Wave functions and correlation functions for GKP strings from integrability

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We develop a general method of computing the semi-classical correlation functions of heavy string states in $AdS_5 \times S_5$ including the contributions of the vertex operators. Our method is based on the the integrable structure of the system and requires only the knowledge of the local behavior of the saddle point configuration around each vertex insertion point. Thus the method can be applied to cases even if the precise forms of the vertex operators are not known. As an important application, we compute the three-point functions of the large spin limit of the Gubser-Klebanov-Polyakov (GKP) strings in AdS_3 spacetime. We obtain finite three-point functions with the expected dependence of the target space boundary coordinates on the conformal dimension and the spin.

Two- and three-point functions together constitute the building blocks of conformally invariant field theories (CFT): The former determines the spectrum of the theory and the latter dictates its dynamics. Although such properties have long been known, the recent development of the string theory has given a fresh look to these quantities. The AdS/CFT correspondence, a duality between CFT and the string theory on Anti-de Sitter spacetime (AdS), relates these two building blocks to another two building blocks. Namely, two point functions of CFT are believed to encode the spectrum of a string on AdS whereas three point functions are believed to describe the interaction of such strings.

Vigorous studies during the past fifteen years have revealed much on the two point functions of the best studied example, the duality between $\mathcal{N} = 4$ super Yang-Mills and the superstring on $AdS_5 \times S^5$. However, the study on three point functions is still in its infancy as the calculation is more involved than that of two point functions both technically and conceptually. First it is technically demanding since various sophisticated techniques, developed in the field of integrability, are not directly applicable. Second it is conceptually more elaborate as it requires the knowledge on the "wave functions" in addition to the knowledge on the spectrum. The wave function on the gauge theory carries the information on the exact form of the single trace operator and the wave function on the string theory side encodes the shapes and the motions of the string. It is now known that the evaluation of the wave functions is necessary to correctly reproduce the spacetime dependence of three point functions both in the calculation of the gauge theory and in the calculation of the string theory. Given the importance of the wave functions in the calculation of three point functions, it is natural to hope that the study on three point functions will indirectly deepen our understanding on the interrelation of the wave functions on both sides of the duality and make it more clearer how the string emerges from the gauge invariant operator. This is actually one of the motivations to study three point functions in the context of the AdS/CFT correspondence.

Let us now discuss the string-theoretical calculation of three point functions more in detail. If the AdS/CFT correspondence is correct, the correlation function of CFT is calculated by the worldsheet correlation function of the dual string theory and the strong coupling limit of the gauge theory is equivalent to the classical limit of the string theory. In this limit, the worldsheet correlation function is dominated by a saddle point and is constituted of two parts; the action, $S[X_*]$, and the vertex operators, $V_i[X_*]$, both evaluated on the saddle point solution.

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle \sim V_1[X_*] V_2[X_*] V_3[X_*] e^{-S[X_*]} \quad X_*\text{: saddle point trajectory.}$$

One of the main practical difficulties in the evaluation of three point functions is that it is difficult to construct the precise form of the vertex operator. In our paper [1], we overcome this difficulty by constructing the wave functions in terms of the action-angle variables. As is well-known in the classical analytical mechanics, the classical limit of the wave function can be obtained by exponentiating the solution of the Hamilton-Jacobi (H-J) equation.

$$\Psi \sim \exp\left(\frac{i}{\hbar} W\right) , \quad W\text{: the solution to the H-J equation.}$$

Although solving the Hamilton-Jacobi equation is itself difficult, it dramatically simplifies if we can construct action-angle variables:

$$\mathrm{W} = \textstyle\sum_j J_j \theta_j \, , \quad J_j\text{: action variable} \, , \quad \theta_j\text{: angle variable.}$$

Fortunately, a method to construct the action-angle variables for the current problem is already known. The method is first developed by Sklyanin [2] and goes under the name of *Sklyanin's magic recipe*. For details on the application of the method to the construction of wave functions, please refer to our second paper [1]. One intriguing feature of the method is that the construction of the action-angle variables utilizes the spectral curve of the classical string, which has proven to be of extreme importance in solving the spectrum problem (or equivalently, the two-point functions). Thus, we hope that the further study will uncover the coherent whole picture of the duality which unifies the calculations of the two-point functions and the three point functions in harmony.

References

[1] Y. Kazama and S. Komatsu, *Wave functions and correlation functions for GKP strings from integrability*, arXiv:1205.6060

[2] E. K. Sklyanin, *Separation of variables - new trends*, Prog. Theor. Phys. Suppl. **118**, 35 (1995), arXiv:solv-int/9504001.