Lepton Flavor Structure from Point Interactions in an Extra Dimension

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Makoto Sakamoto (Kobe Univ.)
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Mysteries of the Standard Model

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♦ Generation

Who ordered the same packages which possess the same quantum numbers in the Standard Model ?

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♦ Mass hierarchy

Why so different the masses of quarks/leptons are in the Standard Model ?

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♦ Mass hierarchy

Why so different the masses of quarks/leptons are in the Standard Model ?

♦ Flavor structure

What determine the small/large mixing pattern of the quark/lepton sector.

Purpose

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- **Generation**
- **Mass hierarchy**
- **Flavor structure**

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in the context of higher dimensional gauge theories.

Setting

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- **5d gauge theories on a circle**

  with
  - ★ 5d fermions (one generation)
  - ★ 5d Higgs & singlet scalar

Setting

♦ 5d gauge theories on a circle

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★ 5d Higgs & singlet scalar

Gauge fields

$$W^a_{MN}(x,y)$$
$$B_{MN}(x,y)$$

Higgs & singlet

$$H(x,y)$$
$$\Phi(x,y)$$

Fermions

$$\begin{pmatrix} u(x,y) \\ d(x,y) \end{pmatrix} \quad \begin{pmatrix} \nu(x,y) \\ e(x,y) \end{pmatrix}$$

$$u'(x,y) \quad \nu'(x,y)$$
$$d'(x,y) \quad e'(x,y)$$

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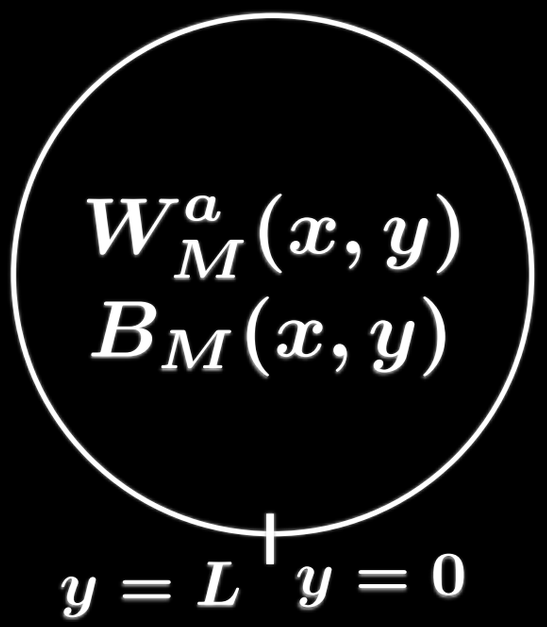
  ★ **Gauge fields do not feel point interactions.**



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  ★ Gauge fields do not feel point interactions.

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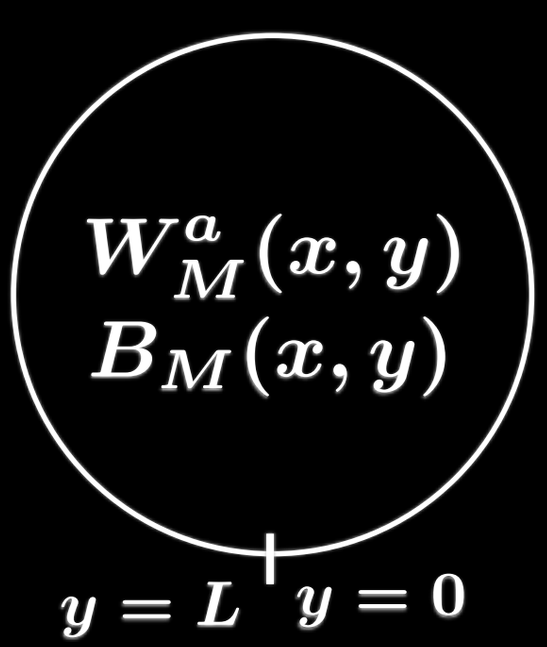

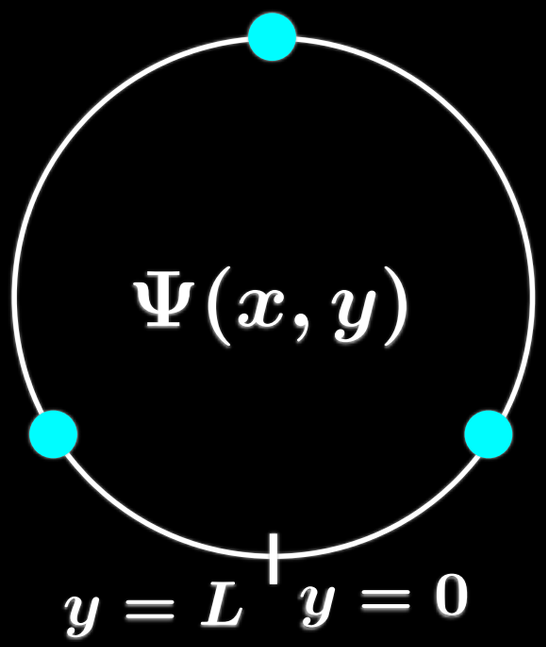

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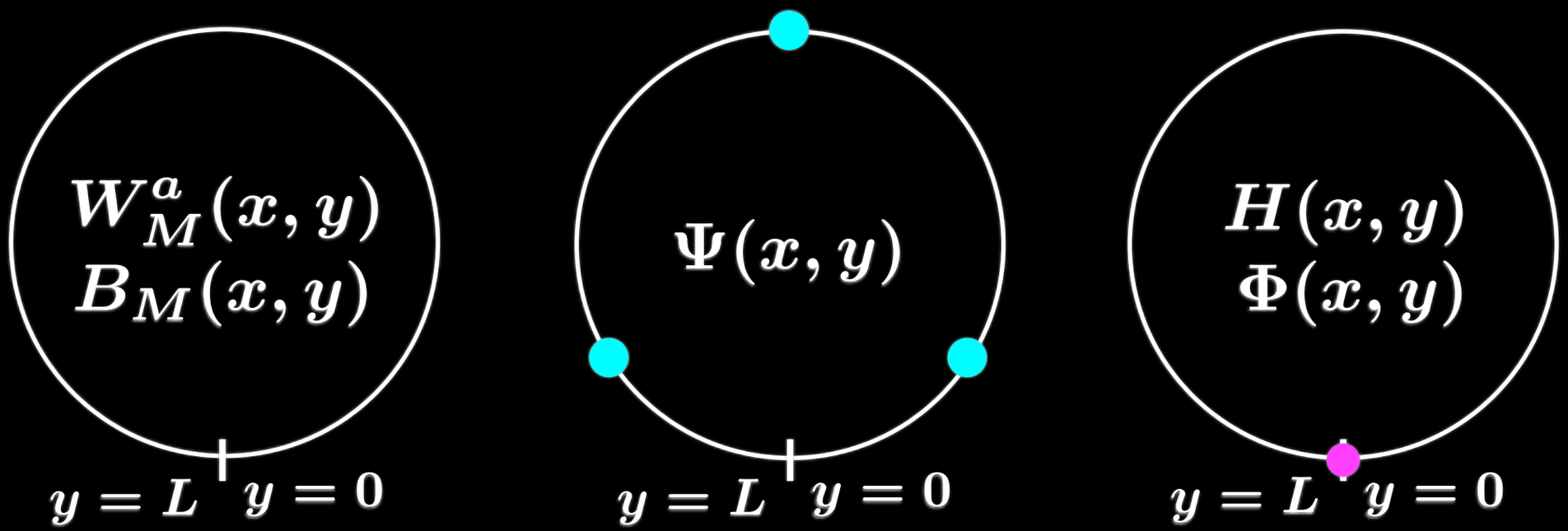

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  ★ Fermions feel several point interactions.
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- **Impose boundary conditions**

  ★ No flow of the probability current through the point interactions

  compatible with { ★ 5d gauge invariance
  ★ action principle etc.

Boundary conditions

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♦ Gauge fields

★ Periodic boundary condition

$$\begin{cases} A_M(L) = A_M(0) \\ \partial_y A_M(L) = \partial_y A_M(0) \end{cases}$$

Boundary conditions

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★ Periodic boundary condition

$$\begin{cases} A_M(L) = A_M(0) \\ \partial_y A_M(L) = \partial_y A_M(0) \end{cases}$$

♦ Higgs field

★ Twisted boundary condition

$$H(y+L) = e^{i\theta} H(y)$$

Boundary conditions

♦ Singlet scalar

★ Robin boundary condition

$$\begin{cases} \Phi(0) + L_+ \partial_y \Phi(0) = 0 \\ \Phi(L) - L_- \partial_y \Phi(L) = 0 \end{cases} \quad (-\infty \leq L_\pm \leq +\infty)$$

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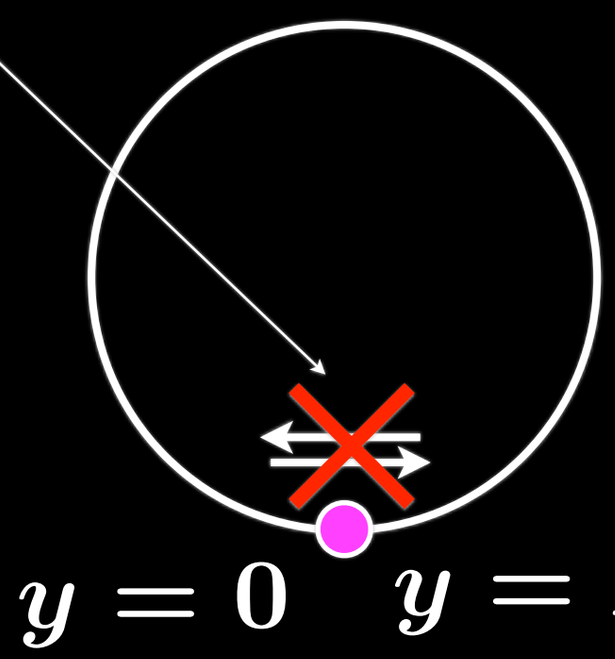
No flow of the probability current



Boundary conditions

♦ Fermion

★ Dirichlet boundary condition

$$\Psi_R(y) = 0 \ @ \text{ point interactions}$$
$$\text{or}$$
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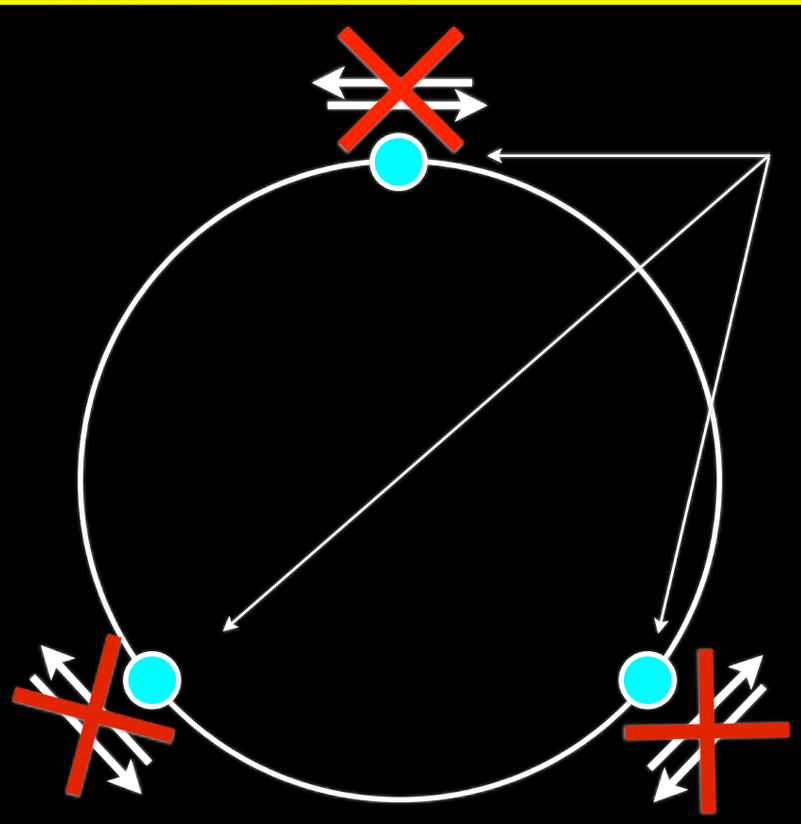

No flow of the probability current

Generation

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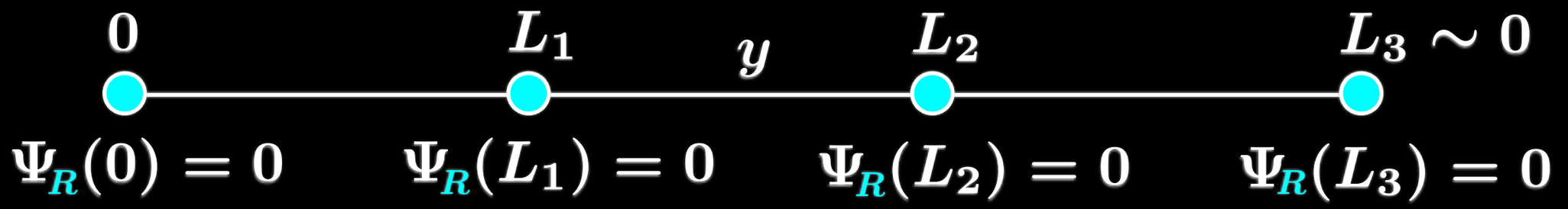

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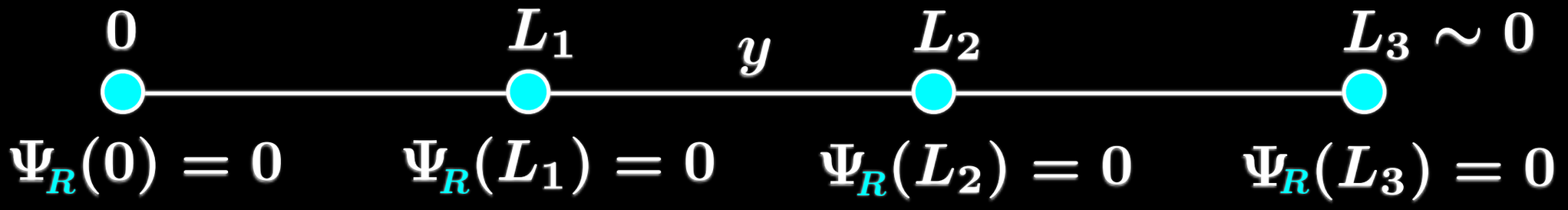

$$\Psi(x,y) = \sum_{i=1}^{3} \psi_{0,L}^{(i)}(x)\mathcal{G}_0^{(i)}(y) + \cdots$$

Generation

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$$(-\partial_y + M_F)\mathcal{G}_0^{(i)}(y) = 0$$

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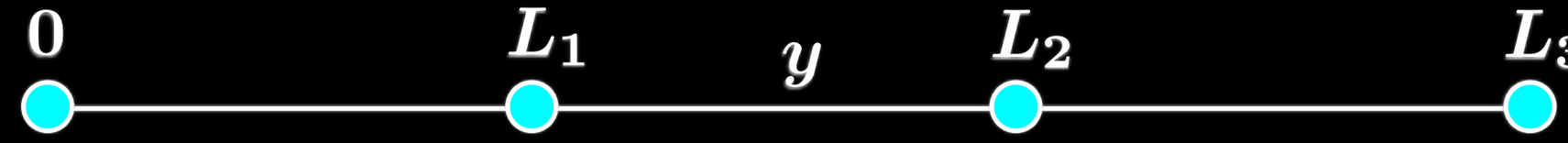

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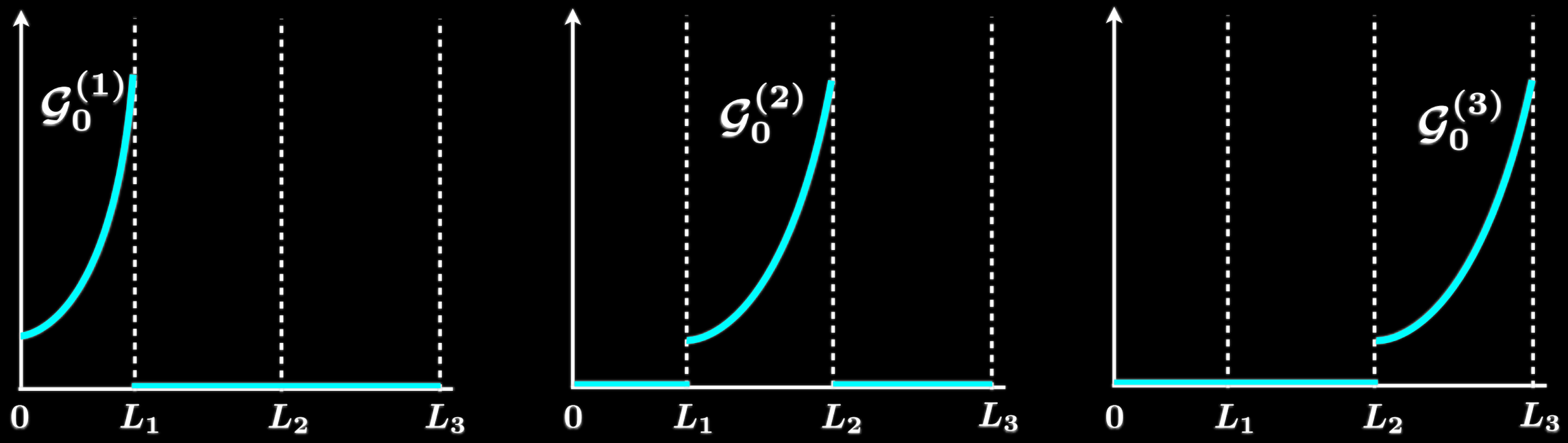

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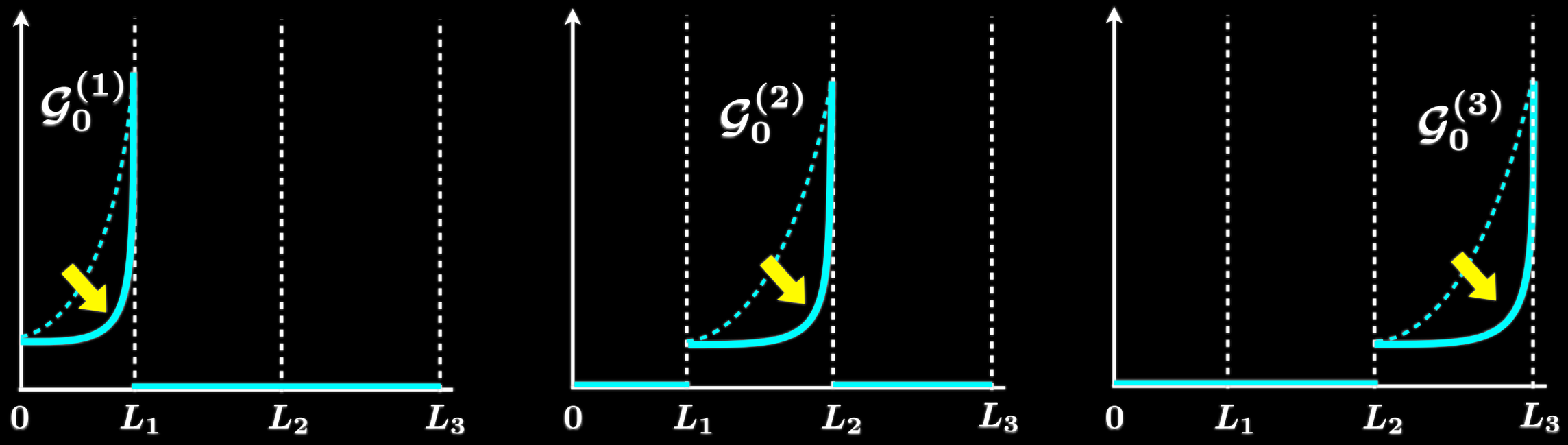
$M_F \to$ Large



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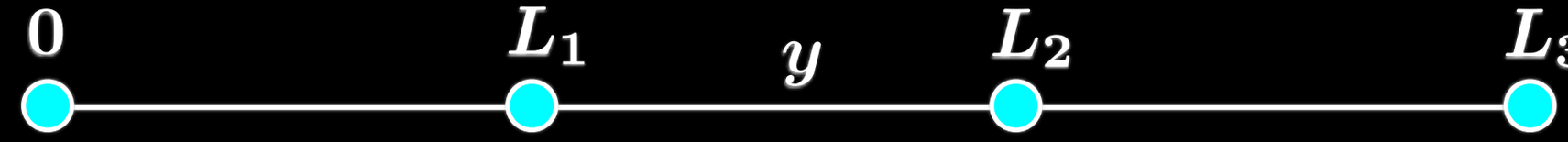

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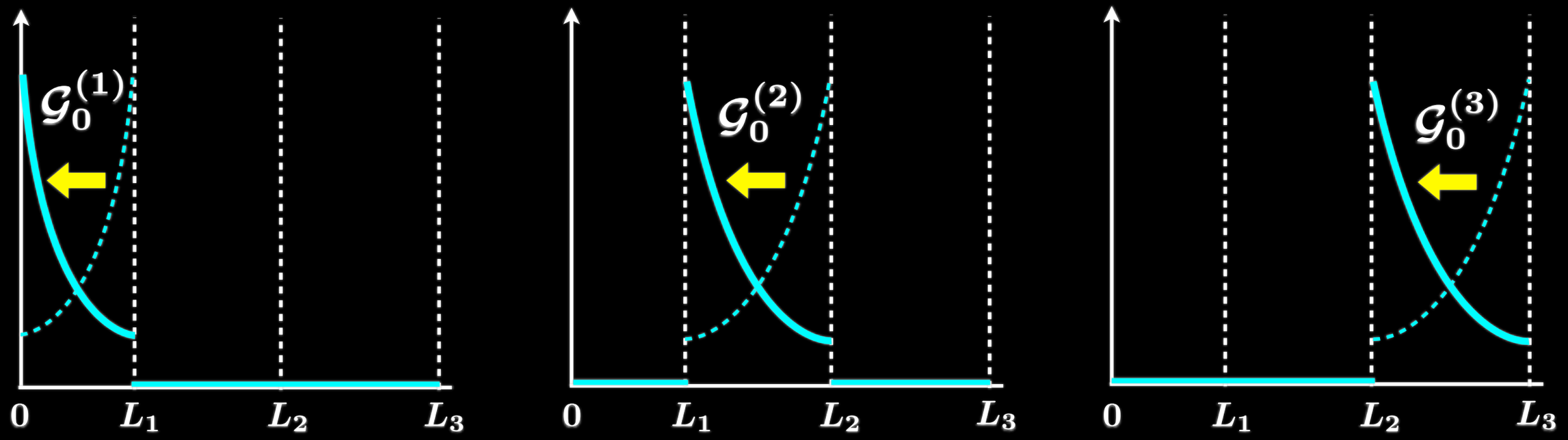
$M_F<0$



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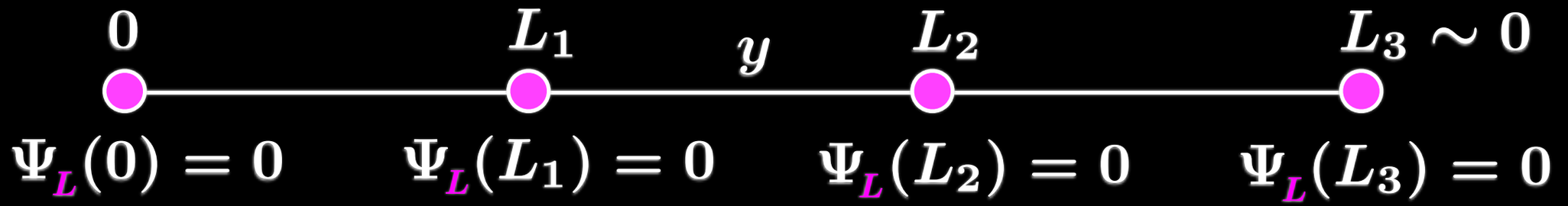

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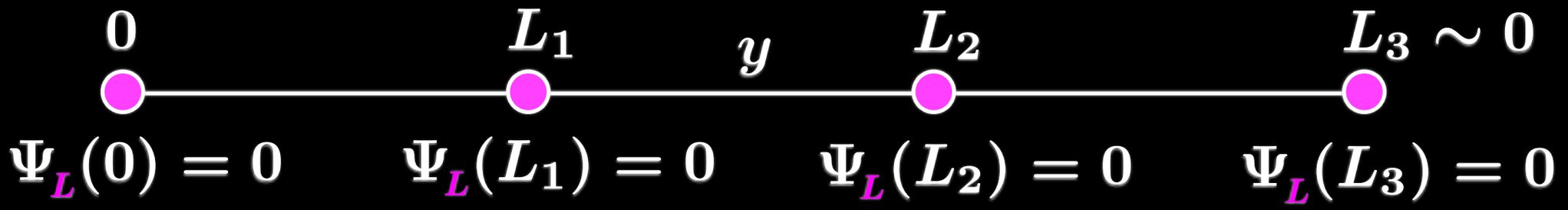

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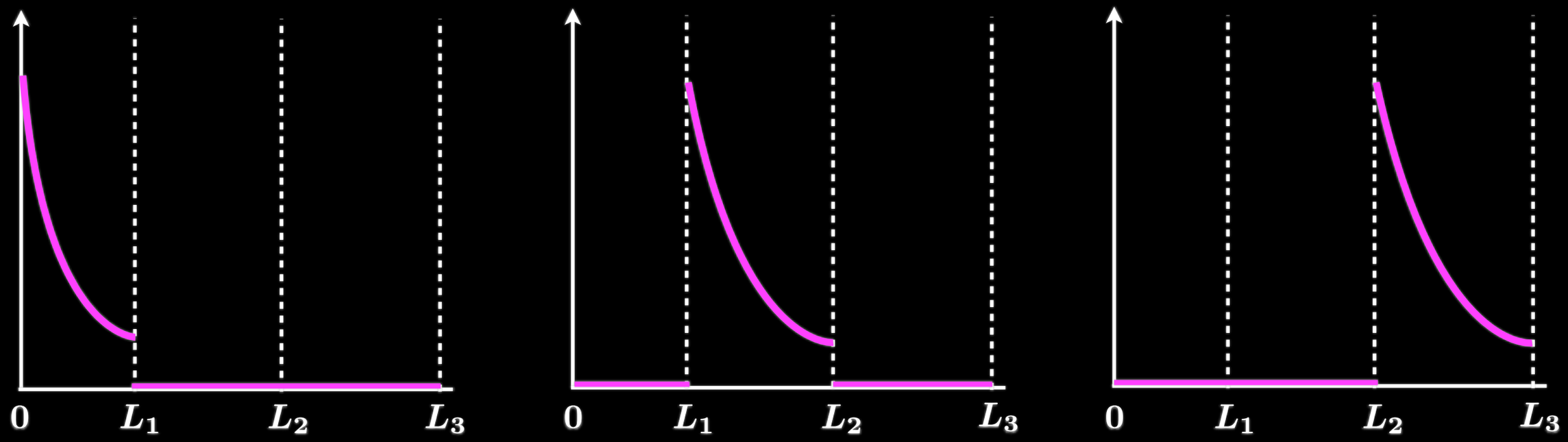

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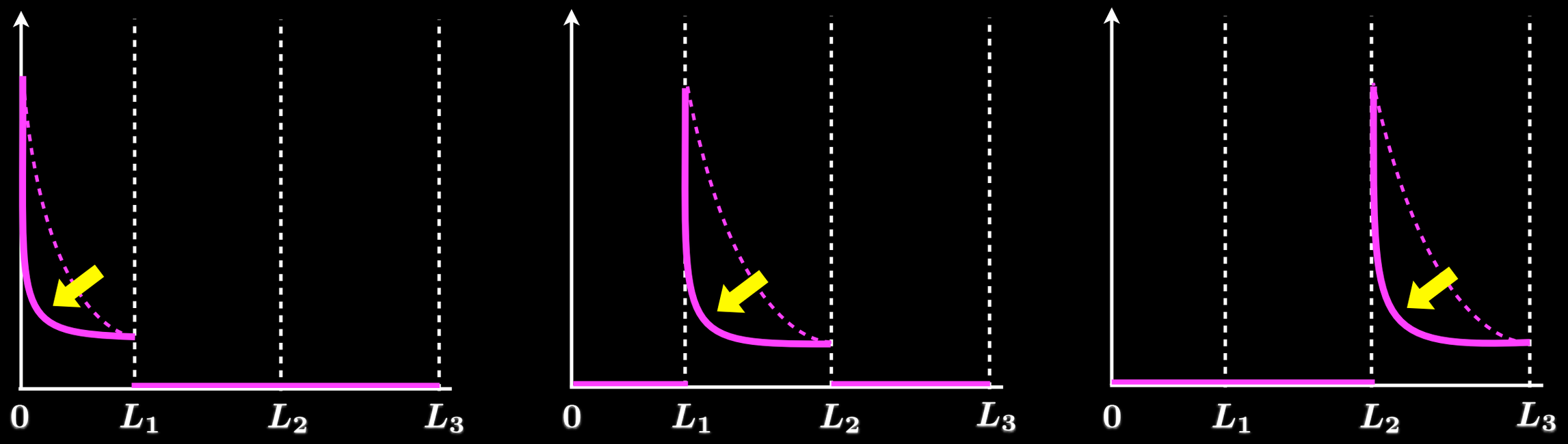

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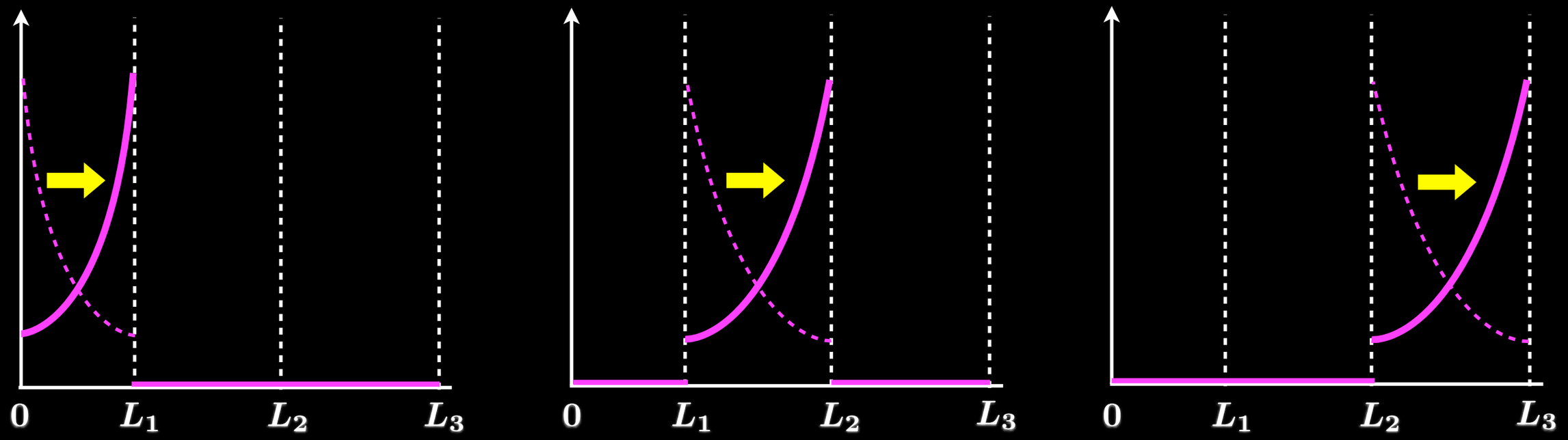

Quark sector

PTEP (2013) 023B07, arXiv : 1301.7253 [hep-ph]

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$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

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The terms $\Phi(y)\bar{\Psi}'\Psi$, $H(y)\bar{\Psi}'\Psi$

are forbidden by the following Z_2 **symmetry.**

$$\Phi(y) \to -\Phi(y) \quad , H(y) \to -H(y)$$

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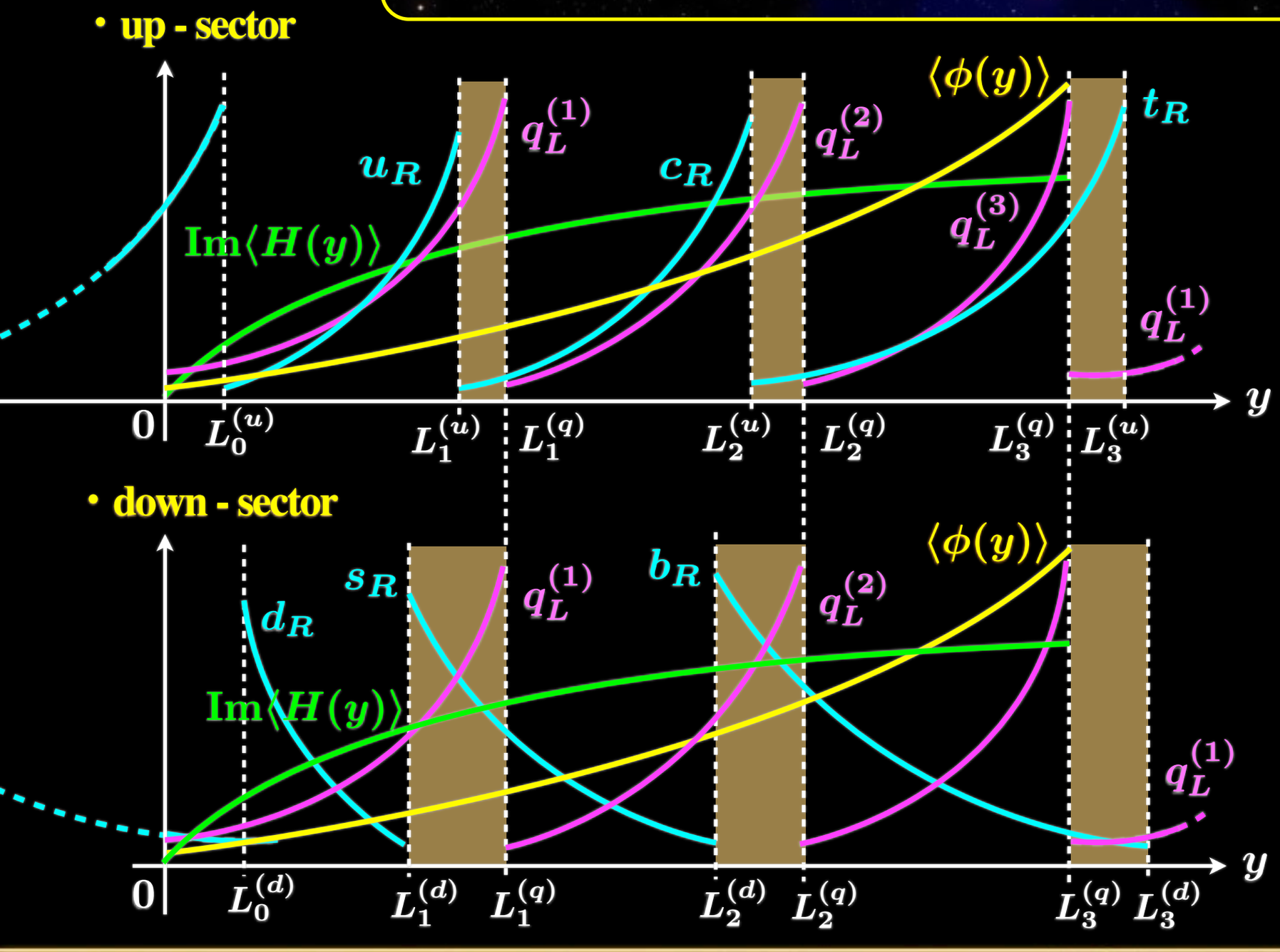

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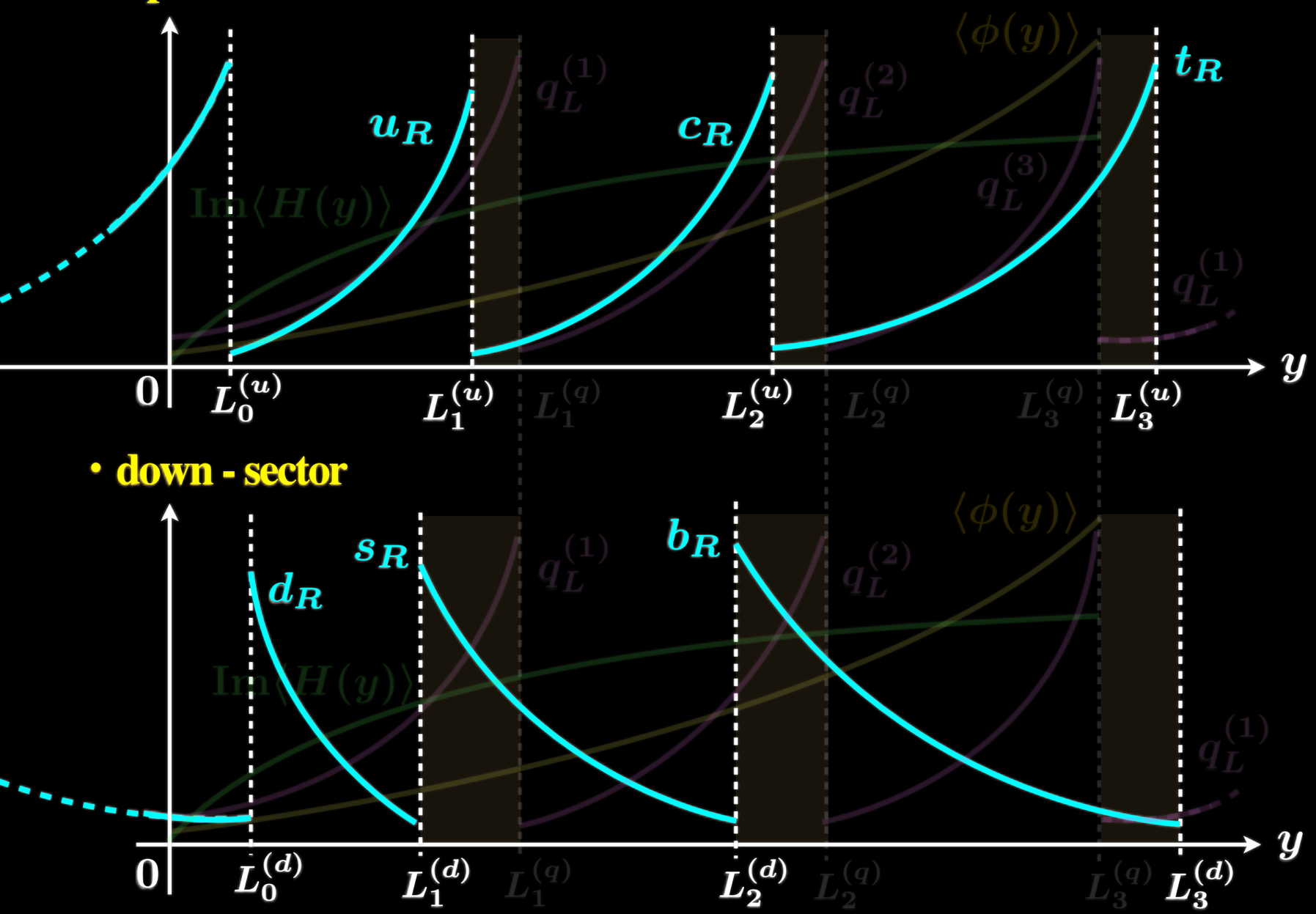

★ Three generations via point interactions

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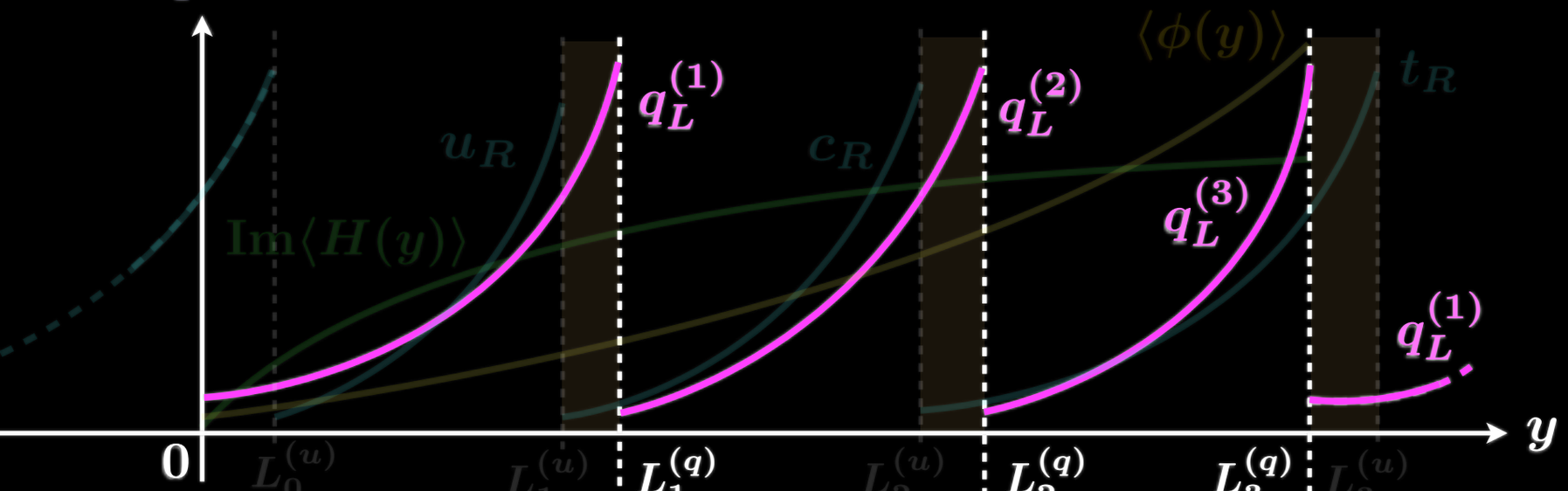
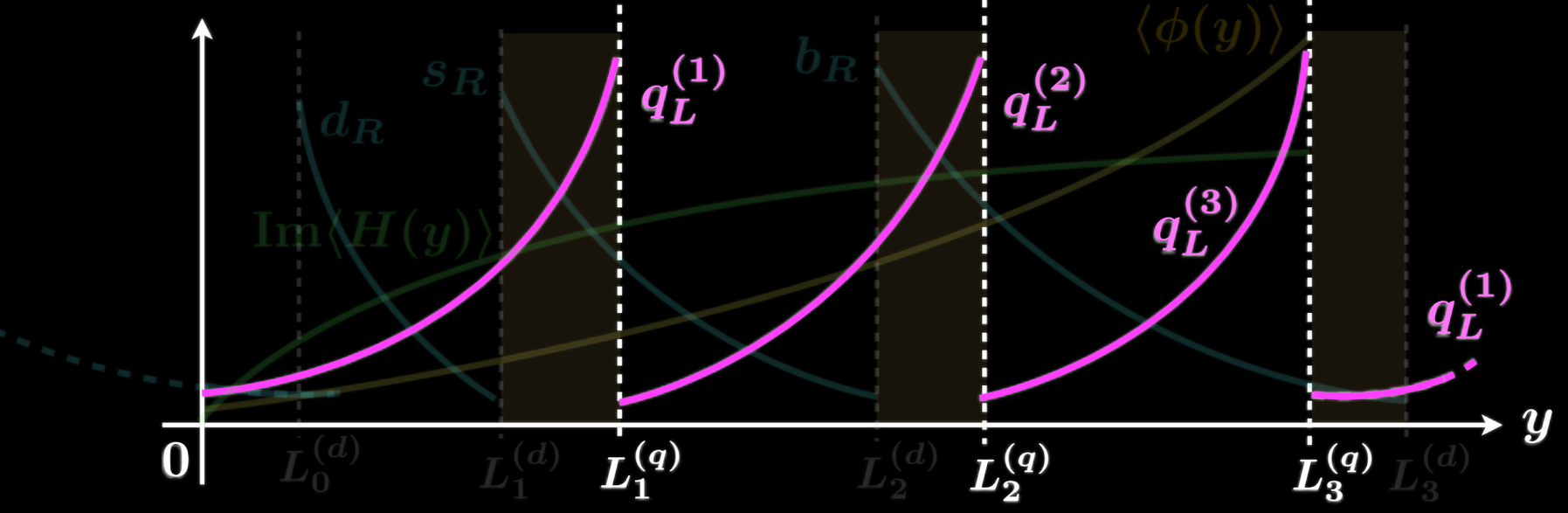
• up - sector



• down - sector



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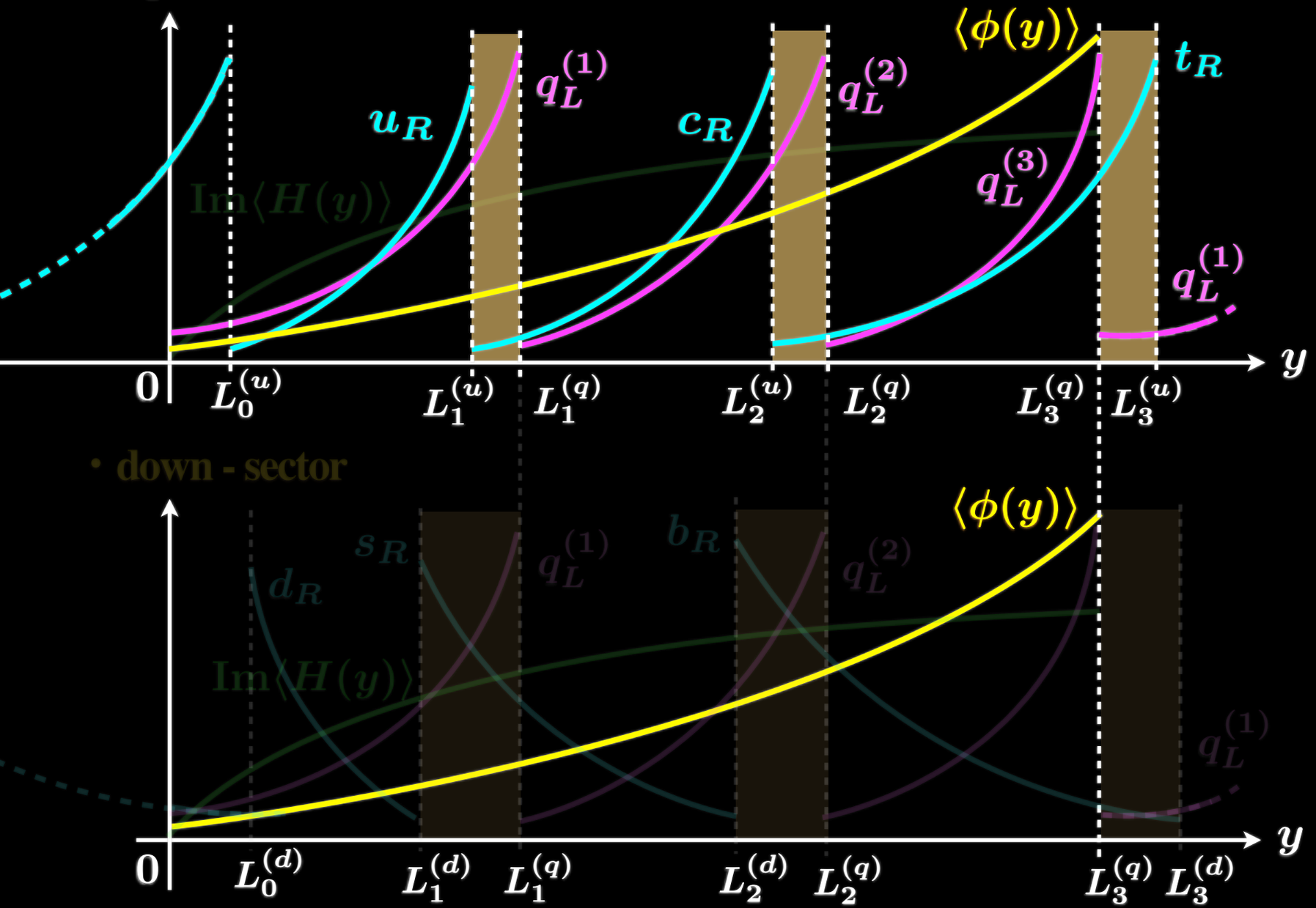

★ Three generations via point interactions

★ Mass hierarchy from $\langle \Phi(y) \rangle$ via the Robin BC

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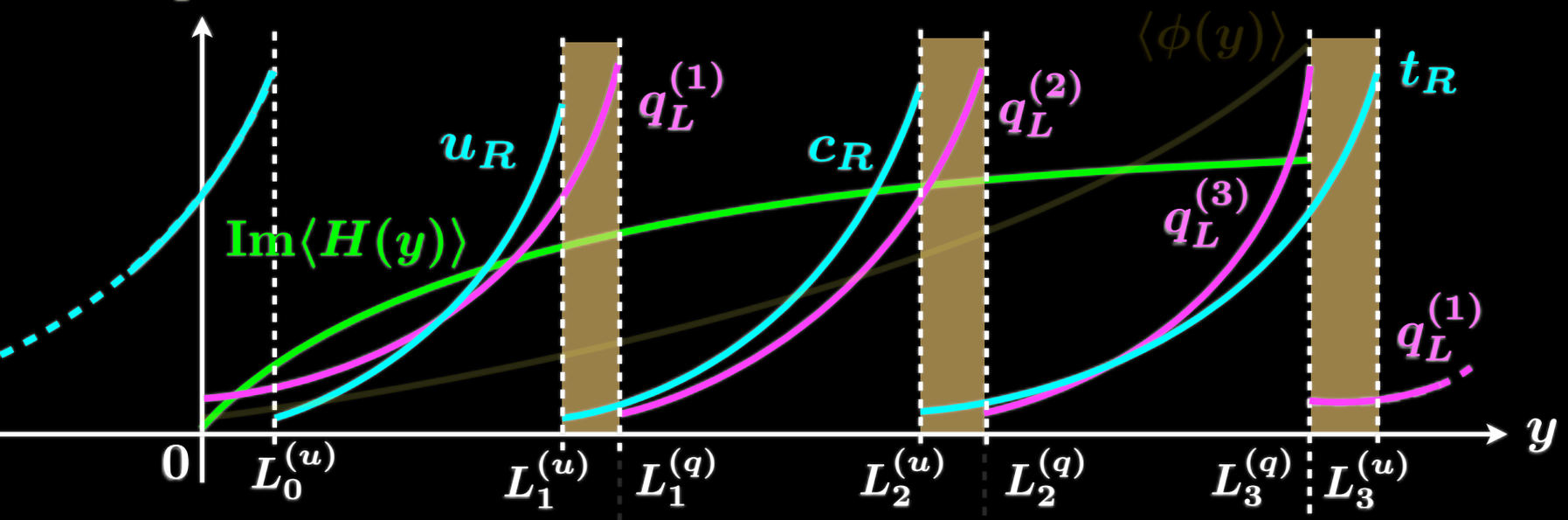

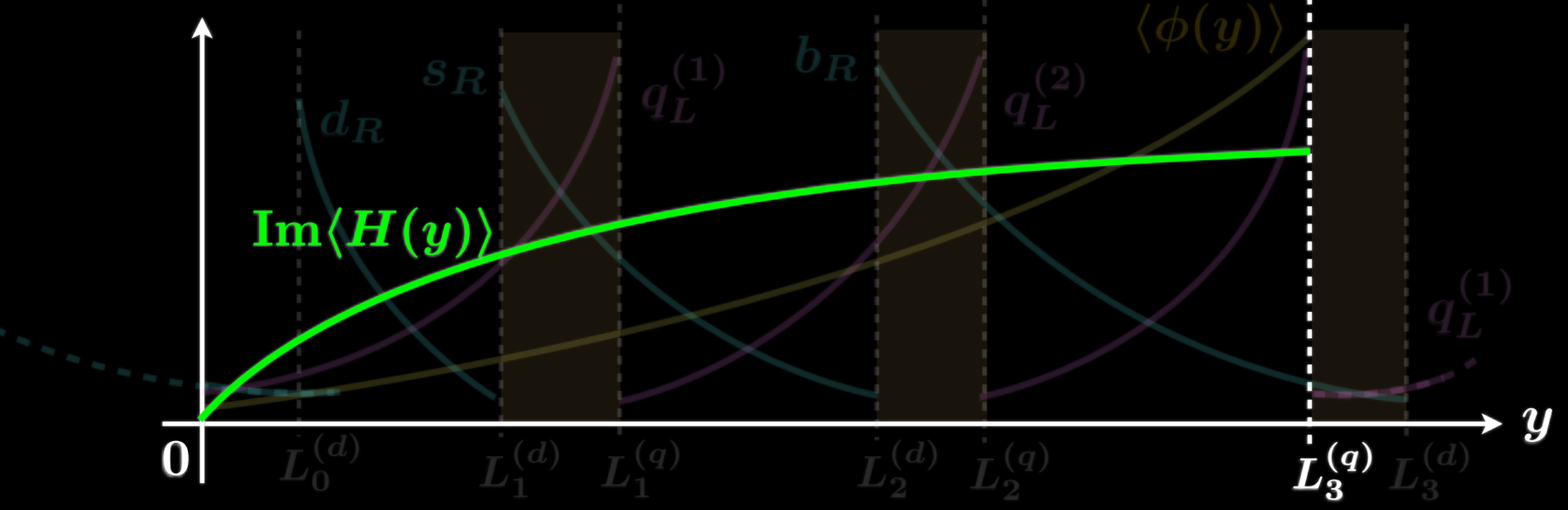

★ Three generations via point interactions

★ Mass hierarchy from ⟨Φ(y)⟩ via the Robin BC

★ CP phase from ⟨H(y)⟩ via the twisted BC

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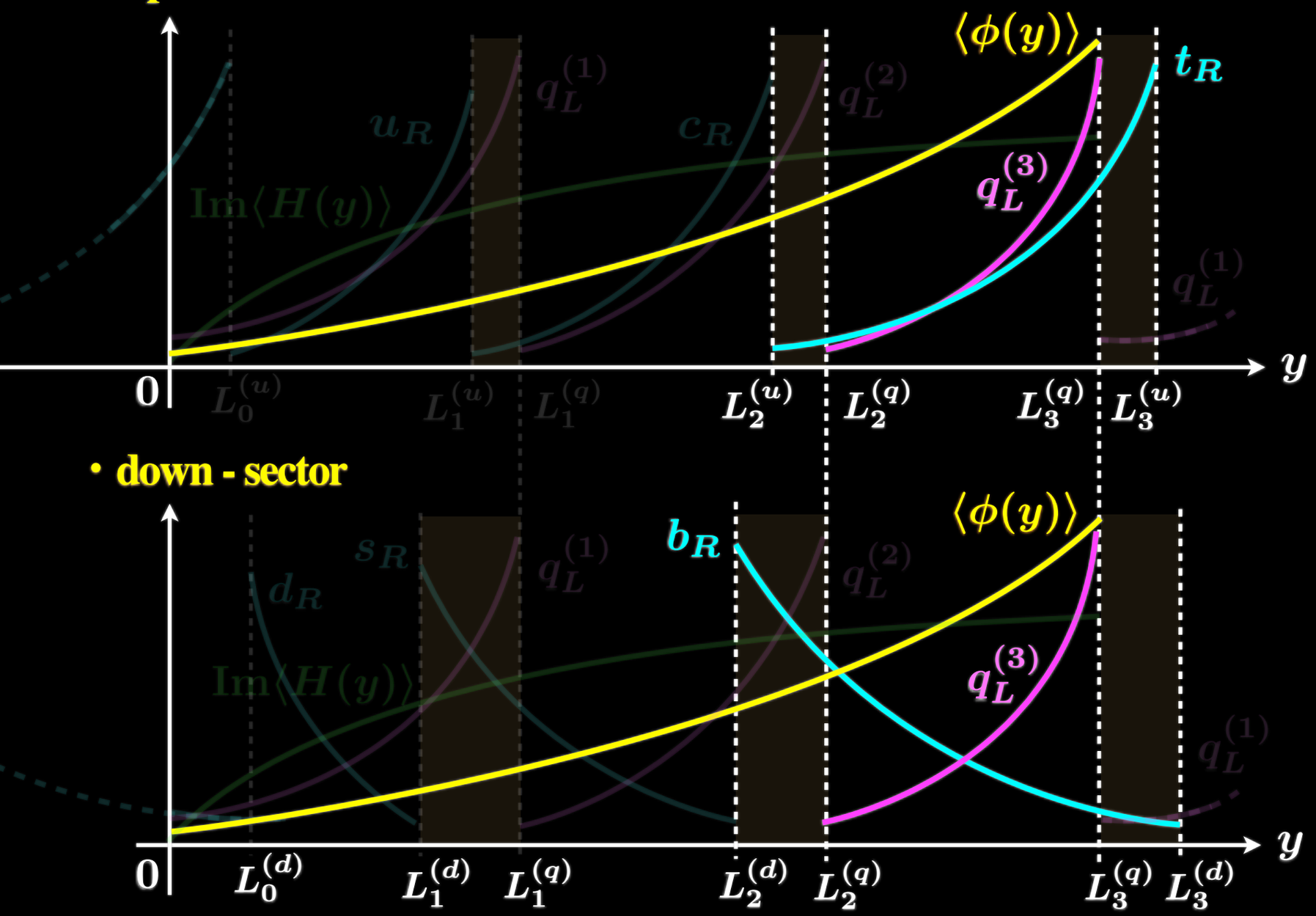

★ Three generations via point interactions

★ Mass hierarchy from $\langle \Phi(y) \rangle$ via the Robin BC

★ CP phase from $\langle H(y) \rangle$ via the twisted BC

★ $M_t > M_b$ from the configuration

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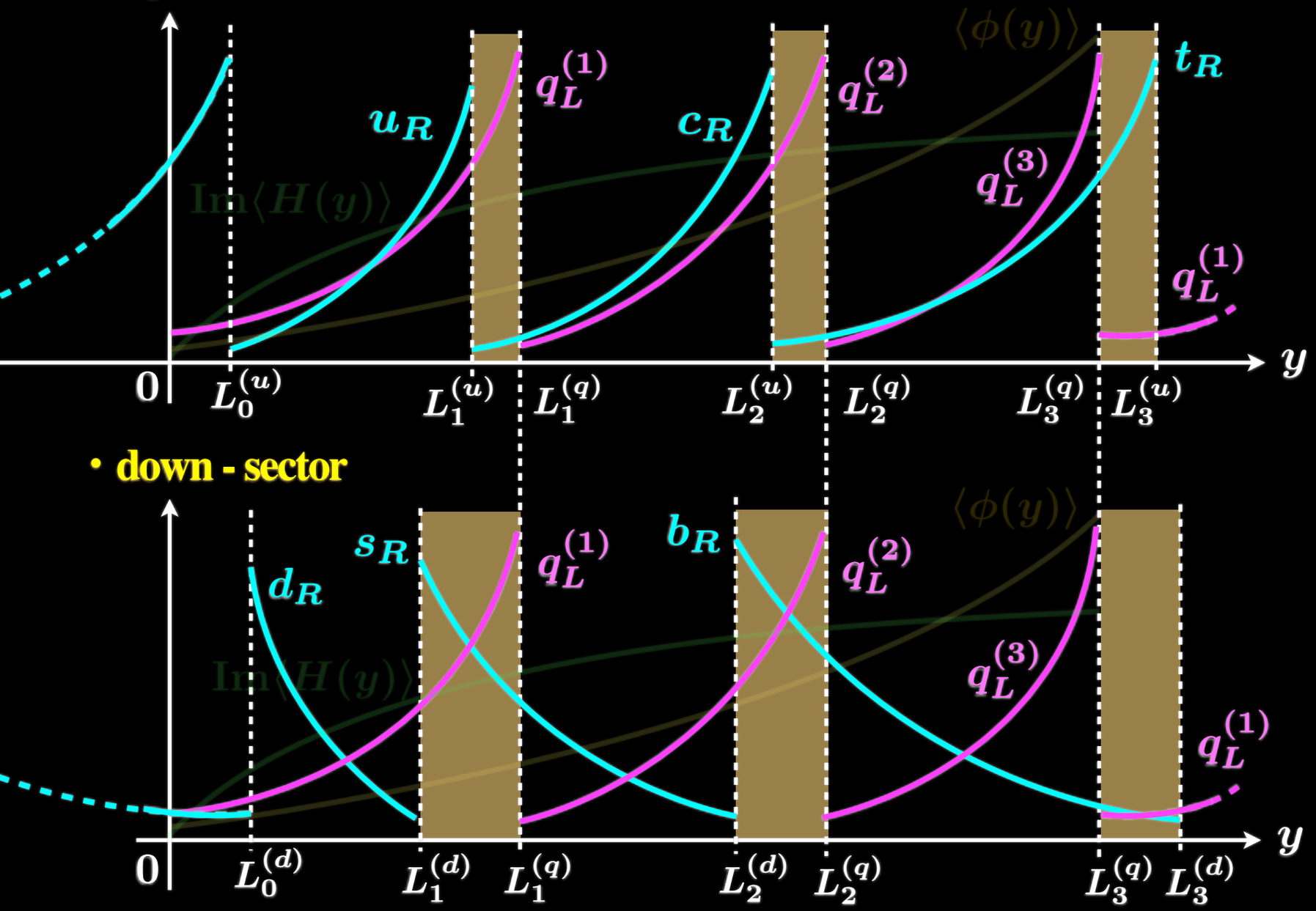

★ Three generations via point interactions

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★ Small mixing compares with lepton - sector

Quark sector

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- **We found a parameter set which reproduces experimental values within 15%**

$$\frac{m_{\rm up}^{\rm (ours)}}{m_{\rm up}^{\rm (exp.)}} = 0.897 \qquad \frac{m_{\rm charm}^{\rm (ours)}}{m_{\rm charm}^{\rm (exp.)}} = 0.978 \qquad \frac{m_{\rm top}^{\rm (ours)}}{m_{\rm top}^{\rm (exp.)}} = 1.00$$

$$\frac{m_{\rm down}^{\rm (ours)}}{m_{\rm down}^{\rm (exp.)}} = 1.02 \qquad \frac{m_{\rm strange}^{\rm (ours)}}{m_{\rm strange}^{\rm (exp.)}} = 1.07 \qquad \frac{m_{\rm bottom}^{\rm (ours)}}{m_{\rm bottom}^{\rm (exp.)}} = 1.00$$

$$\frac{|V_{\rm CKM}^{\rm (ours)}|}{|V_{\rm CKM}^{\rm (exp.)}|} = \begin{bmatrix} 0.997 & 1.06 & 0.906 \\ 1.06 & 0.997 & 0.902 \\ 0.957 & 0.900 & 1.00 \end{bmatrix} \qquad \frac{J^{\rm (ours)}}{J^{\rm (exp.)}} = 0.865$$

Lepton sector

Lepton sector

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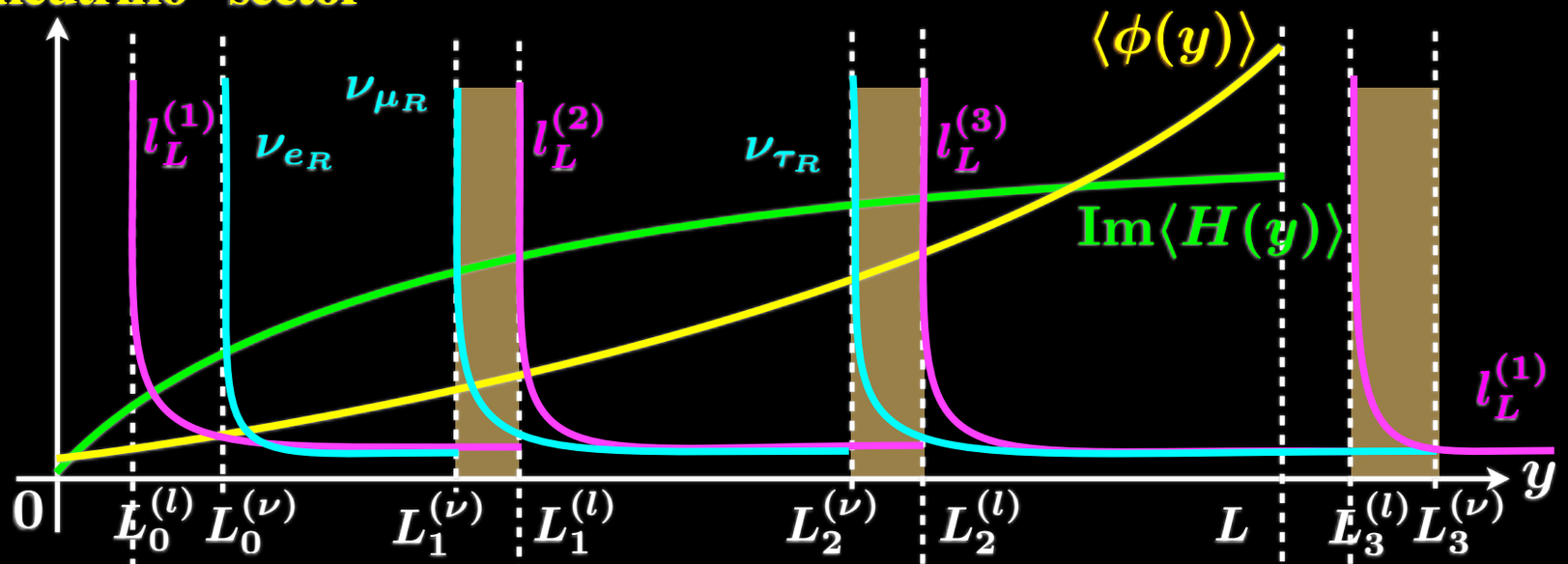
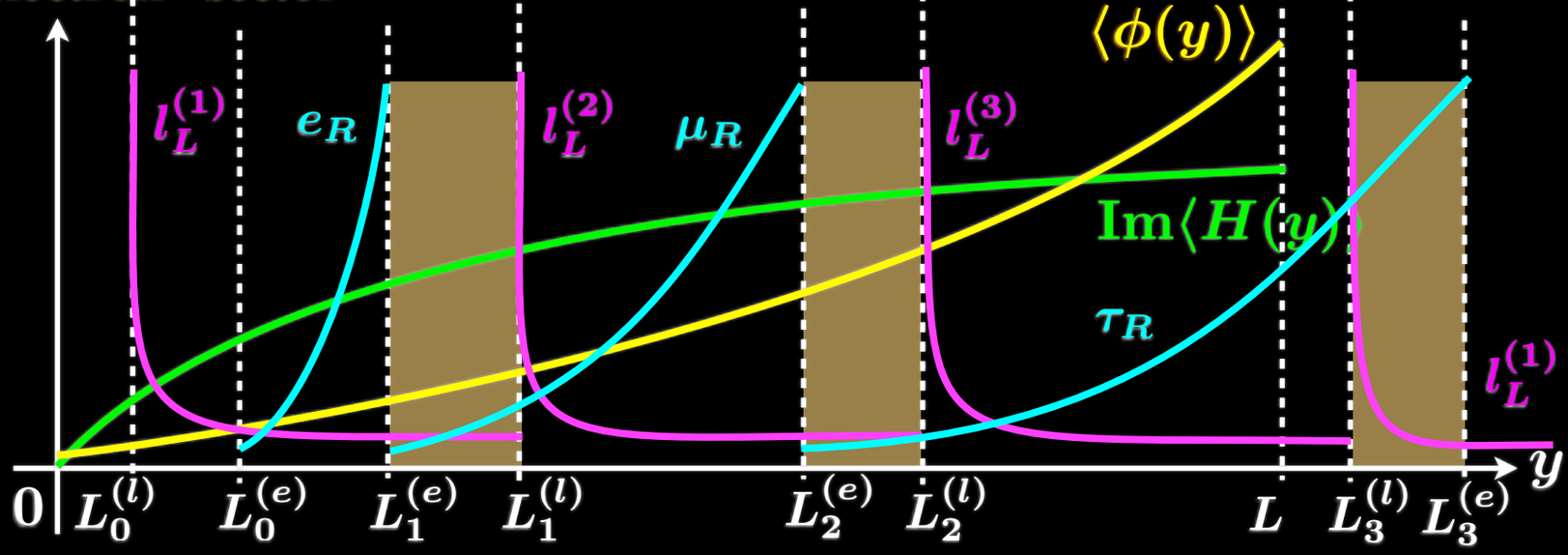
• neutrino - sector



• electron - sector



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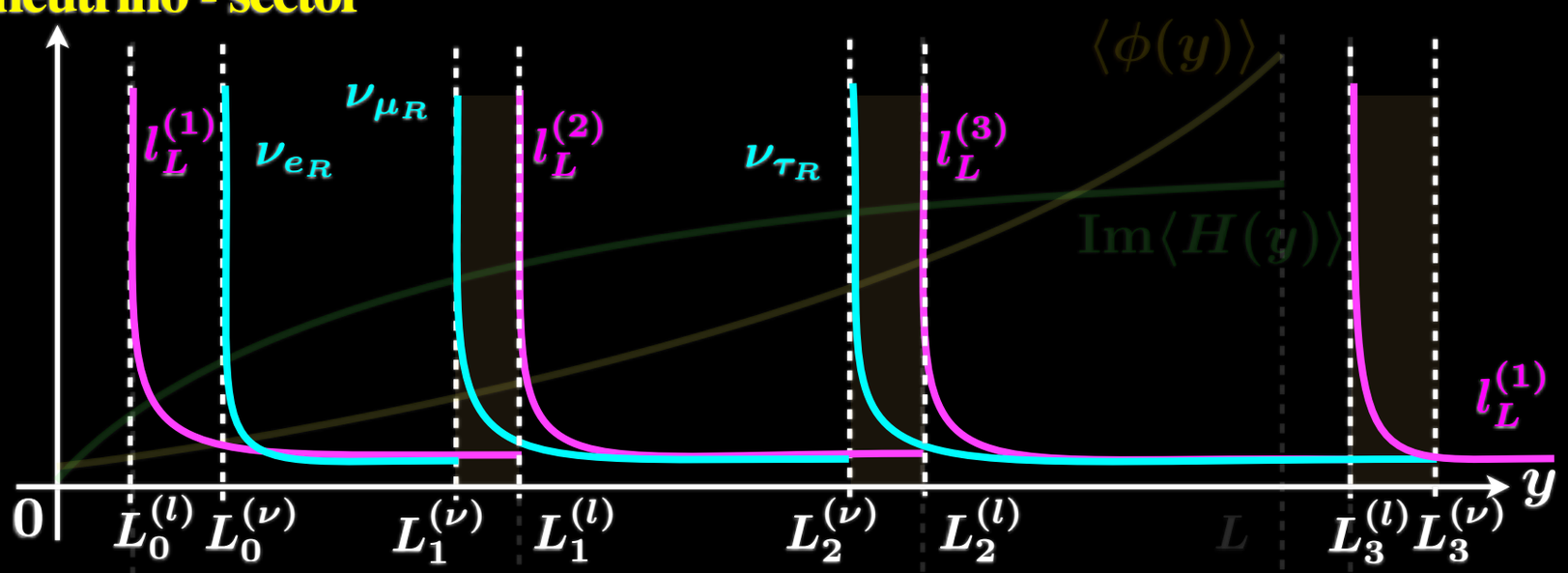
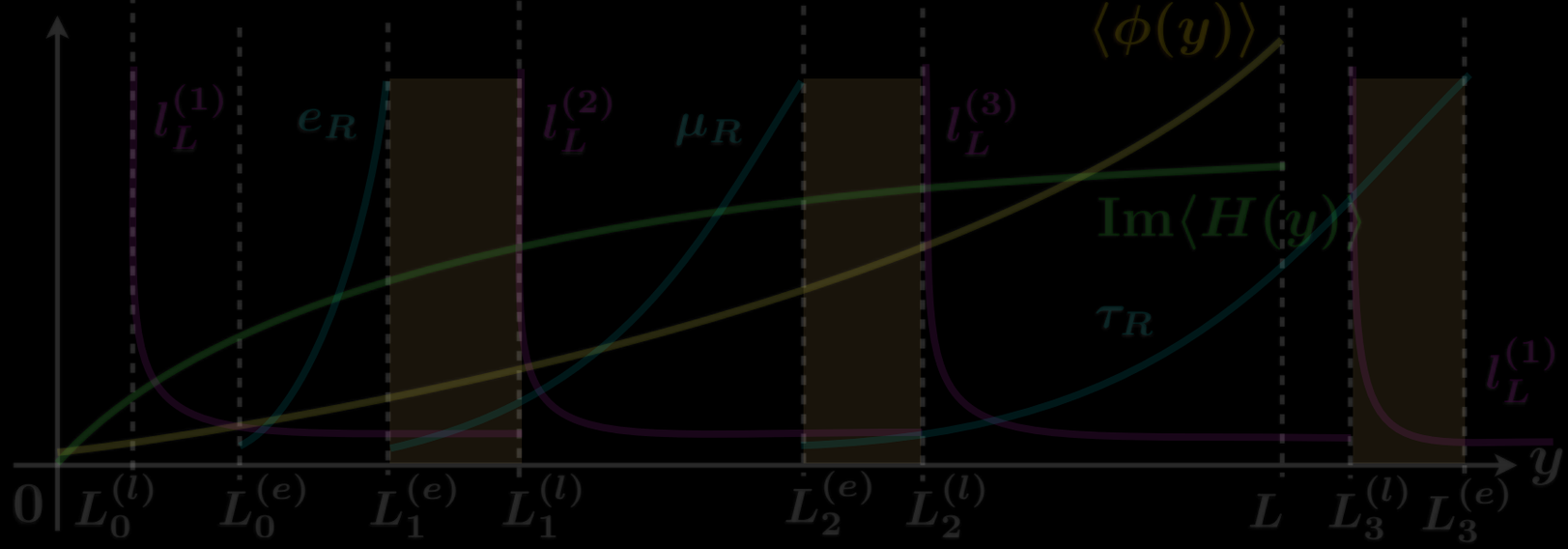


★ Large bulk masses to produce tiny neutrino masses

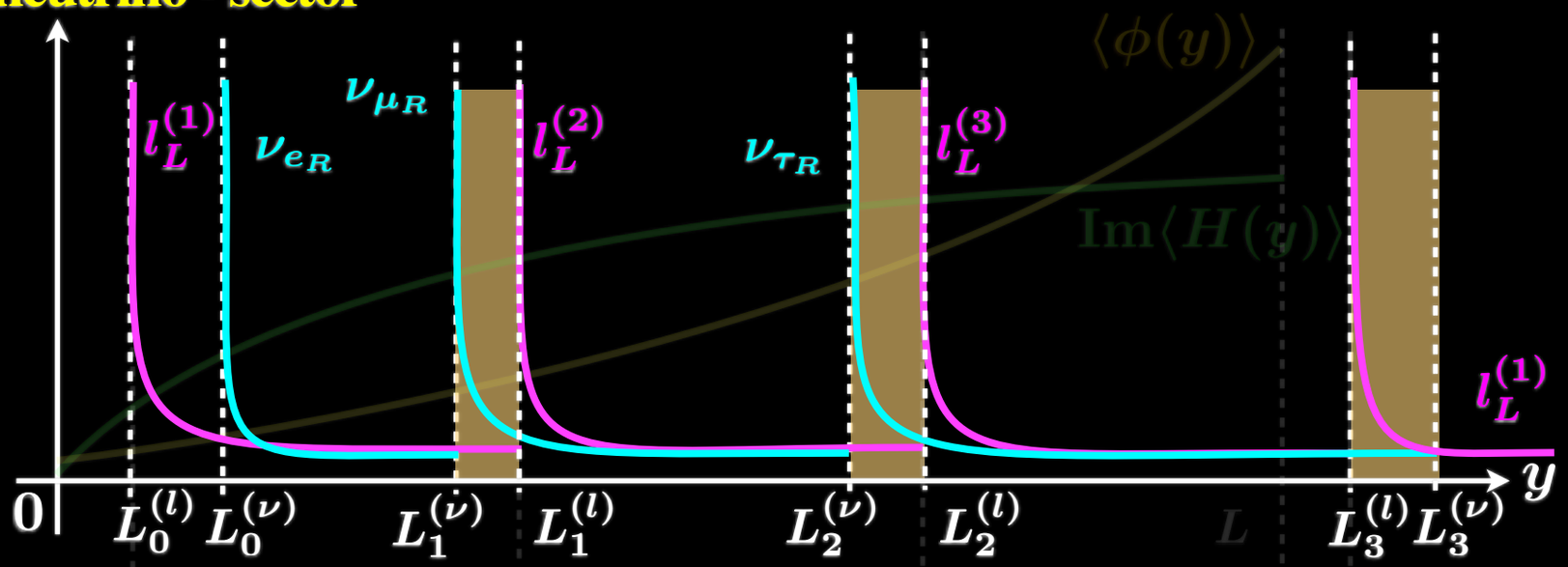
• electron - sector



Lepton sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

• neutrino - sector



★ Large bulk masses to produce tiny neutrino masses

★ Large mixing from large bulk masses

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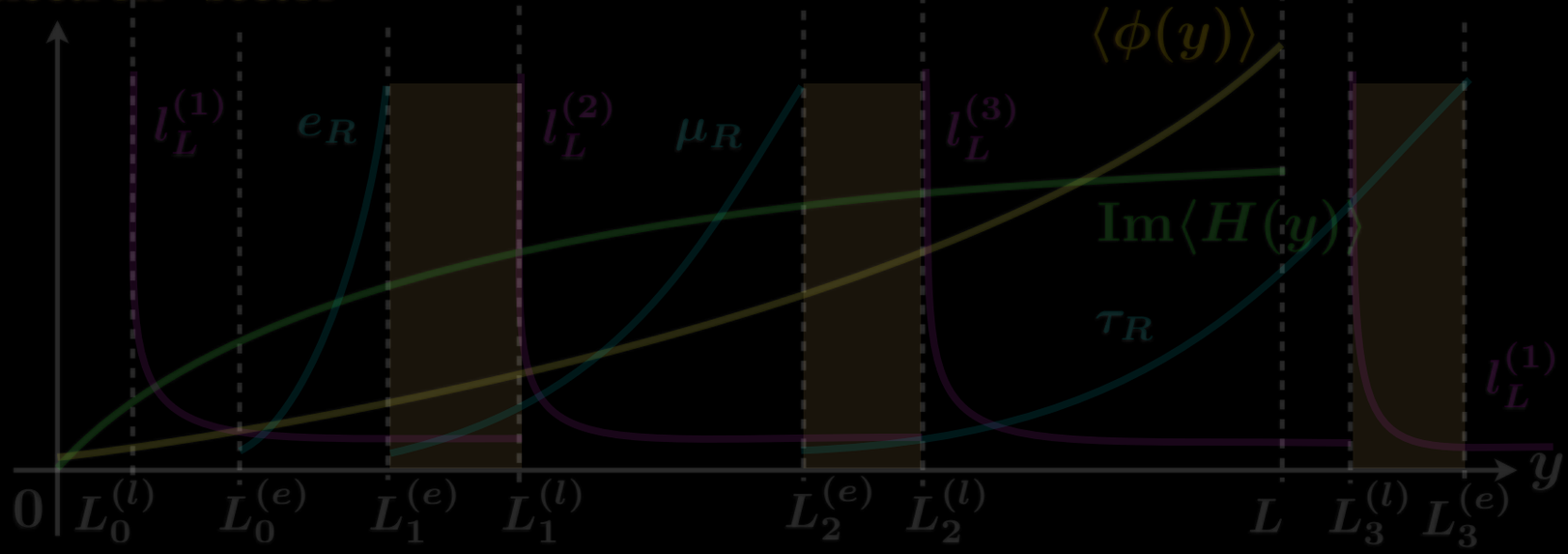

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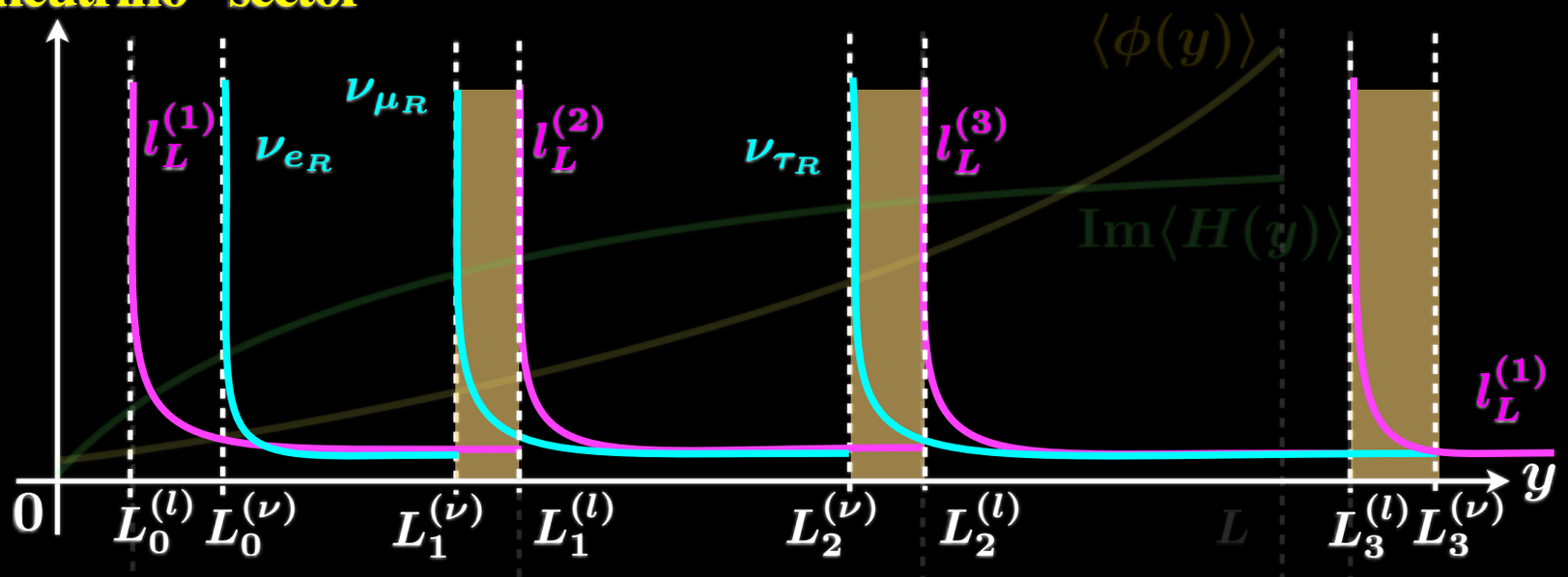

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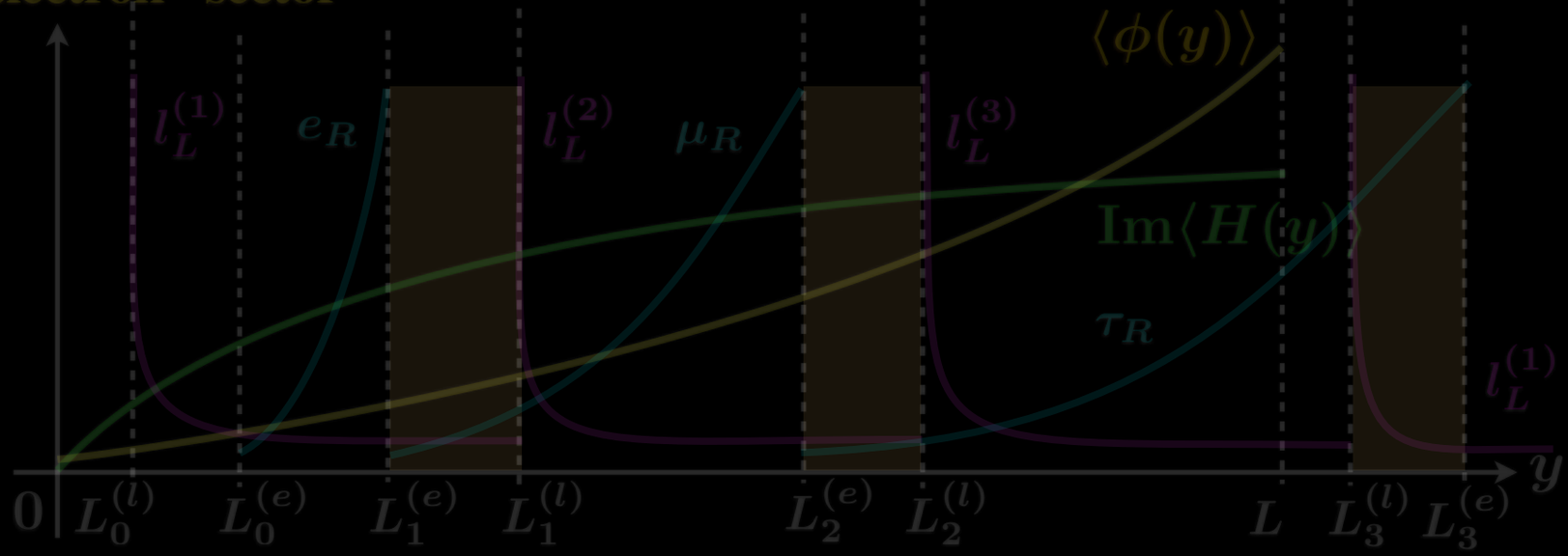

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★ Diagonal components might be compatible with off-diagonal one

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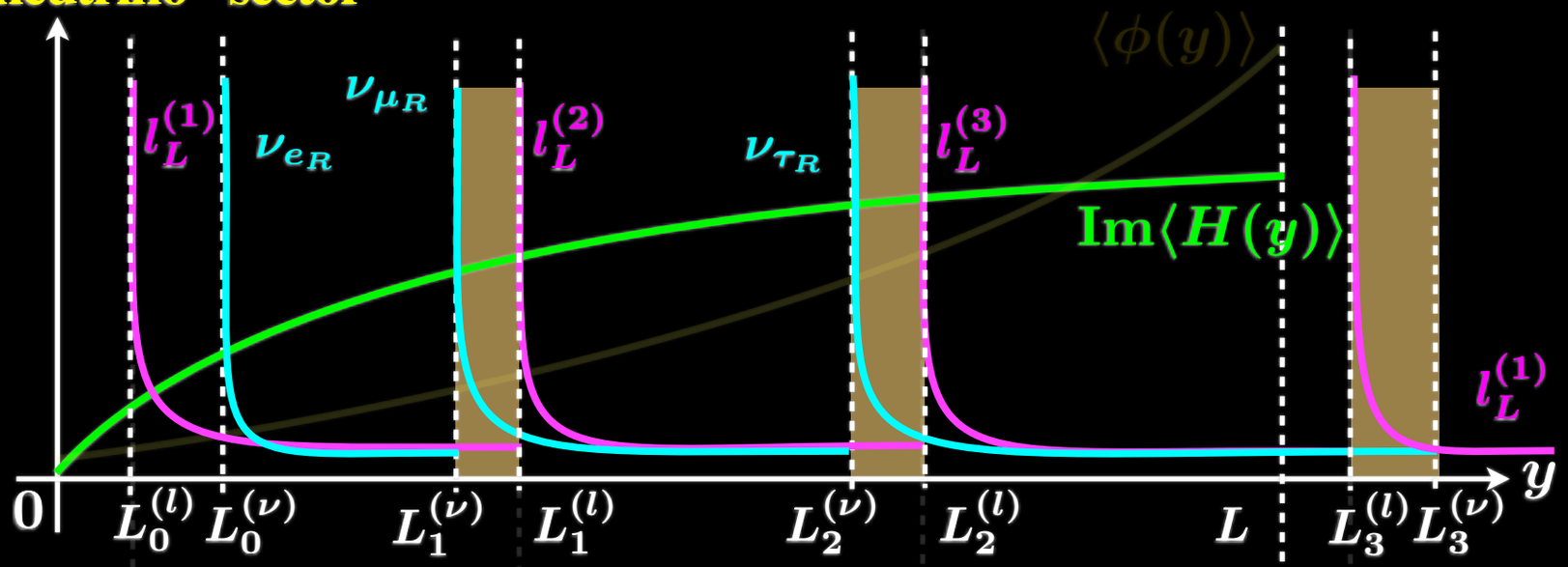
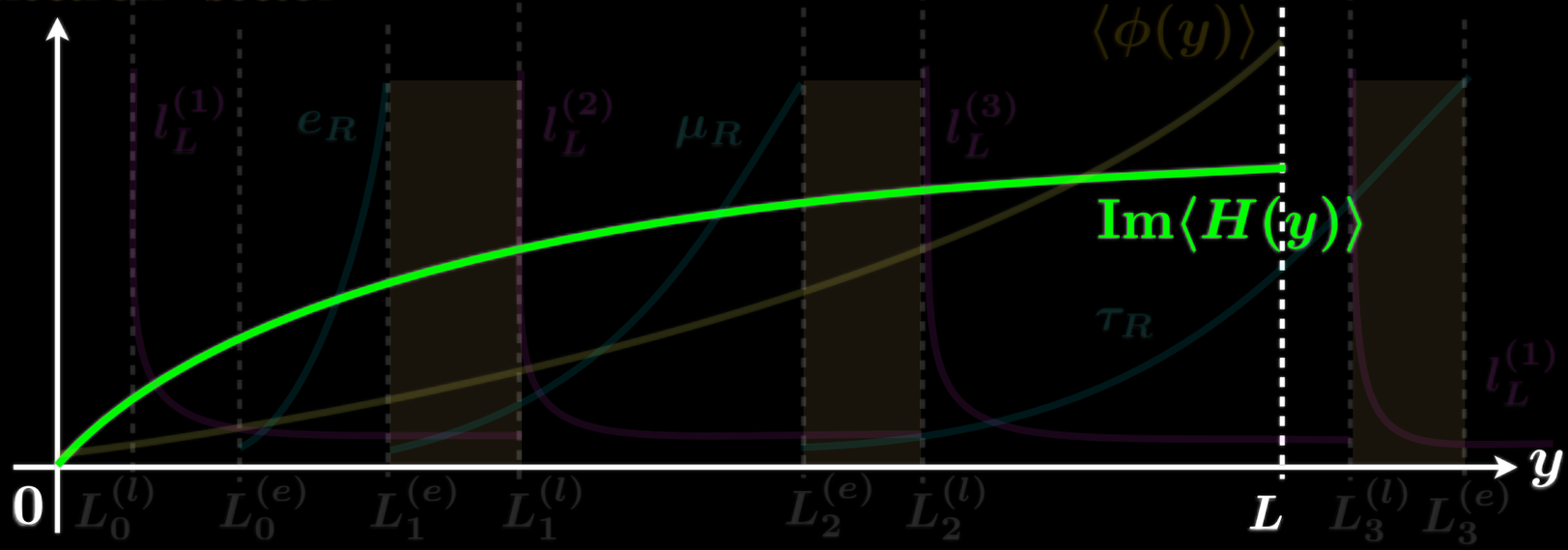


• electron - sector



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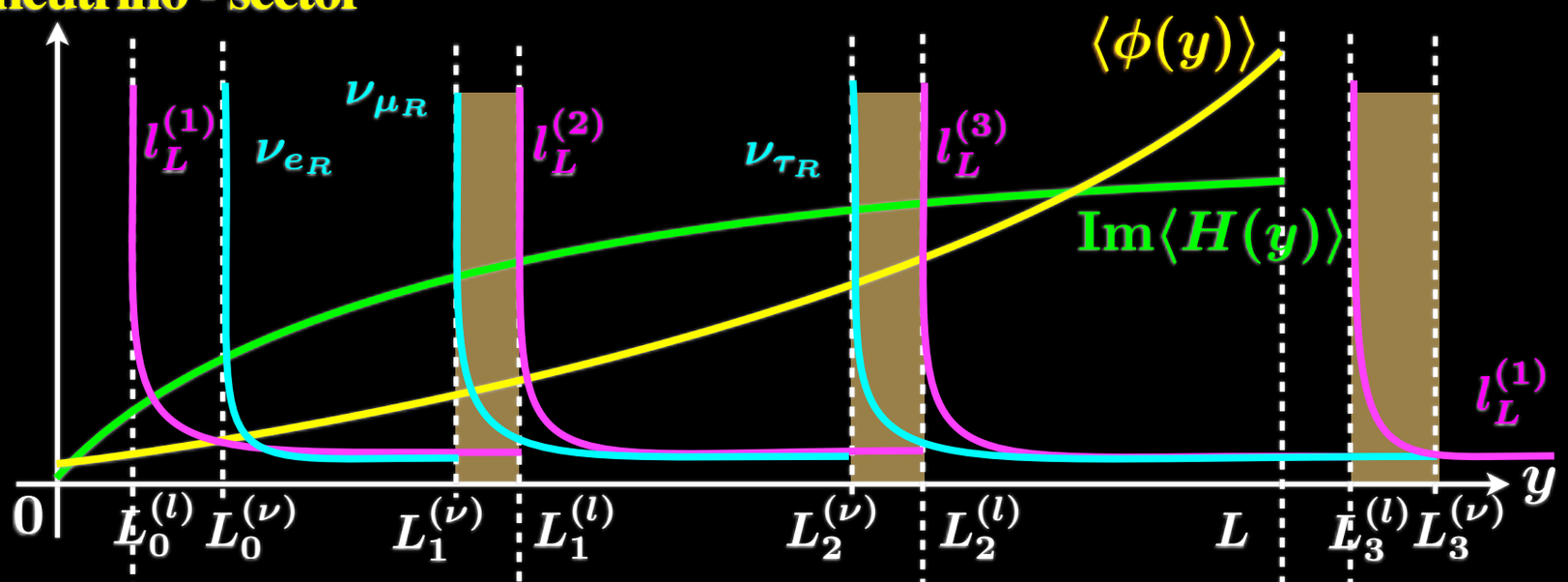

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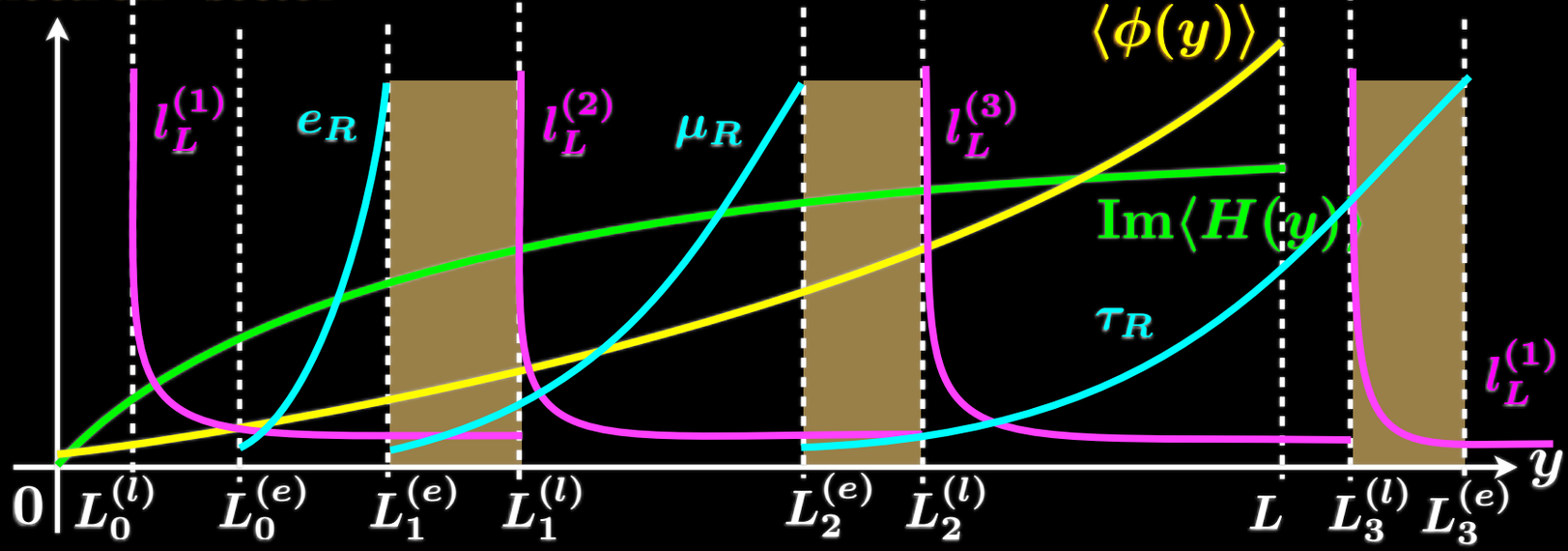

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Summary and Discussion

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5d gauge theories on a circle

with point interactions

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↓ **The low energy effective theory**

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The low energy effective theory

4d gauge theories

Summary and Discussion

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The low energy effective theory

4d gauge theories

- **Generations**
- **Mass hierarchy**
- **CP phase**
- **Large mixing**

Challenges for the future

- ♣ **Reproduce the Lepton Sector**
- ♣ **Point interactions with warped metric**
- ♣ **Radion stability with point interactions**
- ♣ **FCNC phenomenology**

⋮