Constraints on a class of classical solutions in open string field theory

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in collaboration with

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Constraints on a class of classical solutions in open string field theory

cf. 増田さんのトーク，馬場くん、小路田さんのポスター

KBc subalgebra

Constraints on a class of classical solutions

in open string field theory

consistency of boundary states

Constraints on a class of classical solutions

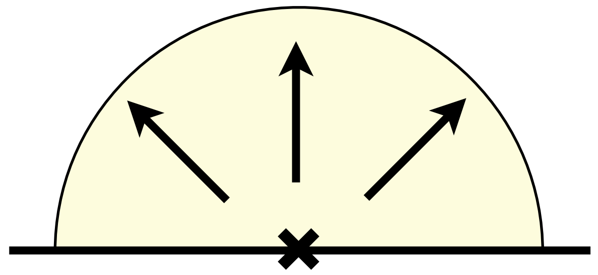
in open string field theory

Plan of my talk

- open SFT & KBc subalgebra
- boundary states from classical solutions
- implication from consistency of boundary states

Open bosonic SFT

- string field Ψ

world-sheet description: 

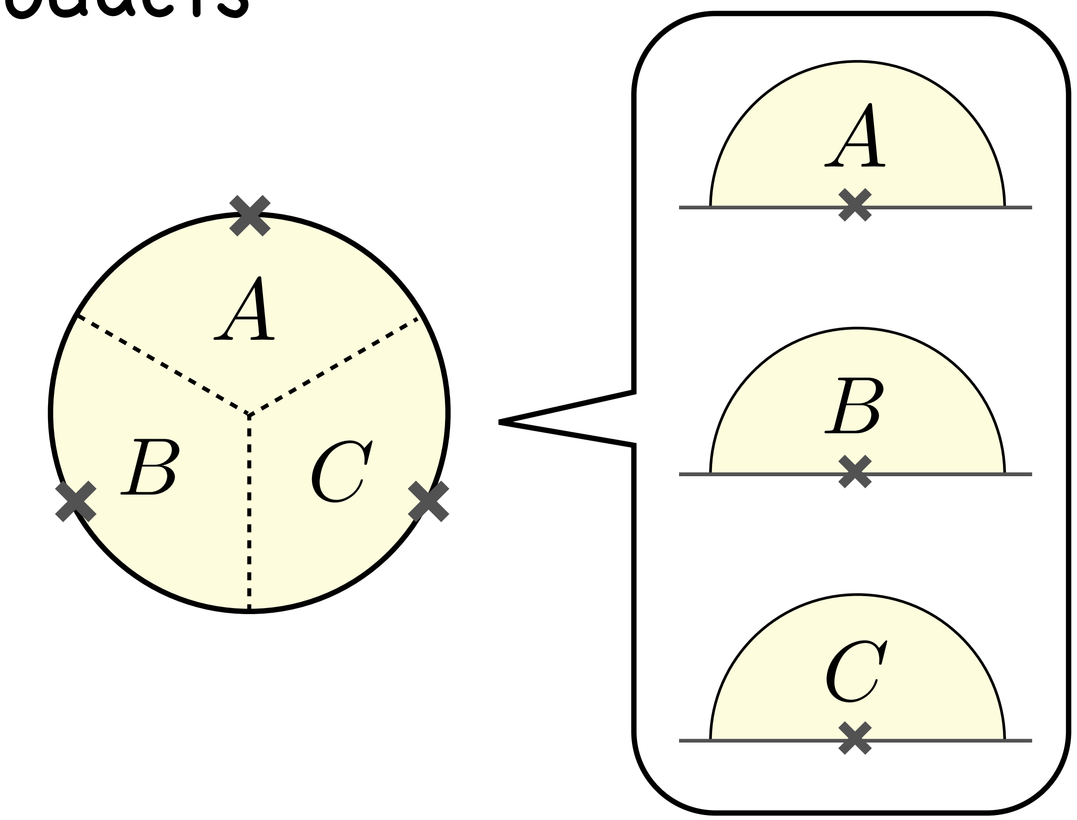
action: $S = \frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle$

eom: $Q\Psi + \Psi * \Psi = 0$

Open bosonic SFT

Witten's star products

$$\langle A, B * C \rangle =$$

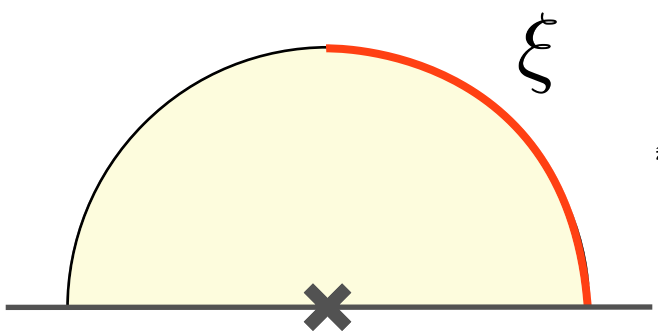


star product was an obstacle

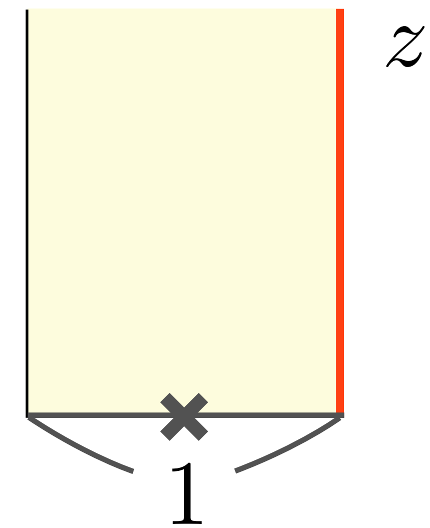
to construct classical solutions analytically

Open bosonic SFT

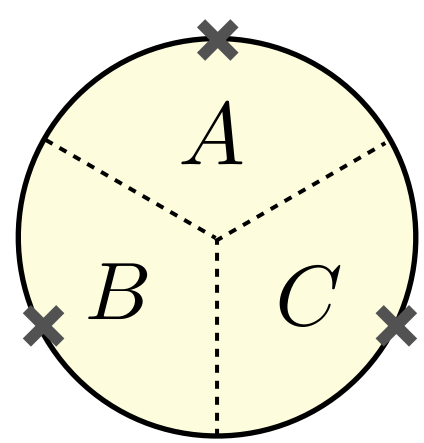
sliver frame



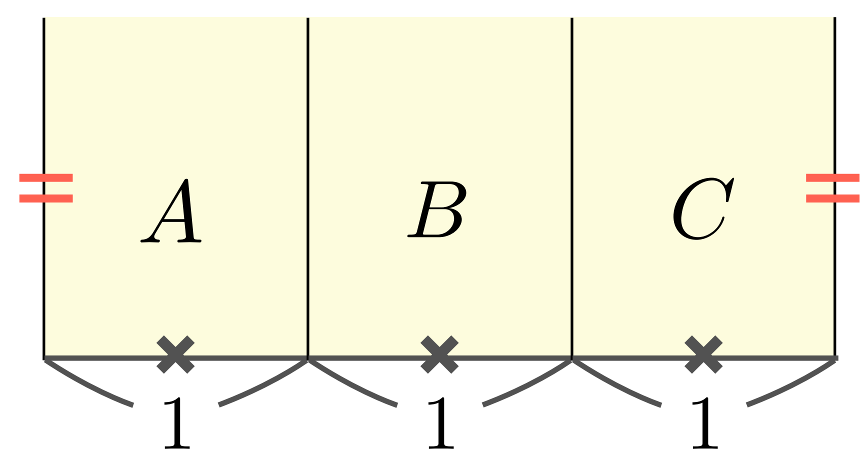
$$z = f(\xi) = \frac{2}{\pi}\arctan\xi$$



$$\langle A, B * C \rangle =$$

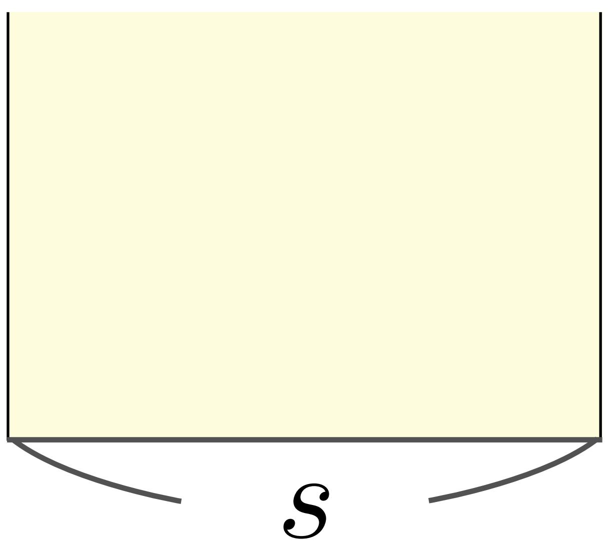


$=$



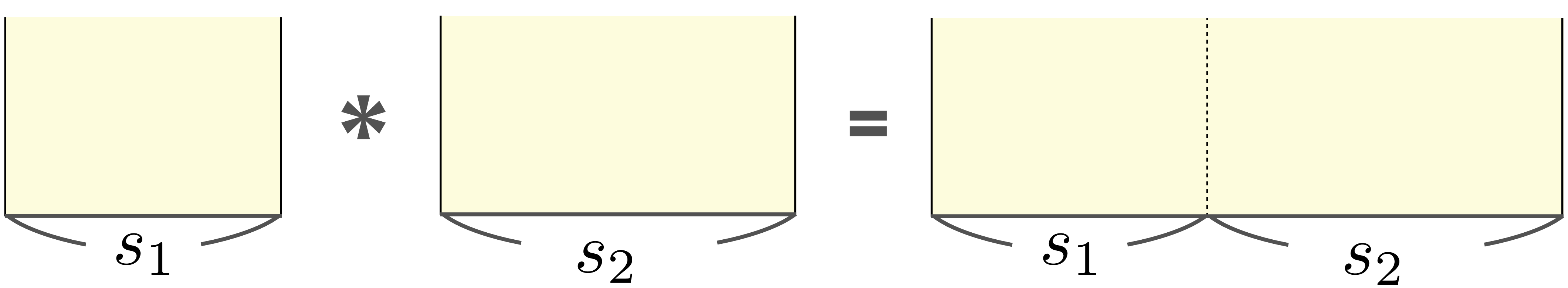
KBc subalgebra

wedge state

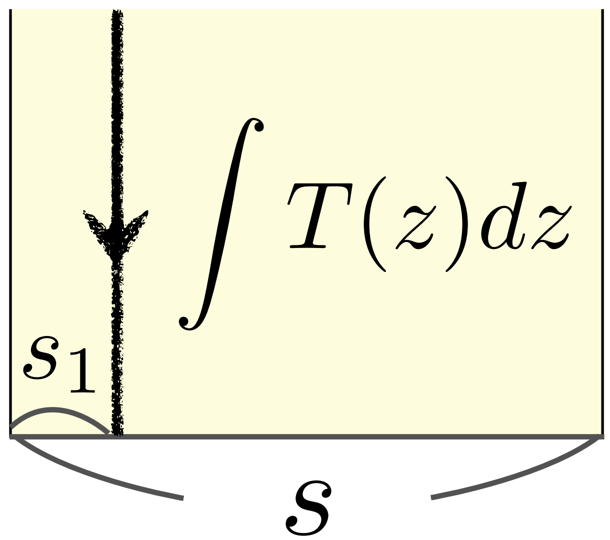


$$= e^{sK}$$

$$e^{s_1 K} * e^{s_2 K} = e^{(s_1+s_2)K}$$



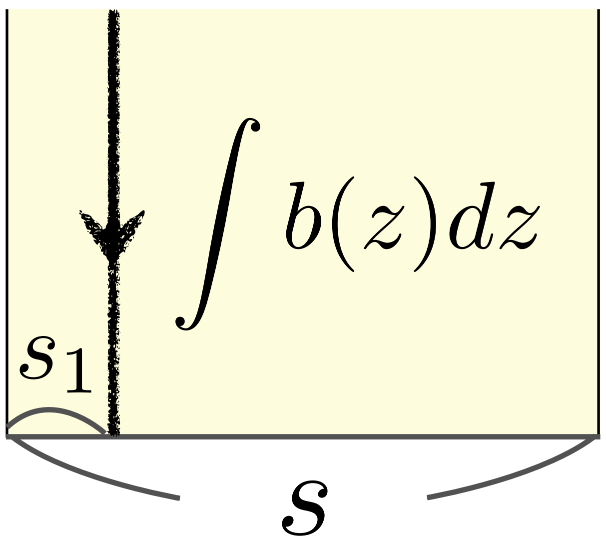
KBc subalgebra



$$= \quad e^{s_1 K} K e^{(s-s_1)K}$$

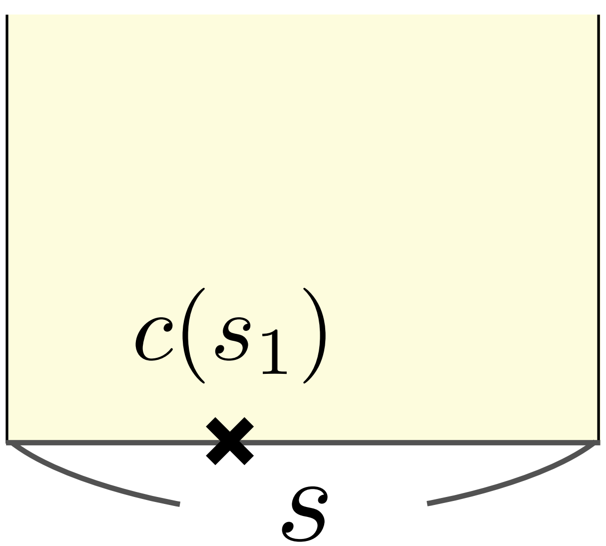
the state K = generator of wedge states

KBc subalgebra



$$= e^{s_1 K} B e^{(s-s_1)K}$$

$$B^2 = 0,\ QB = K\,,\ [K, B] = 0$$



$$= e^{s_1 K} c\, e^{(s-s_1)K}$$

$$c^2 = 0,\ \{B, c\} = 1,\ Qc = cKc$$

KBc subalgebra

$$B^2 = c^2 = 0,\ \{B, c\} = 1,\ QB = K,$$
$$QK = 0,\ Qc = cKc,\ [K, B] = 0$$

- simple algebraic relations

  solutions for tachyon condensation [Schnabl '05, Erler-Schnabl '09]

- can be defined without specifying D-brane configuration

  at the perturbative vacuum

→ in the universal sector of open SFT cf. [Sen '99]

tachyon vacuum, multiple D-branes???

Motivation

Okawa's formal solution: $\Psi = F(K)c\dfrac{KB}{1-F(K)^2}cF(K)$

- F(K) is some function of the state K
- formally satisfies the equation of motion

systematic studies:

cf. 増田さんのトーク，小路田さんのポスター

- energy [Erler '06]
- energy & gauge invariant observables [Murata-Schnabl '11]

  → proposal for multiple D-brane solutions

on their regularization [Hata-Kojita '11, Murata-Schnabl '11, Masuda '12]

no satisfactory regularizations are known

- boundary states [Takahashi '11] → subtleties for multiple D-branes

Motivation

would like to discuss systematically

which class of solutions are in the KBc subalgebra

↓

extending Takahashi's calculation of boundary states,

we calculated those for generic string fields of ghost # 1:

↓

using consistency of obtained boundary states,

we discuss possible boundary states in the KBc subalgebra

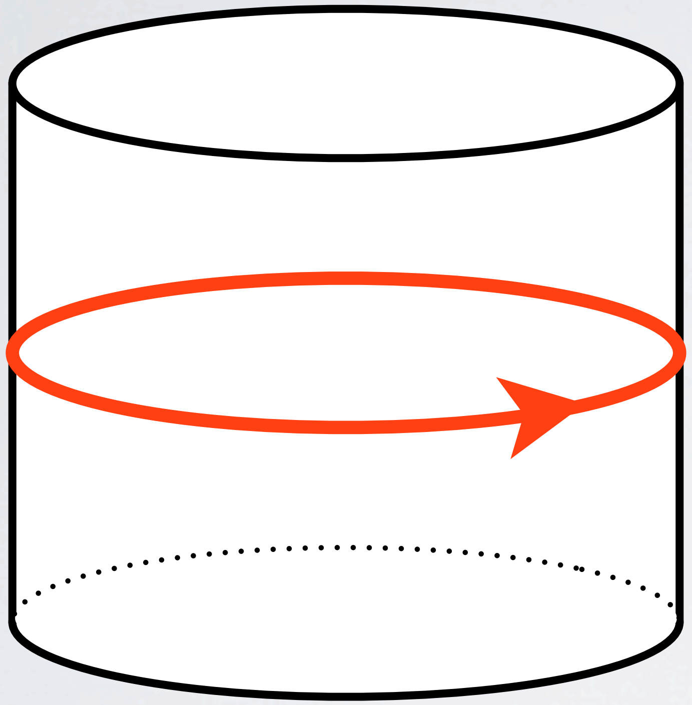
Plan of my talk

- open SFT & KBc subalgebra ✔
- boundary states from classical solutions
- implication from consistency of boundary states

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

basic idea



open string loop

using the propagator $\mathcal{P}$ in open SFT

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

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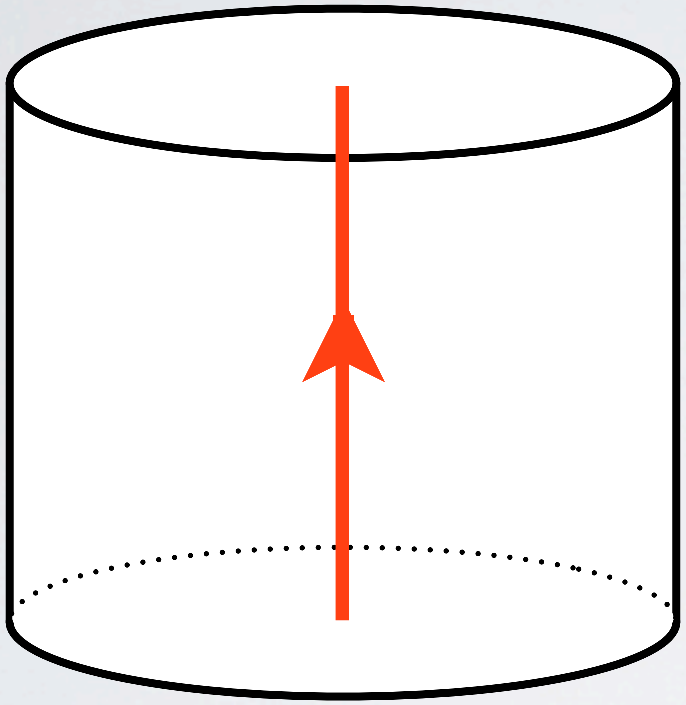

open string loop

using the propagator $\mathcal{P}$ in open SFT

$\longleftrightarrow$ closed string propagation

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

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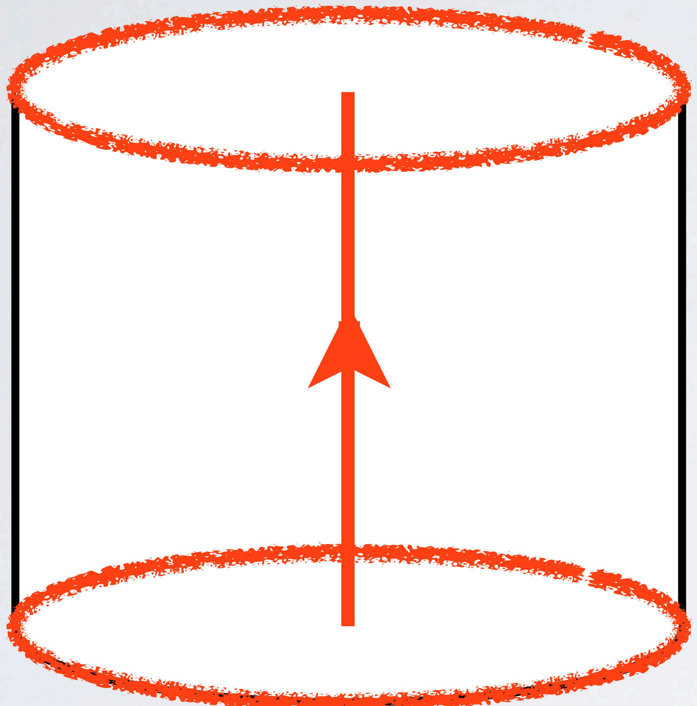

open string loop

using the propagator $\mathcal{P}$ in open SFT

⟷ closed string propagation

boundary state $|B\rangle$

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

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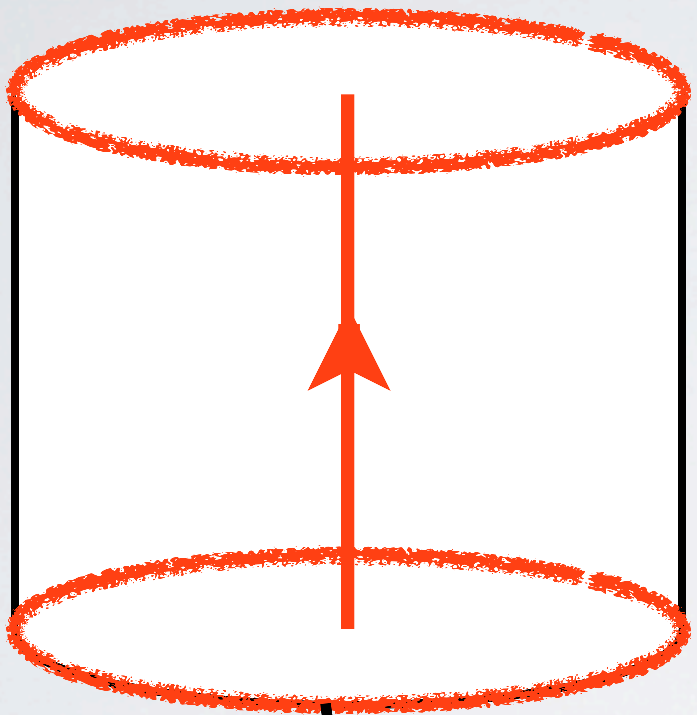


open string loop

using the propagator $\mathcal{P}$ in open SFT

⟷ closed string propagation

boundary state $|B\rangle$

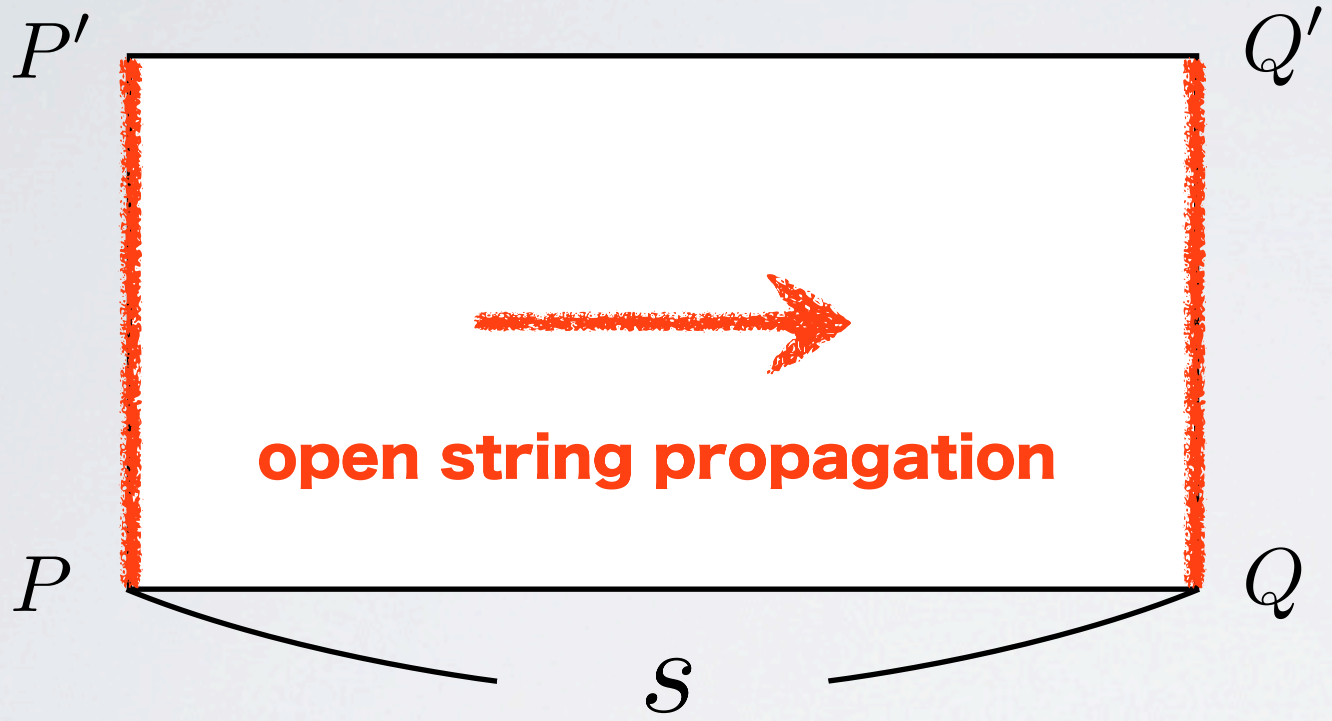
propagator $\mathcal{P}_*$ around a classical solution Ψ

⟶ boundary state corresponding to the solution!?

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

boundary state for the perturbative vacuum



open string propagator strip $e^{-s\mathcal{L}}$ with $\mathcal{L} = \{Q, \mathcal{B}\}$

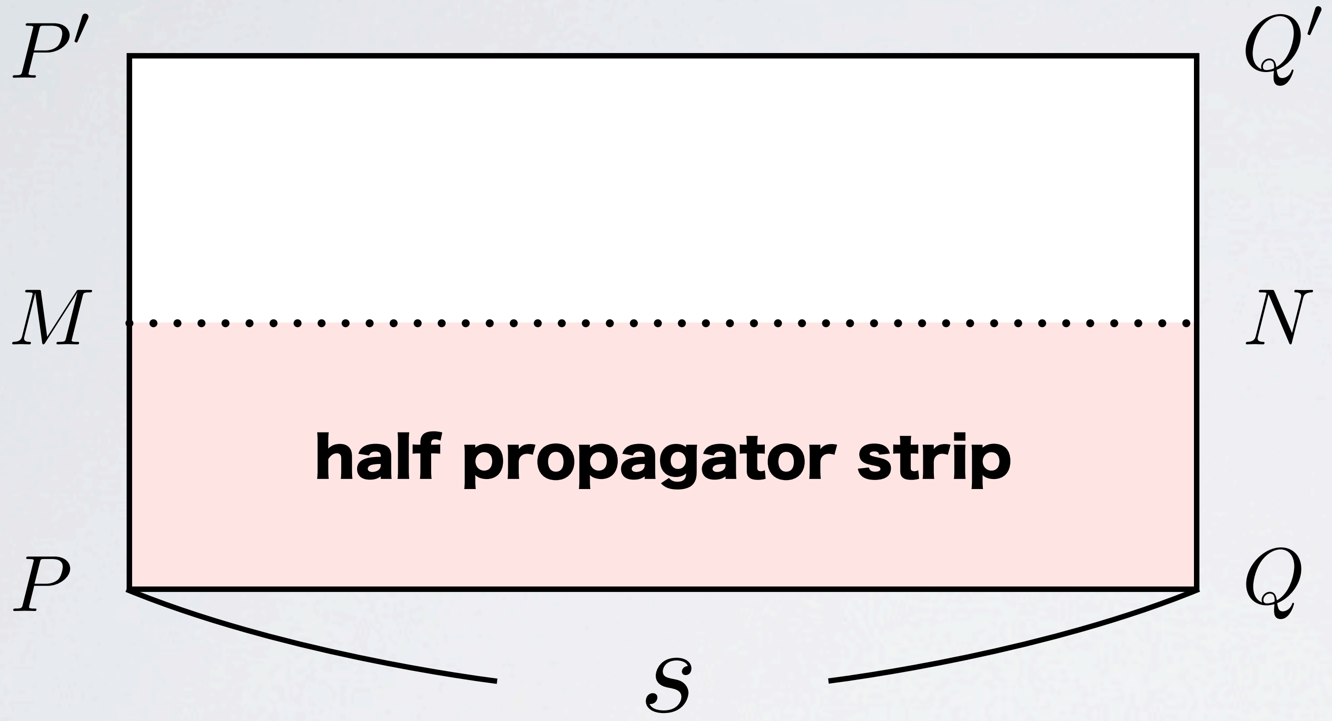
① gauge condition: $\mathcal{B}\Psi = 0$ with $\mathcal{B} = \oint \frac{1}{2\pi i} v(z) b(z)$

② parameter s : propagation length

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

\# boundary state for the perturbative vacuum



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Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

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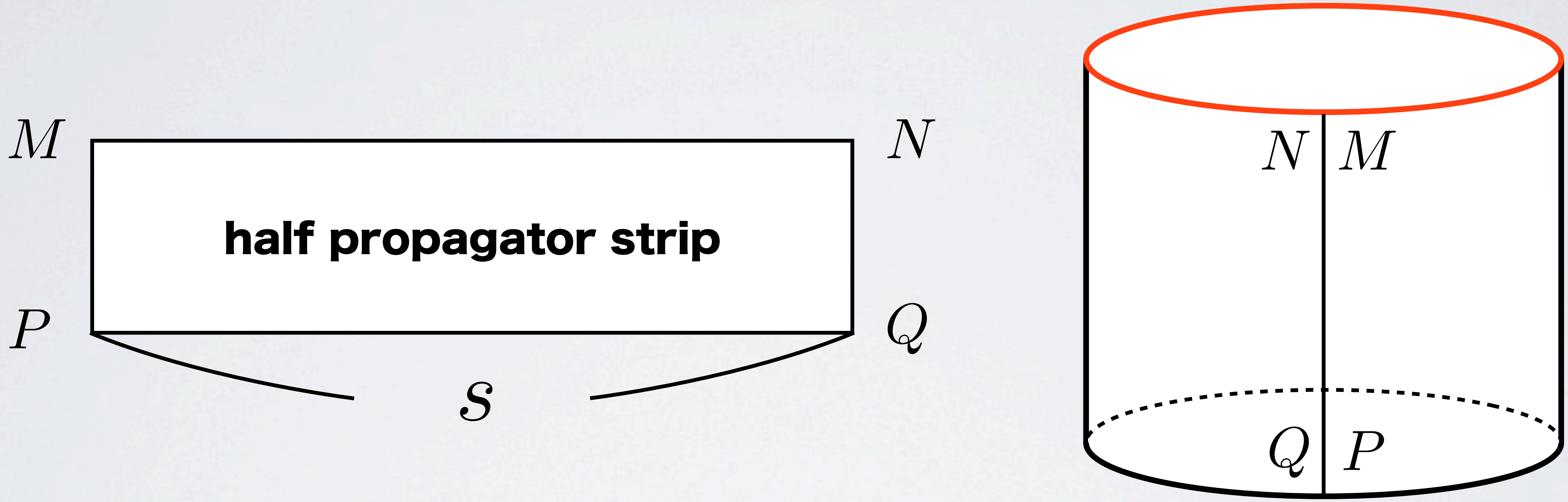
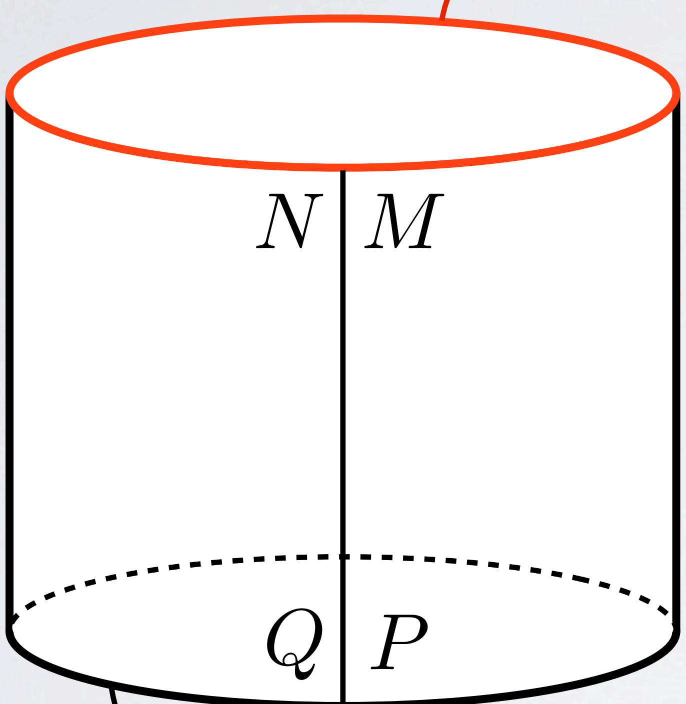


Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

boundary state for the perturbative vacuum

closed string boundary (midpoint propagation)



$$e^{-\frac{\pi^2}{s}(L_0+\tilde{L}_0)}|B\rangle$$

closed string propagation

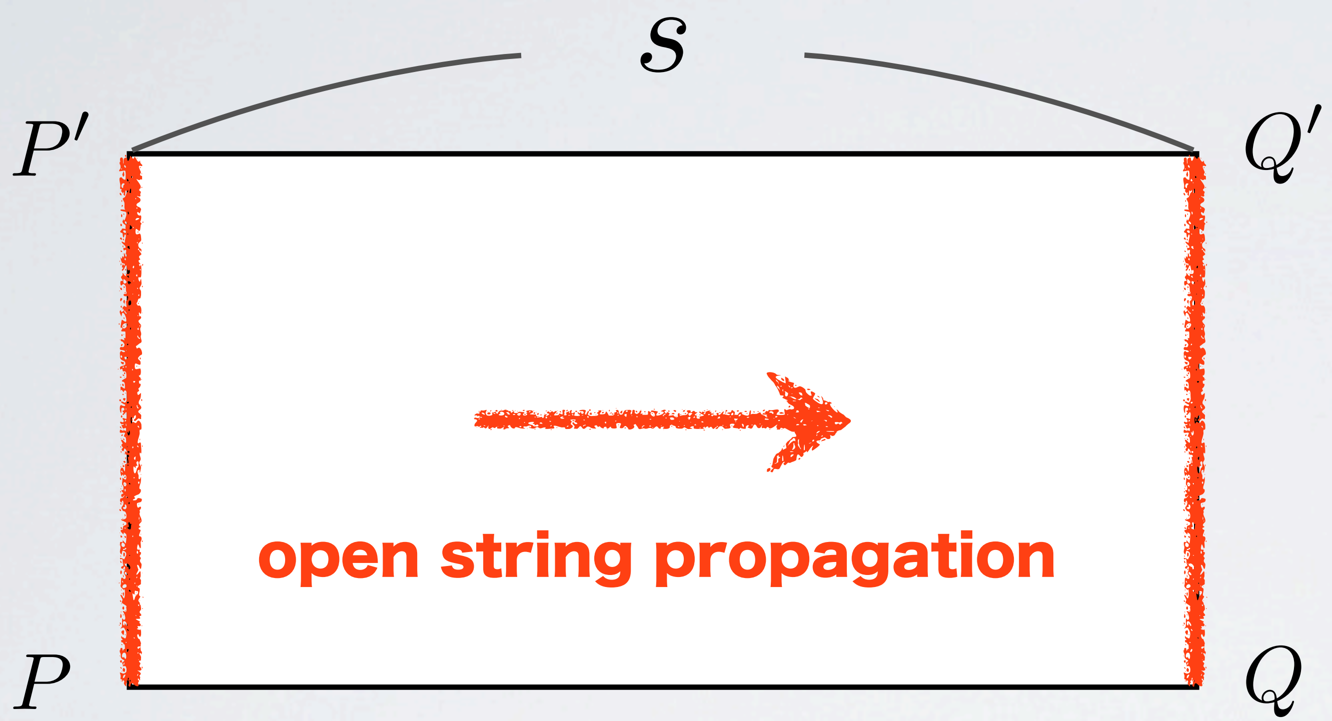
open string boundary

original boundary condition → **boundary state**

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

deformed propagator $\mathcal{P}_*$



open string propagator strip $e^{-s\mathcal{L}}$ with $\mathcal{L} = \{Q, \mathcal{B}\}$

propagator strip around a solution Ψ :

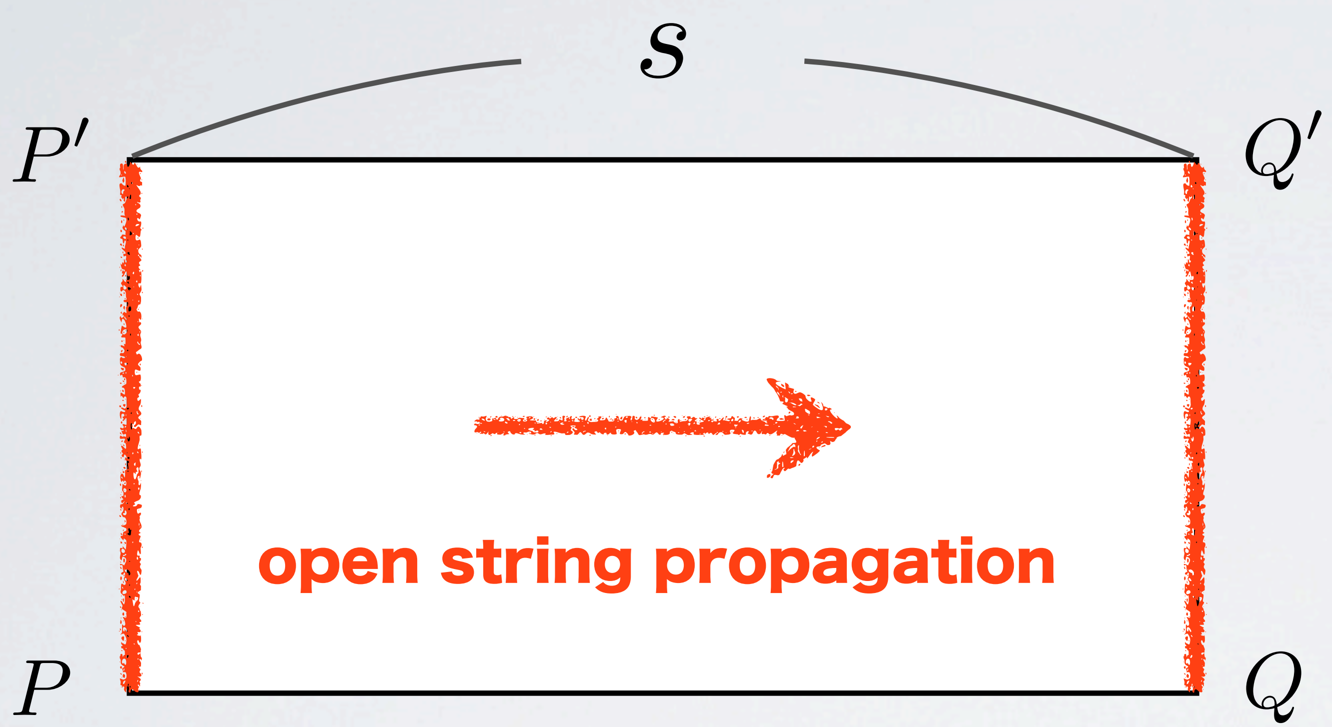
$$e^{-s\mathcal{L}_*} \text{ with } \mathcal{L}_* = \{Q_*, \mathcal{B}\} = \mathcal{L} + \{\mathcal{B}, \Psi\}$$

$$Q_* = Q + \{\Psi, \cdot\ ]$$

Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

deformed propagator $\mathcal{P}_*$



cf. point particle

interaction with external fields

open string propagator strip $e^{-s\mathcal{L}}$ with $\mathcal{L} = \{Q, \mathcal{B}\}$

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Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

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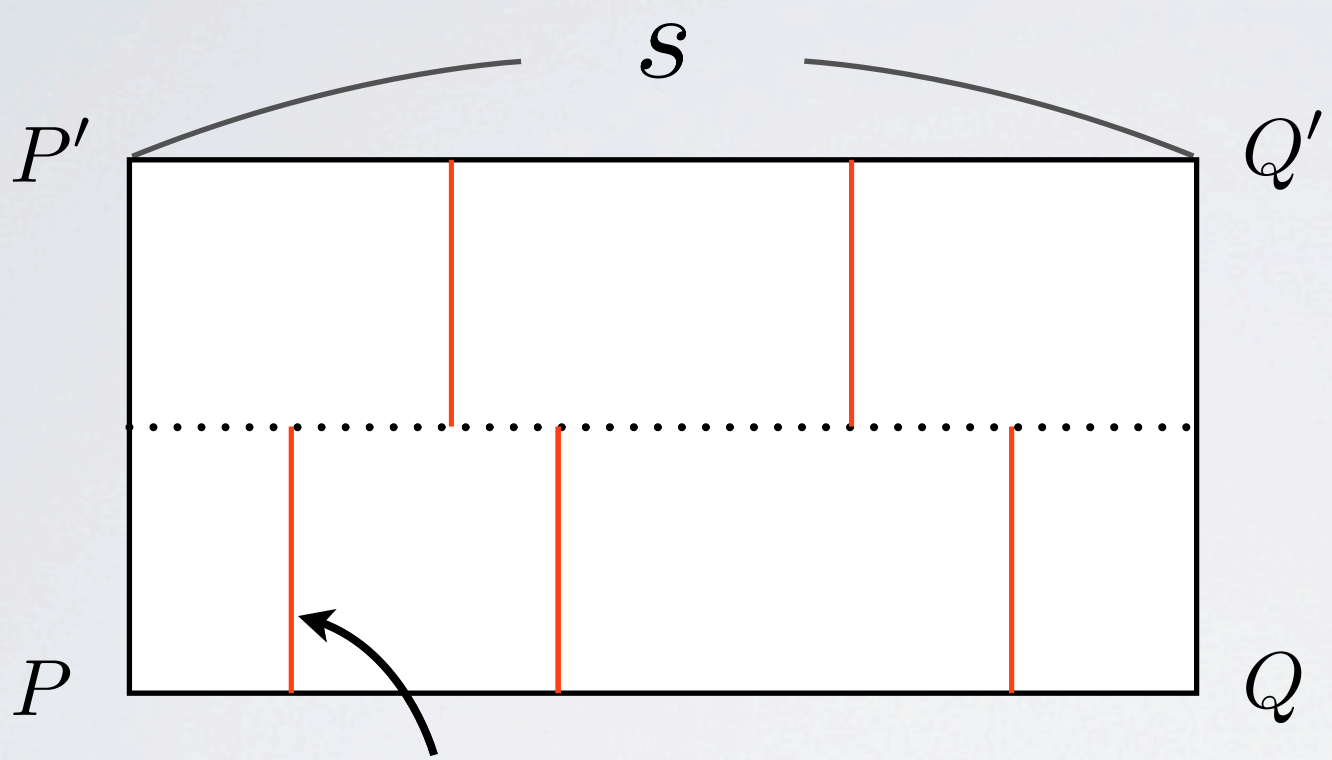

insertion of the solution Ψ

cf. point particle

interaction with external fields

open string propagator strip $e^{-s\mathcal{L}}$ with $\mathcal{L} = \{Q, \mathcal{B}\}$

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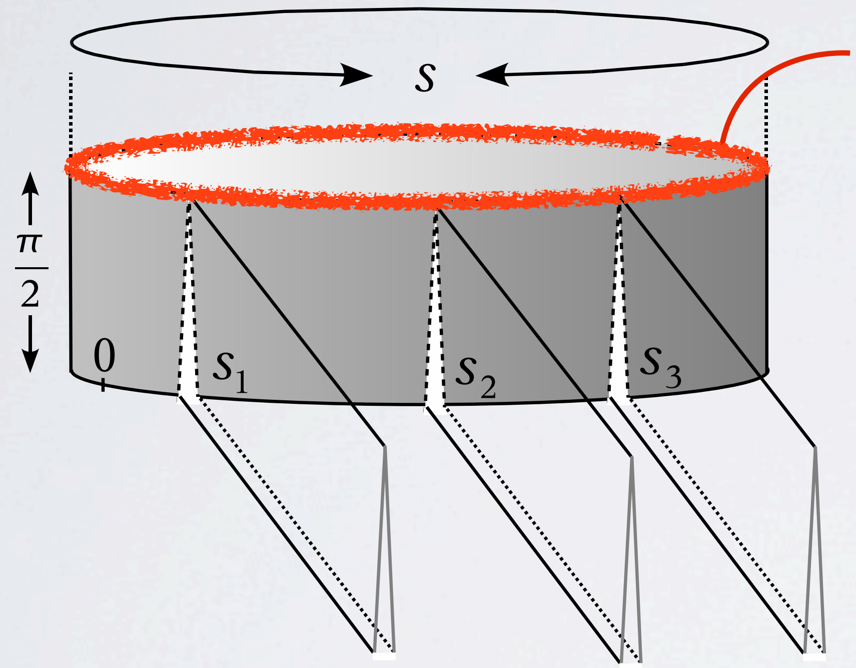
Boundary states from open string fields

[Kiermaier-Okawa-Zwiebach '08]

deformed half propagator



closed string boundary

$$e^{-\frac{\pi^2}{s}(L_0+\tilde{L}_0)}|B_*(\Psi)\rangle$$

- $|B_*(\Psi)\rangle \sim |B_*\rangle$
- independent of the choice of gauge condition and the parameter s

eom

Plan of my talk

- open SFT & KBc subalgebra ✔
- boundary states from classical solutions ✔
- implication from consistency of boundary states

Boundary states in the KBc subalgebra

applying this construction of boundary states,

we calculated those for generic string fields of ghost # 1:

$$\Psi = \sum_i F_i(K)\, cB\, G_i(K) c\, H_i(K)$$

$\{B, c\} = 1,\ B^2 = 0$

we assume

- F_i etc. are defined by superpositions of wedge states:

$$F_i(K) = \int_0^\infty f_i(s) e^{sK} \qquad \text{ex. } \frac{1}{1-K} = \int_0^\infty e^{-s} e^{sK}$$

- $f_i(s)$ damps fast enough for large s

(small contribution from wedge states of large width)

Boundary states in the KBc subalgebra

general form of the boundary states

$$|B_*(\Psi)\rangle = \frac{e^{(x+1)s} - e^{ys}}{e^s - 1}|B\rangle$$

with
$$\begin{cases} x = \sum_i G_i(0)\left(\frac{1}{2}F_i(0)H_i(0) + F_i'(0)H_i(0)\right) \\ y = \sum_i G_i(0)\left(\frac{1}{2}F_i(0)H_i(0) - F_i(0)H_i'(0)\right) \end{cases}$$

- proportional to $|B\rangle$
- c-number factor $\mathcal{N} = \frac{e^{(x+1)s} - e^{ys}}{e^s - 1}$ ~ # of D-branes
- information of Ψ is encoded in x and y
- non-trivial s-dependence

Boundary states in the KBc subalgebra

s-independence of boundary states

Ψ satisfies the eom $\longrightarrow$ $|B_*(\Psi)\rangle$ is s-independent

use the s-independence as a necessary condition

for Ψ to satisfy the equation of motion !

$$|B_*(\Psi)\rangle = \frac{e^{(x+1)s} - e^{ys}}{e^s - 1}|B\rangle$$

Boundary states in the KBc subalgebra

classification using s-independence

$$|B_*(\Psi)\rangle = \frac{e^{(x+1)s} - e^{ys}}{e^s - 1}|B\rangle$$

s-independent only in the following three cases !

- $|B_*(\Psi)\rangle = |B\rangle$ for $x = y = 0$

  ex. perturbative vacuum

- $|B_*(\Psi)\rangle = 0$ for $x = y - 1 = \text{arbitrary}$

  ex. tachyon vacuum

- $|B_*(\Psi)\rangle = -|B\rangle$ for $x = -1\,,\ y = 1$

  one ghost D-brane!? cf. [Okuda-Takayanagi '06]

※ no multiple D-branes in our class !

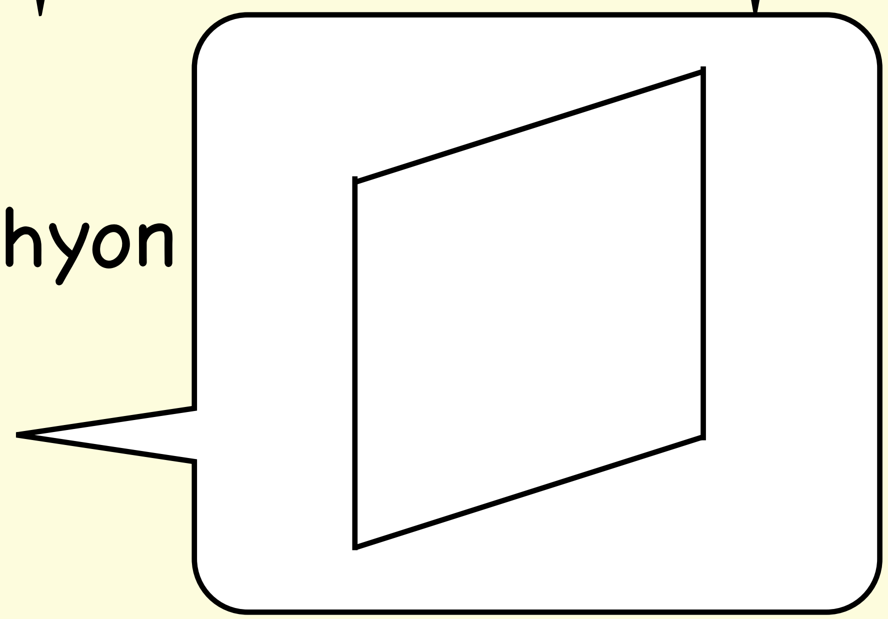
Ghost brane solutions!?

we found a solution reproducing $|B_*(\Psi)\rangle = -|B\rangle$

$$\Psi_{ghost} = \sqrt{\frac{1-pK}{1-qK}}\, c\, \frac{(1-qK)}{p-q}\, Bc \sqrt{\frac{1-pK}{1-qK}} + \sqrt{\frac{1-qK}{1-pK}}\, c\, \frac{(1-pK)}{q-p}\, Bc \sqrt{\frac{1-qK}{1-pK}}$$

- satisfy the eom
- energy density = 2 (that of tachyon

perturbative vacuum	0
tachyon vacuum	-1
ghost brane!?	-2



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perturbative vacuum —— 0

tachyon vacuum —— -1

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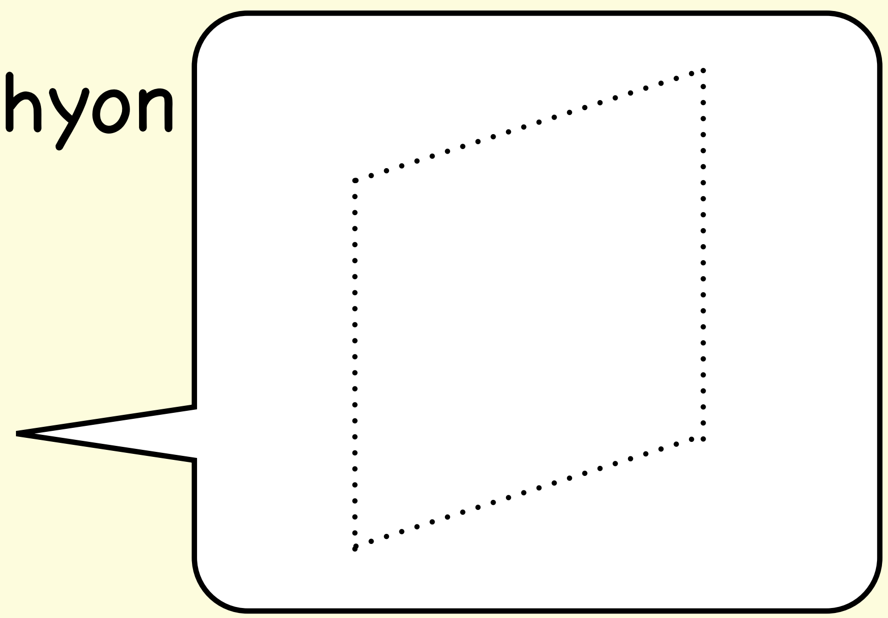

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- satisfy the eom
- energy density = 2 (that of tachyon vacuum)

perturbative vacuum ——— 0

tachyon vacuum ——— -1

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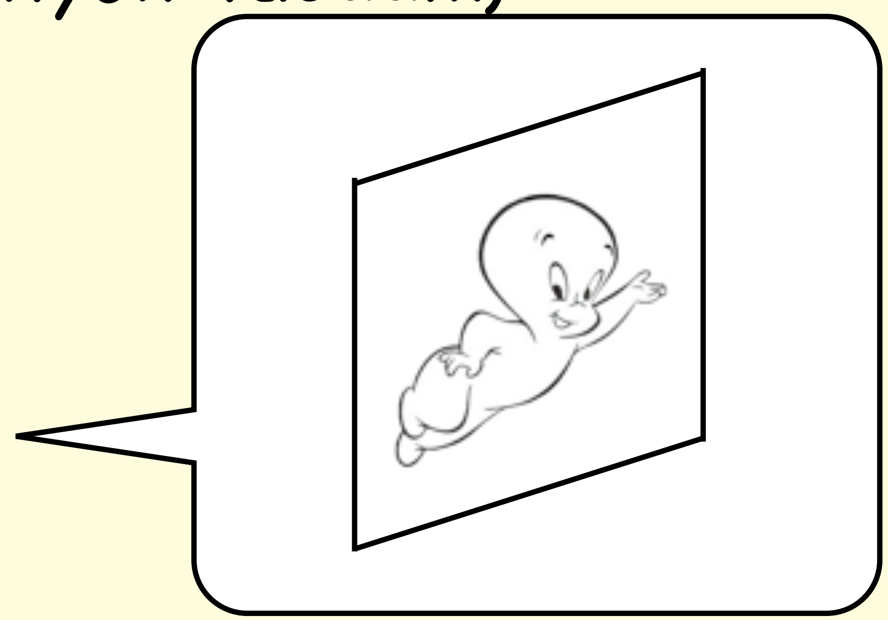

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perturbative vacuum	0
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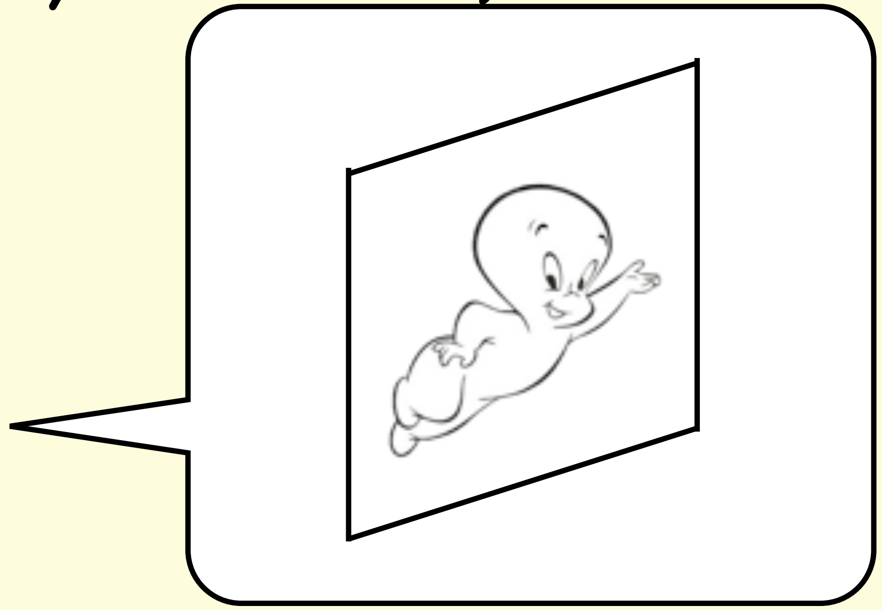

- $|B_*(\Psi)\rangle = -|B\rangle$
- some other consistency checks have been done

※ what's this? "physical" solution?? ghost D-brane???

Summary

calculated boundary states

for generic string fields in the KBc subalgebra

$$|B_*(\Psi)\rangle = \frac{e^{(x+1)s} - e^{ys}}{e^s - 1}|B\rangle \left[\begin{array}{l} x = \sum_i G_i(0) \left(\frac{1}{2} F_i(0) H_i(0) + F_i'(0) H_i(0) \right) \\ y = \sum_i G_i(0) \left(\frac{1}{2} F_i(0) H_i(0) - F_i(0) H_i'(0) \right) \end{array} \right.$$

s-independence of boundary states

⟶ restricted to $|B_*(\Psi)\rangle = \pm|B\rangle,\ 0$

propose a candidate for the ghost brane solution

On multiple D-brane solutions

no multiple D-brane solutions in our class

→ need to relax our regularity conditions

proposed multiple D-brane solutions [Murata-Schnabl '11]

$$\Psi = \frac{1}{K} c \frac{K^2}{K-1} Bc \qquad \left(\frac{1}{K} = -\int_0^\infty e^{sK} \right)$$

- singular and require some regularization
- large contribution from wedge states of large width

※ we used Schnabl gauge propagator to derive our formula

- subtlety for wedge states of large width
- need to improve our discussion

THANK YOU!!