Higher Derivative Corrections to the effective theory of an Non-Abelian Vortex

大橋 圭介 (大阪市立大学)

共同研究者

衛藤 稔 (山形大)

藤森 俊明 (台湾大)

新田 宗土 (慶応大)

坂井 典佑 (東京女子大)

§1. Introduction

- Non-Abelian Vortex [Hannany・Tong, 2003] [Auzzi, et. al, 2003]
  - Abrikosov-Nielsen-Olesen (ANO) Vortex
    ⟶ color-flavor locked vac. of $U(N)$ gauge theory with N flavor
  - ½ BPS config. in 4-dim. $\mathcal{N}=2$ theory
  - Orientational zero mode

$$\mathbb{C}P^{N-1} = \frac{SU(N)_{C+F}}{SU(N-1)\times U(1)} \qquad \begin{pmatrix} \text{ANO} & 0 \\ 0 & \mathbb{1}_{N-1} \end{pmatrix}$$

Namba-Goldstone zero mode

Exact correspondence of BPS spectra

(Kimura, Fujimori's talk)

- 4-d bulk theory

$\mathcal{N}=2$, $U(N)$ gauge theory
$+ N$ flavors

- 2-d vortex worldsheet theory

$\mathcal{N}=(2,2)$ $\mathbb{C}P^{N-1}$ sigma model.

+ higher derivative corrections

↑ today's talk

vortex ⟷ Yang-Mills Instantons

correction?

vacua ⟷ lumps (σ-model instanton)

holomorphic map
$\mathbb{C} \longrightarrow$ 2-cycle in $\mathbb{C}P^{N-1}$

· Derivative Correction

example a particle on $\mathbb{R}^2$

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{\lambda}{2}(r_0^2 - r^2)^2$$

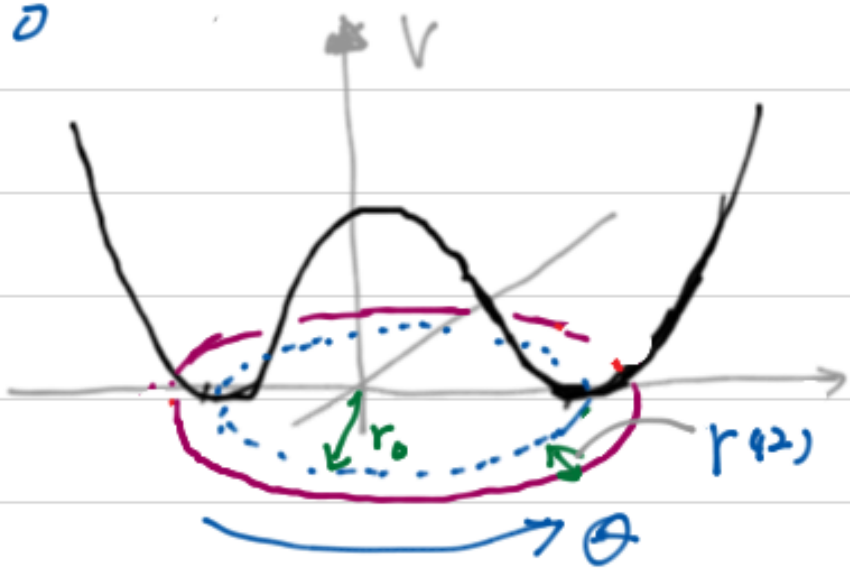
vac. $r = r_0$, θ : moduli parameter

$$\rightarrow L_{eff} = \frac{m}{2} r_0^2 \dot{\theta}^2 \qquad \text{with} \quad \theta = \theta(t)$$

derivative correction $\quad r = r_0 + \overbrace{\underbrace{r^{(2)}}_{O((\frac{d}{dt})^2)} + \underbrace{r^{(4)}}_{O((\frac{d}{dt})^4)}}^{\text{massive modes}} + \cdots$

$$\rightarrow L^{(4)} = m r_0 r^{(2)} \dot{\theta}^2 - \frac{\lambda}{2}(r^{(2)})^2 r_0^2$$

e.o.m of $r^{(2)}$ $\longrightarrow$ $r^{(2)} = \dfrac{m}{\lambda r_0}\dot{\theta}^2$



$$\Rightarrow \mathcal{L}^{(4)} = \frac{m^2}{2\lambda}\dot{\theta}^4$$

higher derivative correction for Orientational zero mode for U(2)

$$S = \underbrace{\frac{4\pi}{g^2}\int d^2x \, \frac{|\partial_\alpha b|^2}{(1+|b|^2)^2}}_{= S^{(2)}} + \boxed{S^{(4)}} + S^{(6)} + \cdots$$

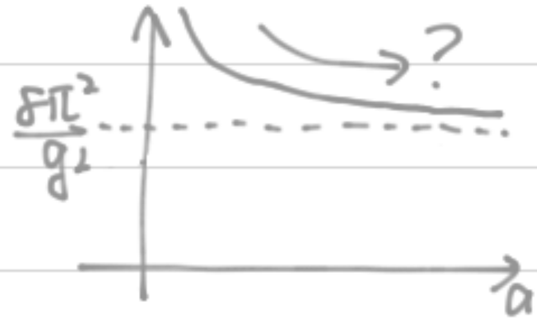
$b \in \mathbb{C}P^1$ F.S metric $\alpha = 1, 2.$

Single lump (instanton) solution

$$b = \frac{x^1 + i x^2}{a} \longleftarrow \text{size moduli} \in \mathbb{C}$$

$$S = \frac{8\pi^2}{g^2} + S^{(4)}(a) + \cdots$$

$$= \frac{8\pi^2}{g^2} + \frac{\text{const.}}{a^2} + \cdots\cdots$$



Instability??

Our main result

$$\boxed{\text{We checked the correction vanishing!}}$$

§2. Non-Abelian Vortices

- 4-dim $\mathcal{N}=2$ $U(N)$ gauge theory + N hypermultiplets in fun. rep.

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + D_\mu H (D^\mu H)^\dagger - \frac{g^2}{4}\left(HH^\dagger - v^2 \mathbb{1}_N \right)^2 \right]$$

gauge coupling const.

scalar field $(N \times N)$

FI para.

- vacuum. $\overset{U(N)_C \quad SU(N)_F}{H = v\,\mathbb{1}_N}$ (color-flavor locking vacuum)

$$U(N)_C \times SU(N)_F \to SU(N)_{C+F}$$

- BPS equation $(z = x^3 + i x^4)$

$$D_{\bar z} H = 0, \qquad i F_{z\bar z} = \frac{g^2}{4}\left(v^2 - HH^\dagger \right)$$

with a tension for static config.

$$T = -v^2 \int dz \wedge d\bar z \, \mathrm{Tr}\, F_{z\bar z} = 2\pi v^2 k$$

$k \in \mathbb{Z}_{\geq 0}$

- BPS solutions

$$H = S^{-1} H_0(z)$$ ← "moduli matrix" holomorphic in z

$$A_{\bar{z}} = -i S^{-1} \partial_{\bar{z}} S$$

with $S \in GL(N, \mathbb{C})$,

- master equation for $\Omega \equiv S S^{\dagger}$

$$\partial_{\bar{z}}(\Omega \partial_z \Omega^{-1}) = \frac{g^2 v^2}{4}(H_0 H_0^{\dagger} - \mathbb{1}_N)$$

- Single vortex sol

$$H_0(z) = \begin{pmatrix} \mathbb{1}_{N-1} & -\vec{b} \\ 0 & z - Z \end{pmatrix}$$

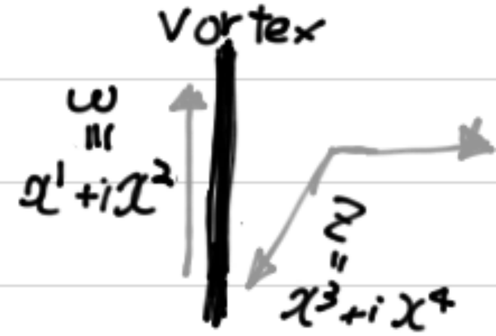
$\vec{b}$: inhomogeneous coord. of $\mathbb{C}P^{N-1}$

Z: position of vortex

$$\Rightarrow F_{z\bar{z}} = F_{z\bar{z}}^{ANO} \times \frac{1}{1+|\vec{b}|^2} \begin{pmatrix} \vec{b} \otimes \vec{b}^{\dagger} & \vec{b} \\ b^{\dagger} & 1 \end{pmatrix}$$

- Derivative expansion and effective action

$(\partial_\alpha,\ \alpha=1,2)$



- $H(x^\mu) = H^{(0)} + \boxed{H^{(2)}} + O(\partial_\alpha^4)$

  $A_{\bar z}(x^\mu) = A^{(0)}_{\bar z} + \boxed{A^{(2)}_{\bar z}} + O(\partial_\alpha^4)$

  $A_\alpha(x^\mu) = A^{(1)}{}_\alpha + O(\partial_\alpha^3)$

- the zero-th order solution.

  $\{\phi^i\}$: complex coordinates on the moduli space

  $Z,\ \vec b$

  $\phi^i \Rightarrow$ chiral (super) field $\Phi^i(x^\alpha)$ $(Z(x^\alpha), \vec b(x^\alpha) \ldots)$

  $H^{(0)} = H^{sol}(x^{3,4},\ \phi(x^{1,2}))$

  $A^{(0)}_{\bar z} = A^{sol}_{\bar z}(x^{3,4},\ \phi(x^{1,2}))$

- the first order $\Longleftarrow$ Gauss' law equation

$$A^{(1)}_{\alpha=1,2} = i\left(\delta_\alpha S^\dagger S^{-1} - S^{-1}\delta^\dagger_\alpha S\right)$$ [Eto. et al. 2006]

$$\delta_\alpha \equiv \partial_\alpha \phi^i \frac{\partial}{\partial \phi^i}, \quad \delta^\dagger_\alpha \equiv \partial_\alpha \bar{\phi}^i \frac{\partial}{\partial \bar{\phi}^i}$$

- the 2-nd order effective action

$$S^{(0)}_{eff} = -\underbrace{T}_{\text{vortex tension}} \int d^2x$$

$$S^{(2)}_{eff} = \int d^2x \left[\frac{T}{2}\partial_\alpha Z \partial^\alpha \bar{Z} + \frac{4\pi}{g^2} g^{FS}_{i\bar{j}} \partial^\alpha b^i \partial_\alpha \bar{b}^j\right]$$

with Fubini-Study metric

$$g^{FS}_{i\bar{j}} = \frac{\partial}{\partial b^i}\frac{\partial}{\partial \bar{b}^j} \log\left(1+|\vec{b}|^2\right)$$

[Hannany-Tong, 2003], [Gorsky-Shifman-Yung 2004]

§3. Higher derivative corrections

- the 2-nd order solution

$$\Psi^{(2)} \equiv \begin{pmatrix} H^{(2)} \\ A_{\bar{z}}^{(2)} \end{pmatrix}$$ ← contribution from massive modes

should be orthogonal to gauge. and physical zero modes

$$\Delta \Psi_i = 0 \qquad \Delta \equiv \begin{pmatrix} iD_{\bar{z}} & -H^{(0)} \\ \frac{e^2}{4} H^{(0)\dagger} & iD_z \end{pmatrix}$$

solution

$$\longrightarrow \quad \partial_\alpha \phi^i \Psi_i = \begin{pmatrix} D_\alpha H^{(0)} \\ F_{\alpha \bar{z}}^{(1)} \end{pmatrix}$$

E.o.M for $\Psi^{(2)}$

Lagrange multiplier

$$4\Delta^\dagger \Delta \Psi^{(2)} + \mathcal{D}_\alpha (\partial^\alpha \phi^i \Psi_i) = \lambda^i \Psi_i$$

We fortunatelly find

cov. deriv. on the moduli space

$$\Delta\Psi^{(2)} = \frac{i}{4}\begin{pmatrix} \frac{4}{g^2 v}\partial_\alpha\phi^i\partial^\alpha\phi^j\, S^\dagger\left[\nabla_i\frac{\partial}{\partial\phi_j}(\bar{\partial}_\Delta\Omega\,\Omega^{-1})\right]H_0^{\dagger-1} \\ i\,\partial_\alpha\phi^i\partial^\alpha\bar{\phi}^j\, S^{-1}\left[\frac{\partial}{\partial\bar{\phi}^j}\left(\Omega_\Delta\frac{\partial}{\partial\phi^i}\Omega_\Delta^{-1}\right)\right]S\end{pmatrix}$$

- the 4-th order effective action for $\mathbb{C}P^{N-1}$

$$S^{(4)}_{\rm eff} = \frac{4\pi C}{g^4 v^2}\int d^2x\left(g^{FS}_{i\bar{j}}\,\partial_\alpha b^i\partial_\beta\bar{b}^j\right)\left(g^{FS}_{k\bar{l}}\,\partial^\alpha b^k\partial^\beta\bar{b}^l\right)$$

$\alpha,\beta = 1,2$ $\quad C = 0.830707..$

· Euclidean action with $\mathbb{C}P^1$ ($U(2)$ case)

$$w = x^1 + i x^2$$

$$S_{\text{eff}}^{(2)+(4)} = \frac{4\pi}{g^2}\int d^2x \left[\frac{|\partial_w b|^2 + |\partial_{\bar{w}} b|^2}{(1+|b|^2)^2} + 4c\,\frac{|\partial_w b\, \partial_{\bar{w}} b|^2}{(1+|b|^2)^4} \right]$$

$$= \frac{16\pi}{g^2}\underbrace{\int d^2x \left[1 + 4c\,\frac{|\partial_w b|^2}{(1+|b|^2)^2} \right] \frac{|\partial_{\bar{w}} b|^2}{(1+|b|^2)^2}}_{\geq 0} + \frac{8\pi^2}{g^2} k$$

instanton (lump) number

$$k = \frac{i}{2\pi}\int \frac{db \wedge d\bar{b}}{(1+|b|^2)^2} \in \mathbb{Z}$$

The lower bound saturated by

holomorphic map $\underset{\sim}{b(w)}$ $\quad (\partial_{\bar{w}} b = 0)$

$$\Rightarrow \boxed{S_{\text{eff}}^{(4)}\Big|_{b=b(w)} = 0}$$

§4. Conclusion

- General Formula for
  the 4-th order derivative corrections
  to the non-Abelian vortex effective action

- Concrete result for
  the single vortex effective action.

- The instanton (lump) solutions and
  the monopole (kink) solutions do not
  accept any correction
  from higher derivative terms.
  (at least in the 4-th order)