On supersymmetric interfaces in string theory

Yuji Satoh
(University of Tsukuba)

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1. Introduction

☆ Conformal (world-sheet) interface :

natural extension of conformal boundary

cond. mat. w/ boundary

D-brane

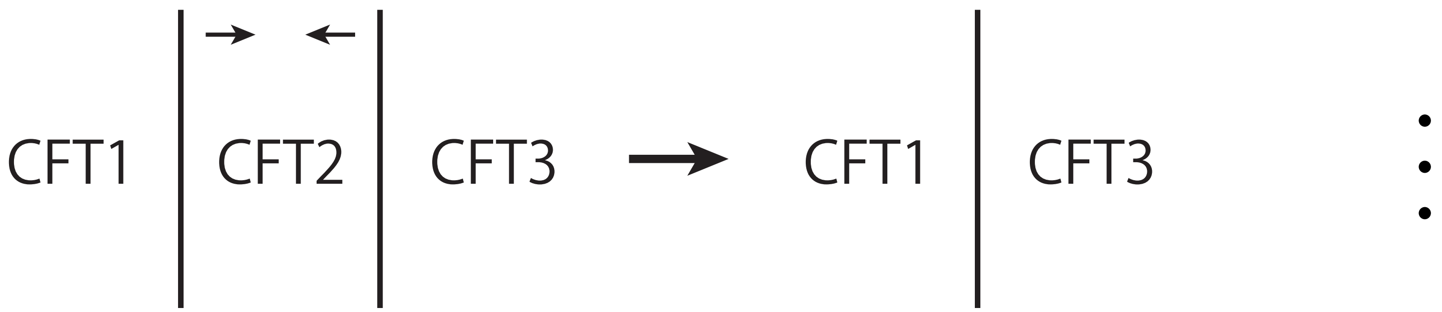
cond. mat. w/defect

??

- may play an interesting role in
  - condensed matter phys.
  - CFT
  - string theory

In fact,

- transform set of D–branes to another [Graham–Watts '03]
- "generator" of RG flow [cf. Gaiotto '12] [Graham–Watts '03]
- "generator" of symmetry (duality) of RCFT
  [Frohlich–Fuchs–Runkel–Scweigert '04, '07]
- target–space interpretation as "bi–brane"
  in G x G (WZW model) [Fuchs–Schweigert–Waldorf '07]
- fusion of interfaces [Bachas–Brunner '07]
  ≈ spectrum generating "algebra" of string theory?
  [cf. Geroch group, U–duality group]



However,

- not fully understood, especially, in string theory

- generally, only 1 Virasoro is preserved

  $\Rightarrow$ ghost problem if embedded in string theory?

  [Bachas-de Boer-Dijkgraaf-Ooguri '01]

- conformal interface in full string world-sheet had not been discussed

Motivation of this work is simple :

- to construct interface in string theory [fixed genus]
- to study its basic properties
- to address issue of ghost, if possible

To avoid subtleties concerning ghosts, work w/ Green-Schwarz formulation in light-cone gauge

2. Conformal interfaces

World-sheet conformal interface

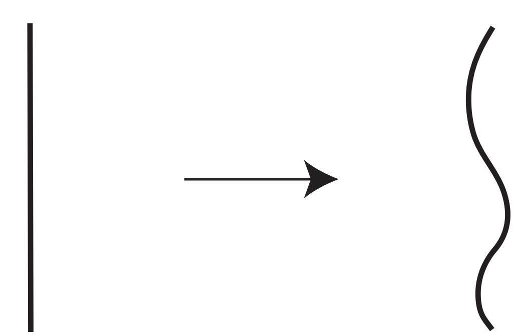
- consider 1 dim. defect/interface in 2dim. world-sheet
- condition to keep conformal Ward id.

$$T_1(z) - \tilde{T}_1(\bar{z}) \approx T_2(z) - \tilde{T}_2(\bar{z})$$

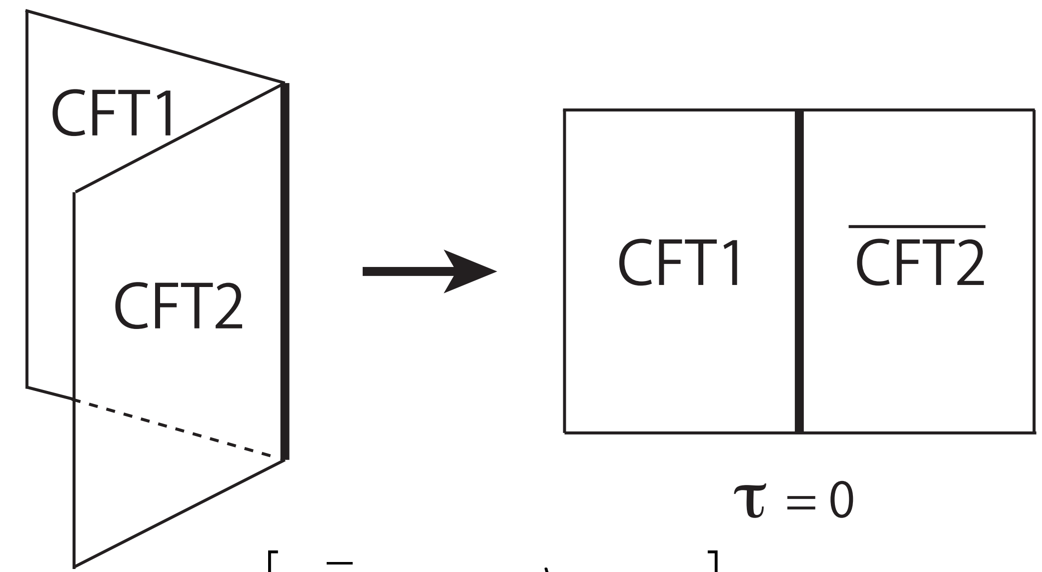
[along interface]

- when $T_1(z) \approx T_2(z)\,, \quad \tilde{T}_1(\bar{z}) \approx \tilde{T}_2(\bar{z})$

$\Rightarrow$
- interface : freely deformed
- called topological interface



(Un)folding trick



$[\ \bar{\ }\ : \tau \to -\tau\ ]$

$(\alpha_n^2, \tilde{\alpha}_n^2) \to (-\tilde{\alpha}_{-n}^2, -\alpha_{-n}^2)$

- a way to construct interface : unfold conformal boundary

- take b.d. state in CFT1 $\otimes$ CFT2

$$|\mathcal{B}\rangle = \sum_{i,j} c_{ij} |\mathcal{B}_i\rangle_1 \otimes |\mathcal{B}_j\rangle_2$$

s.t. $$(L_n^1 + L_n^2 - \tilde{L}_{-n}^1 - \tilde{L}_{-n}^2)|\mathcal{B}\rangle = 0$$

then, $$\mathcal{I} = \sum_{i,j} c_{ij} |\mathcal{B}_i\rangle_1 \cdot {}_2\overline{\langle \mathcal{B}_j|}$$

satisfies $$(L_n^1 - \tilde{L}_{-n}^1)\mathcal{I} = \mathcal{I}(L_n^2 - \tilde{L}_{-n}^2)$$ $\Leftarrow$ interface

3. Susy boundary states for GS string

Type II GS string

- notation (IIB):

$$\alpha_n^I,\ \tilde{\alpha}_n^I\ (I = 1, ..., 8):$$ right, left bosonic modes

$$S_n^a,\ \tilde{S}_n^a\ (a = 1, ..., 8):$$ $so(8)$ spinor modes

$$\gamma^I = \begin{pmatrix} 0 & \sigma^I \\ \bar{\sigma}^I & 0 \end{pmatrix} :$$ 8-dim. gamma matrices

- supercharges

linear susy $$Q^a := \sqrt{2p^+} S_0^a$$

non-linear susy $$Q^{\dot{a}} := \frac{1}{\sqrt{p^+}} \sigma^I_{a\dot{a}} \sum_n S^a_{-n} \alpha^I_n$$

D-branes [Green-Gutperle '96]

- GS string in l.c.-gauge: not a CFT

- Conformal boundary states

$\Rightarrow$ space-time susy boundary states s.t.

$$(Q^a + iM_{ab}\tilde{Q}^b)|\mathcal{B}\rangle = (Q^{\dot{a}} + iM_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}})|\mathcal{B}\rangle = 0$$

$$\Rightarrow |\mathcal{B}(M)\rangle = \mathcal{C}\prod_{n=1}\exp\left[\frac{1}{n}M_{IJ}\alpha^I_{-n}\tilde{\alpha}^J_{-n} - iM_{ab}S^a_{-n}\tilde{S}^b_{-n}\right]|\mathcal{B}\rangle_{\mathrm{b0}}|\mathcal{B}\rangle_{\mathrm{f0}}$$

[zero mode]

$$M_{KL} = \left(e^{\omega_{IJ}\Sigma^{IJ}}\right)_{KL},\ M_{\alpha\beta} = \left(e^{\frac{1}{2}\omega_{IJ}\gamma^{IJ}}\right)_{\alpha\beta} = \begin{pmatrix} M_{ab} & 0 \\ 0 & M_{\dot{a}\dot{b}} \end{pmatrix}$$

- b.d. cond. $(\alpha^I_n - M_{IJ}\tilde{\alpha}^J_{-n})|\mathcal{B}\rangle = (S^a_n + iM_{ab}\tilde{S}^b_{-n})|\mathcal{B}\rangle = 0$

- In GS, "conformal" $\Rightarrow$ "space–time supersymmetric"

Conformal Interface $\Rightarrow$ Susy Interface

4. Susy interface for GS string

- we would like to find susy interface defined by

$$(Q_1^a + iR^1_{ab}\tilde{Q}_1^b)\mathcal{I} = \mathcal{I}(Q_2^a + iR^2_{ab}\tilde{Q}_2^b)$$

$$(Q_1^{\dot{a}} + iR^1_{\dot{a}\dot{b}}\tilde{Q}_1^{\dot{b}})\mathcal{I} = \mathcal{I}(Q_2^{\dot{a}} + iR^2_{\dot{a}\dot{b}}\tilde{Q}_2^{\dot{b}})$$

W. S. 1 | W. S. 2

for some $R^A_{ab}\,,\, R^A_{\dot{a}\dot{b}}\quad (A=1,2)$ [IIB case]

- for this,
  - prepare boundary states w/ "doubled" fields

    $(\alpha_n^{AI}, \tilde{\alpha}_n^{AI})\,,\; (S_n^{Aa}, \tilde{S}_n^{Aa})$ [just as an intermediate step]

  - unfold this to interface
  - impose susy cond.

(analog of) topological interface

- after some analysis, obtain, e.g.,

$$\mathcal{I}(M^1, M^2) = \mathcal{C} \prod_{n=1} e^{\frac{1}{n}(\alpha^{1I}_{-n}\alpha^{2I}_{n} + M^1_{KI}M^2_{KJ}\tilde{\alpha}^{1I}_{-n}\tilde{\alpha}^{2J}_{n})} \times e^{S^{1a}_{-n}S^{2a}_{n} + M^1_{ca}M^2_{cb}\tilde{S}^{1a}_{-n}\tilde{S}^{2b}_{n}} \cdot \mathcal{I}_{\mathrm{b0}}\mathcal{I}_{\mathrm{f0}}$$

$$\mathcal{I}_{\mathrm{b0}} = \sum |k^{1I}, \tilde{k}^{1J}\rangle\langle\tilde{k}^{2K}, k^{2L}|$$

$$k^{1I} = k^{2I}, \quad M^1_{IJ}\tilde{k}^{1J} = M^2_{IJ}\tilde{k}^{2J}$$

$$\mathcal{I}_{\mathrm{f0}} = |I\rangle T_{\mathcal{I}}\langle I| + |\dot{a}\rangle T_{\mathcal{I}}\langle\dot{a}|$$

$$T_{\mathcal{I}} = M^1_{PJ}M^2_{PK}|J\rangle\langle K| + M^1_{\dot{p}\dot{b}}M^2_{\dot{p}\dot{c}}|\dot{b}\rangle\langle\dot{c}|$$

$$\Rightarrow \quad \begin{aligned} Q^a_1 &\approx Q^a_2\,, & M^1_{ab}\tilde{Q}^b_1 &\approx M^2_{ab}\tilde{Q}^b_2 \\ Q^{\dot{a}}_1 &\approx Q^{\dot{a}}_2\,, & M^1_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}_1 &\approx M^2_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}_2 \\ S^{1a} &\approx S^{2a}\,, & M^1_{ab}\tilde{S}^{1b} &\approx M^2_{ab}\tilde{S}^{2b} \\ \partial_- X^{1I} &\approx \partial_- X^{2I}\,, & M^1_{IJ}\partial_+ X^{1J} &\approx M^2_{IJ}\partial_+ X^{2J} \\ T_1 &\approx T_2 & \tilde{T}_1 &\approx \tilde{T}_2 \end{aligned}$$

"topological"

5. Properties

- in the following, we
  - concentrate on un–compactified case
  - set for simplicity

$$M^A_{IJ} = \begin{pmatrix} -\mathbf{1}_{p_A+1} & 0 \\ 0 & \mathbf{1}_{7-p_A} \end{pmatrix} \qquad (p_1 < p_2)$$

Target space geometry

- target space geometry is probed by localized states

$$|x\rangle = \int d^8k\, e^{-ik\cdot x}|k\rangle$$

- then,

$$\langle x|\,\mathcal{I}(Y)\,|x'\rangle \;\sim\; \prod_{I=p_2+2}^{8} \delta(x_I - x'_I - Y^I)$$

$\Rightarrow$ interface : localized in a submanifold (bi-brane)

$$x = x' + Y$$

in doubled (transverse) target space $\mathbb{R}^8 \times \mathbb{R}^8 \ni (x, x')$

Coupling through interfaces

- consider massless NS–NS fields $|\zeta\rangle\rangle := \zeta_{IJ}|I\rangle|J\rangle$

  [spinor zero mode]

- coupling is read off from $\langle\langle\zeta|\ \mathcal{I}\ |\zeta'\rangle\rangle$

- e.g., when $p_1 + 1 = p_2 =: p$ (IIB–IIA), this gives

$$-\zeta^*_{p+1\,p+1}\zeta'_{p+1\,p+1} + \zeta^*_{(p+1\,I)}\zeta'_{[p+1\,I]} + \zeta^*_{[p+1\,I]}\zeta'_{(p+1\,I)} + \sum_{I,J\neq p+1} \zeta^*_{IJ}\zeta'_{IJ}$$

$\Longrightarrow$ nothing but Buscher rules (T–duality)

$$g'_{p+1\,p+1} = \frac{1}{g_{p+1\,p+1}}\,, \quad g'_{p+1\,I} = \frac{b_{p+1\,I}}{g_{p+1\,p+1}}\,, \quad b'_{p+1\,I} = \frac{g_{p+1\,I}}{g_{p+1\,p+1}}$$

$$\phi' = \phi - \frac{1}{2}\log g_{p+1\,p+1}$$

Transformation of D-branes

- interface transforms D-branes as

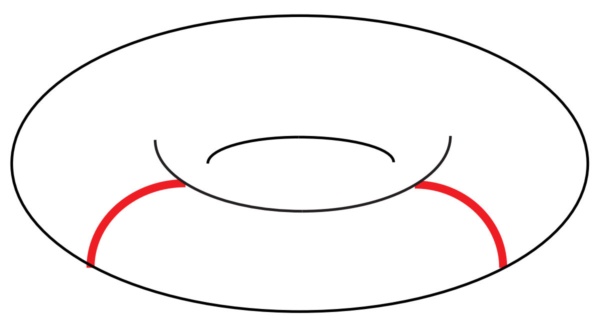
$$|\mathcal{B}'(M')\rangle = \lim_{q\to 1} \mathcal{I}(M^1, M^2)\, q^{L_0^2+\tilde{L}_0^2} |\mathcal{B}(M)\rangle$$

- in the present case, results in SO(8) trans.

$$M' = M(M^2)^T M^1$$

Partition fn. w/ interfaces insertes

- modes coupled to interfaces
- susy breaking, Casimir energy



6. Summary

- we have constructed [fixed genus]
  susy (≈ conformal) interfaces for type II GS string
  - generate T-duality (Buscher rules)
  - interpreted as a submanifold
    in doubled target space (bi-brane)
  - transform (rotate) D-branes
  - partition fn. w/ interfaces, Casimir energy

- correspond to those preserving 2 Virasoros
  $\Rightarrow$ evade ghost problem

Future directions

- NSR formulation ? $\Longrightarrow$ [Bachas-Brunner-Roggenkamp '12]

- more general interfaces ? $\Longrightarrow$ probably, No [Bachas et al.]

- richer algebras among interfaces when compactified

  $\Longrightarrow$ monoid (semi-group) extension of $O(d,d\,|\,\mathbb{Q})$

  [Bachas et al.]

- double field theory ?

- symmetry of string theory ?

  applications ?

$$\vdots$$