# Generalized parton distribution with nonzero skewedness in holographic QCD 

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## 1. Introduction

- Use AdS/CFT for high energy hadron scattering
- Study string scattering on a warped geometry
- Develop preceding work, Brower, Polchinski, Strassler and Tan ['06] (BPST pomeron).

We study DVCS (Deeply Virtual Compton Scattering) in sphere level and large finite 't Hooft coupling


DVCS
and extract Generalized Parton Distribution (GPD), which is non-perturbative parton information.

$$
A_{\mathrm{DVCS}}\left(x, \eta, t, q^{2}\right) \simeq C(\eta / x) \otimes H\left(x, \eta, t, q^{2}\right)
$$

## GPD

## Kinematical parameters of DVCS

$p^{\mu}=\frac{1}{2}\left(p_{1}^{\mu}+p_{2}^{\mu}\right), \quad q^{\mu}=\frac{1}{2}\left(q_{\mu}^{1}+q_{\mu}^{2}\right), \quad \Delta^{\mu}=p_{2}^{\mu}-p_{1}^{\mu}=q_{1}^{\mu}-q_{2}^{\mu}$

$$
x=\frac{-q^{2}}{2 p \cdot q}, \quad \eta=\frac{-\Delta \cdot q}{2 p \cdot q}, \quad t=-\Delta^{2}
$$

skewedness

Forward case (PDF) and nonzero $t$ case have been studied. (RN, Watari ' 11 )
We noticed that PDF/GPD shares qualitative properties of the real QCD.

Remaining problem: nonzero skewedness

## 2. Aspects from Field Theory

OPE can restrict the amplitude

$$
T^{\mu \nu}=i \int d^{4} x e^{-i q x}\left\langle h\left(p_{2}\right)\right| T\left\{J^{\nu}(x / 2) J^{\mu}(-x / 2)\right\}\left|h\left(p_{1}\right)\right\rangle,
$$

$$
i \int d^{4} x e^{-i q x} T\left\{J^{\nu}(x / 2) J^{\mu}(-x / 2)\right\}=\sum_{N} \mathcal{C}_{N \rho_{1} \ldots, \rho_{S}}^{\mu p_{S}}\left(q^{\rho}\right) \mathcal{O}_{N}^{\rho_{1} \ldots \rho_{N}}\left(0 ; q^{2}\right) .
$$

Twist-2 operators give dominant contributions

$$
G^{\mu\left\{\rho_{1}\right.}\left(i \overleftrightarrow{D}^{\rho_{2}}\right) \ldots\left(i \overleftrightarrow{D}^{\rho_{j-1}}\right) G_{\mu}^{\left.\rho_{j}\right\}}
$$

However, when skewedness is nonzero, operators with total derivatives are also twist-2

$$
\partial^{\sigma_{1}} \ldots \partial^{\sigma_{m}}\left[G^{\mu\left\{\rho_{1}\right.}\left(i \overleftrightarrow{D}{ }^{\rho_{2}}\right) \ldots\left(i \overleftrightarrow{D}^{\rho_{j-1}}\right) G_{\mu}^{\left.\rho_{j}\right\}}\right]
$$

Moreover, the anomalous dimension matrix mixes among operators with different number of total derivatives.

After diagonalizing of anomalous dimension, DVCS amplitude is given as

$$
T^{\mu \nu} \simeq \eta^{\mu \nu}\left[\sum_{j} C_{j}(\eta / x) x^{-j}\left(\frac{\Lambda^{2}}{q^{2}}\right)^{\gamma(j) / 2} H_{j}\left(\eta, t, \Lambda^{2}\right)\right](*)
$$

$H_{-} \mathrm{j}$ is the non-perturbative parton information which we want.

$$
H_{j}\left(\eta, t, \Lambda^{2}\right)=\sum_{k=0}^{j} \eta^{j} H_{j, k}\left(t, \Lambda^{2}\right)
$$

$H_{-} j$ is related to the matrix element of the eigen-operators of anomalous dimension.

$$
\left\langle h\left(p_{2}\right)\right| \tilde{O}_{j}^{+\cdots+}\left(\mu^{2}\right)\left|h\left(p_{1}\right)\right\rangle \equiv \sum_{k=0}^{j}\left(\Delta^{+}\right)^{k}\left(p^{+}\right)^{j-k} H_{j, k}\left(t, \mu^{2}\right)
$$

## 3. Gravitational Description

In the case of zero-skewedness, BPST naively covariantized t-channel exchange amplitudes of N (excitation number) $=\mathrm{j}$ (spin) strings polarized in Minkowski Directions.

$$
\begin{gathered}
A_{\text {flat }} \sim \sum_{j}\left(q^{-}\right)^{j} \frac{\left(\eta_{-+}\right)^{j}}{t-\frac{\alpha^{\prime}}{2}(j-2)}\left(p^{+}\right)^{j} \\
\text { AdS } \\
\quad \sum_{j}\left(q^{-}\right)^{j} \frac{\left(g_{-+}\right)^{j}}{\Delta_{j}-\frac{\alpha^{\prime}}{2}(j-2)}\left(p^{+}\right)^{j}
\end{gathered}
$$

However, corresponding to operator mixing in RG flow, the string states may also mixes when skewedness is nonzero.

Our prescription: study in an effective field theory on warped spacetime.

1. We assume there is an action of string field theory whose bilinear terms are of the form:

$$
S_{\text {bilinear }}=\frac{ \pm 1}{2} \int d^{D} x A^{M_{1} \ldots M_{j}}\left(\partial^{2}-M_{N}^{2}\right) A_{M_{1} \ldots M_{j}}
$$

for each component field A
(different Fock states are not mixed, and derivative terms are simply d'Alembertian)
as bosonic open string theory in Siegel gauge.

2, We assume arbitrary local interaction terms.

3, Covariantize the derivatives into one of AdS_5 space.

In this set-up, we studied what contributes as twist-2 (leading at large $q^{\wedge} 2$ ).
a), t-channel exchange of what types of component fields?
b), What type of interaction?
c), t-channel exchange of what eigenmode of Laplacian in AdS?

## Our result

the twist-2 contributions are given only from
a), $\mathrm{N}=\mathrm{j}$, symmetric fields
b), interaction terms of the form

$$
c_{j} \frac{\left(2 \alpha^{\prime}\right)^{j / 2}}{R^{5}} \frac{1}{2!} \int d^{4} x d z \sqrt{-g} A_{m_{1} \ldots m_{j}} \psi \overleftarrow{\nabla^{m_{1}}} \ldots \overleftarrow{\nabla^{m_{j}}} \psi .
$$

(coupling constant is determined in flat limit)
c), transverse and traceless mode in AdS_5

$$
g^{m_{1} m_{2}} A_{m_{1} m_{2} \ldots m_{j}}=0, \quad \nabla^{m_{1}} A_{m_{1} \ldots m_{j}}=0
$$

Eigenmodes which has nonzero z component also contribute.

## Summary and Discussion

We provide a method to calculate $\mathrm{H} \_\mathrm{j}$ in gravity dual.
So far, analytic form for arbitrary j is very complicated.

Our result is consistent:
1), Our result approaches BPST's amplitude in zero skewedness limit.
2), Our result has correctly the form which OPE predicts. (Eq.(*))

