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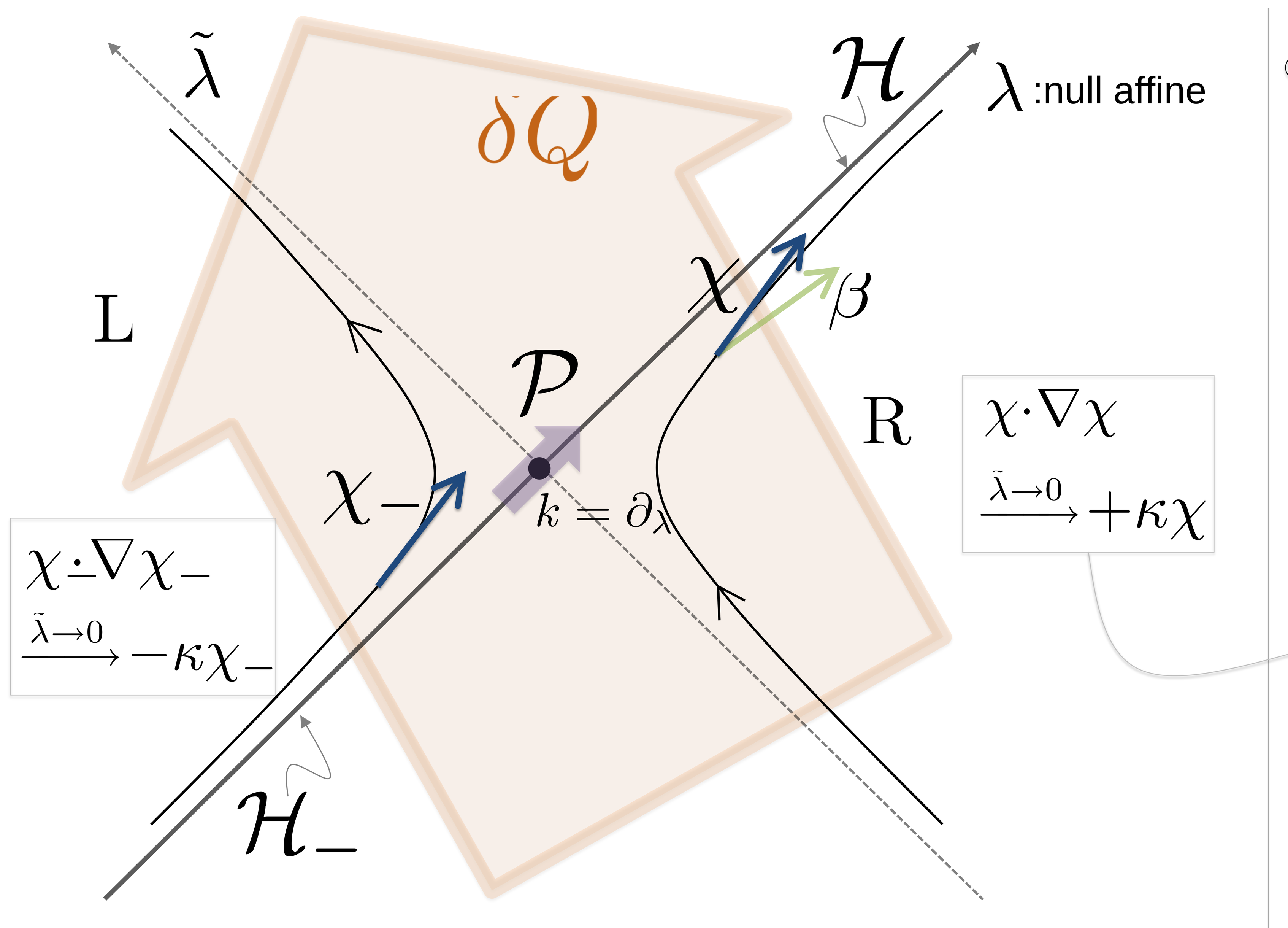
Space-time thermodynamics with a general null hypersurface

嶼田 健悟 with 磯 暁 and 岡澤 晋
（KEK/総研大） skengo@post.kek.jp

Understanding why the unnatural assumptions were necessary in the thermodynamic derivation of the Einstein eq. by Jacobson.

➡ Some possibilities of the interpretation of the e.o.m. of general gravity theory on a general null hypersurface.



Jacobson "derived" the Einstein eq. as the Clausius equality across the local causal horizon $\mathcal{H}_-$.

$$\delta Q = T_U \delta S$$

Unruh temp. $\frac{\kappa\hbar}{2\pi}$

It relates the following thermodynamic quantities measured by observers χ_-.

- Heat into the region L="thermodynamical system with T_U":

$$\delta Q \xrightarrow{\lambda\to 0} -T_{\mu\nu}k^\mu k^\nu(\kappa\lambda)\sqrt{\gamma}d^{n-2}yd\lambda$$

Induced metric on $(n-2)$cross-section of $\mathcal{H}_-$

- Entropy change $\times T_U$:

$$T_U\delta S \rightarrow \frac{\kappa}{8\pi}\theta_{\chi_-}\sqrt{\gamma}d^{n-2}ydt \quad \cdots\cdots (*)$$

$$= \frac{\kappa}{8\pi}(-\lambda R_{\mu\nu}k^\mu k^\nu)\sqrt{\gamma}d^{n-2}yd\lambda$$

Instantaneous equilibrium condition $\theta_k|_{\mathcal{P}} = 0$

$$\nabla_\mu T^\mu_\nu = 0 \implies R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Why observers χ_- behind the horizon $\mathcal{H}_-$?
Difficult of the generalization to general gravity theories.

○ A consideration based on the diff. invariance of the action functional:

$$\delta \mathbf{L} = \epsilon\frac{\mathcal{G}_{\mu\nu}}{16\pi}\delta g^{\mu\nu} + \mathbf{d}\mathbf{\Theta}(g,\delta g)$$

e.o.m $\mathcal{G}_{\mu\nu} = 8\pi T_{\mu\nu}$

Noether current : $\mathbf{J}_\chi = \mathbf{\Theta}(g, \mathcal{L}_\chi g) - i_\chi \mathbf{L}$

$$= \mathbf{d}\mathbf{Q}_\chi + \frac{1}{8\pi}\mathcal{G}^{\mu\nu}\chi_\mu\epsilon_\nu$$

Noether charge (→ Wald entropy)

$$\implies \mathcal{L}_\chi\mathbf{Q}_\chi - \mathbf{d}i_\chi\mathbf{Q} - i_\chi\mathbf{\Theta}(g,\mathcal{L}_\chi g) = \frac{1}{8\pi}\chi^\mu\mathcal{G}^\nu_\mu\chi^\rho\epsilon_{\rho\nu} \quad \cdots\cdots(**)$$

Suppose χ is "uniformly accelerated observer" outside the local causal horizon $\mathcal{H}$.

Especially, in the Einstein gravity

$$\mathcal{L}_\chi\mathbf{Q}_\chi \times dt \xrightarrow{\lambda\to 0} \frac{\kappa}{8\pi}\theta_k\sqrt{\gamma}d^{n-2}yd\lambda \quad = (*)$$

$$\mathbf{d}i_\chi\mathbf{Q}_\chi \times dt \longrightarrow 0$$

$$i_\chi\mathbf{\Theta}(g,\mathcal{L}_\chi g)\times dt \rightarrow \frac{\kappa}{8\pi}(\theta_k - \lambda R_{\mu\nu}k^\mu k^\nu)\sqrt{\gamma}d^{n-2}yd\lambda$$

Holding of the Einstein eq. is equivalent to equating LHS of $(**)$ with heat into the local causal horizon $\mathcal{H}$ from the region R :

$$\delta Q \xrightarrow{\lambda\to 0} +T_{\mu\nu}\chi^\mu k^\nu(\kappa\lambda)\sqrt{\gamma}d^{n-2}ydt$$

$$= +T_{\mu\nu}k^\mu k^\nu(\kappa\lambda)\sqrt{\gamma}d^{n-2}yd\lambda$$

- **The missing terms in Jacobson's derivation** are

$$-i_\chi\mathbf{\Theta} \rightarrow \frac{1}{8\pi}\chi\cdot\partial(-\theta_\chi\sqrt{\gamma})d^{n-2}y - \frac{1}{8\pi}\left(-\frac{n-3}{n-2}\theta_\chi^2 + \sigma_\chi^2\right)\sqrt{\gamma}d^{n-2}y$$

Entropy production δQ_i

bulk viscosity	$\frac{-(n-3)}{8\pi(n-2)}$	shear viscosity	$\frac{1}{16\pi}$

$$-\theta_\beta + \mathcal{O}(\tilde{\lambda}) = -N\gamma^{\mu\nu}\nabla_\mu n_\nu \quad \left(n_\nu = \frac{\beta_\nu}{\sqrt{\beta\cdot\beta}}\right)$$

Brown-York energy surface density

➡ Total energy between Obs. χ and bifurcation surface $\mathcal{P}$? (under consideration)

In the case where $\mathcal{H}$ gets from stationary to stationary, this term does not contribute to integration over $\mathcal{H}$.

- To reach a interpretation of the Einstein eq. on its own, we must deal with such a term directly.

↳ It's possible to vanish by the special choice of Obs. χ.

We can evaluate each terms in $(**)$ for "non-uniformly accelerated" observers χ as follows:

$$i_\chi\mathbf{\Theta}(g,\mathcal{L}_\chi g) \xrightarrow{\lambda\to 0} \frac{\kappa^2}{8\pi}e^{-2c}\lambda'\left[\theta_{k'} - k'\cdot\partial c + \lambda'\left(k'\cdot\partial\theta_{k'} + \frac{1}{n-2}\theta^2_{k'} + \sigma^2_{k'} - \theta_{k'}k'\cdot\partial c\right)\right]\sqrt{\gamma}d^{n-2}y$$

$\chi\cdot\nabla\chi^\mu \xrightarrow{\lambda\to 0} \kappa e^{-c}\chi^\mu$

If we take the observers with $c \to \lambda'\theta_{k'}(\lambda', y) + \mathcal{O}(\lambda'^3)$,

$$= \delta Q_i + \mathcal{O}(\lambda'^3)$$

$$\mathcal{L}_\chi\mathbf{Q}_\chi \xrightarrow{\lambda\to 0} \frac{\kappa}{2\pi}\chi\cdot\partial\left(e^{-c}\frac{\sqrt{\gamma}}{4}\right)d^{n-2}y \equiv T\delta S$$

Modified temperature and entropy
➡ observer-dependent

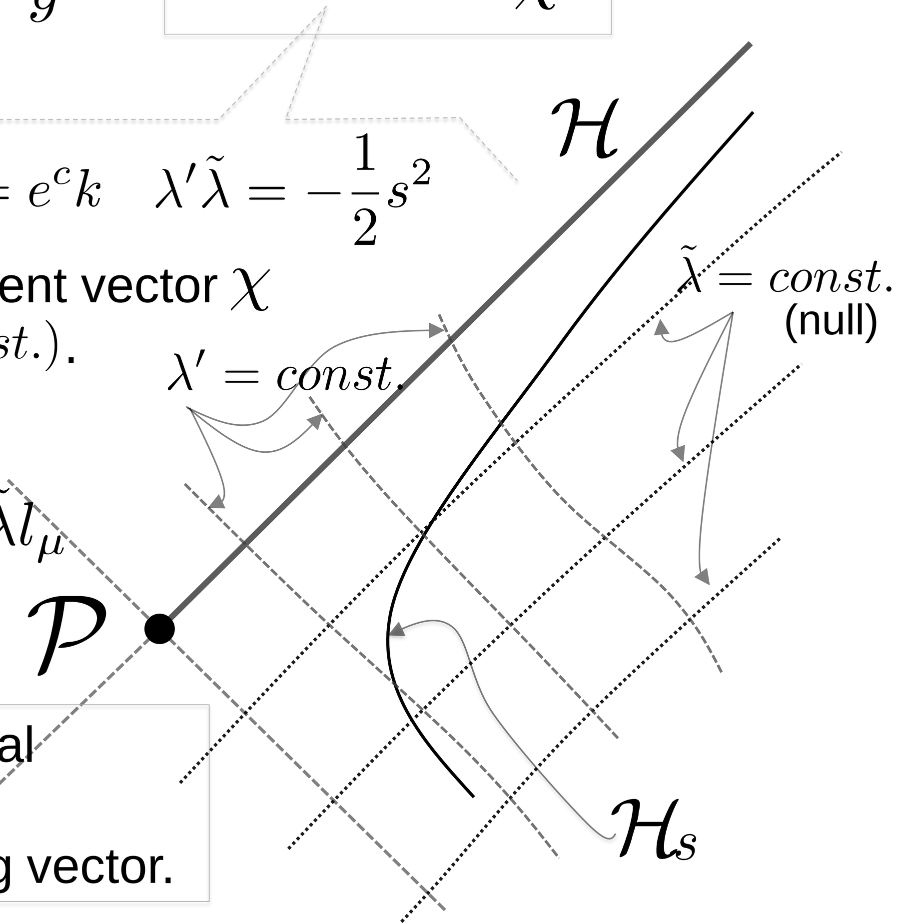
$k_\mu = -\partial_\mu\tilde{\lambda} \quad \frac{\partial\lambda}{\partial\lambda'} = e^c \quad k' \equiv \partial_{\lambda'} = e^c k \quad \lambda'\tilde{\lambda} = -\frac{1}{2}s^2$

Define the normal vector β and a tangent vector χ of time-like hypersurfaces $\mathcal{H}_s(s=const.)$.

$$\beta_\mu = -\kappa\partial_\mu(\lambda'\tilde{\lambda})$$
$$= \kappa(e^{-c}\lambda' + \tilde{\lambda}l\cdot\partial\lambda')k'_\mu + \kappa\tilde{\lambda}l_\mu$$

$$\chi^\mu \xrightarrow{\lambda\to 0} \kappa e^{-c}\lambda' k'^\mu = \beta^\mu|_{\mathcal{H}}$$

There is not a natural normalization of χ which is not a Killing vector.



➡ Entropy balance law : $T\delta S - \delta Q_i = \delta Q$

- Some justification of the modified temperature and entropy is needed.

- This method using Noether charge can be applied to general gravity theories. But it's unlikely that $i_\chi\mathbf{\Theta}$ becomes a familiar form of entropy production.