Instantons
&
Whittaker states of CFT

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based on [H.Kanno, M.T., arXiv:1203.1427]

2012. 7/27 @ YITP

(simple version of) AGT correspondence

4D

Instanton partition function

$$\Updownarrow$$

2D

Whittaker state

(simple version of) AGT correspondence

4D

Instanton partition function

↕

2D

Whittaker state

What is the **Whittaker state** !?

What is the Whittaker state !?

: coherent state of annihilation operators of 2D CFT

Today I will talk on

4D

Instanton partition function

↕

2D

Whittaker state

Today I will talk on

4D

Instanton partition function

↕

2D

~~Whittaker state~~

generalized Whittaker state

6=4+2 :

from M5 to N=2 gauge theories


PEANUT
M 5 s

Gaiotto used me to get N=2 theories

M5 on Cylinder ⟶ 4D Gauge Theory

$\mathbb{R}^4 \times$ 

N_c **M5s** ⟶ $SU(N_c)$

M5 on Cylinder → 4D Gauge Theory

$\mathbb{R}^4 \times$ 

N_c **M5s** → $SU(N_c)$

Quarks ?

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→ **flavors live on the edges**

Flavors via Boundary Conditions

$\mathbb{R}^4 \times$

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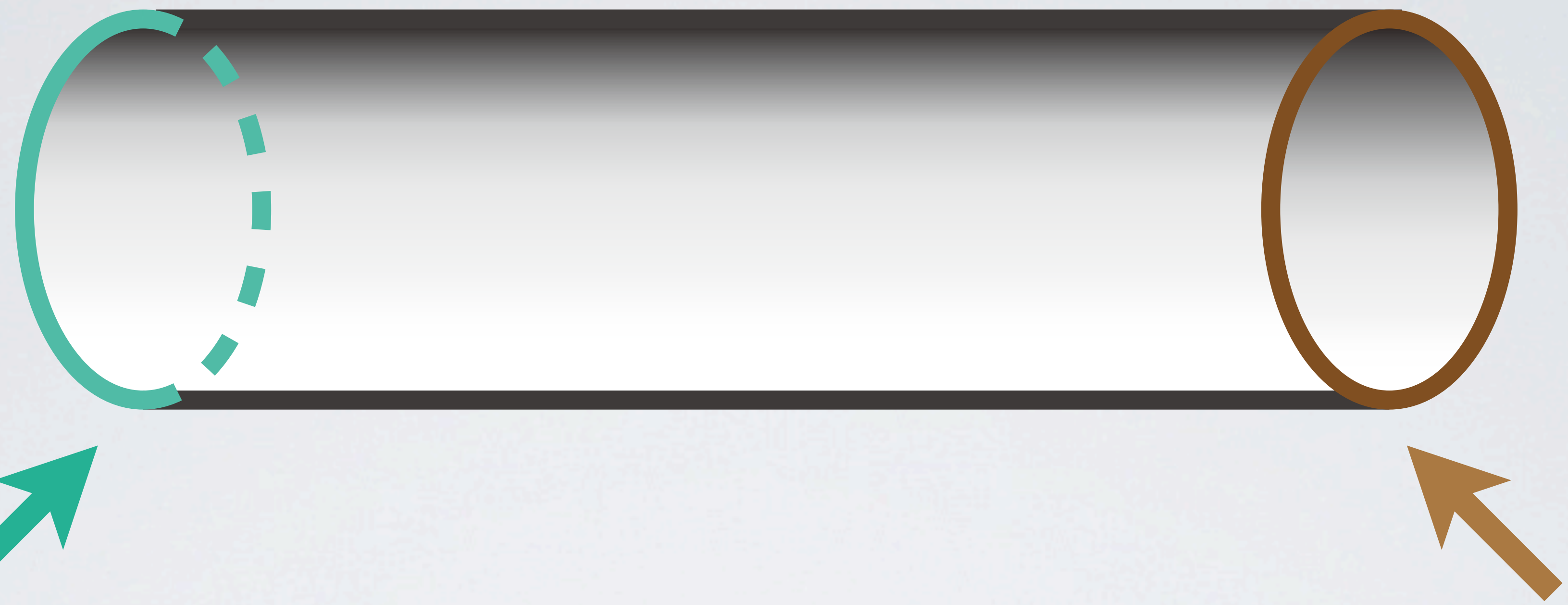
$\mathbb{R}^4 \times$



$N_{\mathbf{Out}}$ quarks

$N_{\mathbf{In}}$ quarks

Flavors via Boundary Conditions

$\Rightarrow$ $N_{\text{Out}} + N_{\text{In}} = N_f$ **susy QCD**

How to describe BCs?

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not easy at all (in M5 language)

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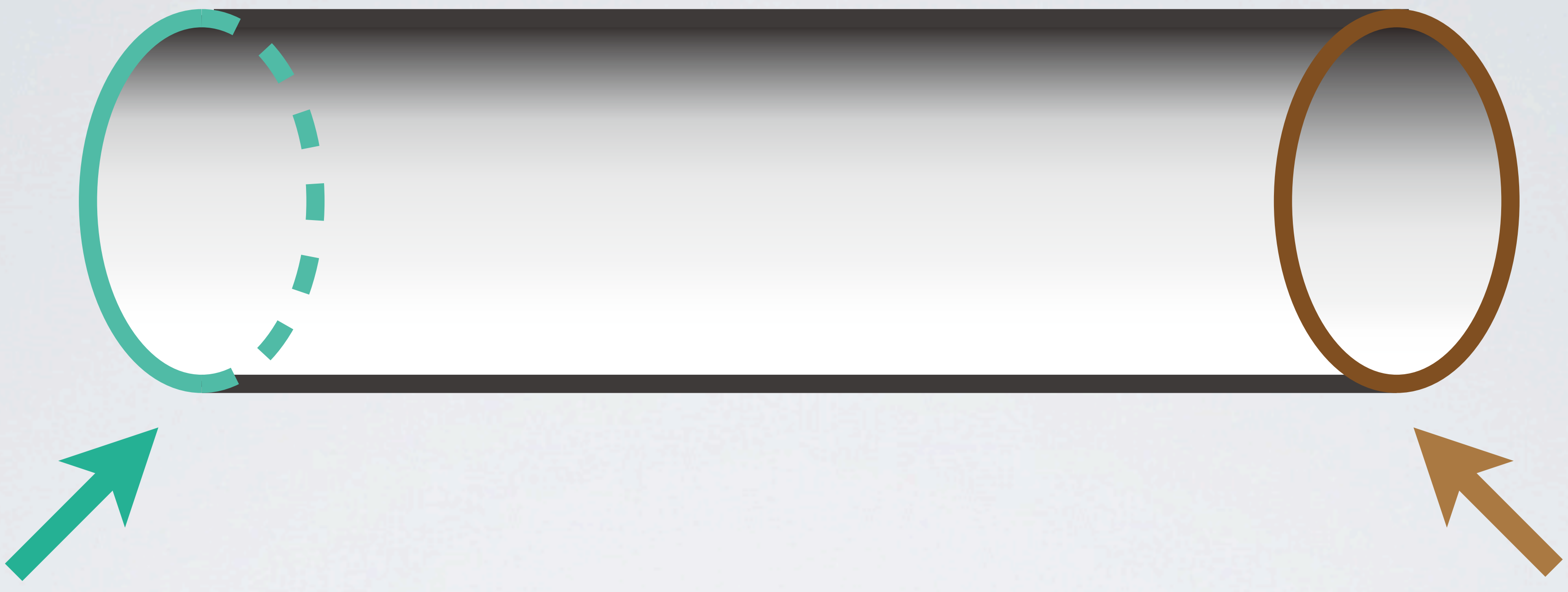
not easy at all (in M5 language)

But, we have a nice description!

2. AGT correspondence

gauge theory via 2d CFT

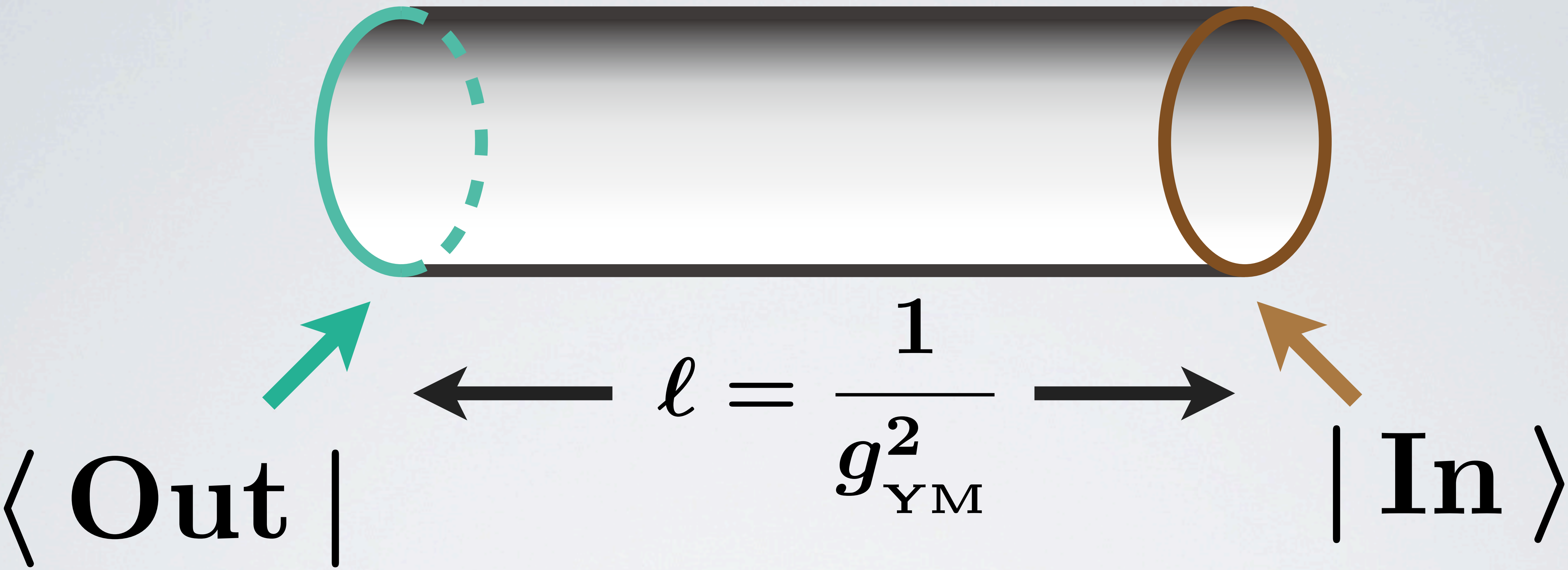
Boundary Condition as a State



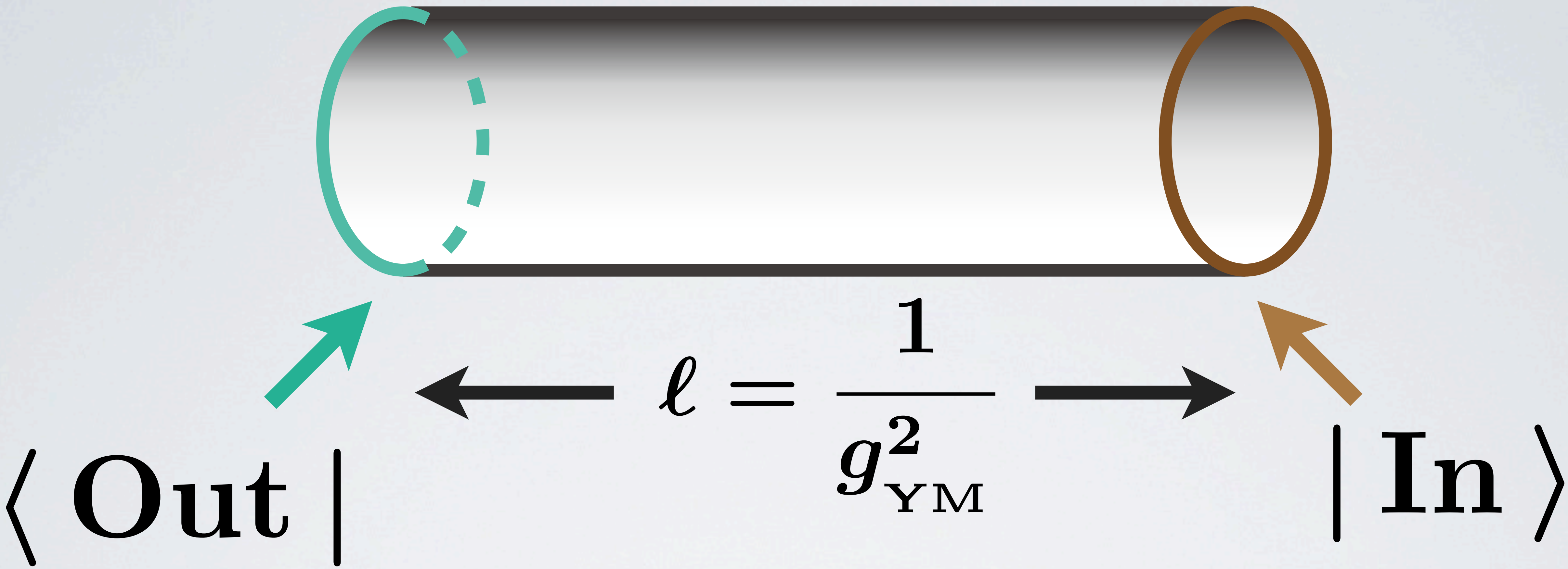
$\langle \,\mathbf{Out}\, |$

$|\, \mathbf{In} \,\rangle$

Gauge Coupling is the Length



Partition function is Matrix Element



$$Z_{4D} = \langle \text{Out} | \Lambda^{2 N_c L_0} | \text{In} \rangle$$

What's the state?

$$[L_n, L_m] = (n - m)L_{n+m}$$

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. . . harmonic oscillators

$L_{1,2,3,\cdots}$: **annihilation op.s** ($\hat{a}$)

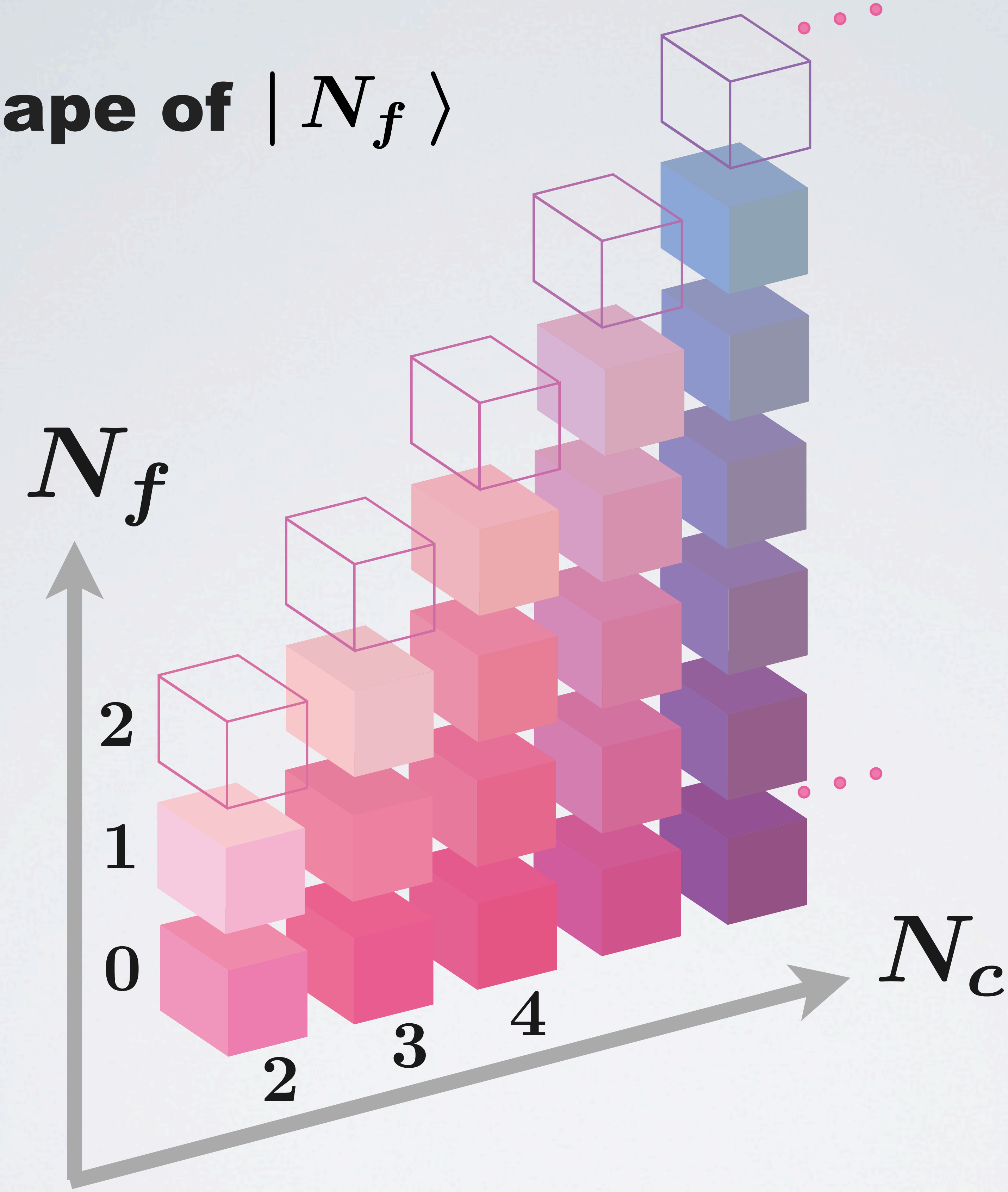
$L_0 = H$ ← **eigenstates**

$L_{-1,-2,-3,\cdots}$: **creation op.s** ($\hat{a}^\dagger$)

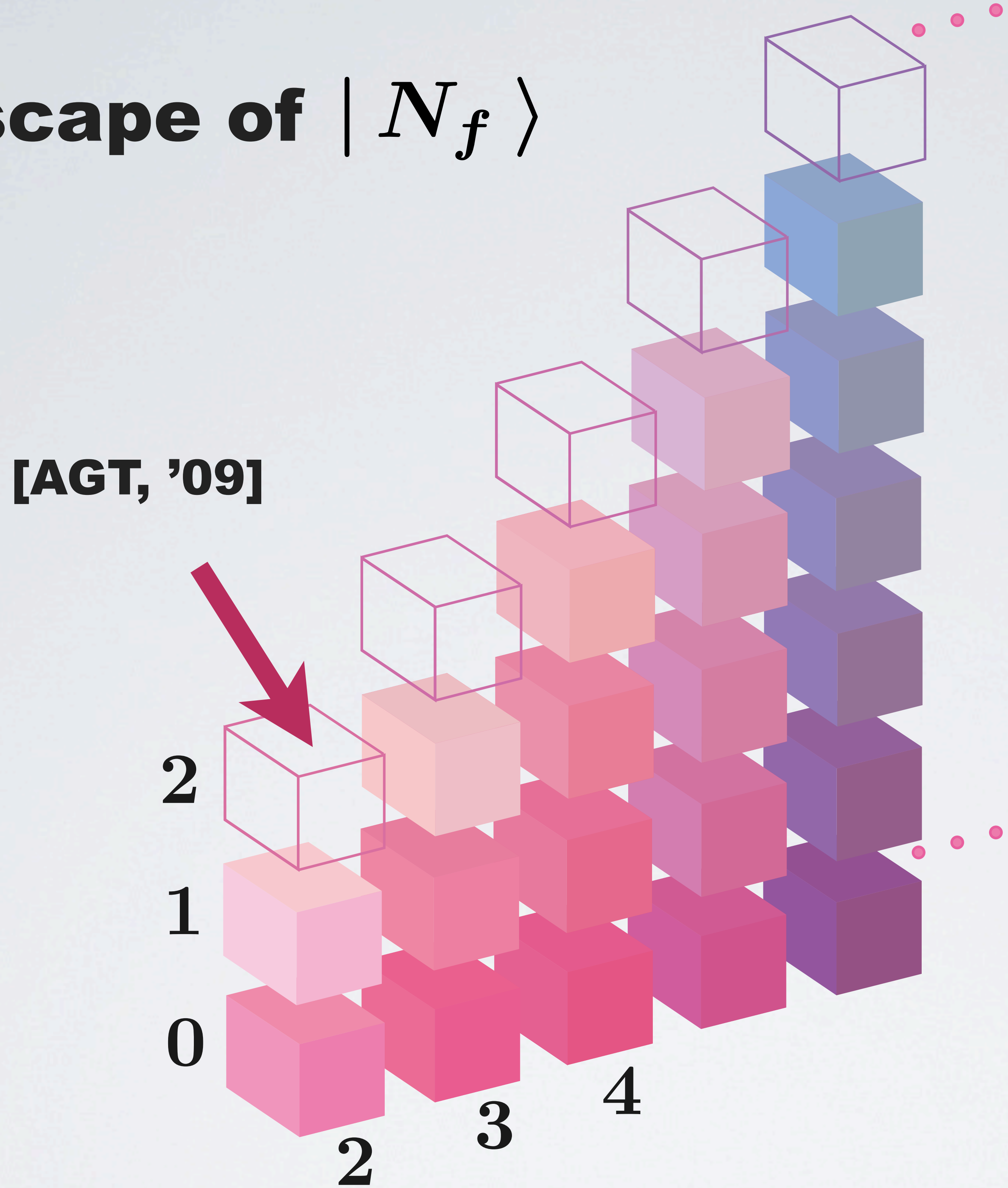
3. flavorful states

Whittaker states for gauge theory

Landscape of $|\, N_f \,\rangle$



Landscape of $|\, N_f \,\rangle$



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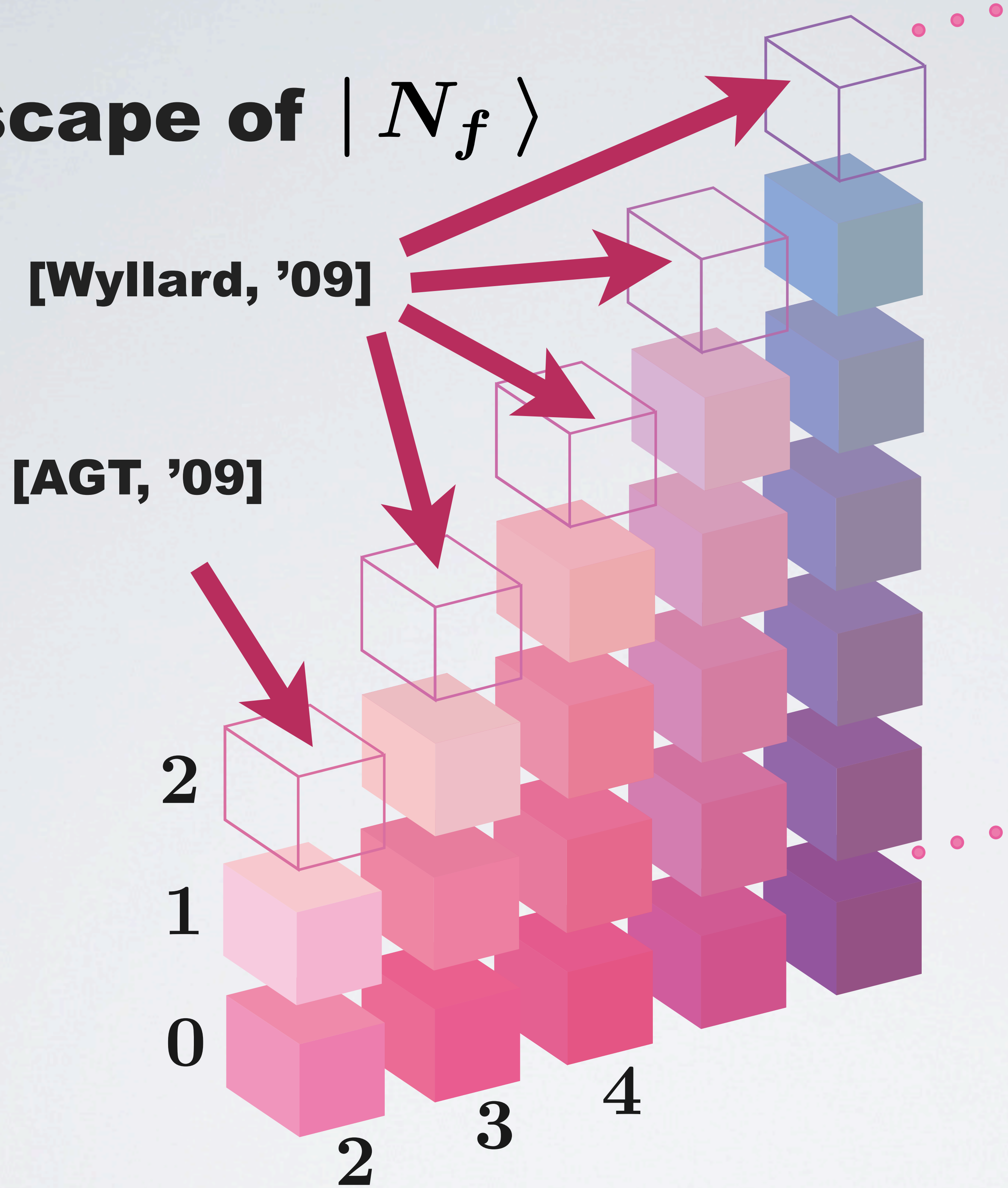
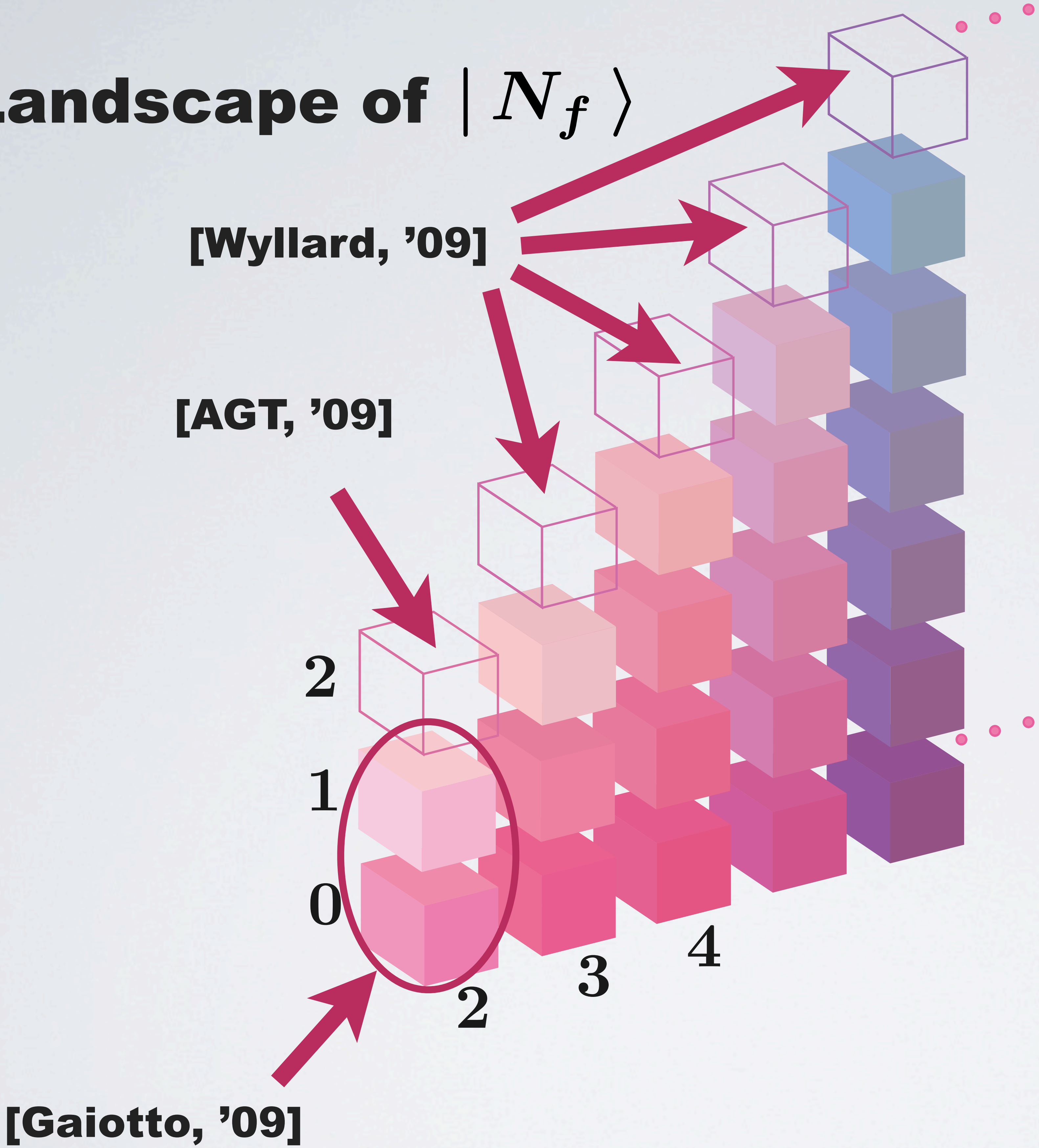


Landscape of $| N_f \rangle$



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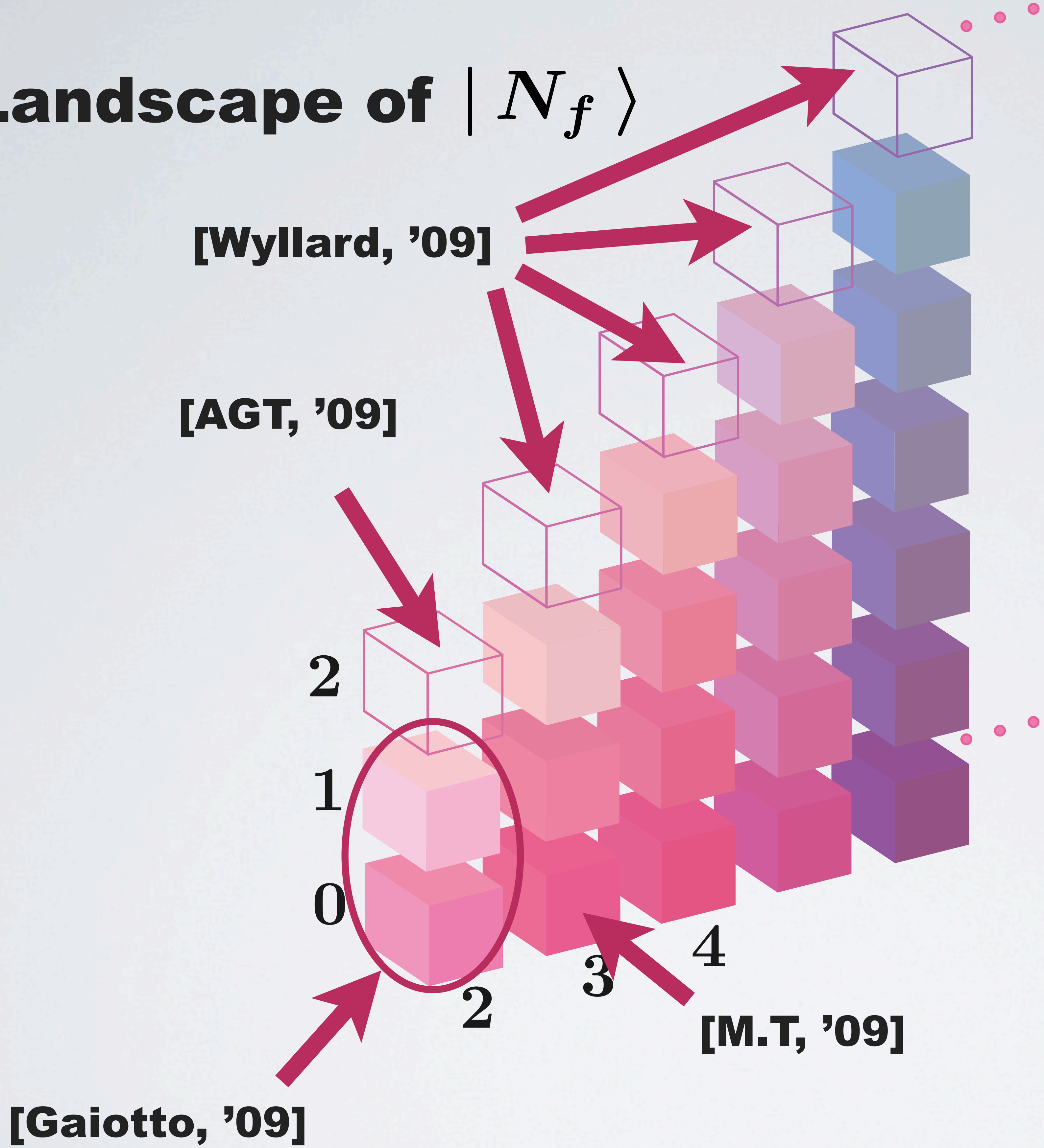

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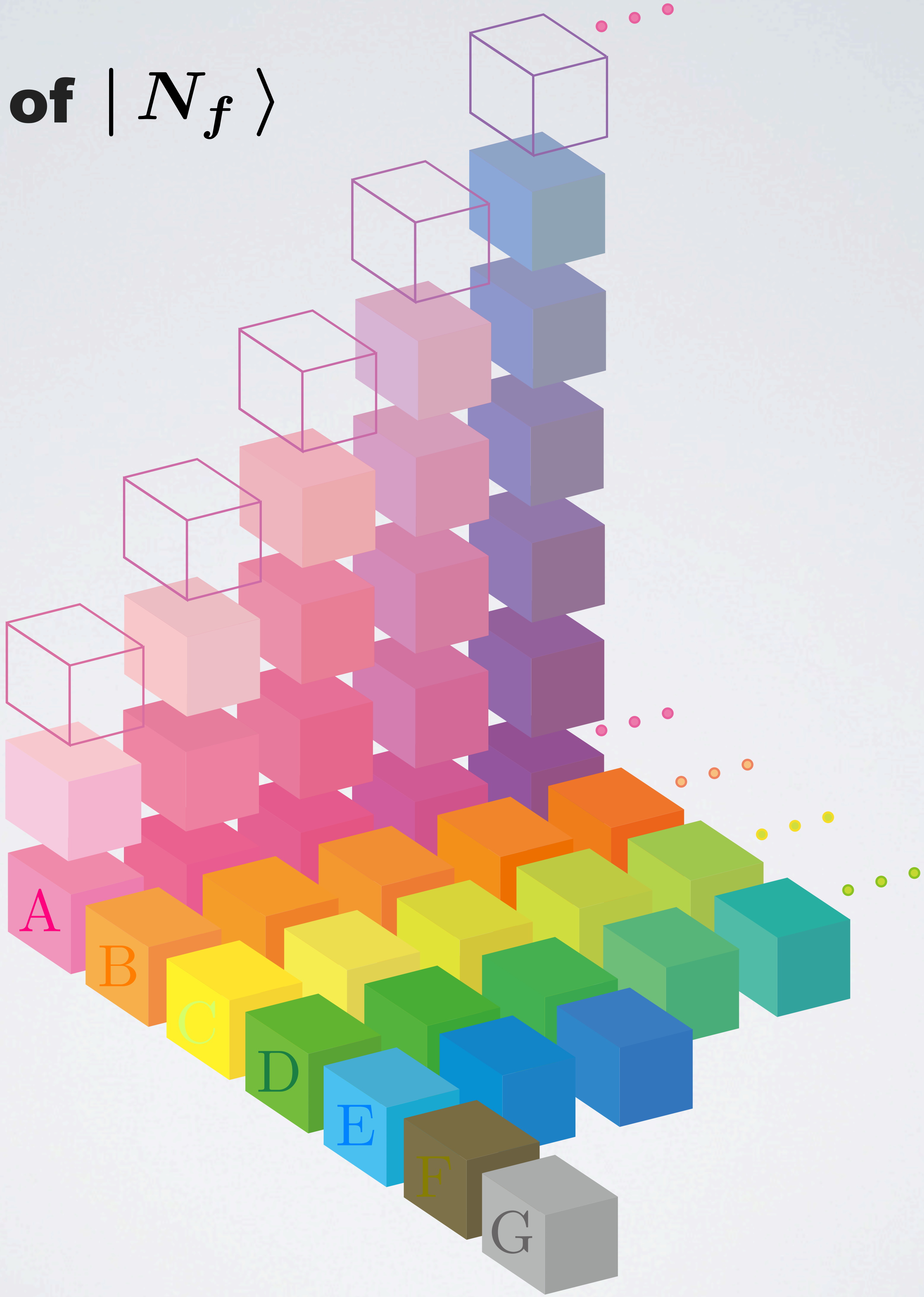

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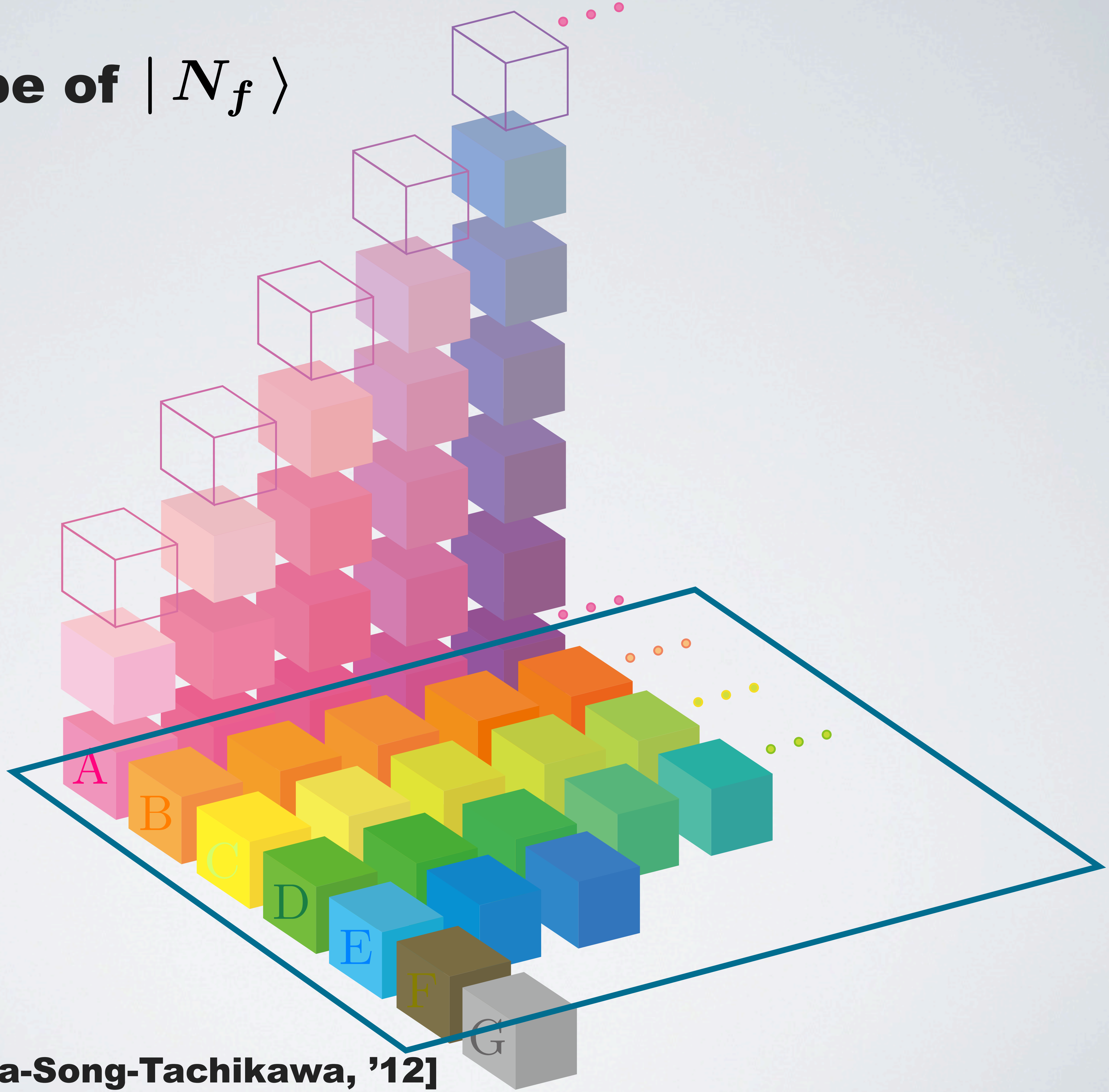
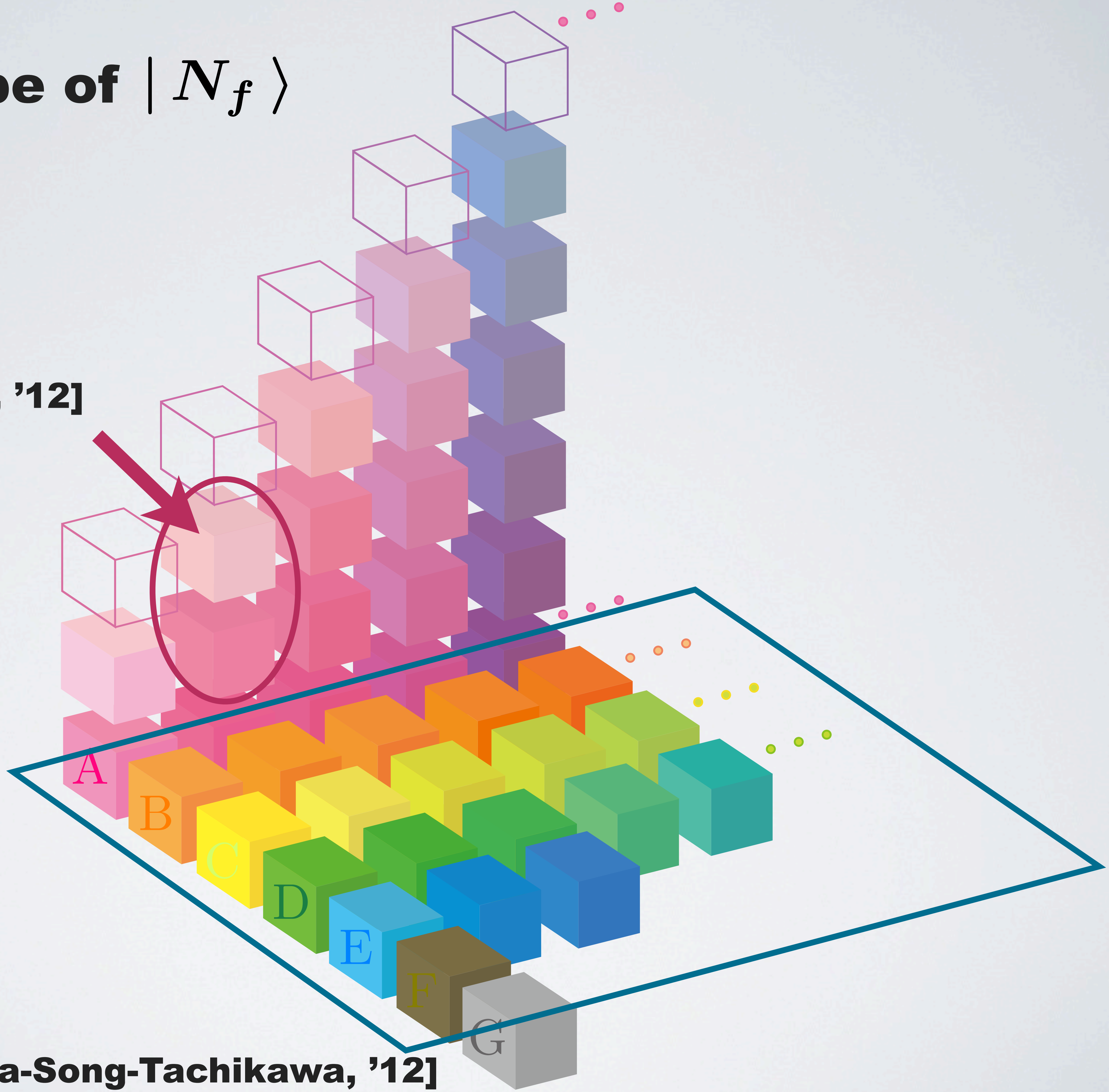


[Keller-Mekareeya-Song-Tachikawa, ’12]

Landscape of $|\, N_f \,\rangle$

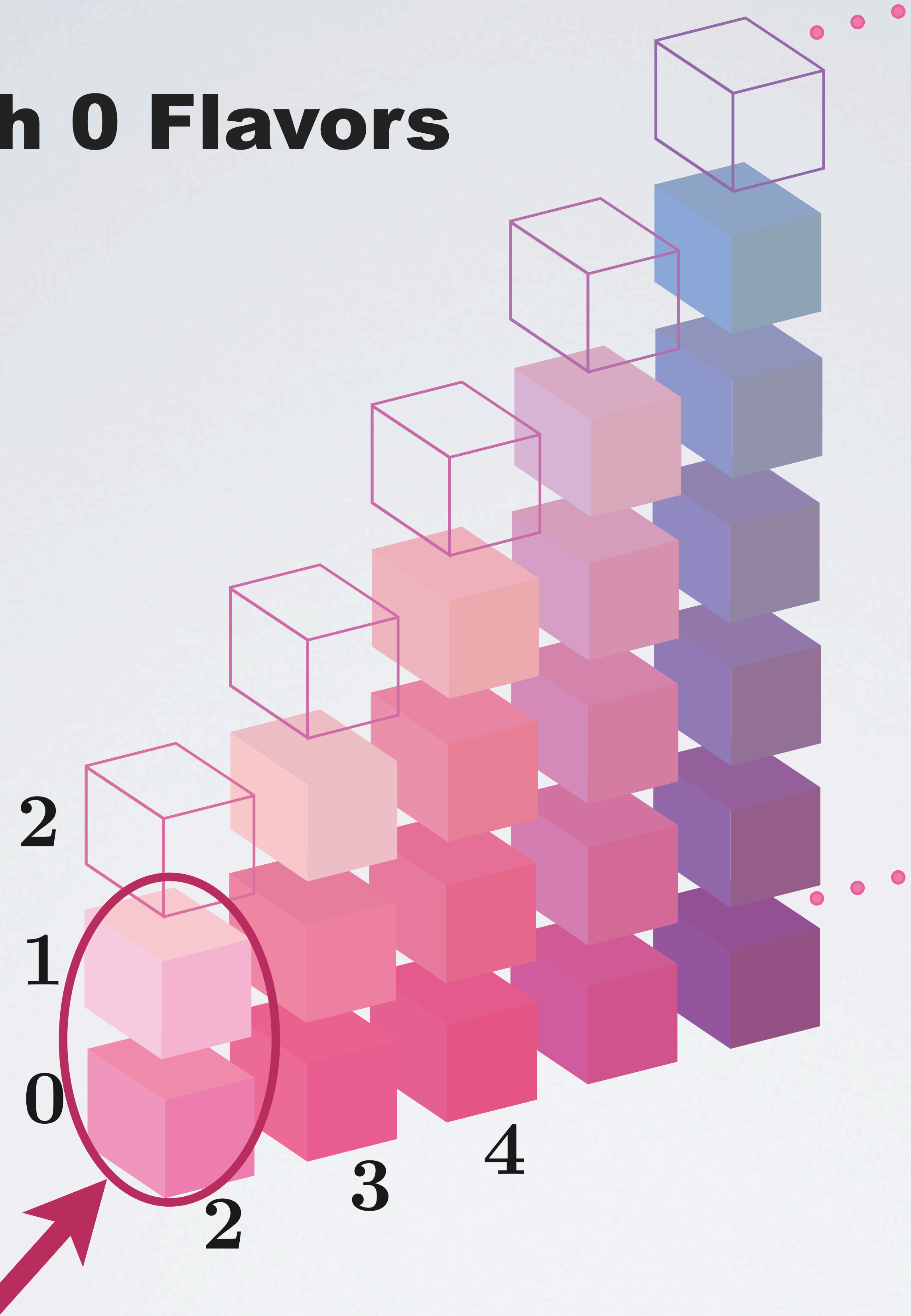
[Kanno-M.T, '12]



[Keller-Mekareeya-Song-Tachikawa, '12]

SU(2)

SU(2) with 0 Flavors



[Gaiotto, '09]

SU(2) with 0 Flavors

$$N_f = 0$$

$$L_1 |\, 0\, \rangle = |\, 0\, \rangle \qquad L_2 |\, 0\, \rangle = 0$$

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* It means 0-flavor, **not** vacuum

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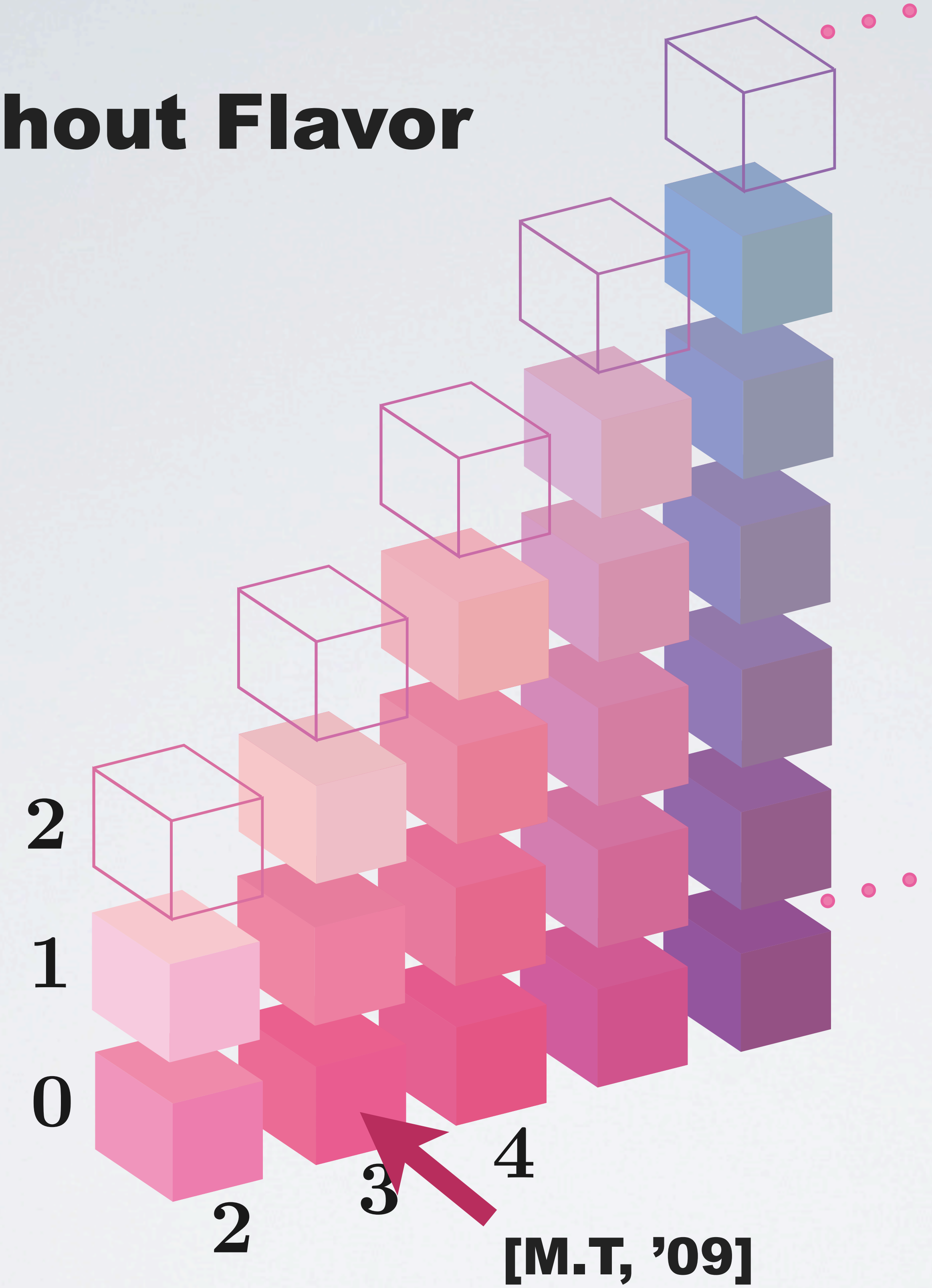
$$\boxed{Z_{SU(2)}^{N_f=0} = \langle 0 | 0 \rangle}$$

SU(3)

SU(3) without Flavor



SU(3) Whittaker state without Flavor

: theory with L_m and W_n

$$[L_m, W_n] = (2m - n)W_{n+m}$$

$$[W_m, W_n] =$$

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$$\boxed{Z_{SU(3)}^{N_f=0} = \langle 0 | 0 \rangle}$$

SU(3) Whittaker states with 0,1 Flavors

$$N_f = 0$$

$$L_1 | 0 \rangle = 0 \qquad W_1 | 0 \rangle = | 0 \rangle$$

$$N_f = 1$$

$$L_1 | 1 \rangle = | 1 \rangle \qquad W_1 | 1 \rangle = m | 1 \rangle$$

SU(3) Whittaker states with 0,1 Flavors

$$Z_{SU(3)}^{N_f=1} = \langle\, 0 \,|\, 1 \,\rangle = \langle\, 1 \,|\, 0 \,\rangle$$

$$Z_{SU(3)}^{N_f=2} = \langle\, 1 \,|\, 1 \,\rangle$$

SU(3) Whittaker states with 0,1 Flavors

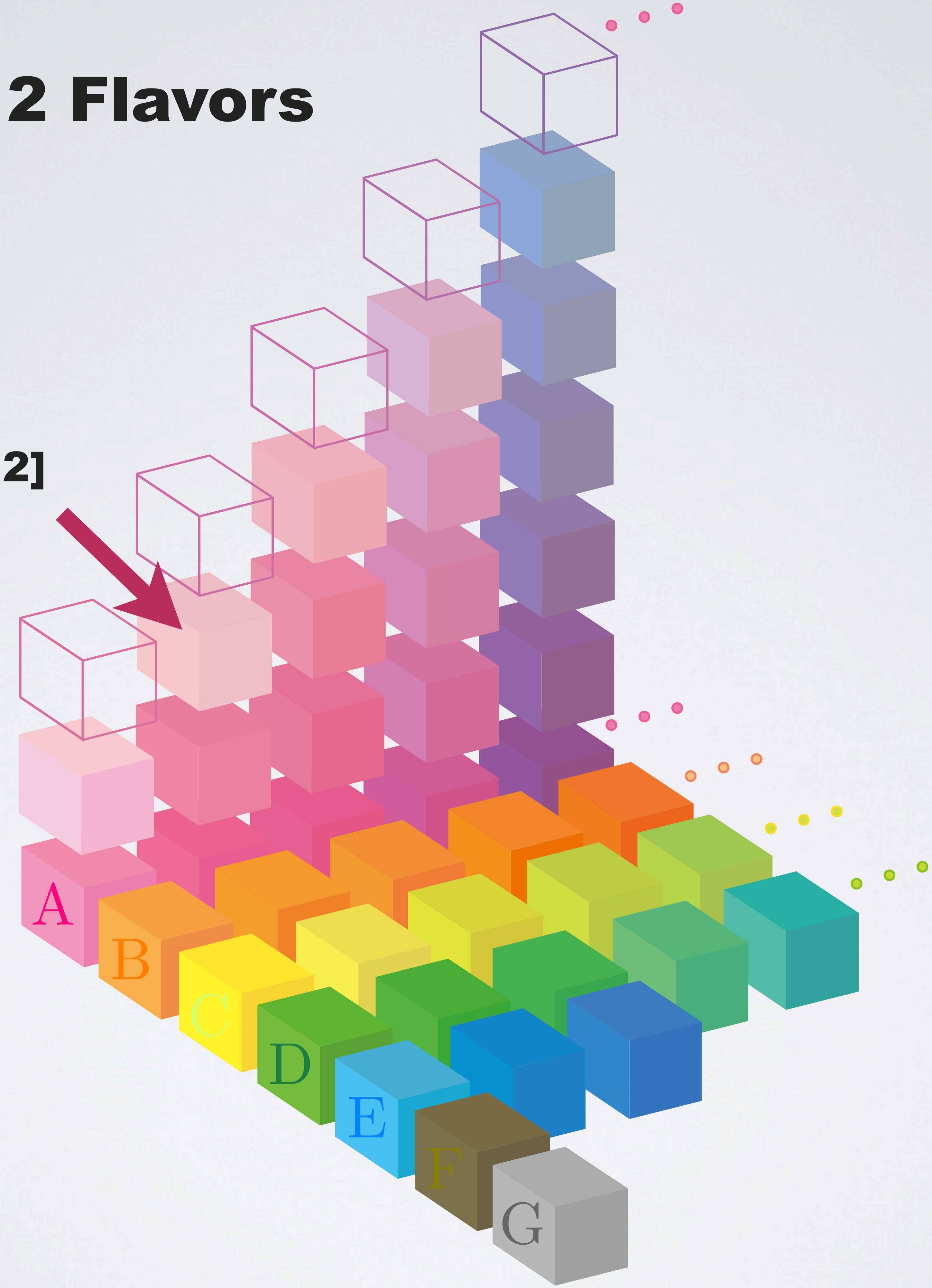
$$Z_{SU(3)}^{N_f=1} = \langle\, 0 \,|\, 1 \,\rangle = \langle\, 1 \,|\, 0 \,\rangle$$

$$Z_{SU(3)}^{N_f=2} = \langle\, 1 \,|\, 1 \,\rangle$$

$$Z_{SU(3)}^{N_f=2} = \langle\, 0 \,|\, 2 \,\rangle = \langle\, 2 \,|\, 0 \,\rangle \ ?$$

SU(3) with 2 Flavors

[Kanno-M.T, ’12]



SU(3) with 2 Flavors → Trouble !?

SU(3) with 2 Flavors $\longrightarrow$ Trouble !?

Qestion.

$|\,2\,\rangle$ **must be** L_1, L_2, W_2, W_3 **eigenstate.**

But

$$W_2 = [L_1, W_1]$$

$$3W_3 = [L_2, W_1]$$

SU(3) with 2 Flavors ⟶ Trouble !?

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$|\,2\,\rangle$ **must be** L_1, L_2, W_2, W_3 **eigenstate.**

But

$$W_2 = [L_1, W_1]$$

$$3W_3 = [L_2, W_1]$$

Answer.

$$(W_1 + L_0)|\,2\,\rangle \propto |\,2\,\rangle$$

$$[L_n, L_0] = nL_n$$

generalized Whittaker states

$\{L_1, L_2\}$

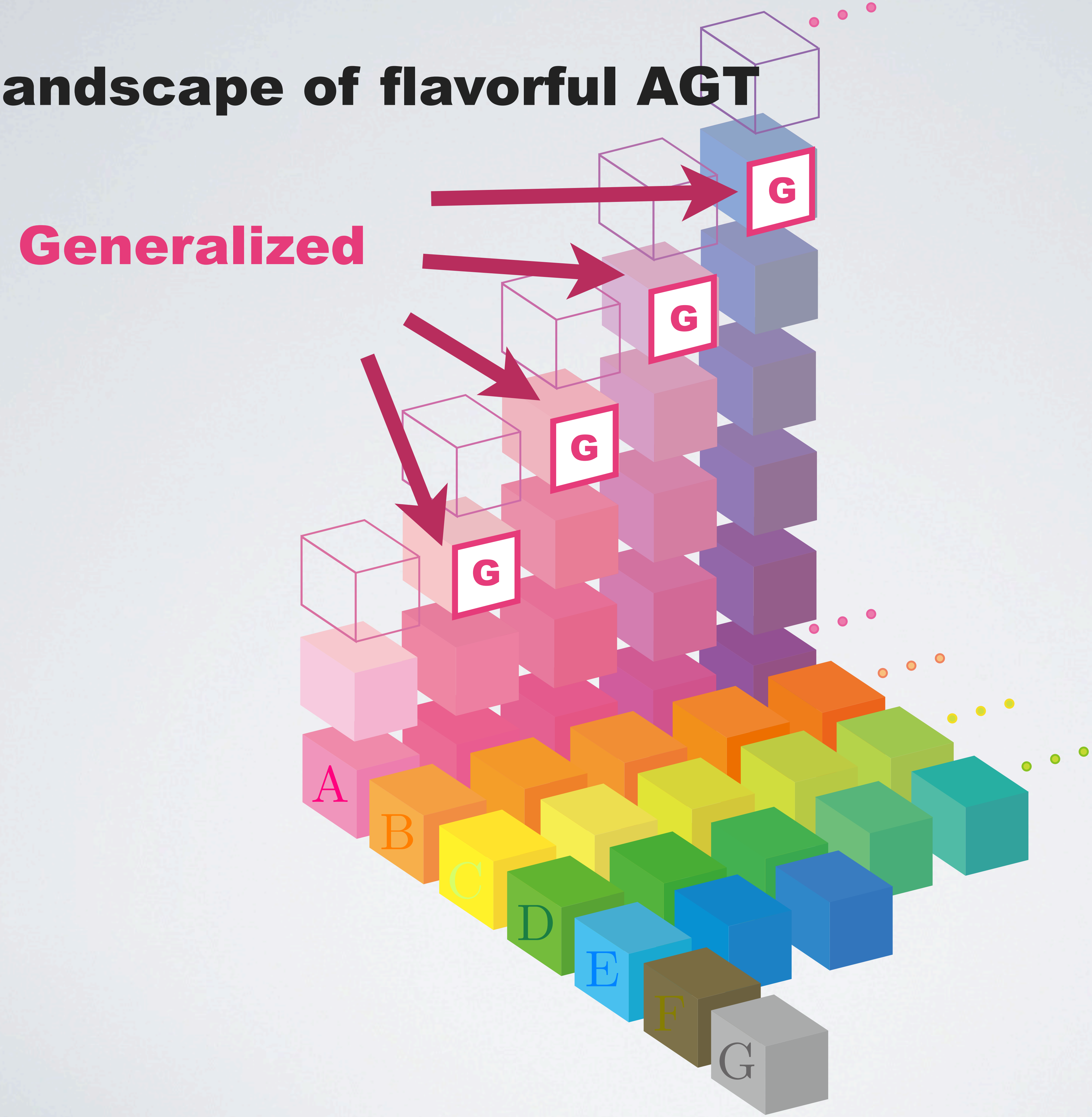
$\{L_0\}$: Cartan

$\{L_{-1}, L_{-2}\}$

$\}$ ← eigenstate of linear combi. of them

This is actually very **ubiquitous** B.C. for M5s !

Landscape of flavorful AGT



4. Summary

“Generalized” is ubiquitous M5 configuration

generalized Whittaker states :

- **flavorful cases of colorful ABCDEFG**
- **surface operators**
- **4D SCFTs**

M5 branes

Gauge Theory

Next step : feedback to M-theory

FIN