 5D Higgs (break the gauge sym.)

$\Phi(x,y)$ ← Gauge singlet scalar field (for fermion mass hierarchy)

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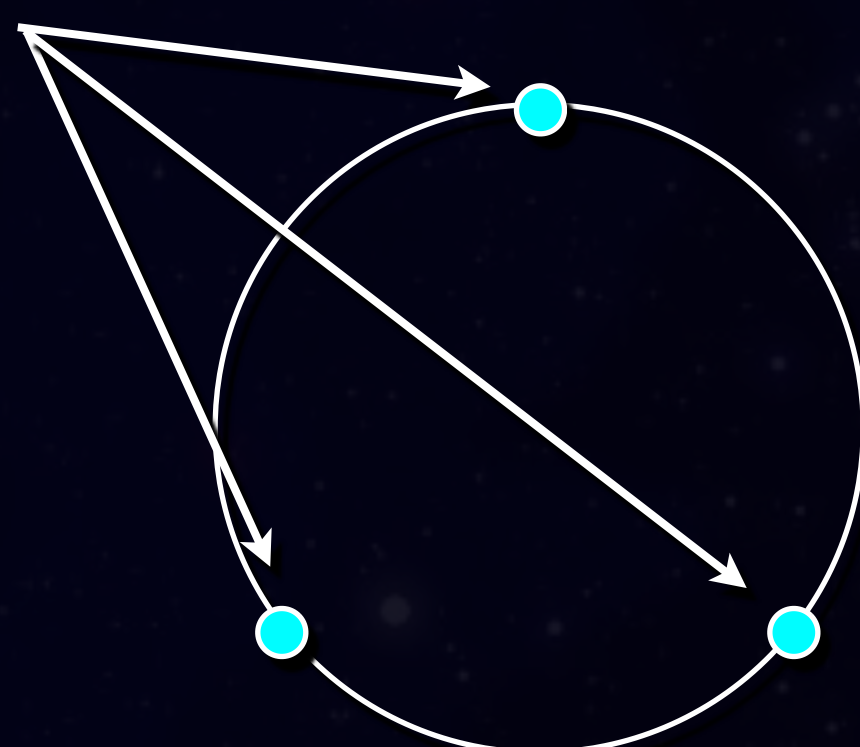

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compatible with several requirement as

★ action principle etc.

Boundary Conditions (BCs)

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◆ Gauge field

$$\begin{cases} \partial_y A_\mu(x,y) = 0 \\ A_y(x,y) = 0 \end{cases} \quad @\ y = L_0, L_3$$

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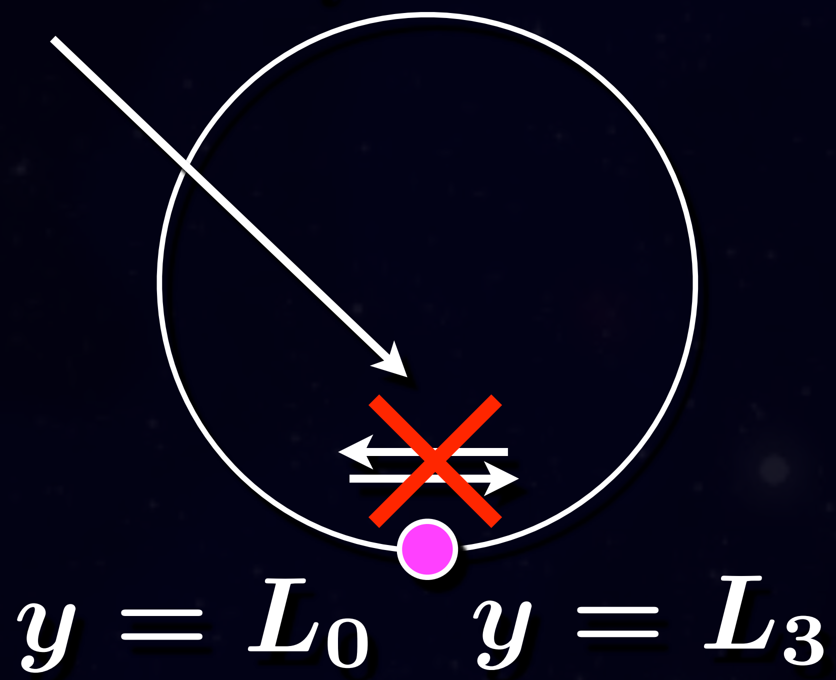

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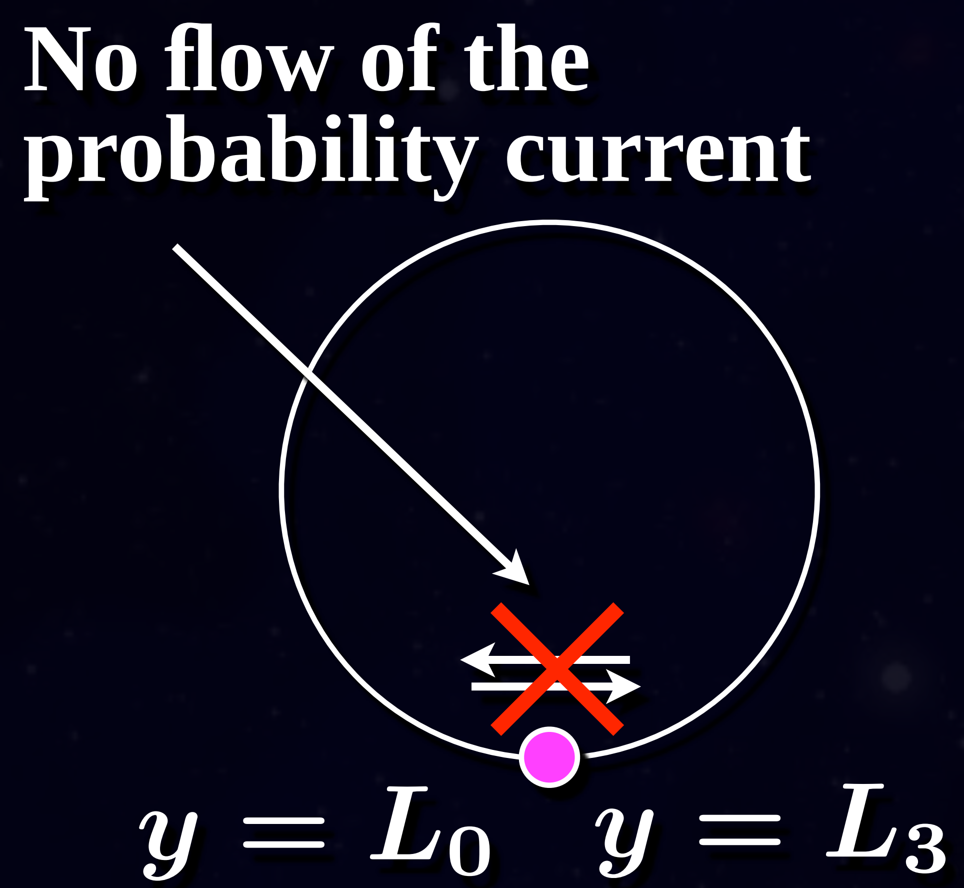
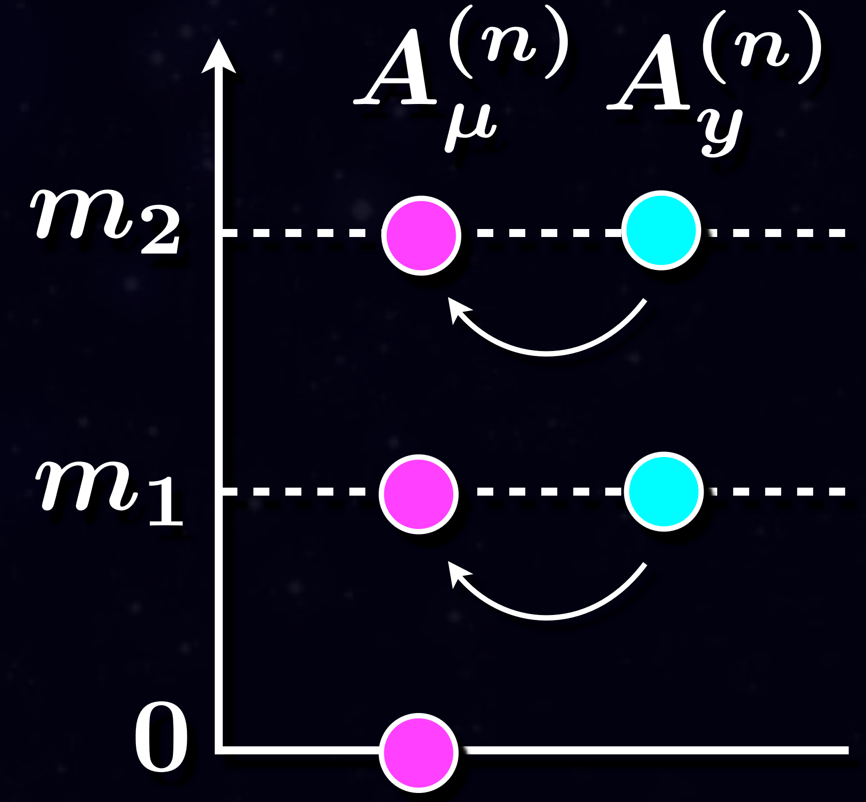


★ **4d spectrum**



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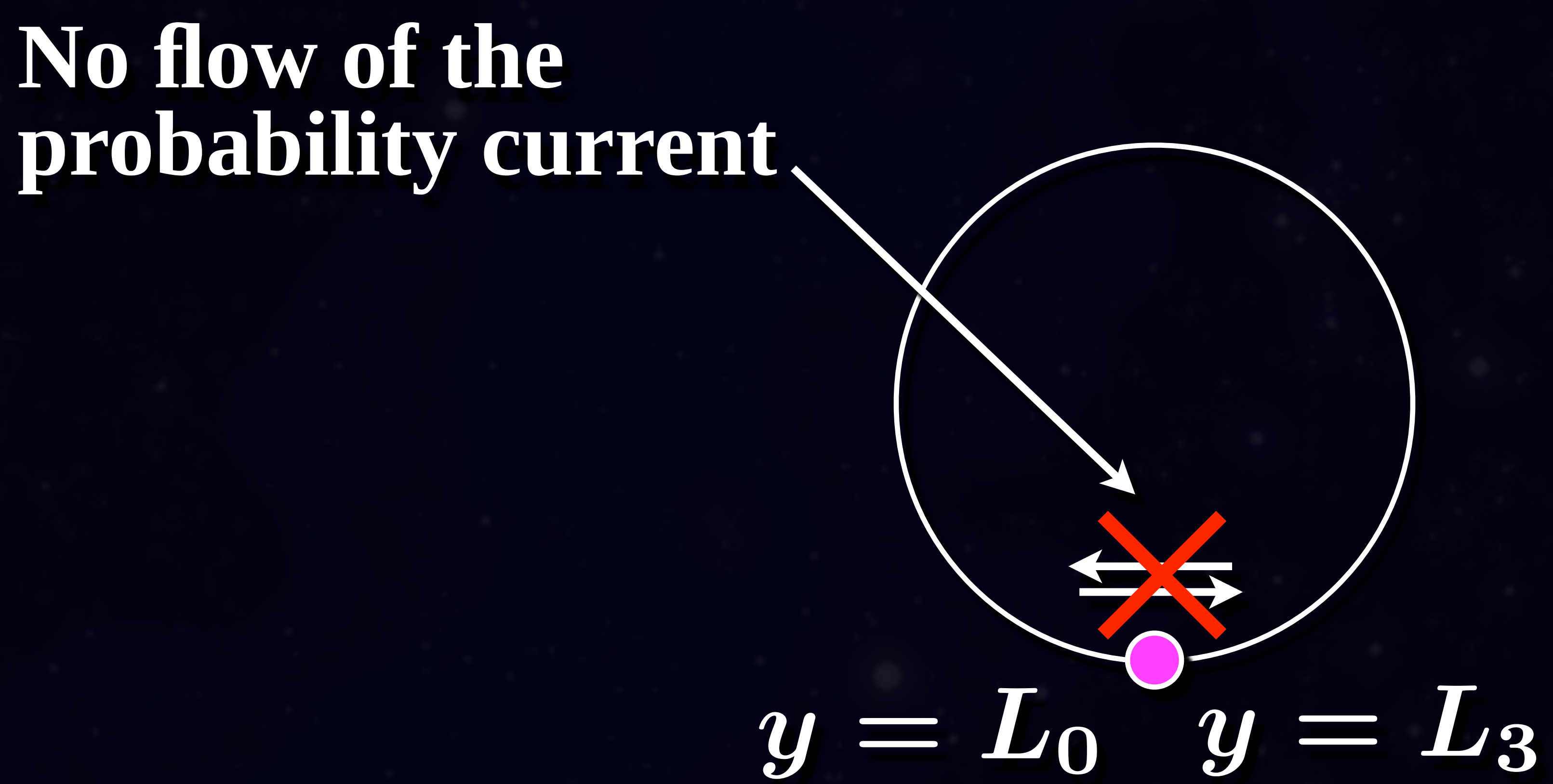

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$$\begin{cases} \Phi(x, L_0) + L_+ \partial_y \Phi(x, L_0) = 0 \\ \Phi(x, L_3) - L_- \partial_y \Phi(x, L_3) = 0 \end{cases}$$

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$y = L_0$ $\quad$ $y = L_3$

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$\Psi_R(x, y) = 0$ **@ point interactions**

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$\Psi_L(x, y) = 0$ **@ point interactions**

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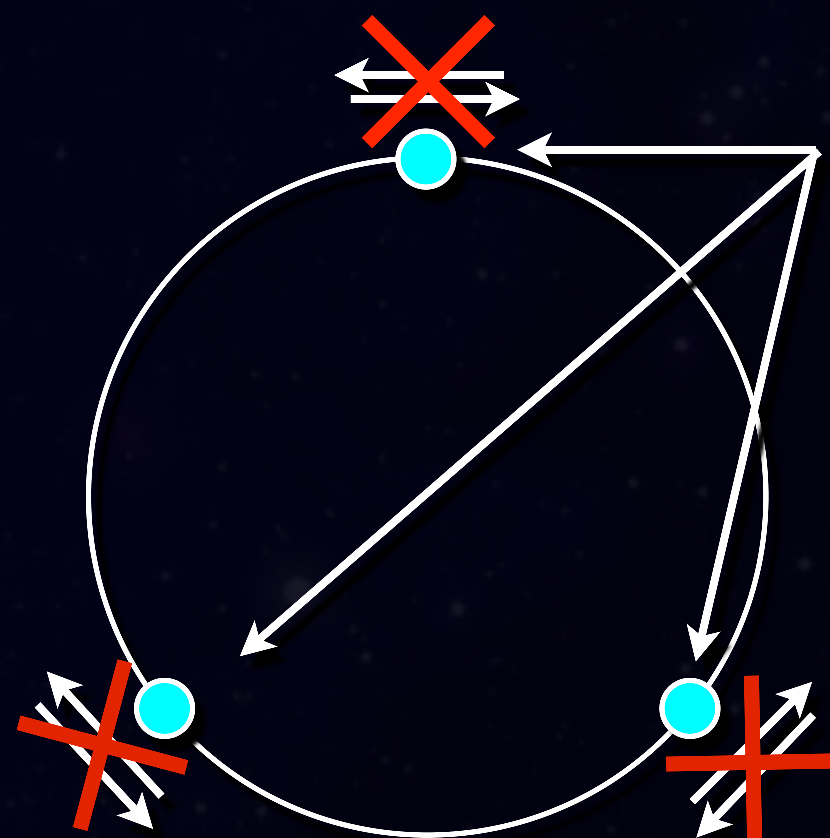

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5d gauge theories on a circle
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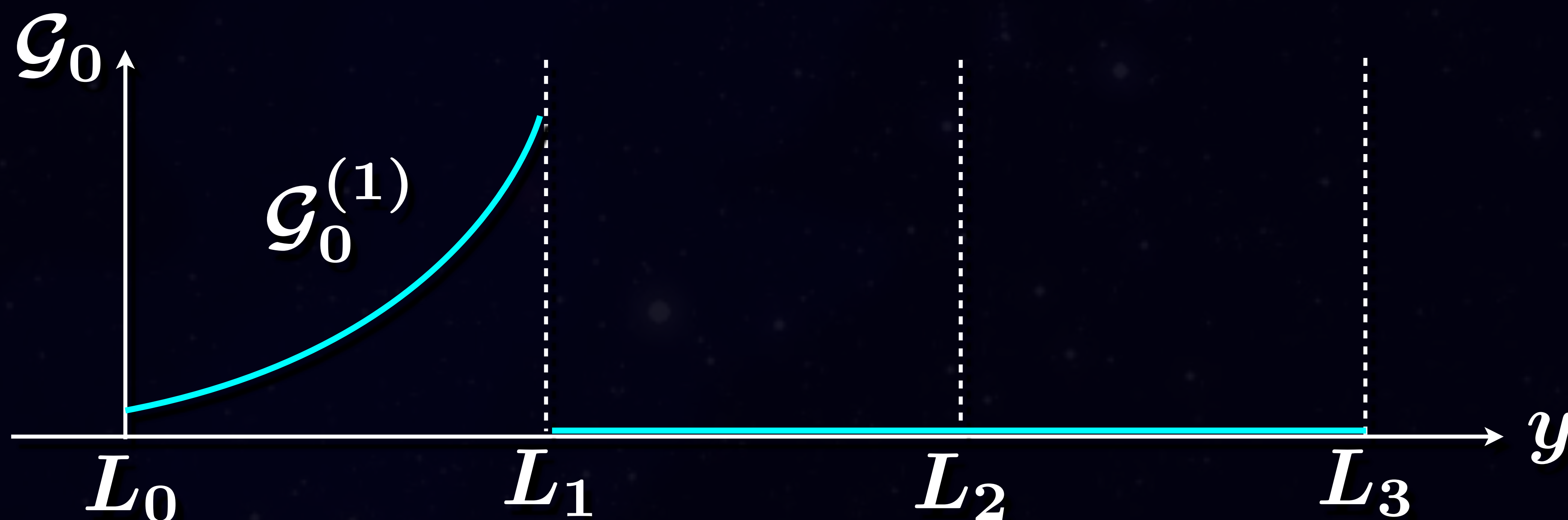

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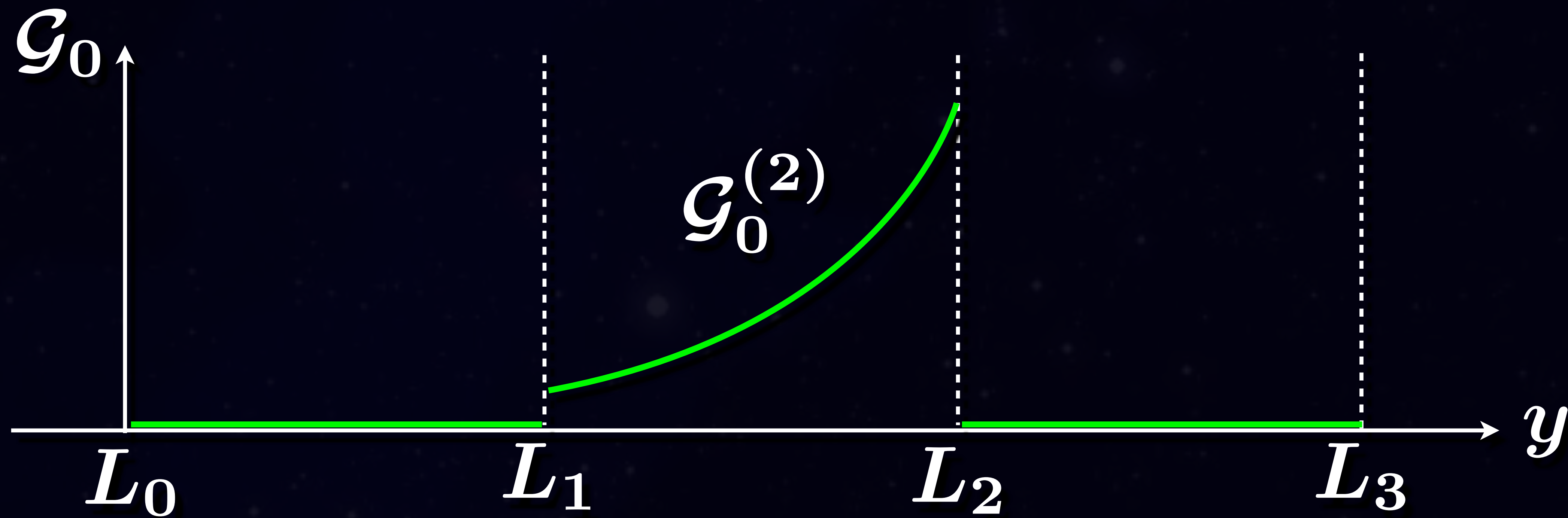

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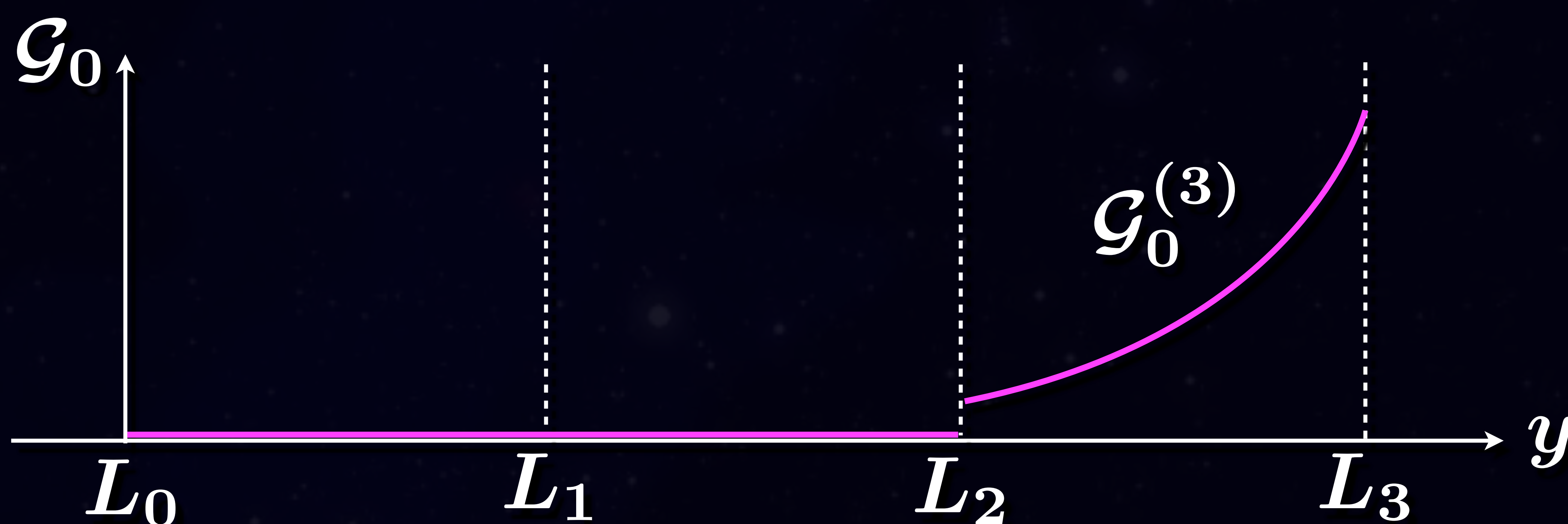

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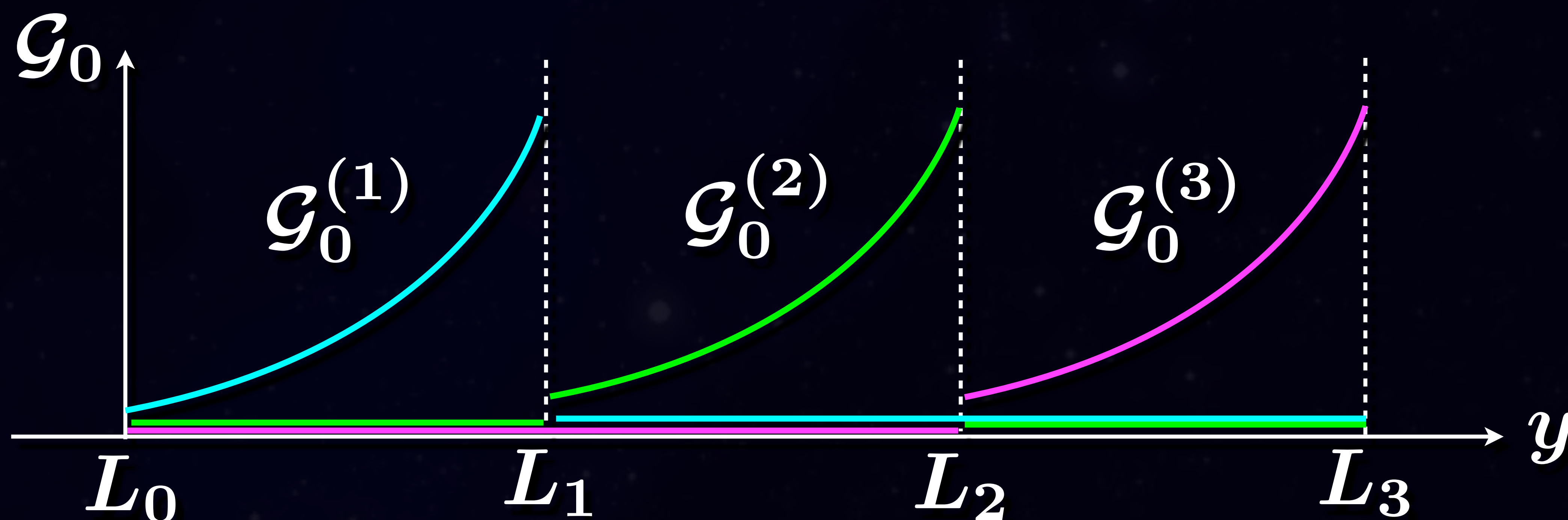

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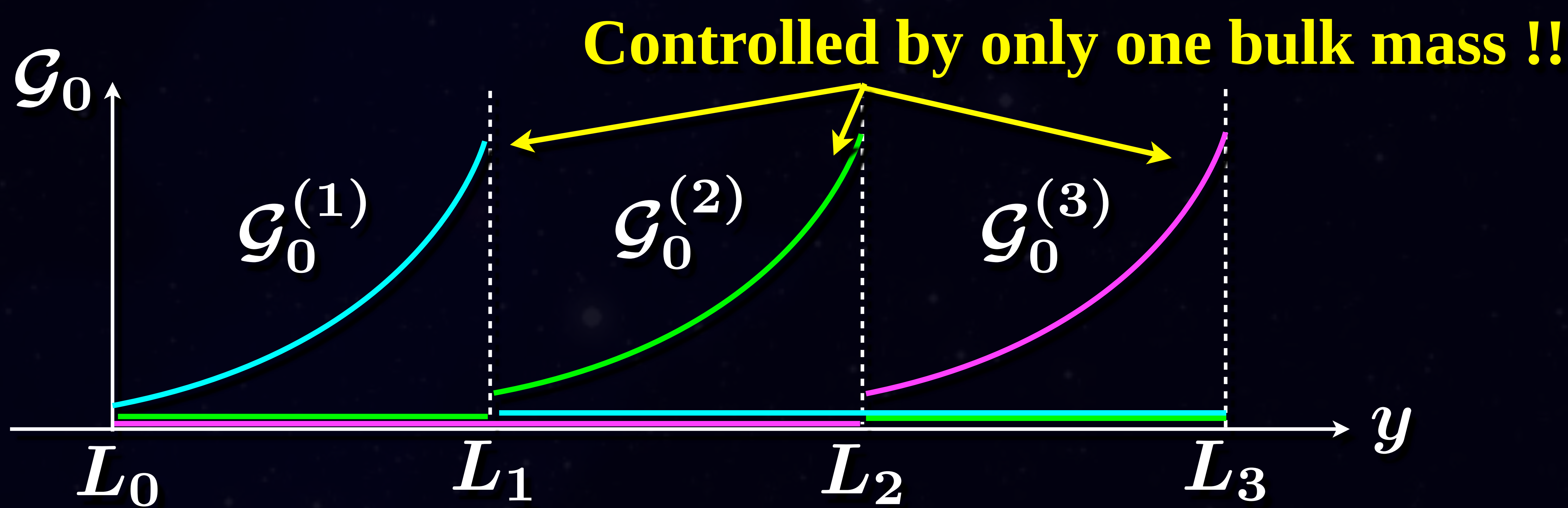

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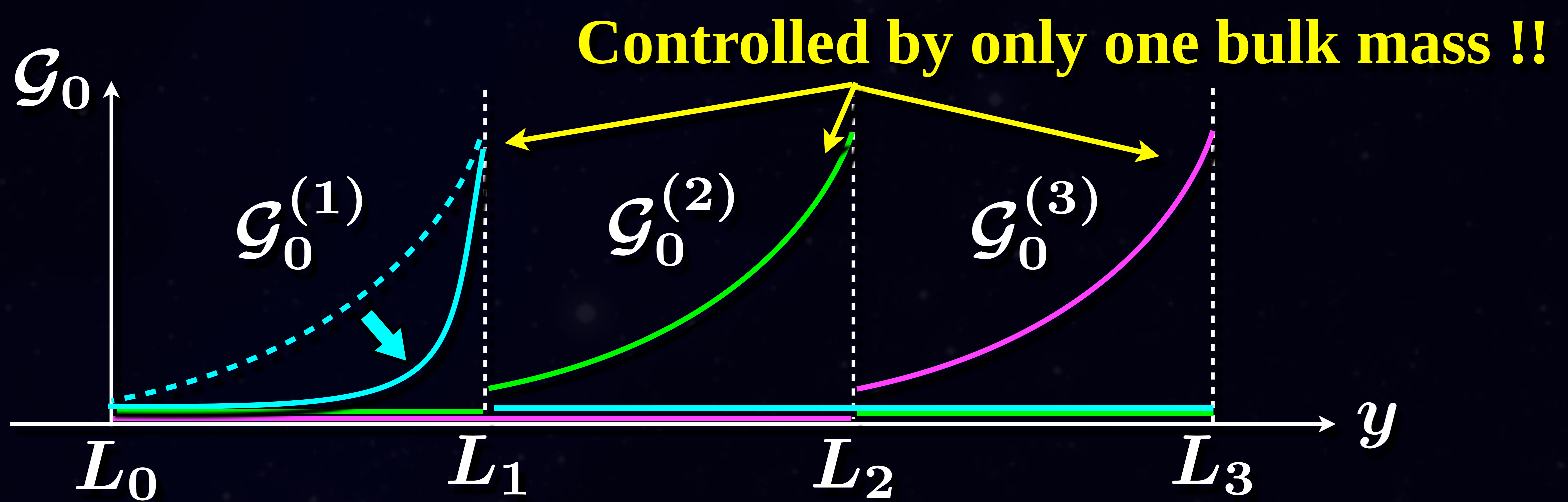

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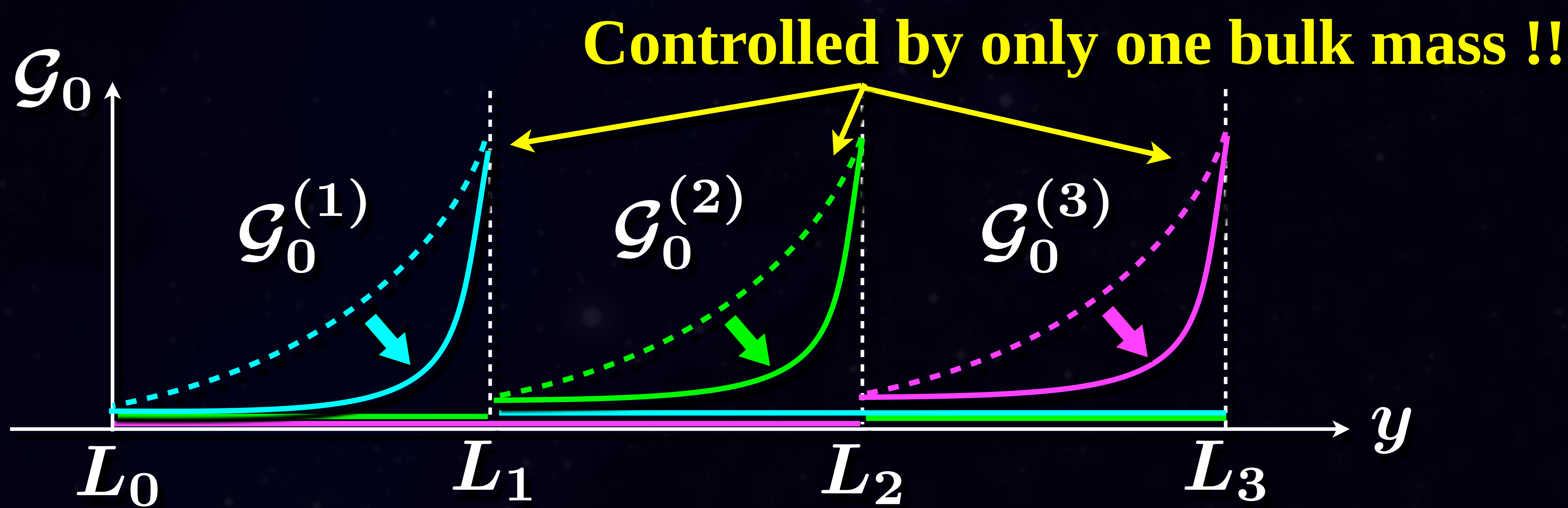

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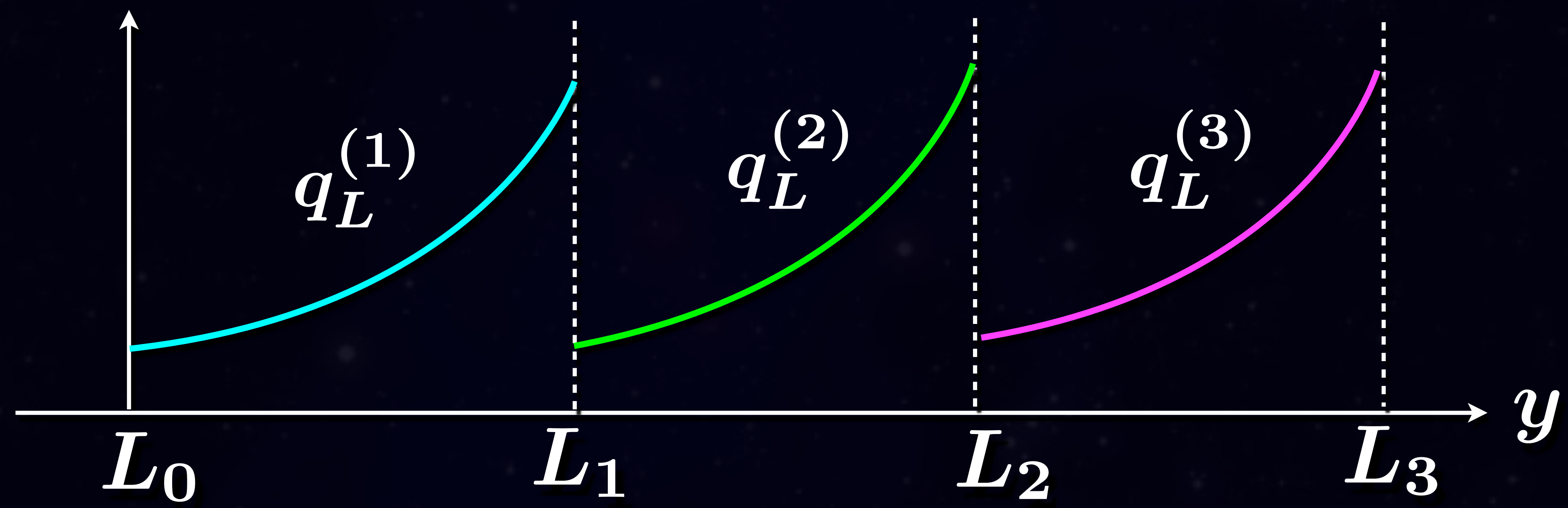

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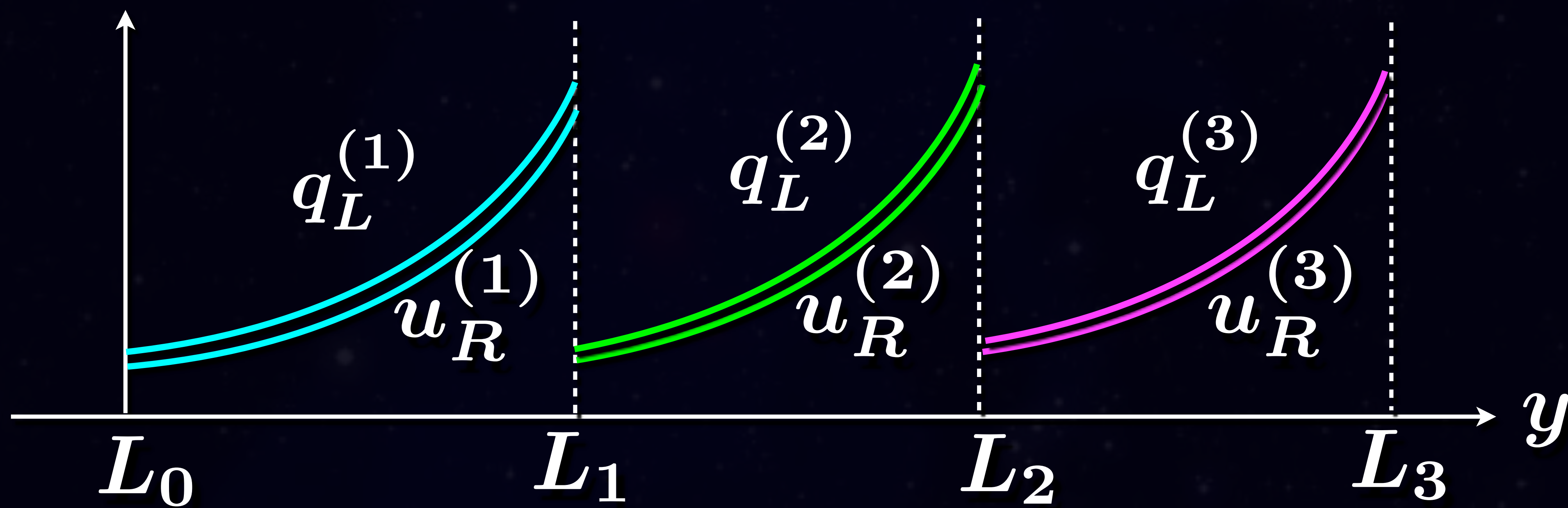

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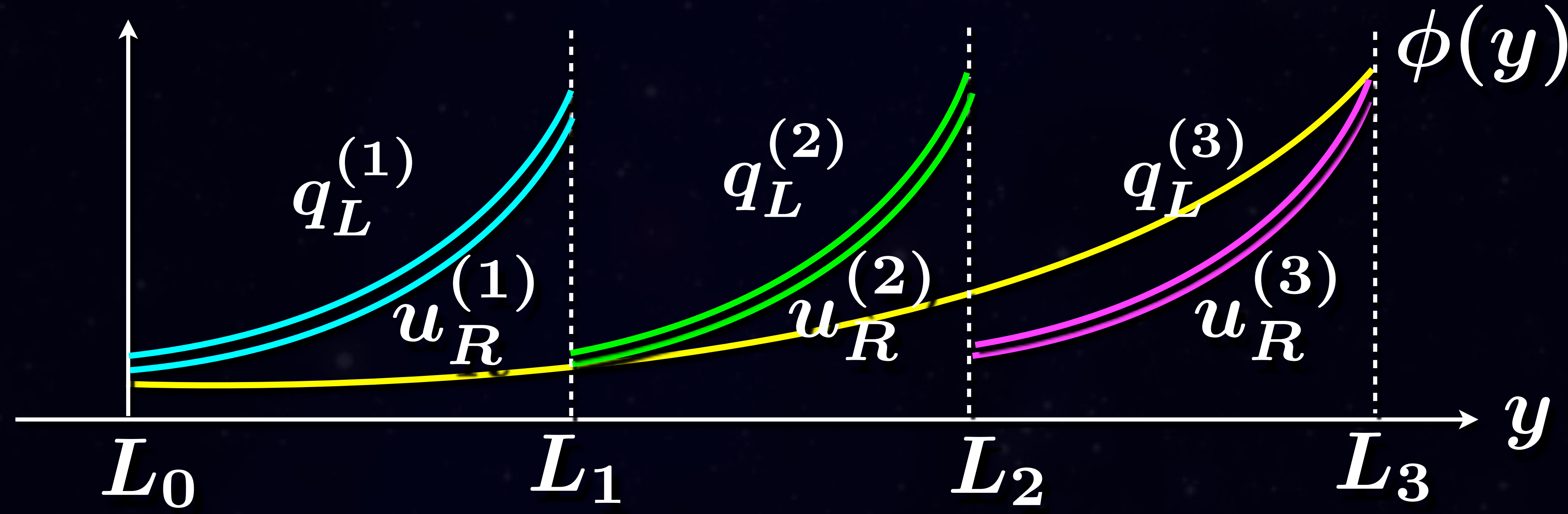

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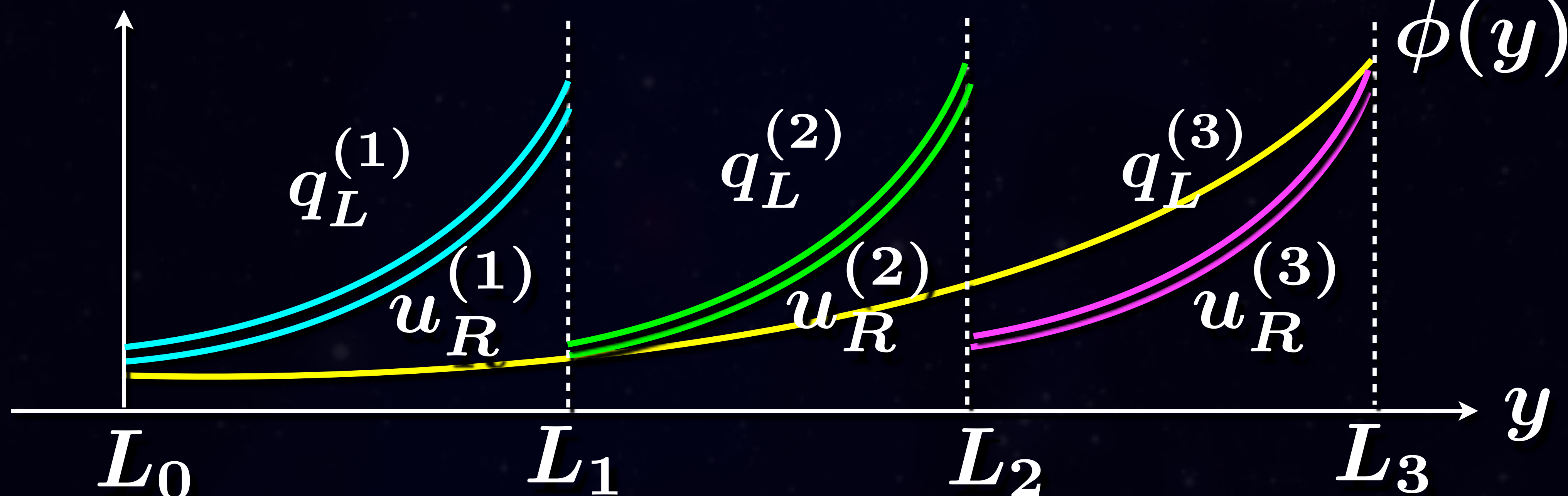

★ Constant VEV **can not produce** mass hierarchies.

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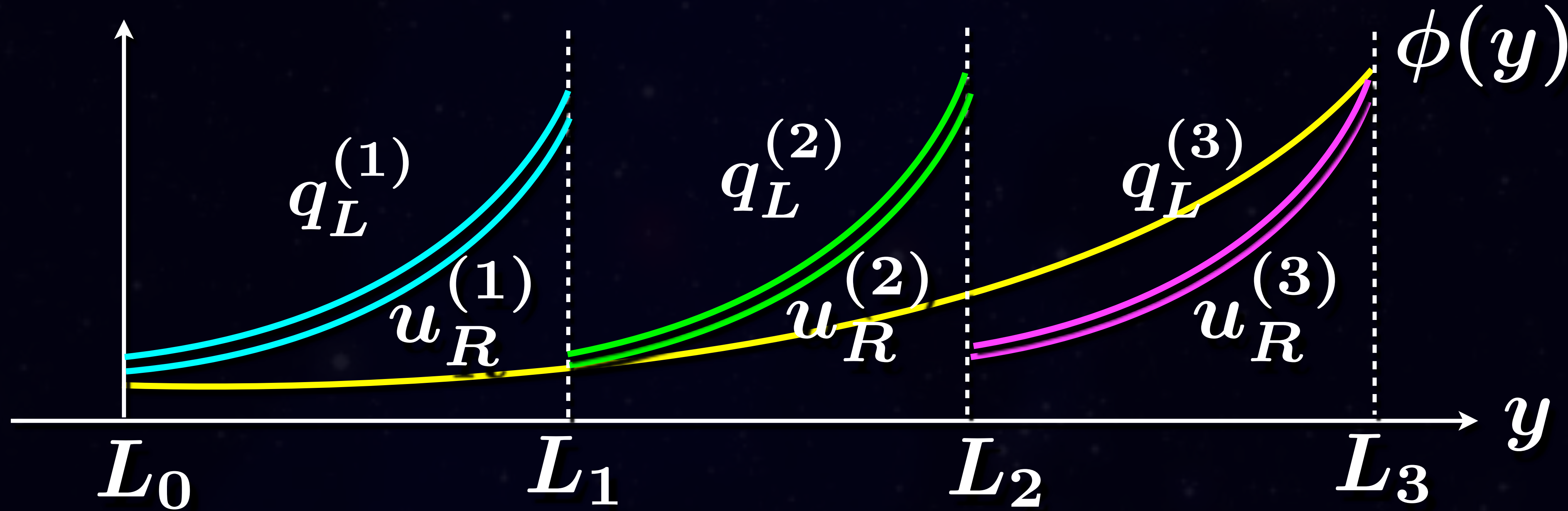

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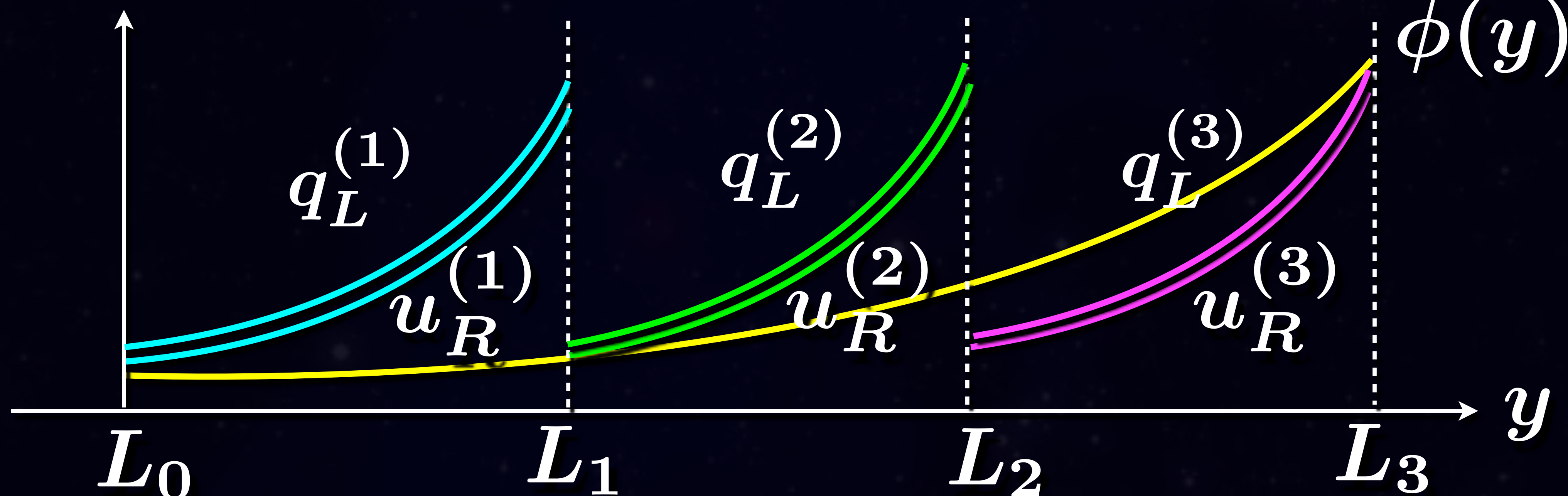

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$$\langle \Phi(x,y) \rangle = \phi(y)$$

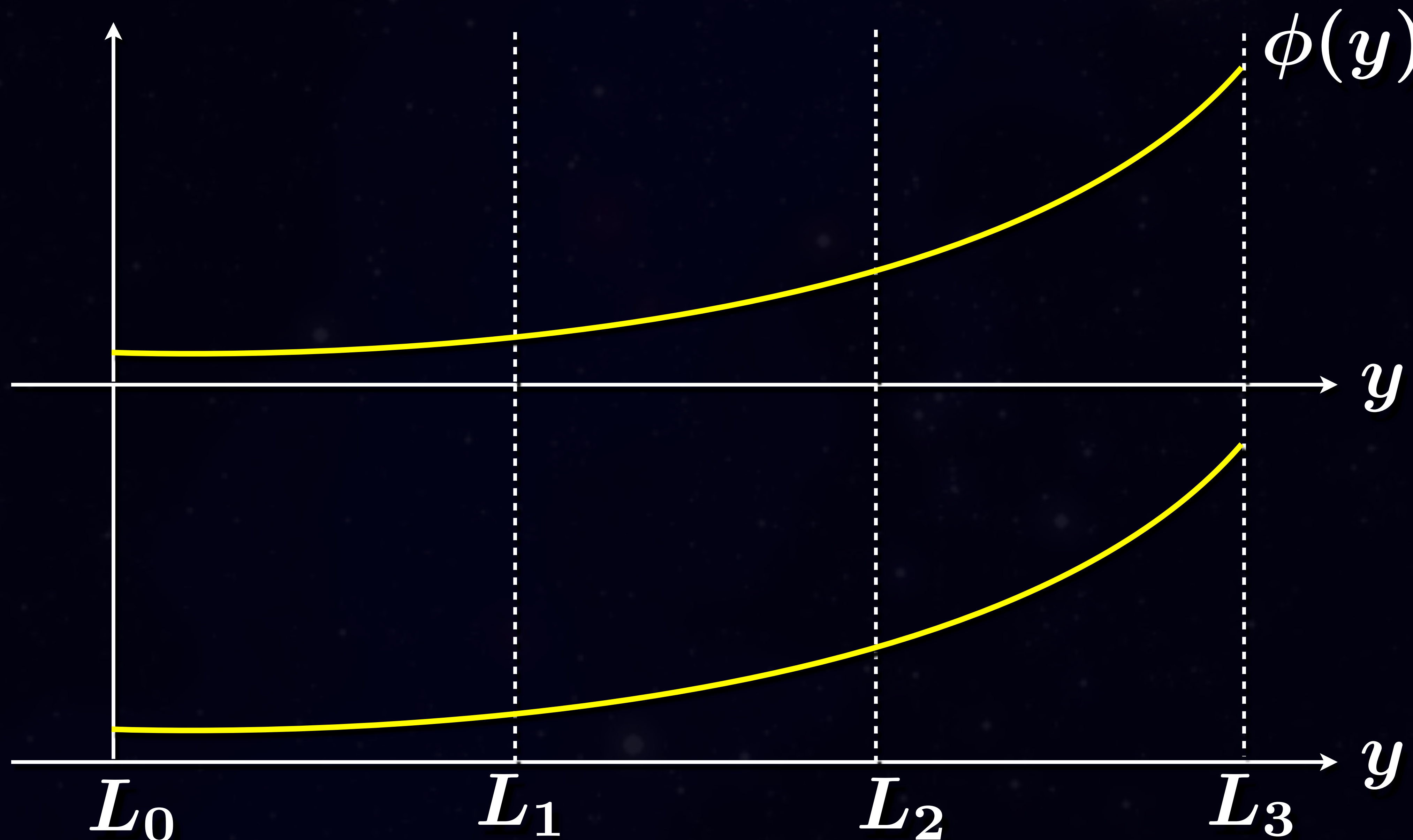
Y.F., T.Nagasawa, S.Ohya and M.Sakamoto, PTP 126 (2011) 841

Mass Hierachy

- **Quark sector**

Mass Hierachy

◆ Quark sector



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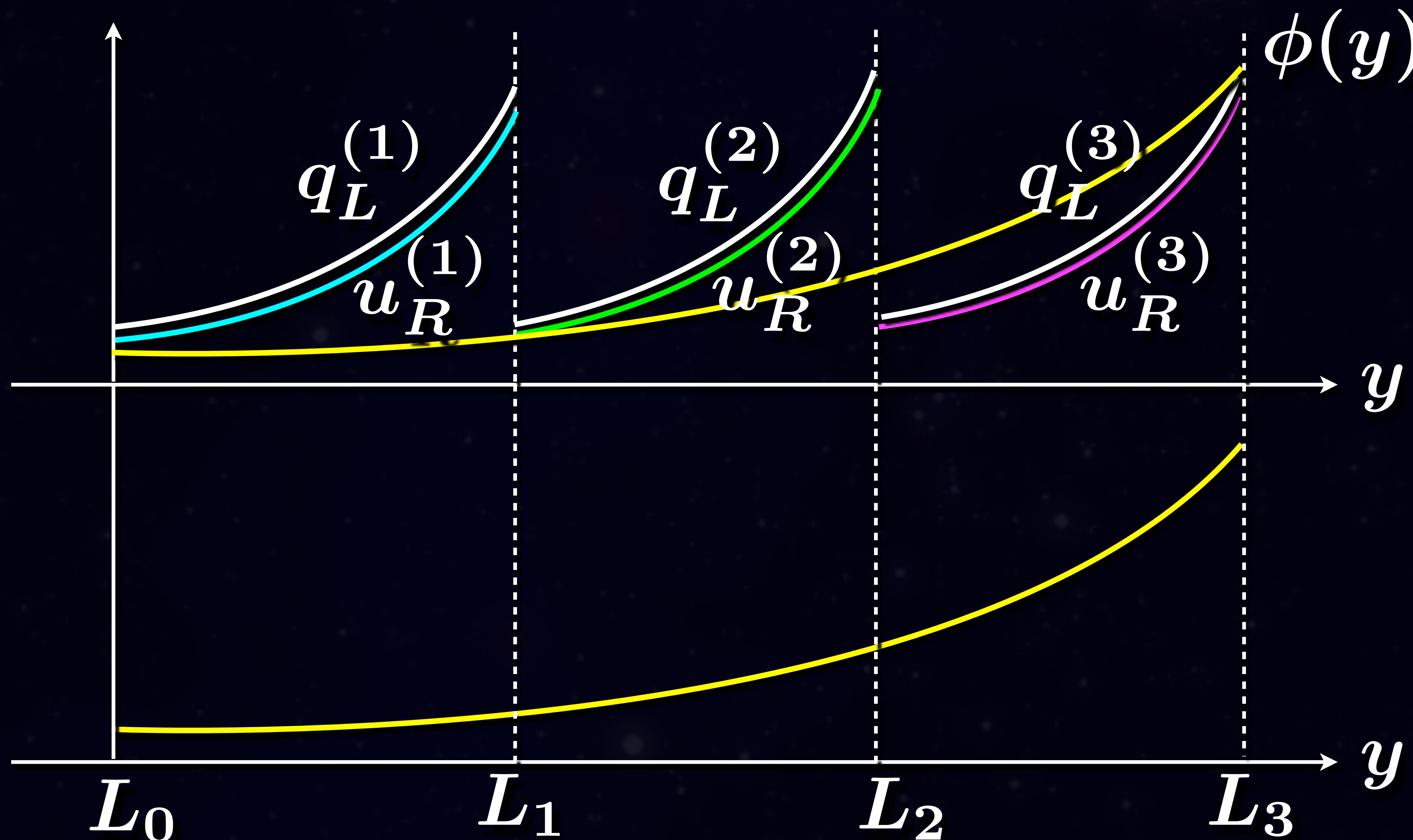

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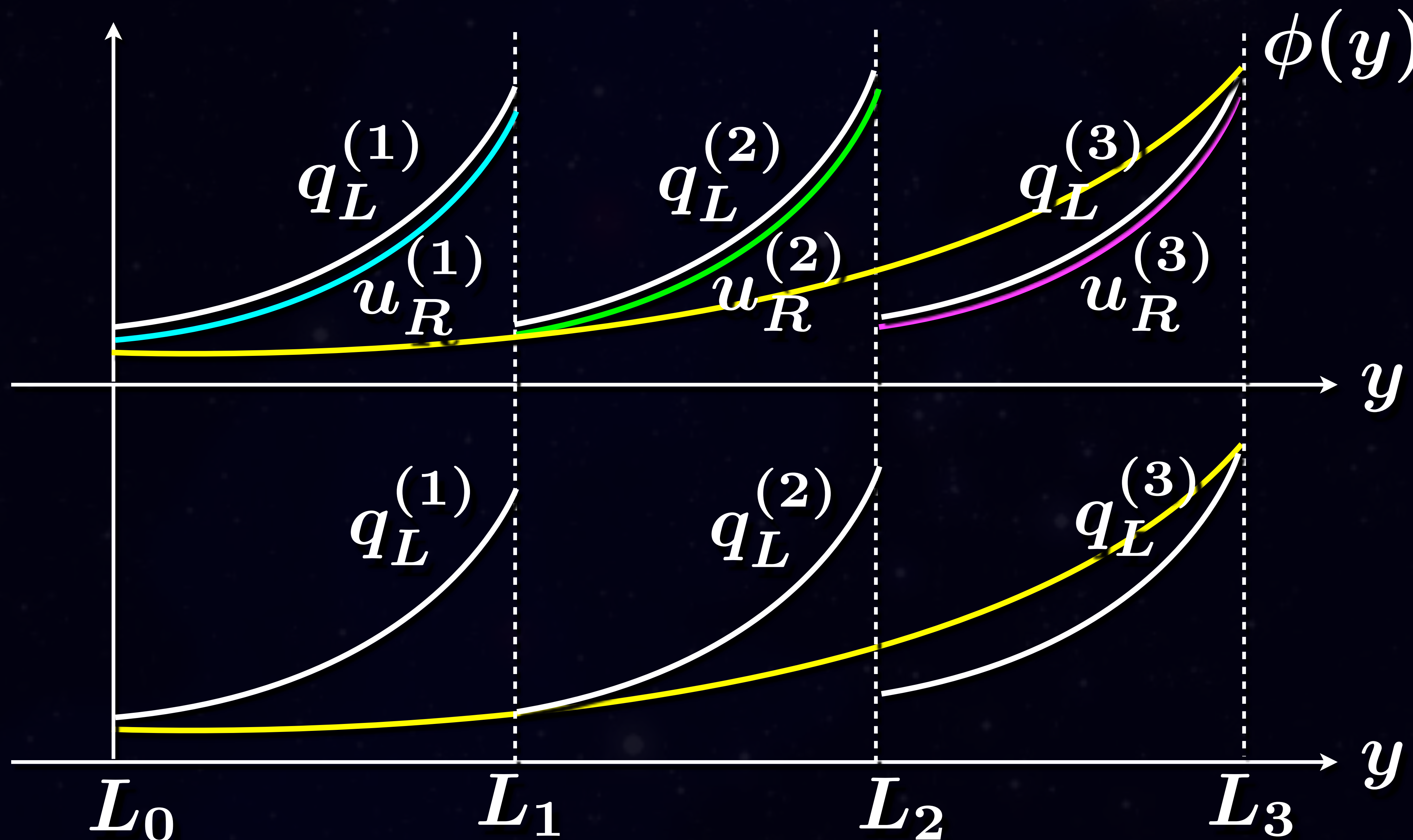

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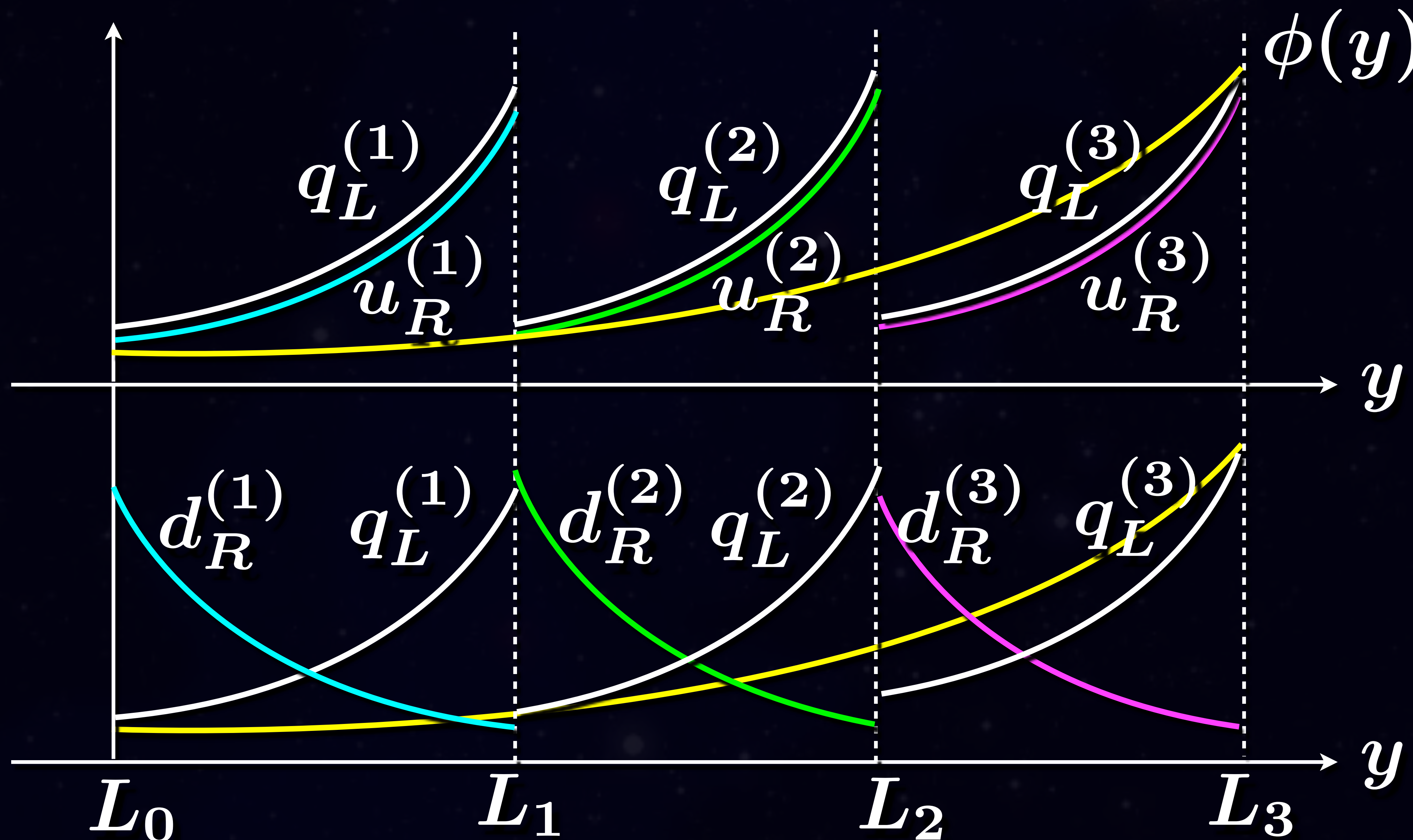

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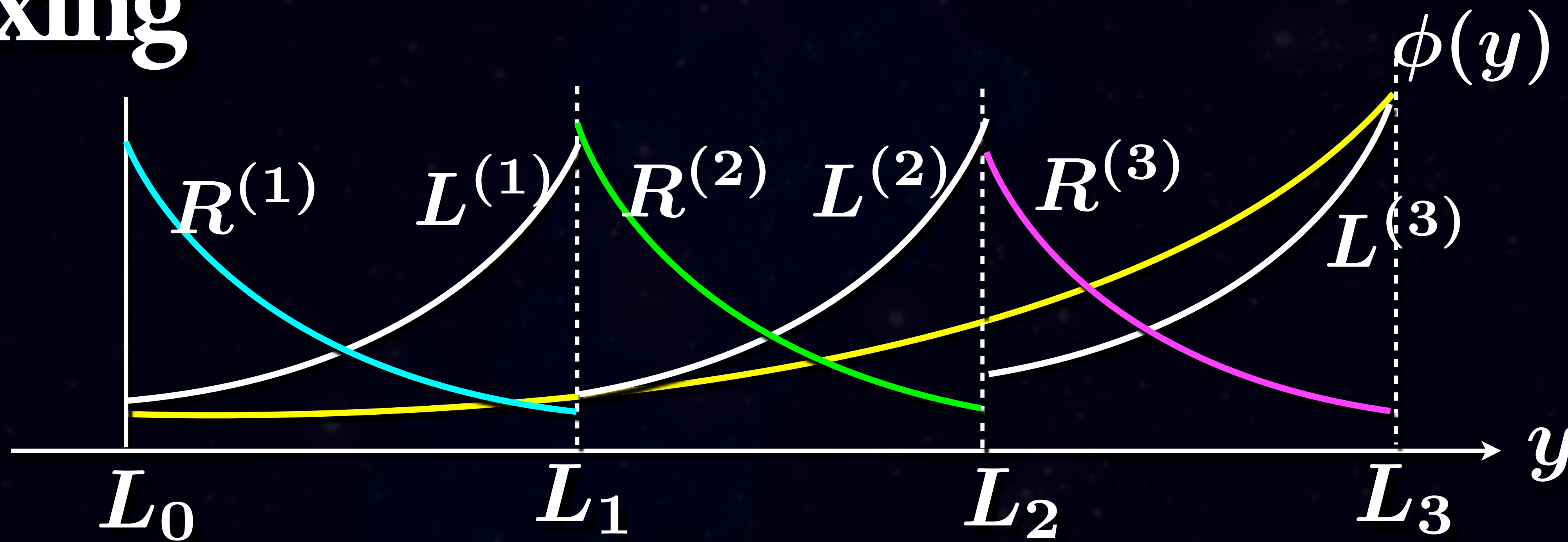

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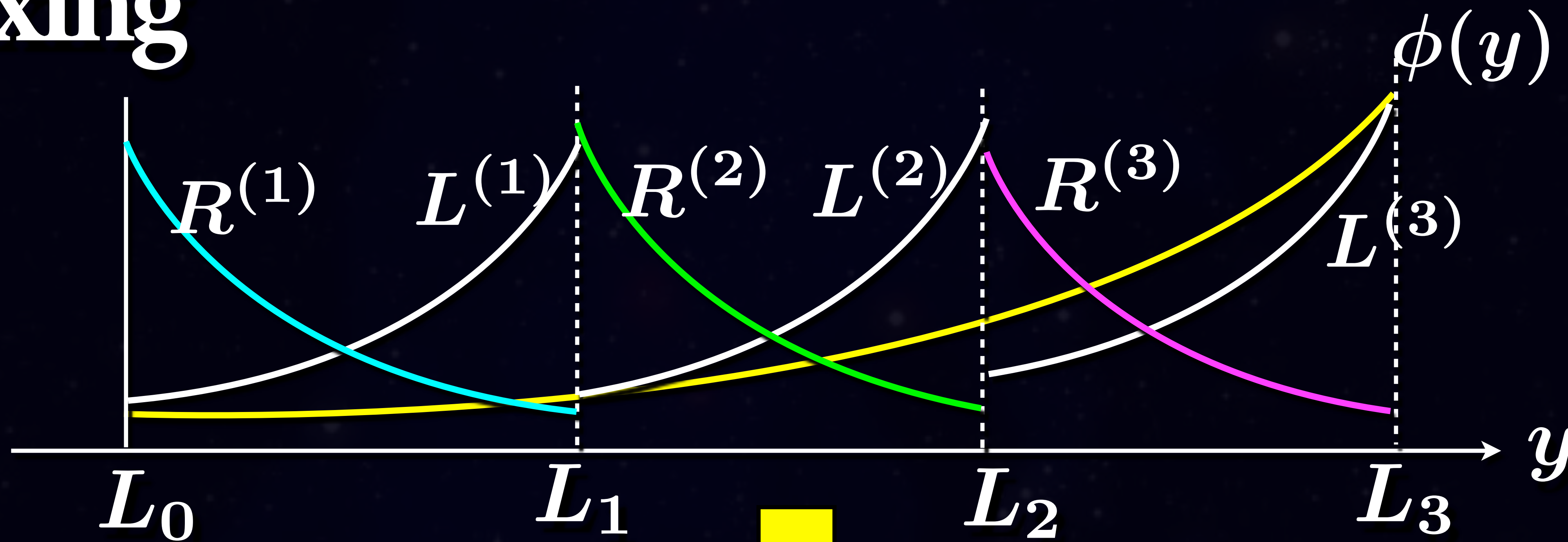

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

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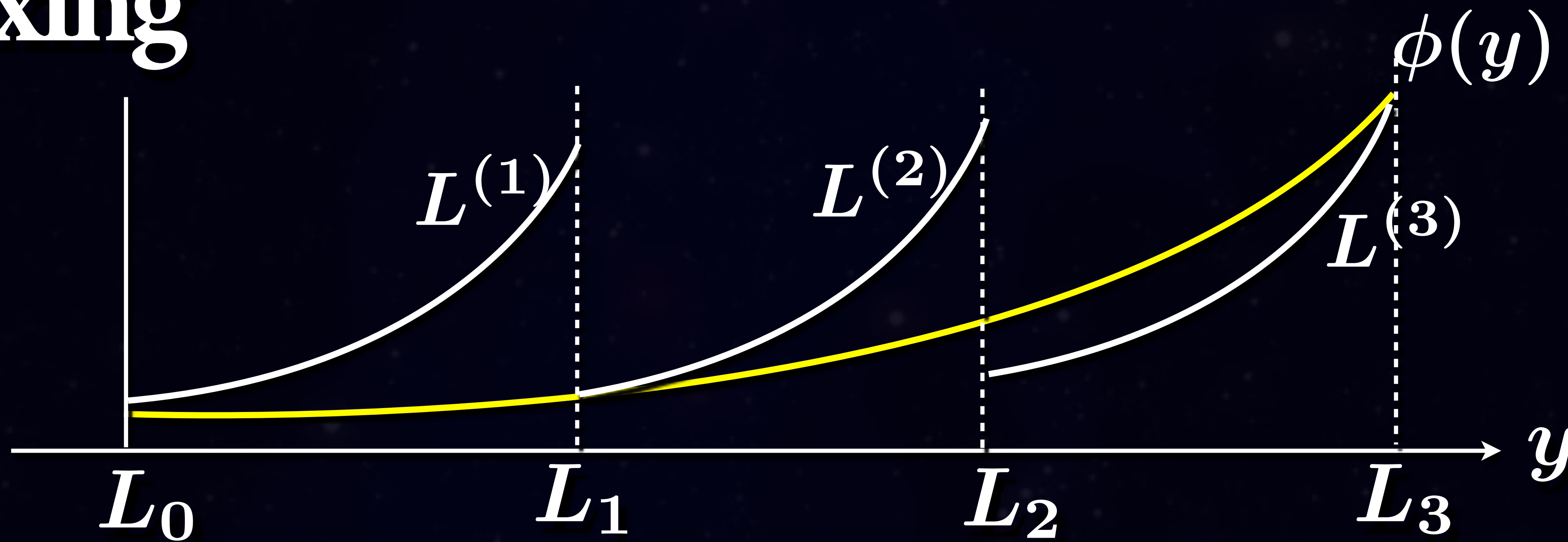

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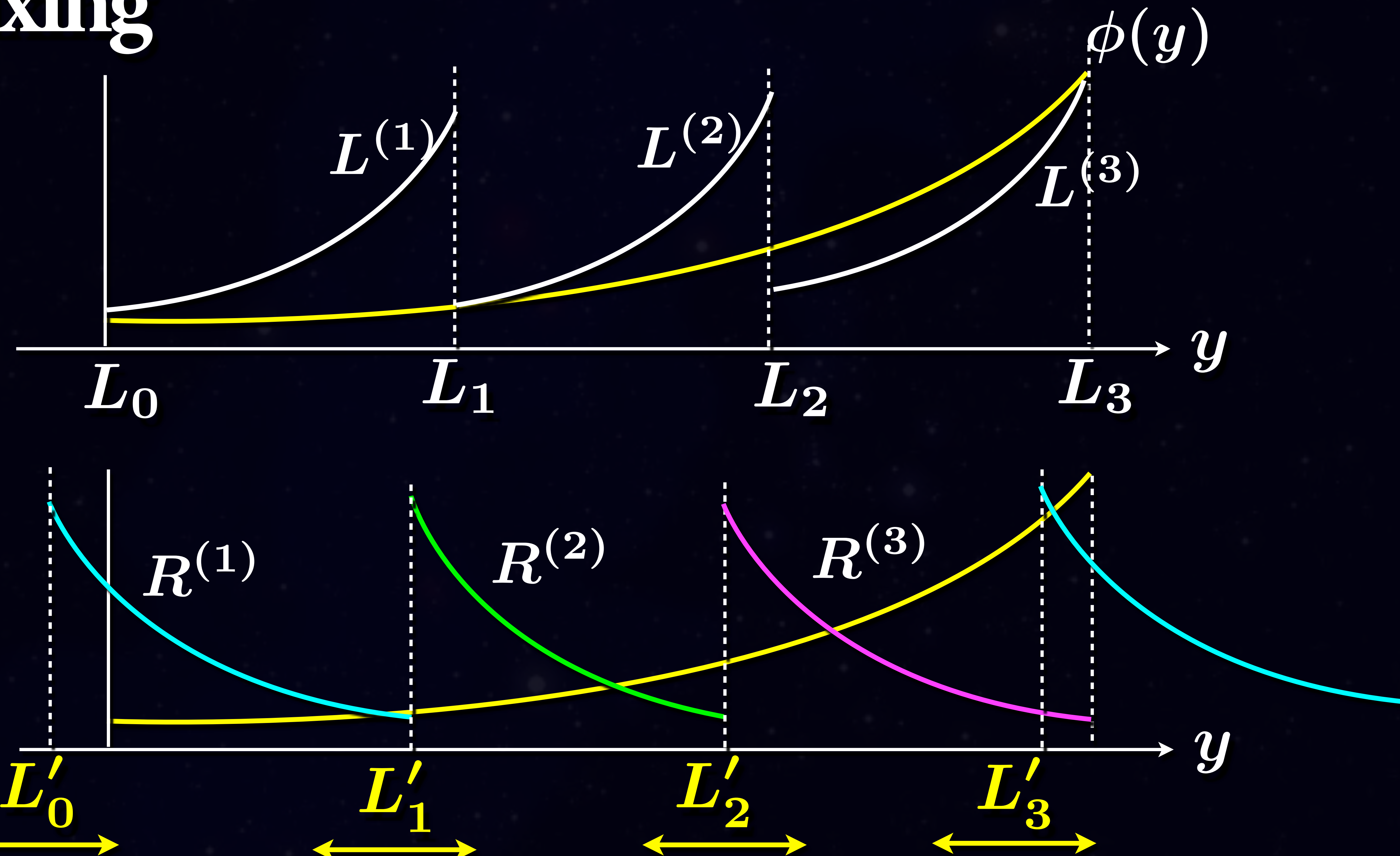

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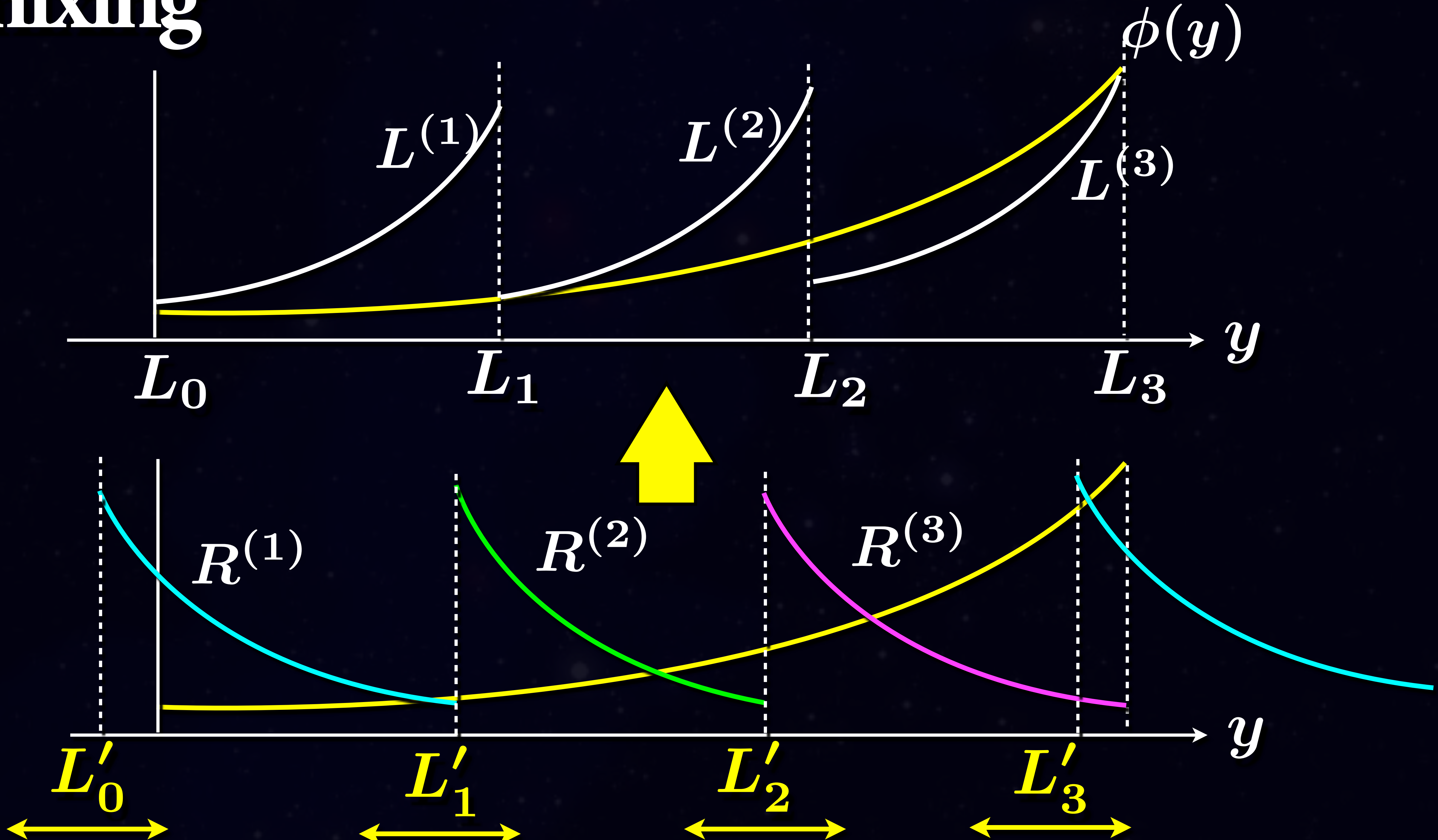

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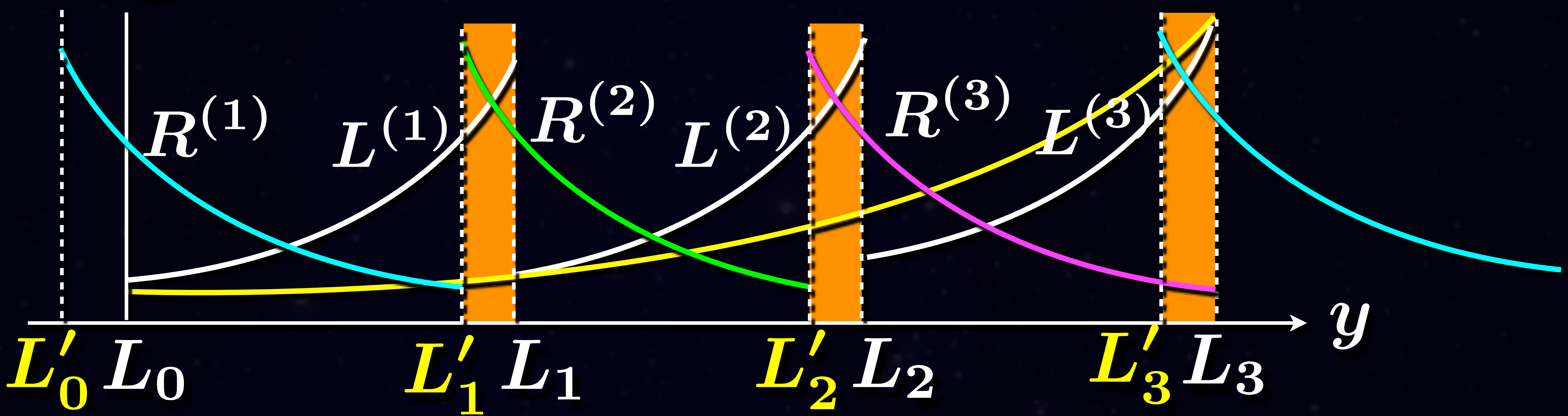

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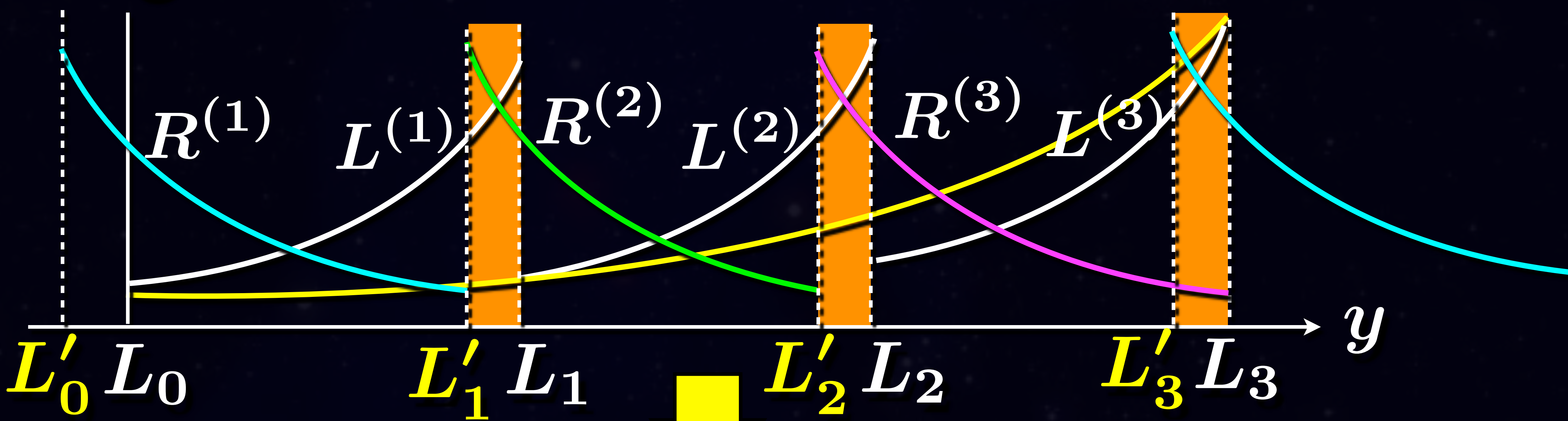

$$M = \begin{pmatrix} m_1 & m_{12} & 0 \\ 0 & m_2 & m_{23} \\ m_{31} & 0 & m_3 \end{pmatrix}$$

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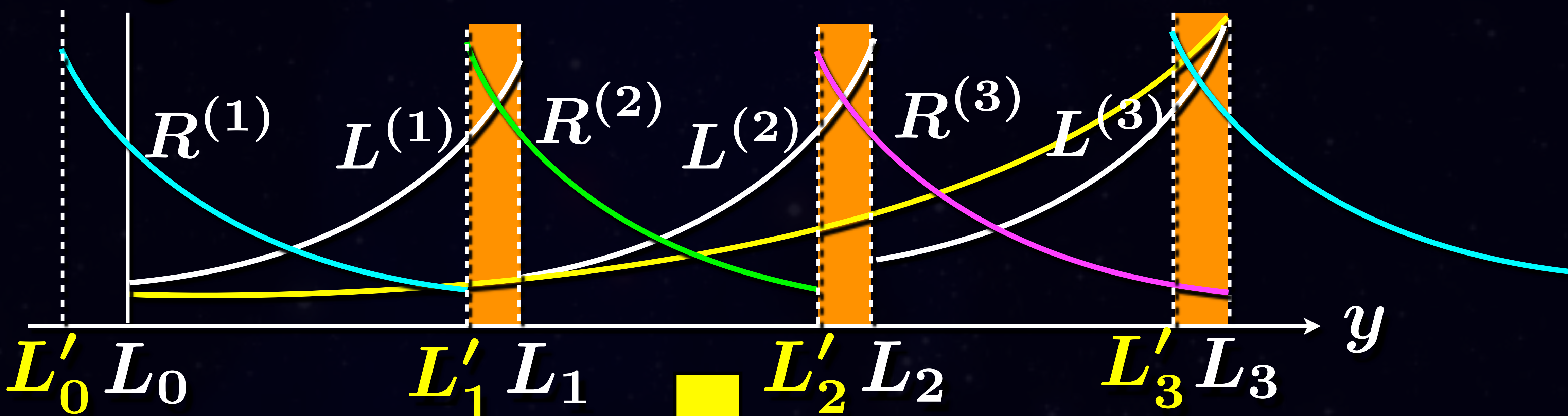

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Source of flavor mixing !!

Flavor Mixing

- Smallness of the neutrino masses lead large mixing structure.

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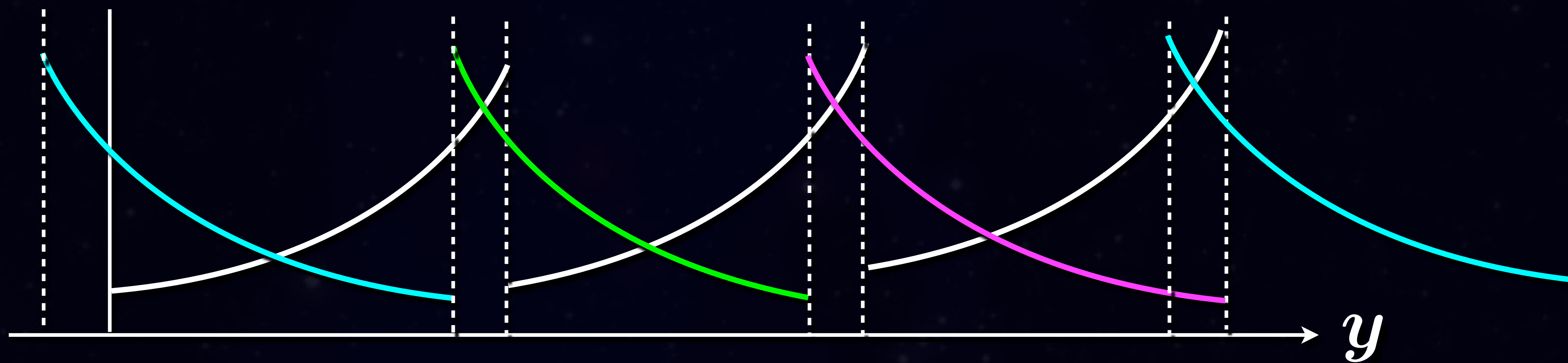

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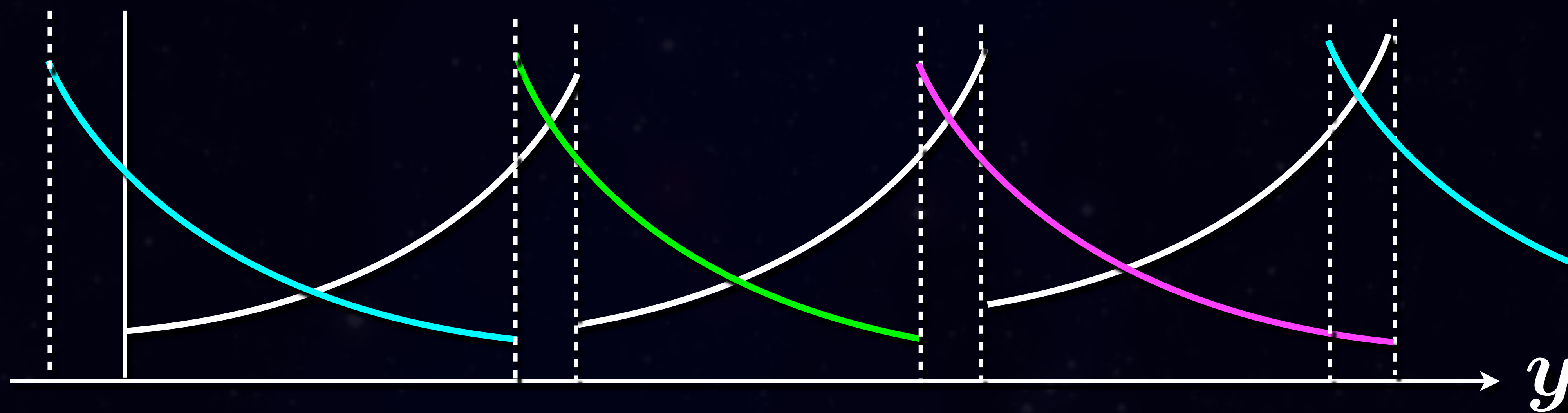

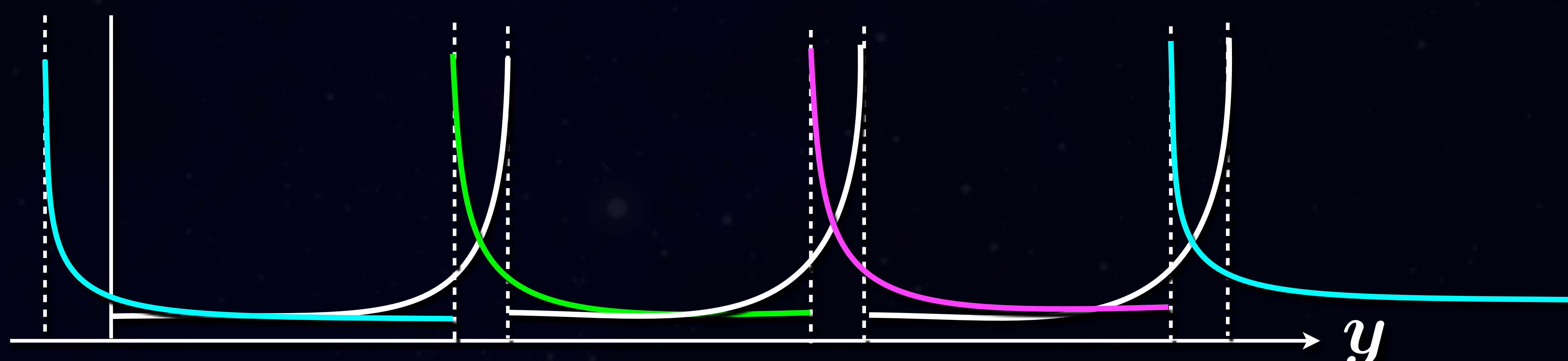

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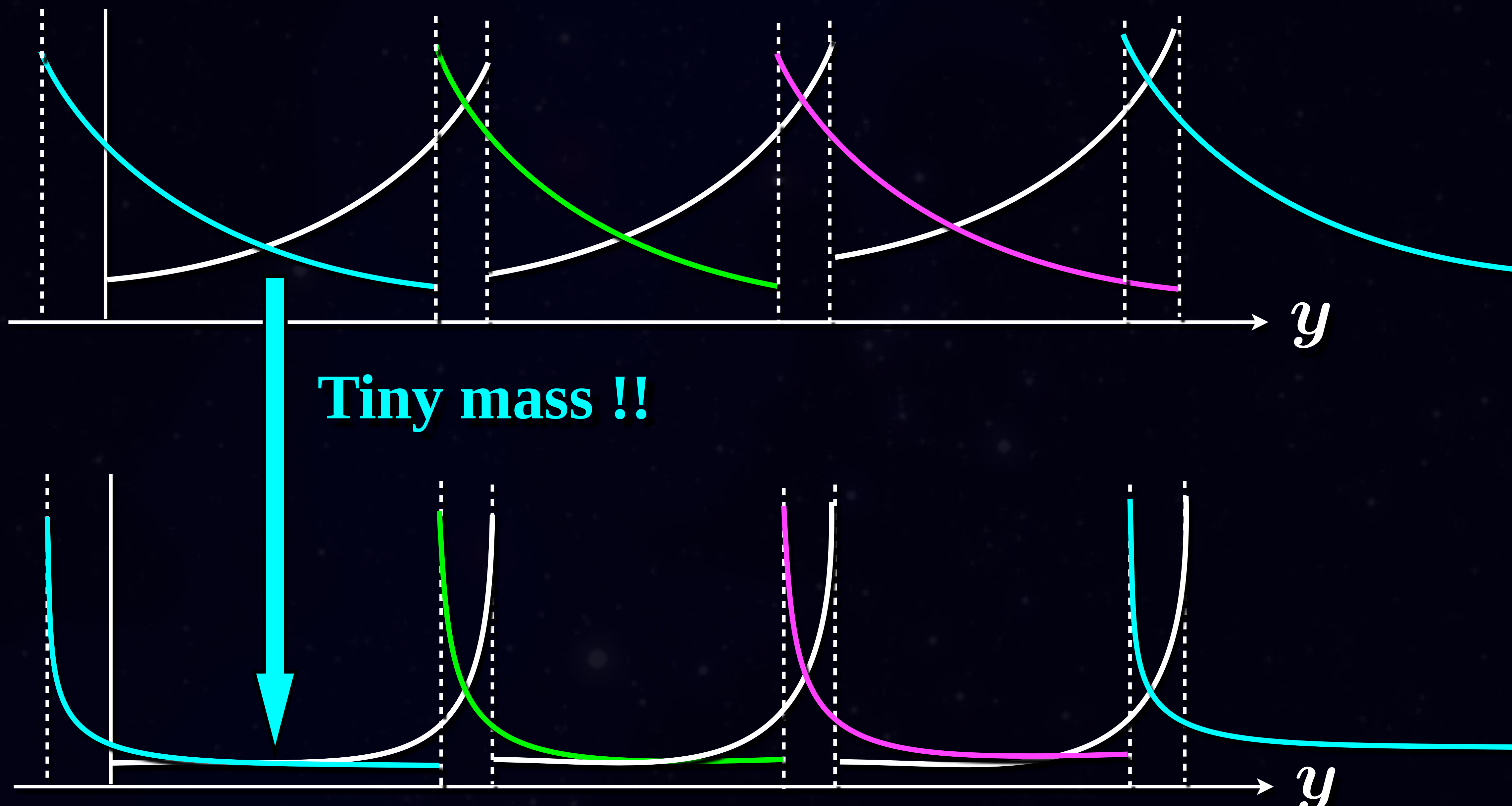

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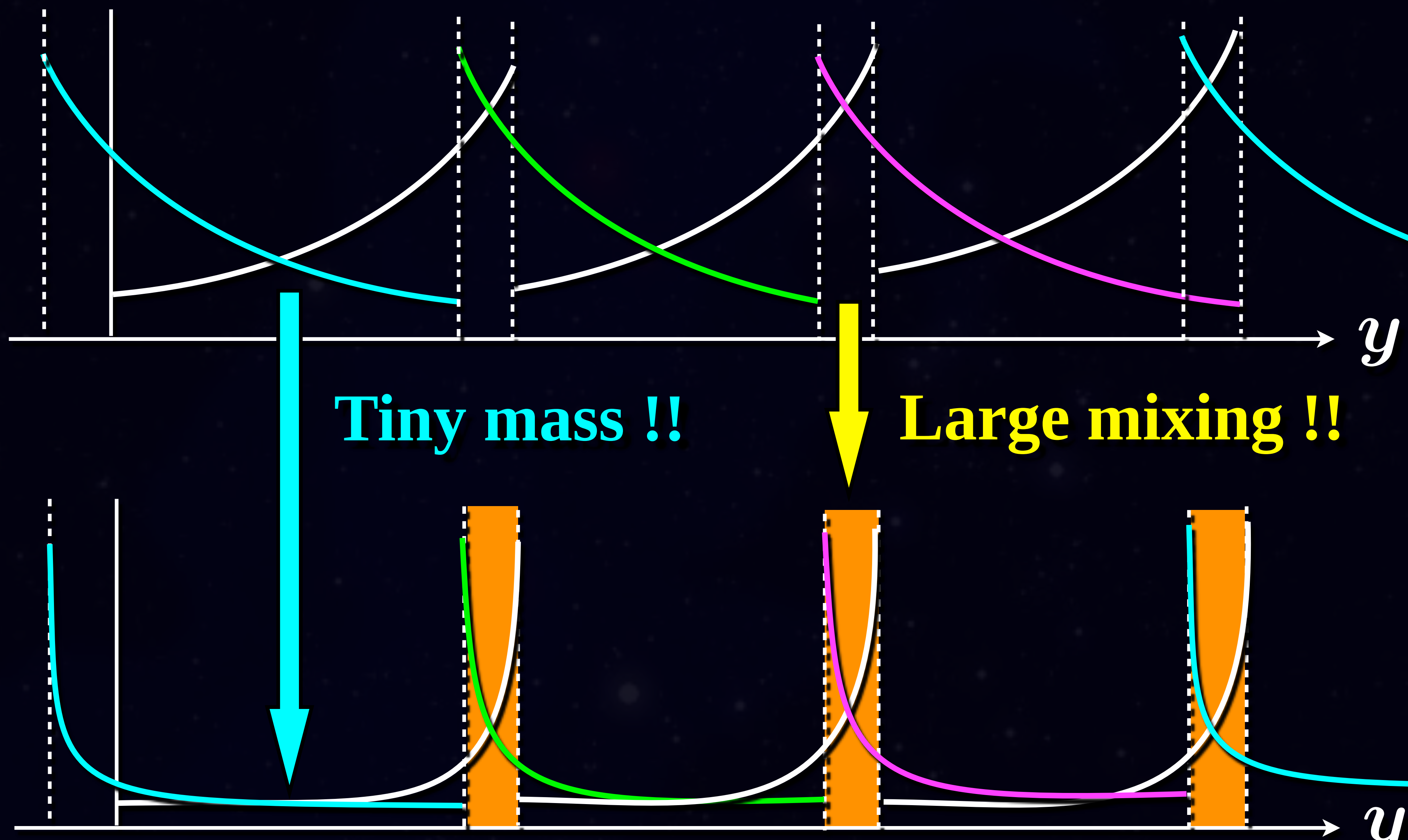

Flavor Mixing II

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- **Minimal extension from an interval can lead the CKM-like matrix.**

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$$|V_{CKM}| = \begin{pmatrix} 0.976 & 0.216 & 0.00313 \\ 0.216 & 0.975 & 0.0498 \\ 0.0138 & 0.0480 & 0.999 \end{pmatrix}$$

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Good agreement !! But....

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↑

60% larger than real CKM.......

Conclusion and Discussion

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5d gauge theories on a circle
with specified boundary conditions

↓ The low energy effective theory

4d gauge theories

+ **Generation**
+ **Large mass hierarchy**
+ **Large/Small mixing**

Conclusion and Discussion

◆ Challenges for the future

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◆ Challenges for the future

- ⋆ Reproduce PMNS matrix
- ⋆ Warped metric
- ⋆ CP phase from BCs
- ⋆ Constraint for extra dim. from FCNC

⋮