Emission spectrum of soft massless states from heavy superstring

Shoichi Kawamoto[1] and Toshihiro Matsuo[2]
Physics Division, NCTS, Taiwan[1], Anan National College of Technology, Japan[2]
E-mail: `kawamoto@yukawa.kyoto-u.ac.jp`[1], `tmatsuo@yukawa.kyoto-u.ac.jp`[2]

We calculate the emission spectrum of soft massless bosons/fermions from heavy open/closed superstrings by use of Green-Schwarz superstring in the light-cone gauge. The initial state is prepared as an averaged state over a fixed high excited level. The emission spectrum is a thermal one at the Hagedorn temperature. We calculate the greybody factors and discuss similarity to those from various black holes.

Introduction One of the characteristic feature of string theory is that the number of excited states grows exponentially. This behavior leads to a characteristic temperature of the theory, the Hagedorn temperature $T_H \propto 1/\sqrt{\alpha'}$, where perturbation theory may break down. Though it is still unclear what actually happens at this temperature, one suggestion is that near this temperature the energy of the system tends to be stored in a single string, rather than distributed to many strings, and thus highly excited heavy strings become dominant. The physics of heavy strings is thus important and interesting when energy scales are near or beyond the string scale. This growing number density also suggests that a single string possesses a very large entropy, $S_{\text{string}} \sim \sqrt{N}$, if the excited level N is large. Susskind has argued that the states of a single string may be in one-to-one correspondence with the microstates of a black hole (BH), and these two descriptions are switched when the string coupling constant comes to a "correspondence point" where the curvature radius of a black hole of the mass of the string becomes the string size[3]. On that point, the entropy of a string qualitatively coincides with that of a black hole of the same mass. Now we may ask a question about a dynamical property of this correspondence. We study the emission spectrum of massless bosonic/fermionic states from a heavy open/closed superstring by use of Green-Schwarz (GS) superstring in the light-cone (LC) gauge, and compare them with that from black holes. The LC gauge GS superstring enables us to carry out the whole calculation explicitly and unambiguously for both boson and fermion emissions. In literature, it has been discussed that fermion and Ramond-Ramond (RR) field emissions are suppressed by $1/N$ in Ramond-Neveu-Schwarz (RNS) superstring[5]. This observation is based on a form of vertex operators but in the Ramond sector there is a potential ambiguity due to the choice of the picture. We suspect that this discussion is not conclusive and try to provide a definite answer. The emission rate from a RR-charged BH has been found to agree with effective string calculation on D-branes of the same charges, including numerical factors[4]. This discovery is a precursor of gauge/gravity correspondence. We thus hope that our calculation also leads to more profound understanding of quantum aspects of BHs through the study here.

Semi-inclusive decay rates We consider a string of excited level $N(\gg 1)$ that eventually decays into a massless state and another heavy state, and observe the energy spectrum of the massless state at an asymptotic infinity. We take the leading order approximation in perturbation theory and do not consider multi-decaying processes. A heavy string may decay into two heavy strings, but such strings may be too heavy to reach us, and we neglect such a process too. We are not interested in the details of an initial state, except its mass (or equivalently the exited level), and consider an averaged initial state at the level. We also sum over final state except the energy we measure. For this semi-inclusive process, the decay rate is given by

$$\Gamma = \frac{d^9k}{M(M-\omega)\omega} P(\Phi_N \to \gamma(k) + \Phi_{N'})\,, \tag{1}$$

where $\Phi(N)$ stands for a state at level N, M is the mass of the initial state, $M \propto \sqrt{N/\alpha'}$, and ω is the energy of the emitted massless state $\gamma(k)$. P denotes the probability of this three-body process, and expressed as

$$P(\Phi_N \to \gamma(k) + \Phi_{N'}) = \frac{1}{\mathcal{G}(N)} \sum_{\Phi|N} \sum_{\Phi|N'} \sum_{\gamma} |\langle \Phi(N')|V(\gamma,k)|\Phi(N)\rangle|^2\,, \tag{2}$$

$\sum_{\Phi|N}$ represents the summation over all the states at level N. $\mathcal{G}(N)$ is the asymptotic number of states for level N which behaves like $e^{\sqrt{N}}$, and $V(\gamma,k)$ is the string vertex operator corresponding to the emitted massless state. In practice, there are exponentially many states at a high level N and most of them are very complicated. Amati and Russo[2] have introduced a nice trick for this calculation; by inserting a level projection operator $\hat{P}_N$ onto the level N states to convert the square of the disc amplitude into the form of a one-loop amplitude. The calculation is straightforward but lengthy, and we pick up the results from [1]. The decay rates are collectively written as

$$\Gamma \simeq \frac{\omega^8 d\omega}{M^2} \frac{\sigma(\omega)}{e^{\beta_H \omega} \mp 1}\,, \tag{3}$$

where a function $\sigma(\omega)$ is different for each case. β_H is the inverse Hagedorn temperature, and this thermal behavior appears due to the averaging over the initial states. $\sigma(\omega)$ for each process is summarized as

$$\text{open from open}: \quad \sigma_{\text{boson}} = \tilde{g}^2 \cdot 1\,, \qquad \sigma_{\text{fermion}} = \tilde{g}^2 \cdot \omega^{-1}\,, \tag{4}$$

$$\text{closed from open/closed}: \quad \sigma^{(\text{cl})}_{NS-NS} = \tilde{g}^4 \omega \rho_- \left(\frac{\beta_H}{2}\right)^2 \rho_-\big(\beta_H\big)^{-1}\,,$$

$$\sigma^{(\text{cl})}_{R-NS} = \tilde{g}^4 \rho_- \left(\frac{\beta_H}{2}\right) \rho_+ \left(\frac{\beta_H}{2}\right) \rho_+\big(\beta_H\big)^{-1}\,, \quad \sigma^{(\text{cl})}_{R-R} = \tilde{g}^4 \omega^{-1} \rho_+ \left(\frac{\beta_H}{2}\right)^2 \rho_-\big(\beta_H\big)^{-1}\,, \tag{5}$$

where $\tilde{g} = g_s N^{1/4}$ is an effective coupling, the subscripts are the species of the massless states, and $\rho_\pm(\beta) = 1/(e^{\beta\omega} \pm 1)$ is the thermal factor for fermions/bosons. We first see that the fermion and R-R field emissions are not suppressed but of the same order in $1/N$ expansion, in contrast to speculation in past literature. Secondly, the emission of massless open states (namely gauge bosons) have the black body spectrum. At the tree level, the massless states are emitted only from the ends of a long open string, and the chance becomes tiny for large N. Thus a heavy string may exhibit a cavity radiation for open massless states. Finally, we point out that these expressions are valid for small ω and the fermion emission is enhanced compared to the boson emission. The mechanism behind this enhancement is unclear at this point.

This $\sigma(\omega)$ factor describes deviation from the pure black body radiation and is called the greybody factor. Interestingly, this greybody factors here take very similar forms of those of RR-charged BH[4]; namely they consist of the same $\rho_\pm$ factor β_H replaced with the Hawking temperature. Again, fermions and RR fields have a discrepancy in a power of ω. It should be interesting to clarify this point. Our setup is completely neutral, due to the averaging, and it is also intriguing how this universal functional dependence appears.

Conclusion We have observed that the semi-inclusive decay rates of a heavy superstring of an averaged initial state exhibits the thermal behavior of the Hagedorn temperature. The greybody factor is of black body type for open string emissions, while it takes a similar form to a R-R charged BH for closed ones. Also, we have confirmed that the fermion emission rate appears at the same order in $1/N$ expansion.

References

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