New geometric interpretation of D-branes and DBI action

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Introduction

[Fradkin, Tseytlin '85]
[Abouelsaood, Callan, Nappi, Yost '87]
[Callan, Lovelace, Nappi, Yost '88]

Dirac-Born-Infeld (DBI) action:

the low-energy effective theory of D-brane

$$S_{\mathrm{DBI}} = \int_{\varphi_\Phi(\Sigma)} \sqrt{\det(\varphi_\Phi^*(g+B) - F)_{ab}}\, dx^0 \wedge \cdots \wedge dx^p$$

We proposed that

DBI action = Generalization of Nambu-Goto action
in the framework of generalized geometry

It is known that

Scalar fields describing transverse displacements
= NG bosons for broken translational symmetries

Broken symmetries are non-linearly realized in NG action.

[Low, Manohar '02]

Under T-duality,

Gauge field $\xleftrightarrow{\text{exchange}}$ Scalar field

We show that

- The gauge field and the scalar fields can be treated on equal footing in the framework of generalized geometry.
- As scalar fields, the gauge field are interpreted as NG boson for broken (constant) B-field gauge transformations.
- The DBI action is invariant under non-linearly realized symmetry for all types of diffeomorphisms and B-field gauge transfomations over the target space.

Outline

Introduction

1. What is generalized geometry
2. D-brane (Dirac structure)
3. Adding a metric and a B-field (generalized metric str.)
4. Transformation law
5. Invariance of the DBI action

Summary

1. What is generalized geometry

What is generalized geometry?

[Courant '90] [Hitchin '03]

Generalized geometry is extension of (usual) differential geometry.

M : D-dimensional manifold (the target space)

Differential geometry

$$TM$$

Diffeomorphism

Generalized Geometry

$$\mathbb{T}M = TM \oplus T^*M$$

Diff. and B-field gauge transf.

Lie derivative:

$$\mathcal{L}_{\underset{\substack{\big\uparrow \\ \text{generators}}}{\underline{u+\xi}}}(v+\eta) = \underbrace{\mathcal{L}_u(v+\eta)}_{\text{Diffeomorphism}} - \underbrace{\imath_v d\xi}_{\substack{\text{B-field gauge} \\ \text{transf.}}}$$

2. D-brane

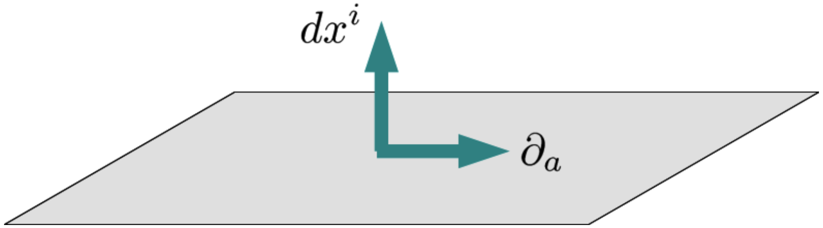
(Dirac structure)

Dp-brane :

(Dirac structure [Courant '90])

A Dp-brane without fluctuations (Φ, A) can be characterized as a subbundle

$$\begin{aligned} L &= \{v^a(x)\partial_a + \xi_i(x)dx^i\} \\ &= \Delta \oplus \mathrm{Ann}(\Delta) \qquad \subset \mathbb{T}M \end{aligned}$$



Examples of D-branes:

TM : D9-brane
T^*M : D(-1)-brane

(M:10 dim manifold)

Dp-brane with fluctuations

Acting a diffeomorphism and a B-field gauge transformation on L , a D-brane with fluctuations can be represented as a subbundle

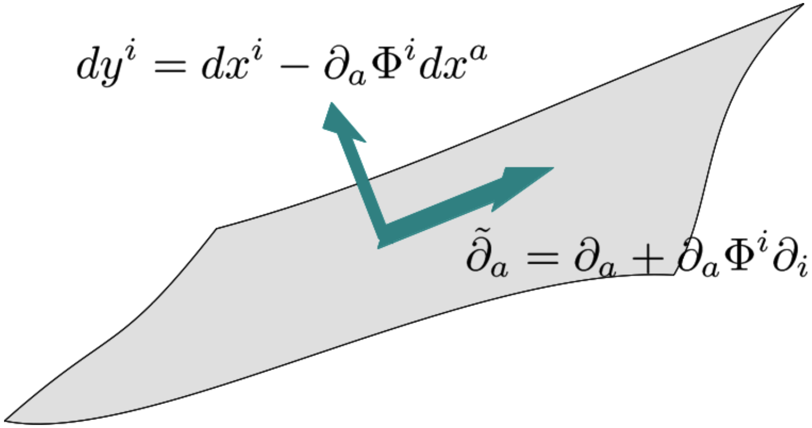
$$L_{\mathcal{F}} = e^{-\mathcal{L}_{\Phi+A}} L \subset \mathbb{T}M$$

$$V_L + \mathcal{F}(V_L) = v^a(x)(\partial_a + \partial_a\Phi^i\partial_i + F_{ab}dx^b) + \xi_i(x)(dx^i - \partial_a\Phi^i dx^a) \quad \in \Gamma(L_{\mathcal{F}})$$

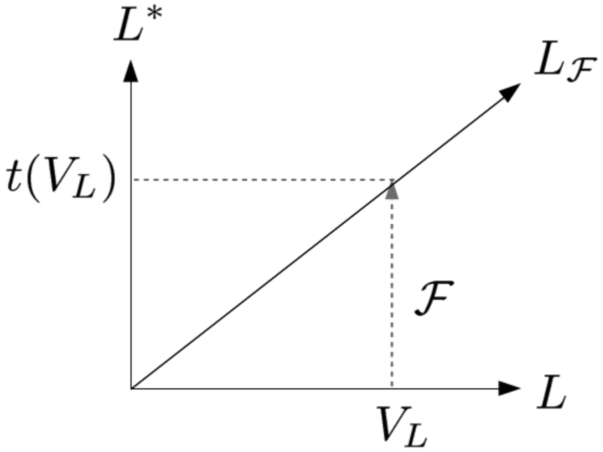
where

$\mathcal{F} = \frac{1}{2}F_{ab}dx^a \wedge dx^b + \partial_a\Phi^i dx^a \wedge \partial_i$: a generalized field strength

Image of $L_{\mathcal{F}}$,



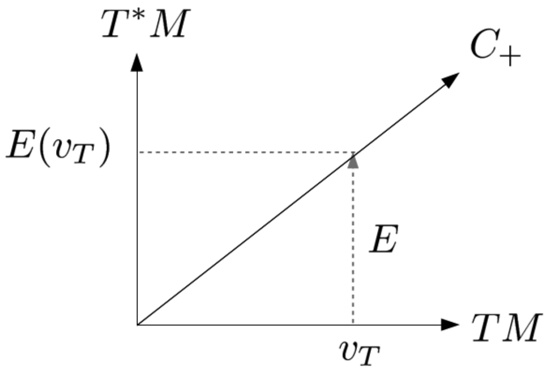
Graph of a map $\mathcal{F} : L \to L^*$,

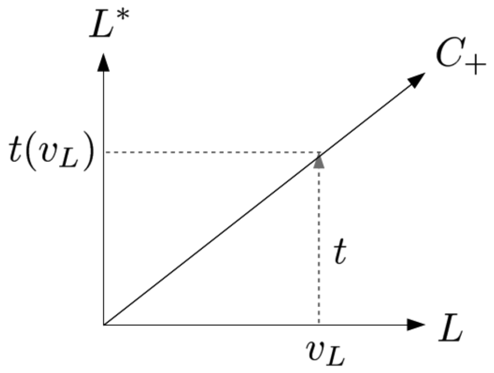


3. Adding a metric and a B-field (generalized metric structure)

Generalized metric

Generalized metric is given by a subbundle $C_+ \subset \mathbb{T}M$ of which sections are described as a graph of $E = g + B : TM \to T^*M$ or as a graph $t : L \to L^*$,





Since these graphs represent the same subbundle, we find the relation

$$\begin{aligned} t^{ij} &= E^{ij}, & t_a{}^j &= -E_{ak}E^{kj}, \\ t^i{}_b &= E^{ik}E_{kb}, & t_{ab} &= E_{ab} - E_{ak}E^{kl}E_{lb} \end{aligned}$$

These relations are similar to the Buscher rule since T-duality exchanges vector fields and 1-forms.

Another definition of the generalized metric

We introduce the operation $G : \mathbb{T}M \to \mathbb{T}M$ given by

$$G = \begin{pmatrix} -Bg^{-1} & g^{-1} \\ g - Bg^{-1}B & g^{-1}B \end{pmatrix}$$

[Gualtieri '03]

Then $C_\pm = \mathrm{Ker}(1 \mp G)$

1. Restriction of G to $L_\mathcal{F}$ gives the induced metric on $L_\mathcal{F}$:

$$s_\mathcal{F} = s - (a - \mathcal{F})s^{-1}(a - \mathcal{F}) \quad \in \Gamma(L_\mathcal{F}^* \otimes L_\mathcal{F}^*)$$

2. Restriction of G to C_+ gives a bulk metric:

$$g \quad \in \Gamma(T^*M \otimes T^*M)$$

s : symmetric part of t

a : anti-symmetric part of t

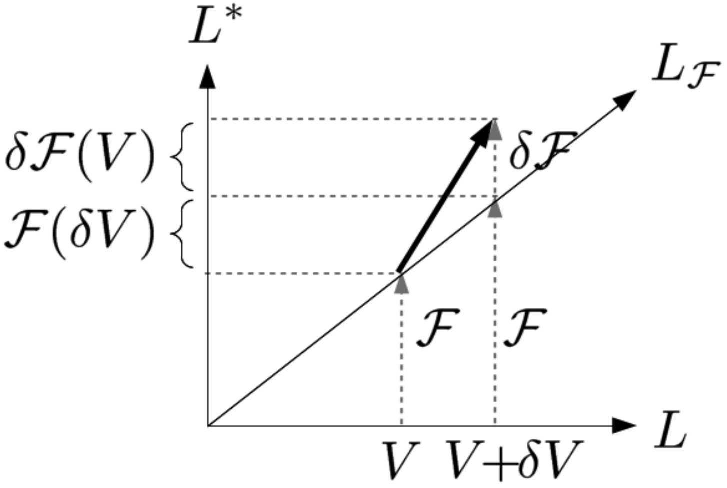
4. Transformation law

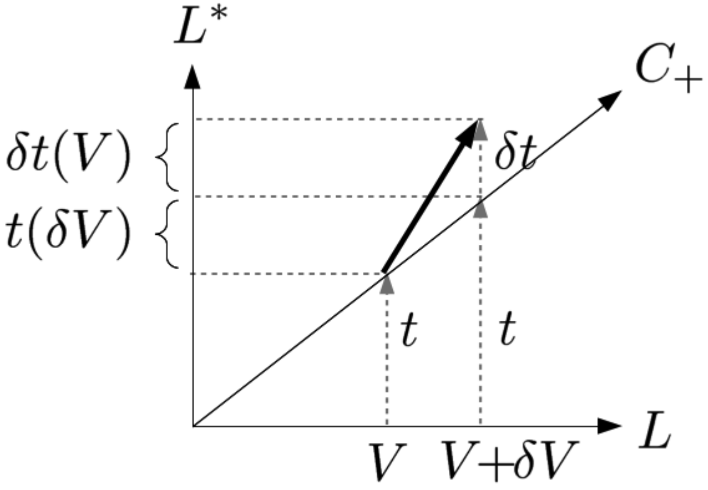
Transformation law

Consider an infinitesimal transformation of a diffeomorphism and a B-field gauge transformation generated by

$$\epsilon = \epsilon_{\|} + \epsilon_{\perp} = \epsilon^a \partial_a + \epsilon^i \partial_i$$
$$\Lambda = \Lambda_{\|} + \Lambda_{\perp} = \Lambda_a dx^a + \Lambda_i dx^i$$

The transformation law for the tensor $\mathcal{F} \in \Gamma(L^* \otimes L^*)$ and $t \in \Gamma(L^* \otimes L^*)$ can be found as the transformation of the subbundles $L_{\mathcal{F}}$ and C_+ .





Transformation law:

$$\begin{aligned}
\delta\mathcal{F}_{ab} &= -\epsilon^M\partial_M\mathcal{F}_{ab} - \partial_a\epsilon^c\mathcal{F}_{cb} - \mathcal{F}_{ac}\partial_b\epsilon^c - \partial_{[a}\Lambda_{k]}\mathcal{F}^k{}_b + \mathcal{F}_a{}^k\partial_{[k}\Lambda_{b]} \\
&\quad -\mathcal{F}_a{}^k\partial_k\epsilon^c\mathcal{F}_{cb} + \mathcal{F}_{ac}\partial_k\epsilon^c\mathcal{F}^k{}_b - \mathcal{F}_a{}^k\partial_{[k}\Lambda_{l]}\mathcal{F}^l{}_b + \partial_{[a}\Lambda_{b]} \\
\delta\mathcal{F}_a{}^j &= -\delta\mathcal{F}^j{}_a = -\epsilon^M\partial_M\mathcal{F}_a{}^j - \partial_a\epsilon^c\mathcal{F}_c{}^j + \mathcal{F}_a{}^k\partial_k\epsilon^j - \mathcal{F}_a{}^k\partial_k\epsilon^c\mathcal{F}_c{}^j + \partial_a\epsilon^j
\end{aligned}$$

$$\begin{aligned}
\delta t_{ab} &= -\epsilon^M\partial_M t_{ab} - \partial_a\epsilon^c t_{cb} - t_{ac}\partial_b\epsilon^c + \partial_{[k}\Lambda_{a]}t^k{}_b + t_a{}^k\partial_{[k}\Lambda_{b]} \\
&\quad -t_a{}^k\partial_k\epsilon^c t_{cb} + t_{ac}\partial_k\epsilon^c t^k{}_b - t_a{}^k\partial_{[k}\Lambda_{l]}t^l{}_b + \partial_{[a}\Lambda_{b]} \\
\delta t_a{}^j &= -\epsilon^M\partial_M t_a{}^j - \partial_a\epsilon^c t_c{}^j + t_a{}^k\partial_k\epsilon^j + \partial_{[k}\Lambda_{a]}t^{kj} \\
&\quad -t_a{}^k\partial_k\epsilon^c t_c{}^j + t_{ac}\partial_k\epsilon^c t^{kj} - t_a{}^k\partial_{[k}\Lambda_{l]}t^{lj} + \partial_a\epsilon^j \\
\delta t^i{}_b &= -\epsilon^M\partial_M t^i{}_b + \partial_k\epsilon^i t^k{}_b - t^i{}_c\partial_b\epsilon^c + t^{ik}\partial_{[k}\Lambda_{b]} \\
&\quad -t^{ik}\partial_k\epsilon^c t_{cb} + t^i{}_c\partial_k\epsilon^c t^k{}_b - t^{ik}\partial_{[k}\Lambda_{l]}t^l{}_b - \partial_b\epsilon^i \\
\delta t^{ij} &= -\epsilon^M\partial_M t^{ij} + \partial_k\epsilon^i t^{kj} + t^{ik}\partial_k\epsilon^j \\
&\quad -t^{ik}\partial_k\epsilon^c t_c{}^j + t^i{}_c\partial_k\epsilon^c t^{kj} - t^{ik}\partial_{[k}\Lambda_{l]}t^{lj}
\end{aligned}$$

$$\begin{aligned}
\delta\mathcal{F}_{ab} &= -\epsilon^M\partial_M\mathcal{F}_{ab} - \partial_a\epsilon^c\mathcal{F}_{cb} - \mathcal{F}_{ac}\partial_b\epsilon^c - \partial_{[a}\Lambda_{k]}\mathcal{F}^k_b + \mathcal{F}_a{}^k\partial_{[k}\Lambda_{b]} \\
&\quad -\mathcal{F}_a{}^k\partial_k\epsilon^c\mathcal{F}_{cb} + \mathcal{F}_{ac}\partial_k\epsilon^c\mathcal{F}^k_b - \mathcal{F}_a{}^k\partial_{[k}\Lambda_{l]}\mathcal{F}^l{}_b + \partial_{[a}\Lambda_{b]} \\
\delta\mathcal{F}_a{}^j &= -\delta\mathcal{F}^j{}_a = -\epsilon^M\partial_M\mathcal{F}_a{}^j - \partial_a\epsilon^c\mathcal{F}_c{}^j + \mathcal{F}_a{}^k\partial_k\epsilon^j - \mathcal{F}_a{}^k\partial_k\epsilon^c\mathcal{F}_c{}^j + \partial_a\epsilon^j
\end{aligned}$$

This transformation law can be rewritten as

$$\begin{aligned}
\delta\mathcal{F}_{ab} &= \tilde{\partial}_{[a}\delta A_{b]} - \delta\Phi^k\partial_k F_{ab} \\
\delta\mathcal{F}_a{}^j &= -\delta\mathcal{F}^j{}_a = \tilde{\partial}_a\delta\Phi^j
\end{aligned}$$

We can read the non-linear transformation law :

$$\begin{aligned}
\delta A_a &= \Lambda_a - \epsilon^c F_{ca} + \Lambda_k\partial_a\Phi^k \\
\delta\Phi^i &= \epsilon^i - \epsilon^c\partial_c\Phi^i
\end{aligned}$$

(Φ^i, A_a) can be interpreted as NG bosons for $\epsilon^i = \text{const.}$ and $\Lambda_a = \text{const.}$

4. Invariance of the DBI action

In this section, we prove invariance of the DBI action under the full diffeomorphisms and B-field gauge transformations on the target space.

Firstly we define the DBI action as an integral over the target space M as

$$S_{\rm DBI} = \int_M \mathcal{L}_{\rm DBI}\ \delta^{(D-p-1)}(x^i - \Phi^i(x^a))\ dx^0 \wedge \cdots \wedge dx^{D-1}$$

$$\mathcal{L}_{\rm DBI} = \det{}^{\frac{1}{4}} g \det{}^{\frac{1}{4}} s_{\mathcal{F}}$$

This action can be rewritten as a well-known form

$$S_{\rm DBI} = \int_\Sigma \sqrt{\det(\varphi_\Phi^*(g+B) - F)_{ab}}\ dx^0 \wedge \cdots \wedge dx^p$$

The proof of this relation are found in our paper.

Transformation rule:

$$\begin{cases} \delta\sqrt{\det s_{\mathcal{F}}} = -\epsilon^M \partial_M \sqrt{\det s_{\mathcal{F}}} + \sqrt{\det s_{\mathcal{F}}}\,\{-\partial_c \epsilon^c + \partial_k \epsilon^k - 2\mathcal{F}_c{}^k \partial_k \epsilon^c\} \\ \delta\sqrt{\det g} = -\epsilon^M \partial_M \sqrt{\det g} + \sqrt{\det g}\,\{-\partial_M \epsilon^M\} \end{cases}$$

$$\delta\mathcal{L}_{\mathrm{DBI}} = -\epsilon^M \partial_M \mathcal{L}_{\mathrm{DBI}} - (\partial_c \epsilon^c + \partial_c \Phi^k \partial_k \epsilon^c)\mathcal{L}_{\mathrm{DBI}}.$$

The delta function is transformed as a density to

$$\delta[\delta^{(D-p-1)}(x^i - \Phi^i)]$$
$$= -\epsilon^M \partial_M [\delta^{(D-p-1)}(x^i - \Phi^i)] - (\partial_k \epsilon^k - \partial_k \epsilon^c \partial_c \Phi^k)\delta^{(D-p-1)}(x^i - \Phi^i).$$

Combining these transformation laws, we get

$$\delta\left[\mathcal{L}_{\mathrm{DBI}}\,\delta^{(D-p-1)}(x^i - \Phi^i)\right] = \underline{-\partial_M\left[\epsilon^M \mathcal{L}_{\mathrm{DBI}}\,\delta^{(D-p-1)}(x^i - \Phi^i)\right]}$$

total derivative

Interpretation as determinant bundles

$$\begin{array}{ll} \sqrt{\det g}\, dx^0 \wedge \cdots \wedge dx^{D-1} & \in \Gamma(\det(T^*M)) \\ \sqrt{\det s_{\mathcal{F}}}\, dx^0 \wedge \cdots \wedge dx^p \wedge \partial_{p+1} \wedge \cdots \wedge \partial_{D-1} & \in \Gamma(\det(L^*)) \end{array}$$

The determinant bundle of each structures are decomposed as

$$\begin{aligned} \det(T^*M) &= \det(\Delta^*) \otimes \det(\mathrm{Ann}(\Delta)), \\ \det(L^*) &= \det(\Delta^*) \otimes \det(\mathrm{Ann}^*(\Delta)) = \det(\Delta^*) \otimes \det{}^{-1}(\mathrm{Ann}(\Delta)) \end{aligned}$$

Then

$$\det(T^*M) \otimes \det(L^*) = \det{}^2(\Delta^*) \otimes \det(\mathrm{Ann}(\Delta)) \otimes \det{}^{-1}(\mathrm{Ann}(\Delta))$$

Therefore the DBI Lagrangian can be characterized as a volume form over a D-brane world-volume as

$$\mathcal{L}_{\mathrm{DBI}} dx^0 \wedge \cdots \wedge dx^p \in \Gamma(\det \Delta)$$

Summary

- D-branes are described as a subbundle (Dirac structure), where scalar fields and a gauge field can be treated on an equal footing.
- As the scalar fields, the gauge fields are interpreted as NG boson for broken B-field gauge transformations.
- We show that the DBI action is invariant under non-linearly realized symmetry.

Open questions

How can we extend these arguments to the following cases?

- Non-Abelian gauge theory
- Ramond-Ramond coupling (especially for A-roof genus)
- The relation with non-geometric fluxes
- M2, M5-brane