Matryoshka Skyrmions, Confined Instantons and (P,Q) Torus Knots

20th August 2013
Field theory and
String theory @ YITP

Keio University
1858
CALAMVS
GLADIO
FORTIOR

Topological
Quantum
Phenomena

Muneto Nitta (Keio Univ.)

①[1] Phys.Rev.D86 (2012) 125004 [arXiv:1207.6958]

[2] Phys.Rev.D87 (2013) 025013 [arXiv:1210.2233]

[3] Nucl.Phys.B872 (2013) 62–71[arXiv:1211.4916]

[4] Phys.Rev.D87 (2013) 066008 [arXiv:1301.3268]

② With **Michikazu Kobayashi** (Kyoto Univ)

[5] Phys.Rev.D87 (2013) 085003 [arXiv:1302.0989]

[6] arXiv:1304.4737 [7] arXiv:1304.6021

[8] Nucl.Phys.B [arXiv:1305.7417]

[9] Phys.Rev. D87 (2013) 125013 [arXiv:1307.0242]

(toward) Unified Understanding of Topological Solitons and Instantons

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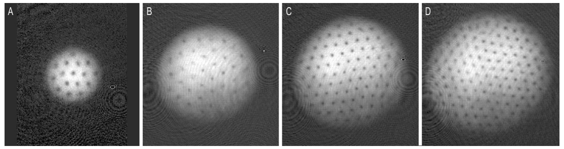
Topological Solitons and Instantons

Field theory & String theory

- non-perturbative effects in (SUSY) field theories
  - **instanton** counting in *d*=3+1 -> SW preporential
  - **vortex** counting in *d*=1+1 -> twisted superpotential
- D-branes (brane within brane, brane ending on brane)
- QCD

Vortices and Other Topological Solitons in Dense Quark Matter
Eto,Hirono, MN &Yasui, arXiv:1308.1535 [hep-ph],PTEP invited162 p

Condensed matter physics



- **vortices** in superconductors, superfluids
- ultracold atomic BEC, BEC/BCS crossover
- quantum turbulence, - phase ordering in non-equibrium
- topological supercond, topological quantum computation
- **skyrmions** in magnets - BKT transition

Cosmology

- monopole/domain wall problem
- cosmic strings

Nuclear&Astrophysics

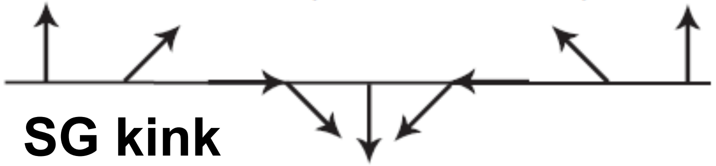
- neutron stars: neutron superfluid, proton supercond

Topological Solitons and Instantons

model \ dim	d=1+1	d=2+1	d=3+1	d=4+1
Textures NLSM	1D Skyrmion (SG kink)	2D Skyrmion (lump)	3D Skyrmion	4D Skyrmion
Defects Gauge theory	Domain wall	Vortex	Monopole	YM instanton

$$d = D + 1 = 1 + 1$$

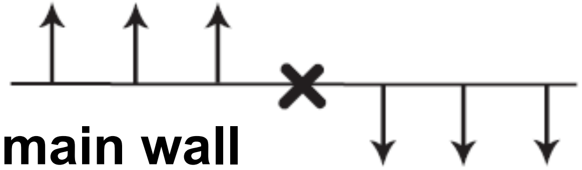
Textures (soft core)



Order parameter is defined everywhere

$$\pi_D\left(M\right) \quad \mathbf{R}^D + \{\infty\} = S^D \longrightarrow M$$

Defects (hard core)



Order parameter is not defined at the core

$$\pi_{D-1}\left(M\right) \quad \partial\mathbf{R}^D = S^{D-1} \longrightarrow M$$

Topological Solitons and Instantons

model ╲ dim	d=1+1	d=2+1	d=3+1	d=4+1
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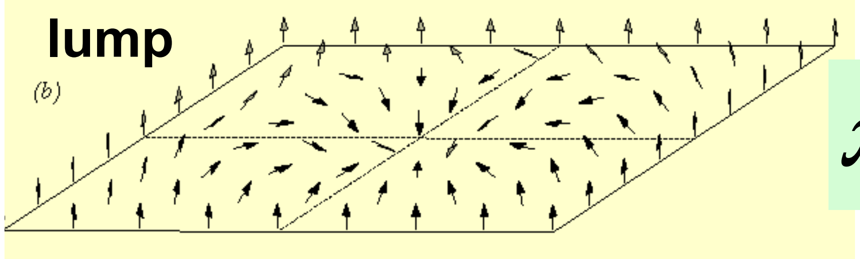
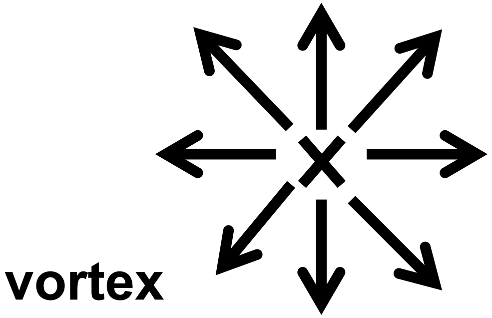
Defects Gauge theory	Domain wall	Vortex	Monopole	YM instanton

$$d = D + 1 = 2 + 1$$



Order parameter is defined everywhere

$$\pi_D\left(M\right) \quad \mathbf{R}^D + \{\infty\} = S^D \to M$$



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Topological Solitons and Instantons

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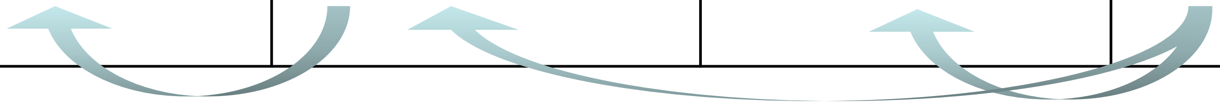


What are relations among them?

Dimensional reduction (old idea)

- YM instantons -> carolons -> BPS monopoles

Harrington-Shepard ('78) etc

- vortices -> domain walls Eto-Isozumi-MN-Ohashi-Sakai('04)
- YM instantons on $\mathbf{H}^2 \times S^2,\ \mathbf{R}^2 \times S^2,\ \ \Sigma \times S^2$ etc

  -> vortices on $\mathbf{R}^2,\ \mathbf{H}^2,\ \Sigma$ Witten('77), Forgacs-Manton('80)

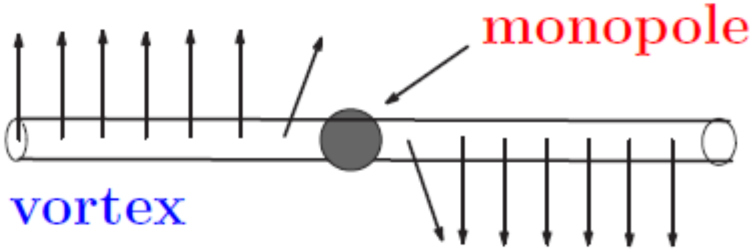
Topological Solitons and Instantons

model \ dim	d=1+1	d=2+1	d=3+1	d=4+1
Textures **NLSM**	**1D Skyrmion** **(SG kink)**	**2D Skyrmion** **(lump)**	**3D Skyrmion**	**4D Skyrmion**
Defects **Gauge theory**	**Domain wall** ←	**Vortex**	**Monopole**	**YM instanton**

What are relations among them? ←-- **vortex**

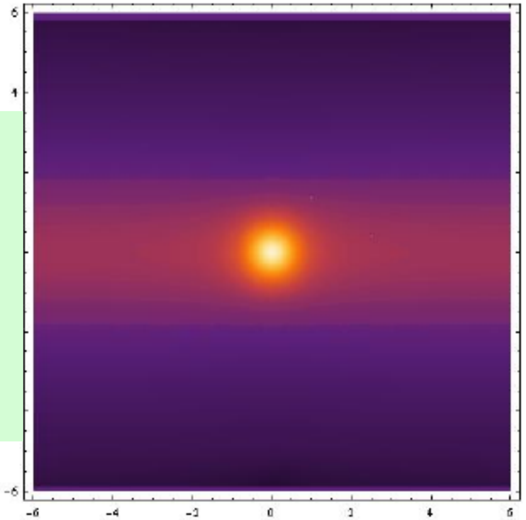
Brane within brane (these ten years)

Confined monopole



domain wall (kink)
on a vortex
= monopole
in d=3+1 bulk

Tong('03), Hanany-Tong, Shifman-Yung ('04)



Numerical solution by Fujimori

Topological Solitons and Instantons

model \ dim	*d*=1+1	*d*=2+1	*d*=3+1	*d*=4+1
Textures **NLSM**	**1D Skyrmion (SG kink)**	**2D Skyrmion (lump)**	**3D Skyrmion**	**4D Skyrmion**
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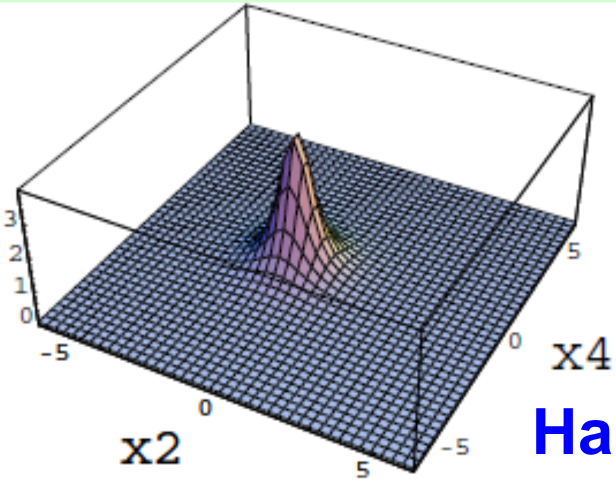
What are relations among them?

<--- **vortex**

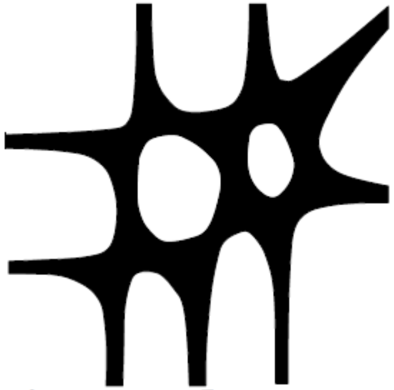
Instanton inside a vortex



lump (2D Skyrmion) on a vortex = instanton-particle in *d*=4+1 bulk

Hanay-Tong, Eto-Isozumi-MN-Ohashi-Sakai('04)

Amoeba



Fujimori-MN-Ohta-Sakai-Yamazaki('08)

Topological Solitons and Instantons

model ╲ dim	d=1+1	d=2+1	d=3+1	d=4+1
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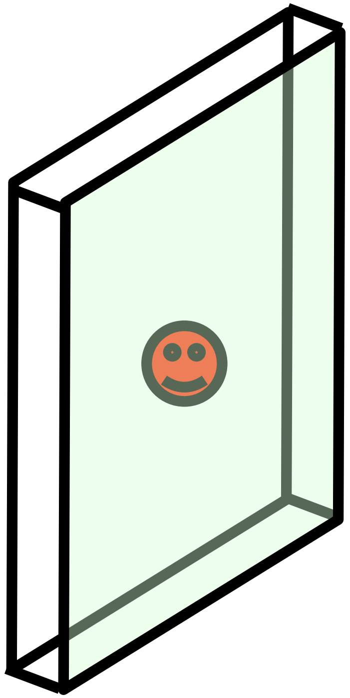
What are relations among them?

<-- wall



Domain wall Skyrmion

(3D) Skyrmion
on a domain wall
= instanton-particle
in d=4+1 bulk

Physical realization for Atiyah-Manton construction of Skyrmion from instanton holonomy

Eto,MN,Ohashi&Tong, PRL('05)

Topological Solitons and Instantons

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What are relations among them?

Other examples?

Today's topic

Topological Solitons and Instantons

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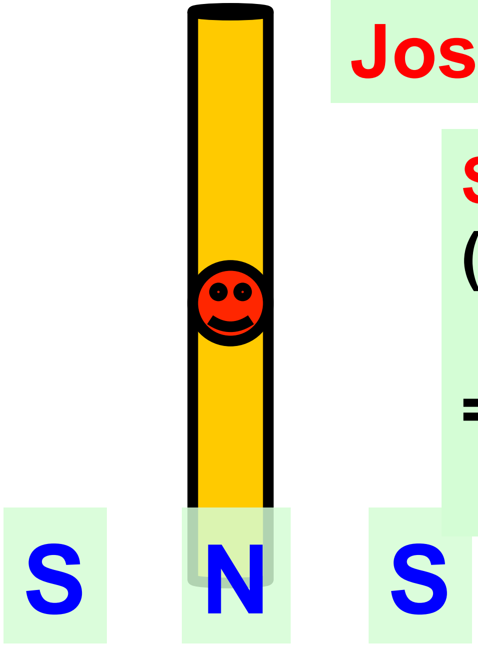
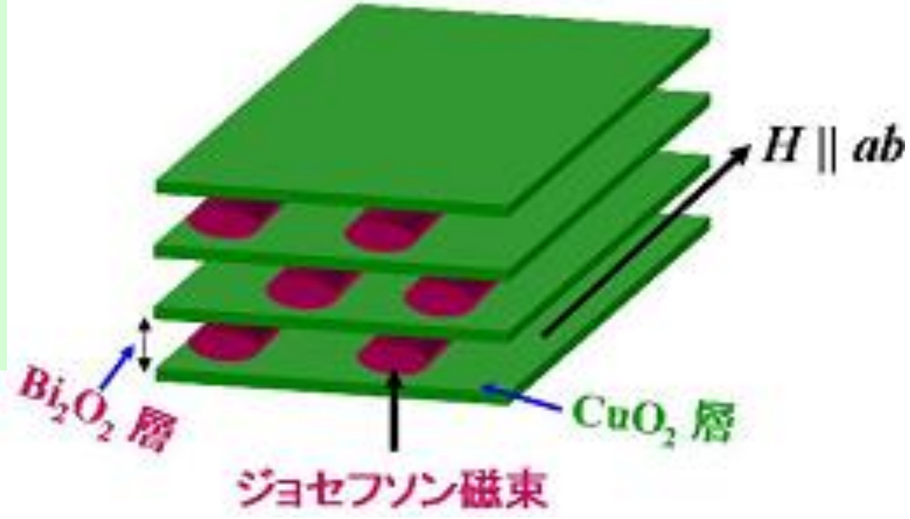
What are relations among them? <-- wall



Josephson vortex

Sine-Gordon kink
(1D Skyrmion)
on a domain wall
= vortex
in d=2+1 bulk

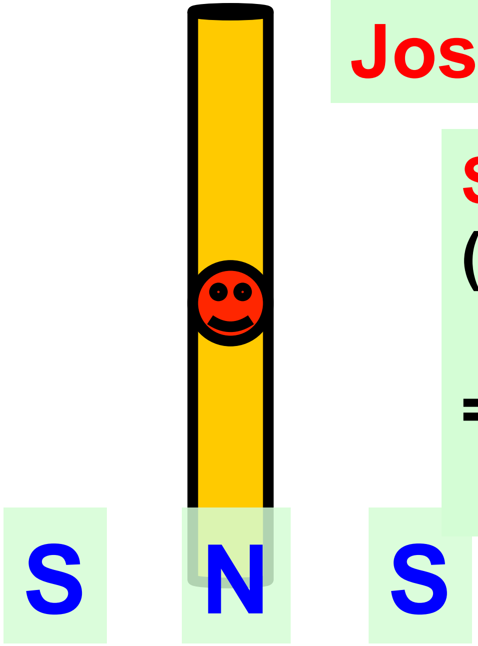
[1] Phys.Rev.D86 (2012) 125004 [arXiv:1207.6958]



Topological Solitons and Instantons

model \ dim	d=1+1	d=2+1	d=3+1	d=4+1
Textures NLSM	1D Skyrmion ← (SG kink)	2D Skyrmion (lump)	3D Skyrmion	4D Skyrmion
Defects Gauge theory	Domain wall	Vortex	Monopole	YM instanton

What are relations among them? ← wall



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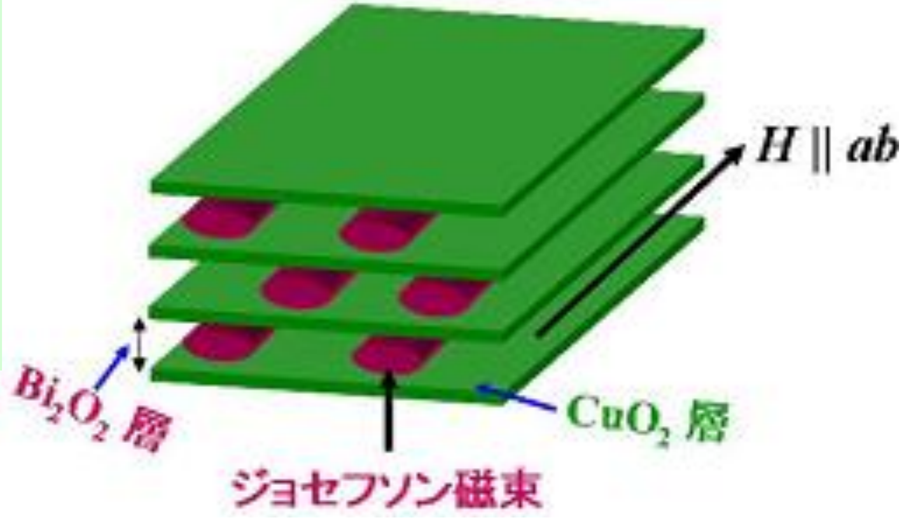

Topological Solitons and Instantons

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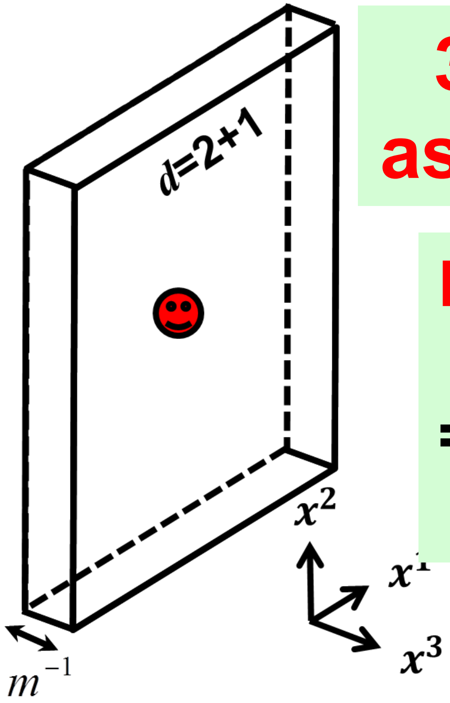
What are relations among them? ← wall



3D Skyrmion
as 2D Skyrmion

[2] Phys.Rev.D87 (2013)
025013 [arXiv:1210.2233]

Lump (baby Skyrmion)
on a domain wall
= 3D Skyrmion
in d=3+1 bulk

Topological Solitons and Instantons

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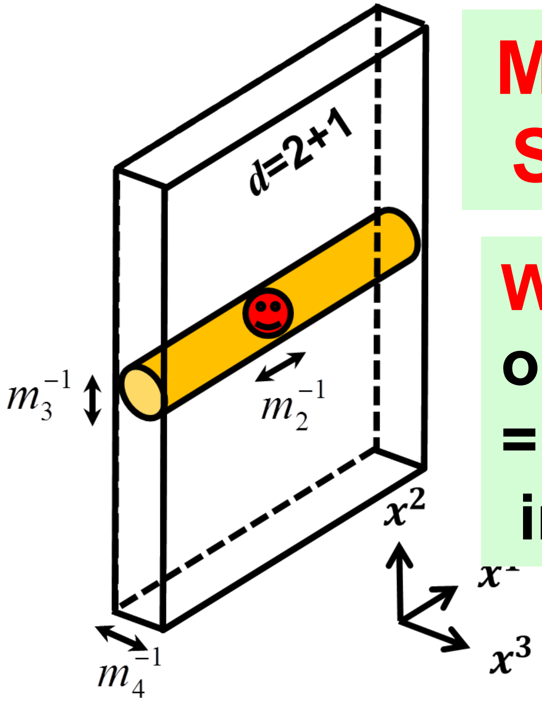
What are relations among them? ← wall



Matryoshka Skyrmions

[3] Nucl.Phys.B872 (2013) 62–71[arXiv:1211.4916]

Wall on wall
on wall …..
= N Dim Skyrmion
in d=N+1 bulk

Topological Solitons and Instantons

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What are relations among them?

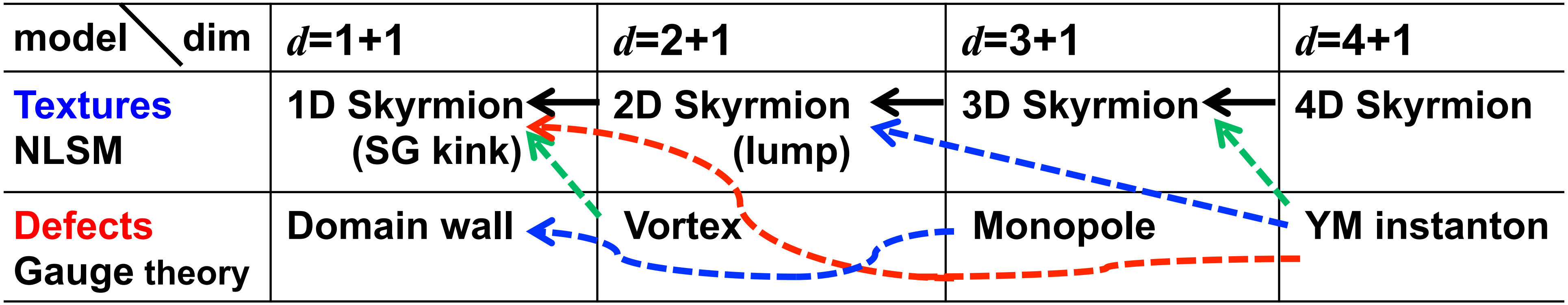
←-- **monopole**

Confined instanton

[4] Phys.Rev.D87 (2013) 066008 [arXiv:1301.3268]

Sine-Gordon kink
on a monopole-string
= instanton-particle
in *d*=4+1 bulk

Topological Solitons and Instantons



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What are relations among them?

<u>**Brane within brane**</u> (these ten years)

← **wall**

←-- **wall**

←-- **vortex**

←-- **monopole**

Summary so far

Plan of My Talk

§1 Introduction

§2 Josephson Vortices and
Matryoshyka Skyrmions

§3 Confined Instantons

§4 (P,Q) Torus Knots

§5 Summary and Discussion

Plan of My Talk

§1 Introduction

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§5 Summary and Discussion

Lower dimensional version of "Matryoshka"

U(1) gauge theory with two Higgs fields $\Phi = (\phi^1, \phi^2)$

$(\phi^{1,2}, \widetilde{\phi}^{1,2} = 0)$ **two hypermultiplets**

d=3+1, $\mathcal{N}$=2 supersymmetry

$(\Sigma, F_{\mu\nu})$ **U(1) vector multiplet**

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{e^2} (\partial_\mu \Sigma)^2 + |D_i \Phi|^2 - V$$

$$V = \frac{e^2}{2} (\Phi^\dagger \Phi - v^2)^2$$

vacua $(\phi^1, \phi^2) = (v, 0) : \ S^3$

semi-local vortex

$$(\phi^1, \phi^2) = v(f(r)e^{i\theta}, g(r))$$

$$A_\theta = \gamma(r)/r \qquad (f, g, \gamma) \rightarrow (1,0,1), \ r \rightarrow \infty$$

Lower dimensional version of "Matryoshka"

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$$V = \frac{e^2}{2} (\Phi^\dagger \Phi - v^2)^2 + \Phi^\dagger (\Sigma \mathbf{1}_2 - M)^2 \Phi$$

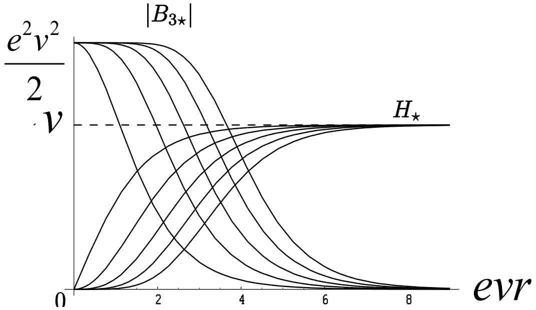
$$M = \mathrm{diag.}(m_1, m_2)$$

SUSY preserving mass deformation

vacua $(\phi^1, \phi^2; \Sigma) = (v, 0; m_1),\ (0, v; m_2)$

domain wall

semi-local vortex shrinks to Abrikosov-Nielsen-Olesen vortex



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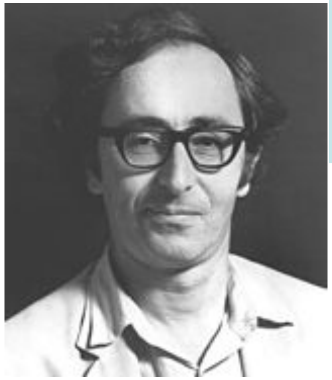
$$V = \frac{e^2}{2} (\Phi^\dagger \Phi - v^2)^2 + \Phi^\dagger (\Sigma \mathbf{1}_2 - M)^2 \Phi \; - \boxed{\beta^2 \Phi^\dagger \sigma_x \Phi}$$

$$M = \mathrm{diag.}(m_1, m_2)$$

SUSY preserving mass deformation



$$\beta^2 \Phi^\dagger \sigma_x \Phi = \beta^2 \left(\phi^{*1} \phi^2 + \phi^{*2} \phi^1 \right)$$

Josephson coupling

SUSY is broken

In the limit $e \longrightarrow \infty$ $\quad \Phi = (\phi^1, \phi^2) = (1, u)/\sqrt{1+|u|^2}$

C*P*

CP^1 model

$$D_x \equiv \frac{u+u^*}{1+|u|^2}$$

$$\mathcal{L} = \frac{\partial_\mu u^* \partial^\mu u - m^2|u|^2}{(1+|u|^2)^2} + \beta^2 D_x$$

SUSY preserving mass deformation

Josephson deformation

$$\mathbf{n} = \Phi^\dagger \sigma \Phi$$

$$\mathbf{n}^2 = 1$$

O(3) NLSM

$$\mathcal{L} = \frac{1}{2}\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n} - m^2(1-n_z^2) + \beta^2 n_x$$

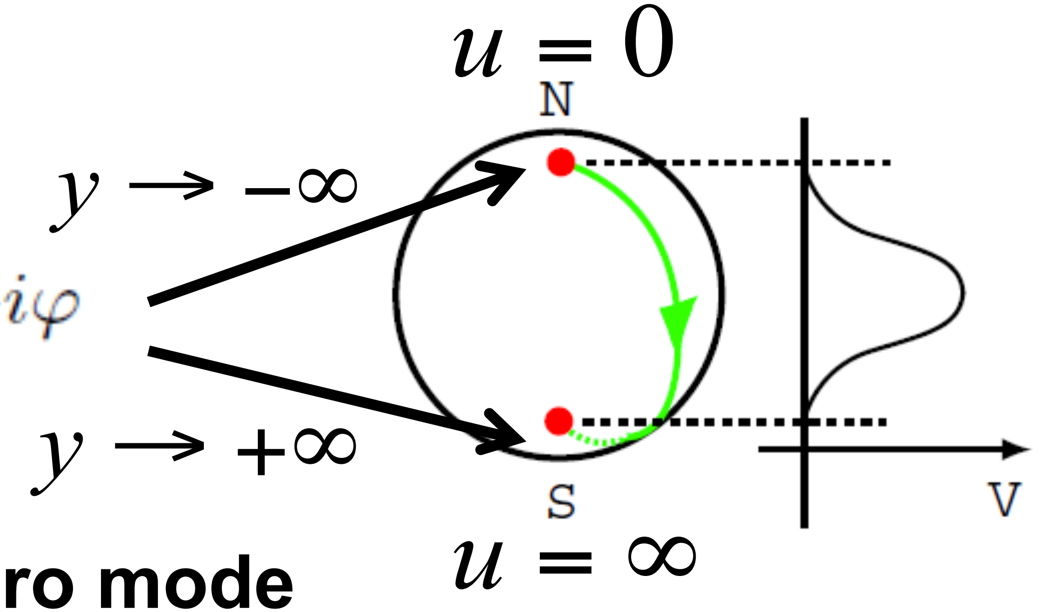
Anisotropic ferromagnets with 2 easy axes

$\beta = 0$ No Josephson

Abraham-Townsend('92)
Arai-Naganuma-MN-Sakai('02)

Domain wall solution

$$u_{dw} = e^{m(y-Y)+i\varphi}$$



Y : translational zero mode

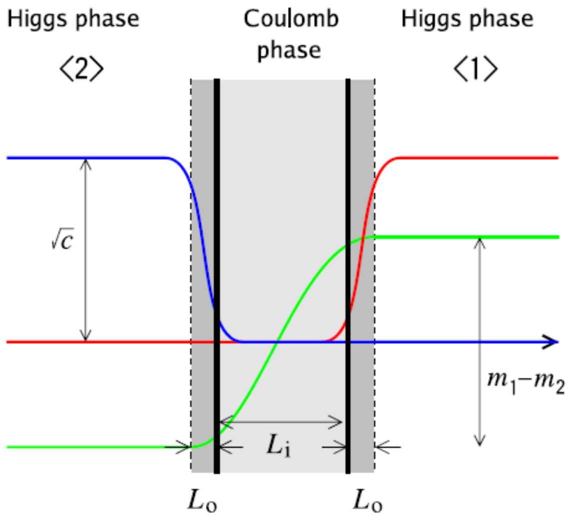
φ : internal U(1) zero mode

Effective theory on a domain wall

$$\mathcal{L}_{\rm dw.eff.} = \int_{-\infty}^{+\infty} dy \frac{e^{2my}}{(1+e^{2my})^2}[(\partial_i Y)^2 + (\partial_i \varphi)^2]$$

$$= \frac{1}{2m}[(\partial_i Y)^2 + (\partial_i \varphi)^2]$$

Arai-Naganuma-MN-Sakai('02)



finite e

$\beta \neq 0 << m$ Josephson

$$\Delta\mathcal{L} = \beta^2 \int_{-\infty}^{+\infty} dy \frac{e^{my+i\varphi} + e^{my-i\varphi}}{1+e^{2my}} = \frac{\pi\beta^2}{m}\cos\varphi$$

Eff. theory on domain wall = sine-Gordon model

$$\mathcal{L}_{\rm dw.eff.} = \frac{1}{2m}[(\partial_i Y)^2 + (\partial_i\varphi)^2 + 2\pi\beta^2\cos\varphi]$$

Sine-Gordon kink

BPS equation $\partial_i\varphi \pm \tilde{\beta}\sin\frac{\varphi}{2} = 0$ $\tilde{\beta}^2 \equiv 2\pi\beta^2$

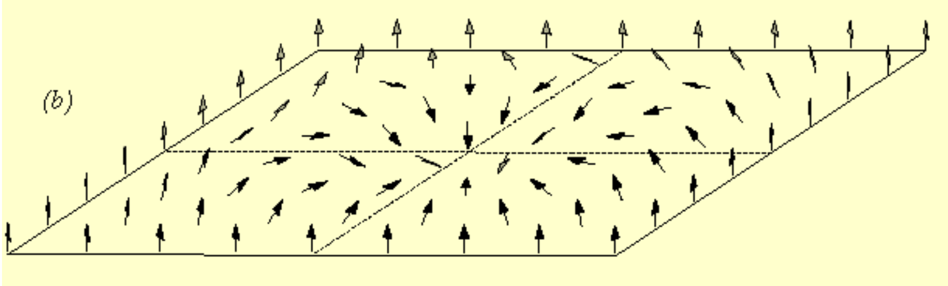
1-kink solution

$$\varphi = 4\arctan\exp\frac{\tilde{\beta}}{4}(x-X) + \frac{\pi}{2}$$

mass

$$T = \frac{4\tilde{\beta}}{m}$$

What does this SG kink look like in the bulk?



Lump charge $\pi_2(\mathbf{C}P^1) = \mathbf{Z}$

$$T_{\rm lump} = \int d^2x \frac{i(\partial_i u^* \partial_j u - \partial_j u^* \partial_i u)}{(1+|u|^2)^2}$$

$$= \oint dx^i \frac{|u|^2}{1+|u|^2}\partial_i \varphi = 2\pi k$$

for k SG-kinks

PRD86 (2012) 125004 [arXiv:1207.6958]

sine-Gordon kink in d=1+1 C*P*¹ wall w.v.
=C*P*¹ lump in d=2+1 bulk(C*P*¹ instanton in d=2+0)

Lump is semi-local vortex for *finite* e

$$T_{\rm vortex} = \int d^2x F_{12} = \oint dx^i A_i = T_{\rm lump}$$

$$A_i = \frac{i}{2}(\Phi^\dagger \partial_i \Phi - (\partial_i \Phi^\dagger)\Phi) = \frac{-i(u^* \partial_i u - (\partial_i u^*)u)}{2(1+|u|^2)}$$

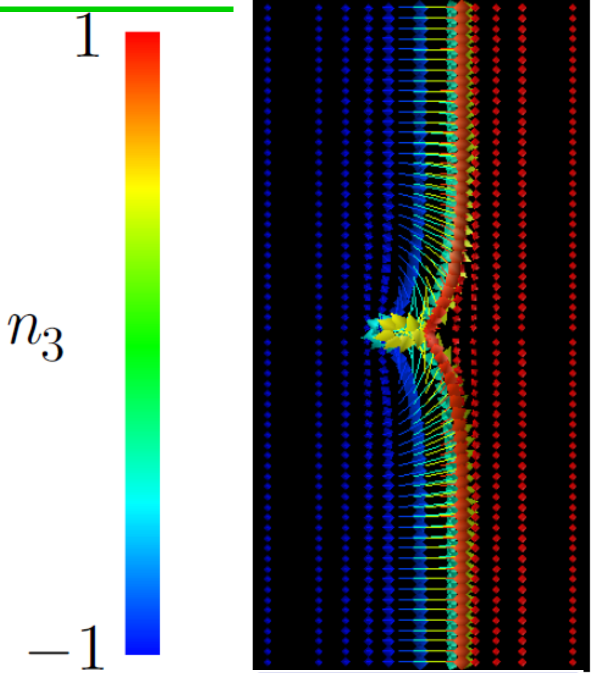
Numerical solutions

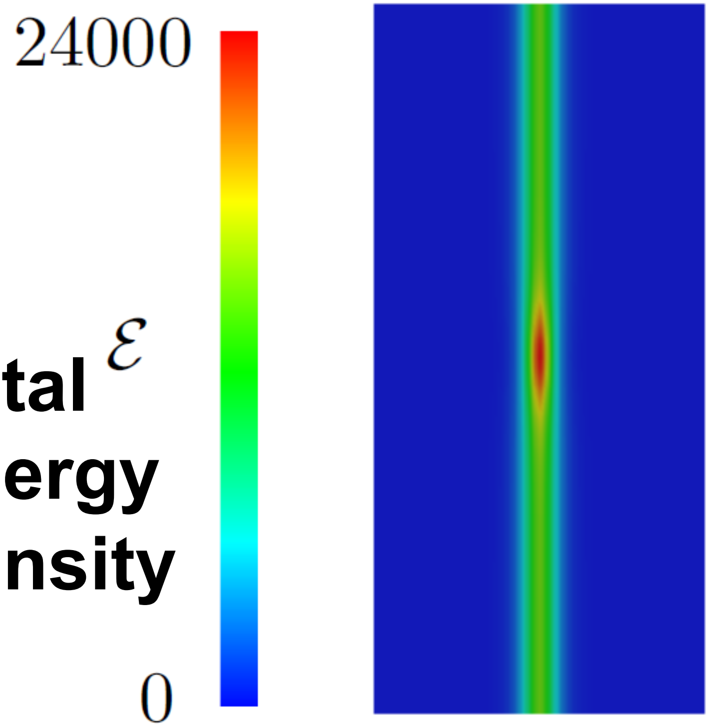
With

M.Kobayashi,

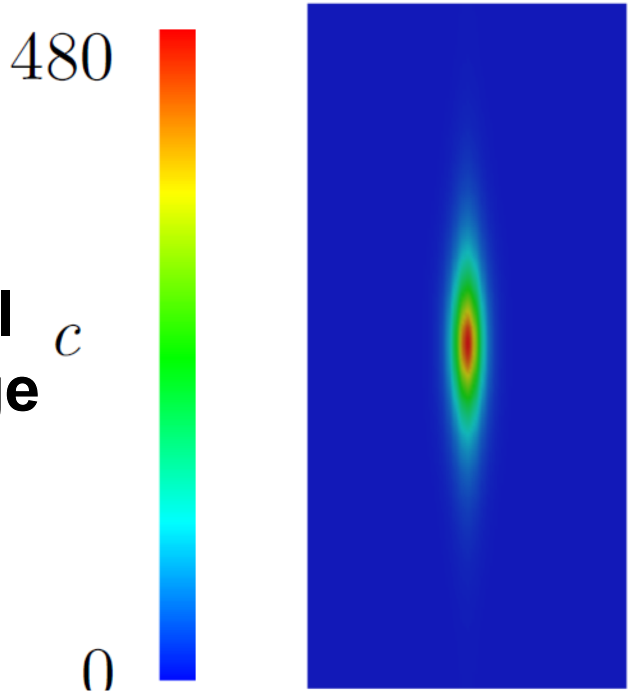
PRD87 (2013) 085003

[arXiv:1302.0989]

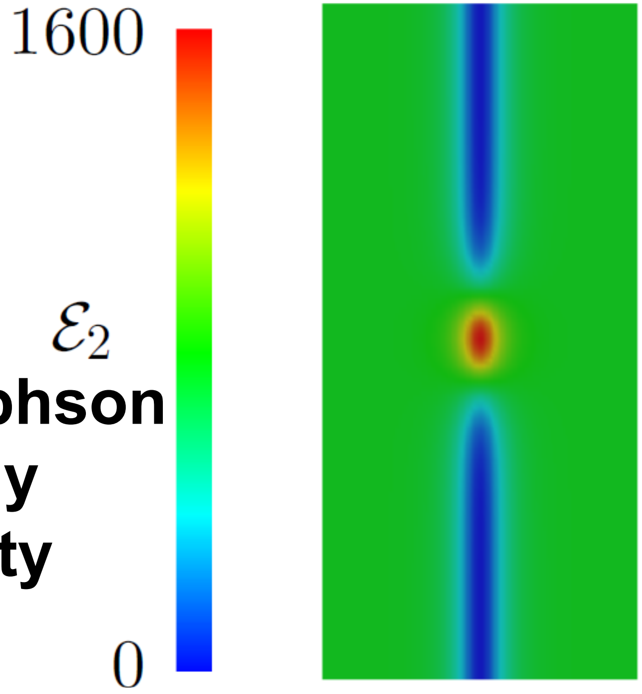




Total energy density



Topological lump charge density



Josephson energy density

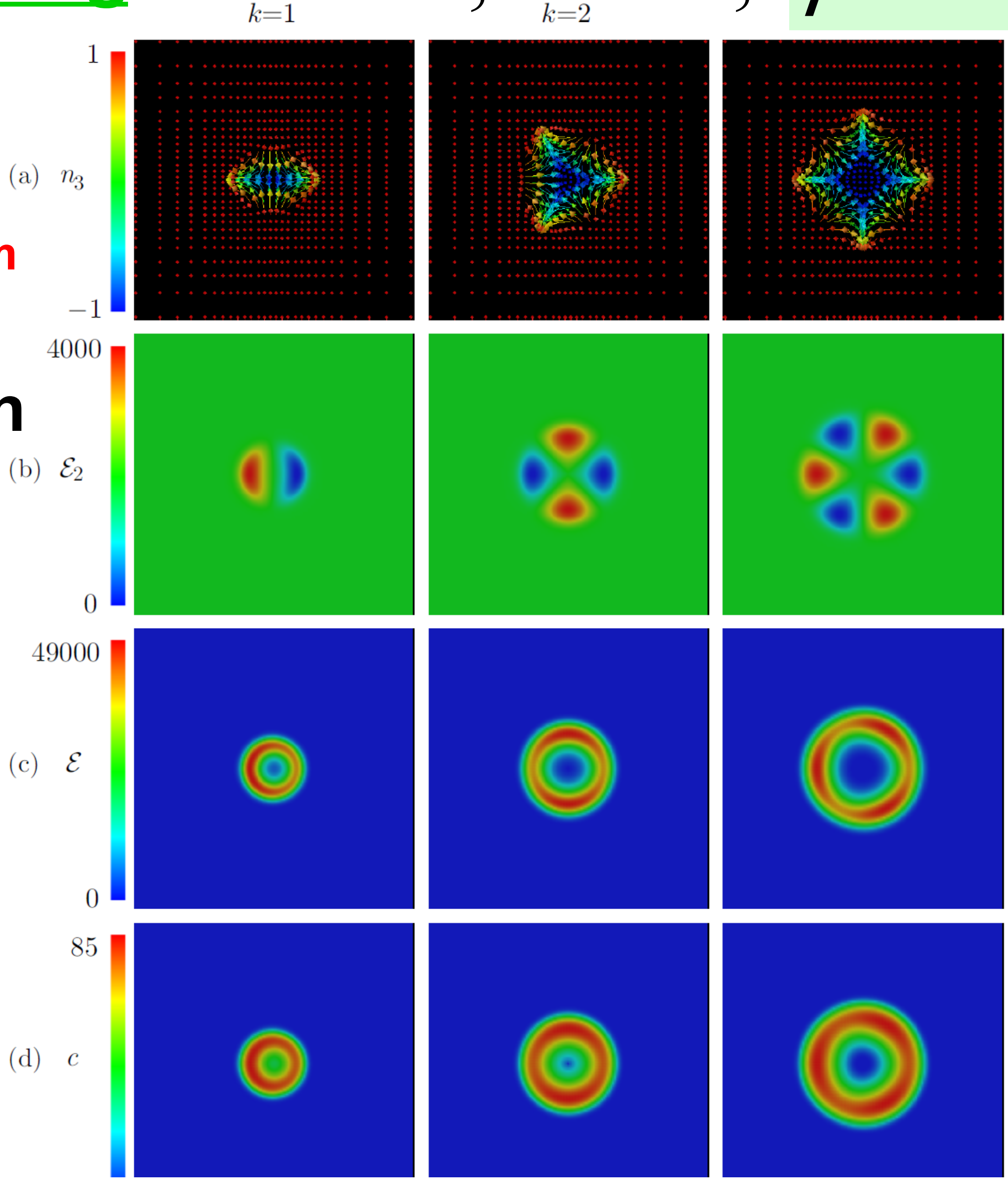
Jewels on wall ring $d=2+1,\ m\neq 0,\ \beta\neq 0$

with M.Kobayashi

PRD87 (2013) 085003

[arXiv:1302.0989]

4 derivative(Skyrme) term
(Baby Skyrme model)



Josephson energy density

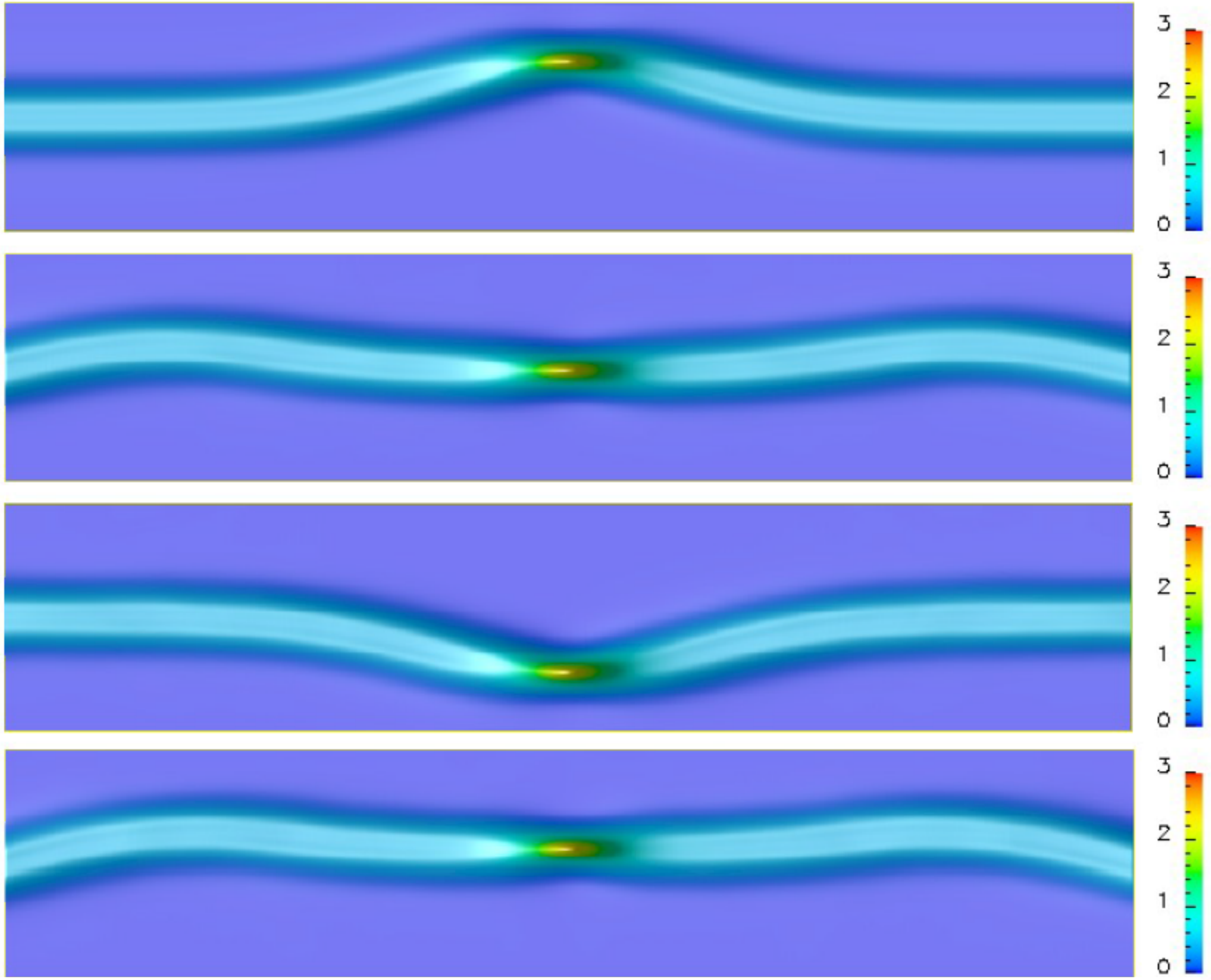
Total energy density

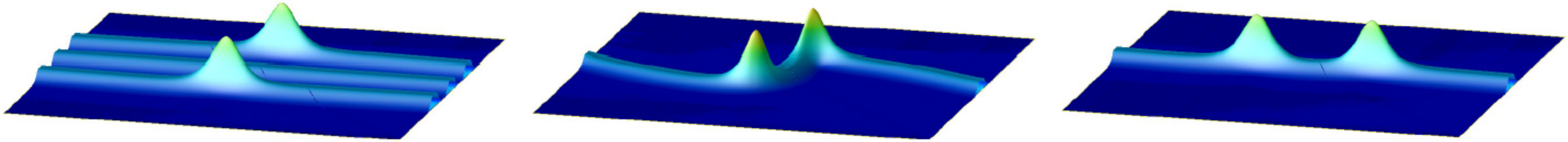
Lump charge density

The dynamics of domain wall Skyrmions

P. Jennings, P. Sutcliffe.

arXiv:1305.2869



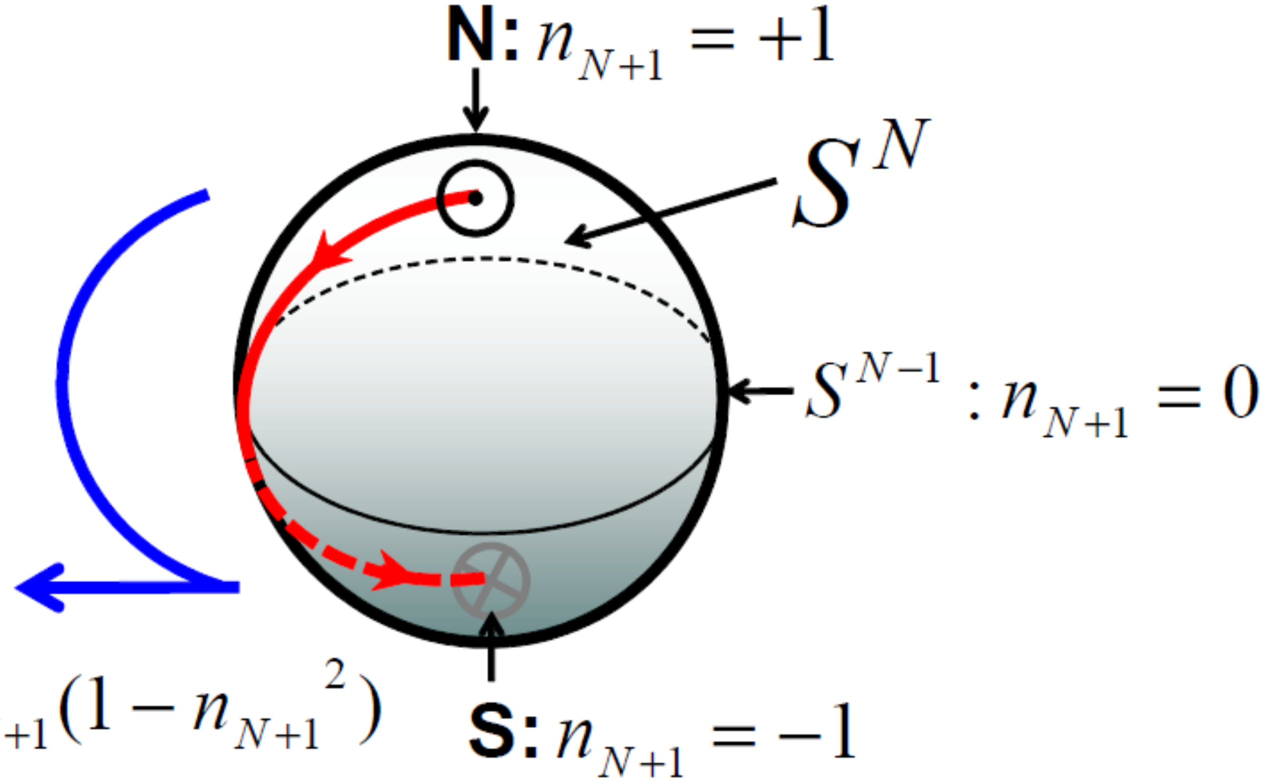


O(*N*+1) NLSM in *d*=*N*+1 dimensions

Nucl.Phys.B872 (2013) 62-71
[arXiv:1211.4916]

$$m_2 \ll m_3 \ll \cdots \ll m_{N+1}$$

$$O(N+1) \xrightarrow{m_{N+1}} O(N) \xrightarrow{m_N} \cdots \xrightarrow{m_{N-k+1}} O(N-k) \xrightarrow{m_{N-k}} \cdots \xrightarrow{m_3} O(2) \xrightarrow{m_2} \{0\}$$



$$V = m_{N+1}^2 (1 - n_{N+1}{}^2)$$

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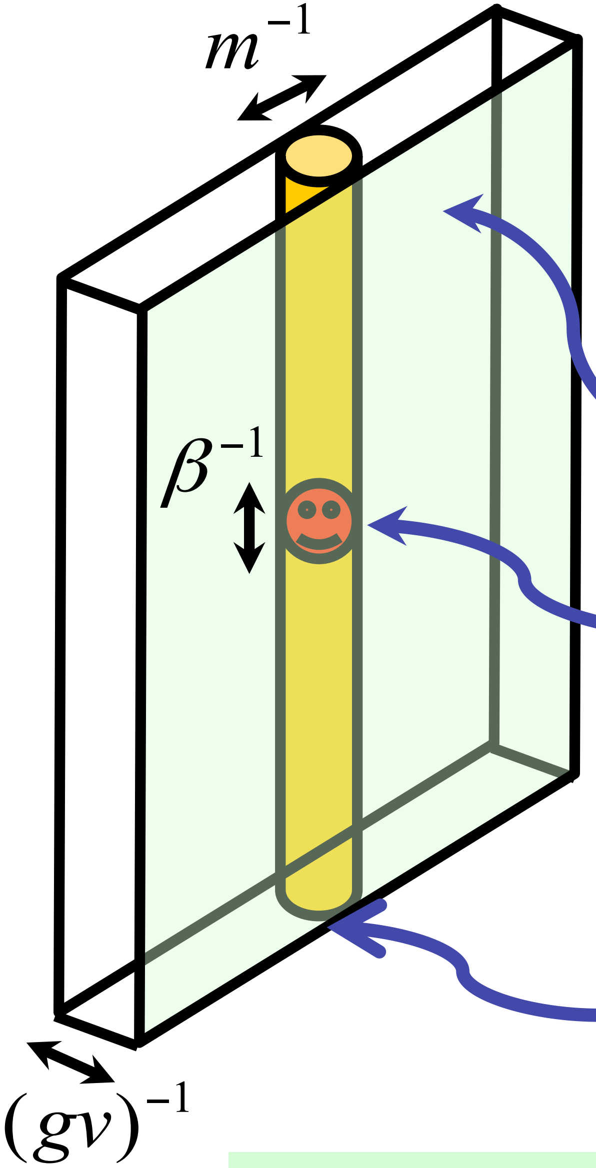
Non-Abelian

d=4+1 bulk

Phys.Rev.D87(2013)066008 [arXiv:1301.3268]



Potential

$$V = g^2 \mathrm{tr}\left(HH^\dagger - v^2 \mathbf{1}_2\right)^2 + \mathrm{tr}\left[H(\Sigma \mathbf{1}_2 - M)^2 H^\dagger\right] - \frac{\beta^2}{v^2}\mathrm{tr}\left(H\sigma_x H^\dagger\right)$$

SUSY mass deformation

NA Josephson deformation

non-Abelian vortex membrane
(d=2+1 world-volume)

Instanton-particle in d=4+1 bulk
= CP^1 lump in d=2+1 vortex w.v.
= sine-Gordon kink
in d=1+1 CP^1 wall w.v.

monopole-string in d=4+1 bulk
= CP^1 wall in d=2+1 vortex w.v.

Instantons confined by monopole strings

U(2) gauge theory with two Higgs fields in fund.rep.

$(H, \widetilde{H} = 0)$ **2 hypermultiplets in fund.rep.**

$(\Sigma, F_{\mu\nu})$ **U(2) vector multiplet** $H : 2 \times 2$

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{2g^2}\mathrm{tr}\,(D_\mu\Sigma)^2 + \mathrm{tr}\, D_i H^\dagger D_i H - V$$

$$V = g^2 \mathrm{tr}\,(HH^\dagger - v^2 \mathbf{1}_2)^2$$

$$+ \mathrm{tr}\,\left[H(\Sigma\mathbf{1}_2 - M)^2 H^\dagger\right]$$

SUSY preserving mass deformation

$$-\frac{\beta^2}{v^2}\mathrm{tr}\,(H\sigma_x H^\dagger)$$

non-Abelian Josephson term

vacuum $H = v\mathbf{1}_2\,,\ \Sigma = M = \begin{pmatrix} m & \\ & -m \end{pmatrix}$

Color-flavor locked vacuum

$U(2)_C \times SU(2)_F \rightarrow SU(2)_{C+F}$

$\beta = 0, m = 0$ No Josephson

Hanany-Tong,
Konishi et.al ('03)

Non-Abelian vortex

We can embed the ANO solution $H^{\mathrm{ANO}}(z), F_{12}^{\mathrm{ANO}}(z)$

$$H = \left(\begin{array}{c|c} H^{\mathrm{ANO}}(z - \underline{z_0}) & \\ \hline & v \end{array}\right), \quad F_{12} = \left(\begin{array}{c|c} F_{12}^{\mathrm{ANO}}(z - \underline{z_0}) & \\ \hline & 0 \end{array}\right)$$

This solution breaks $SU(2)_{\mathrm{C+F}} \to U(1)$

The moduli space of Nambu-Goldstone modes:

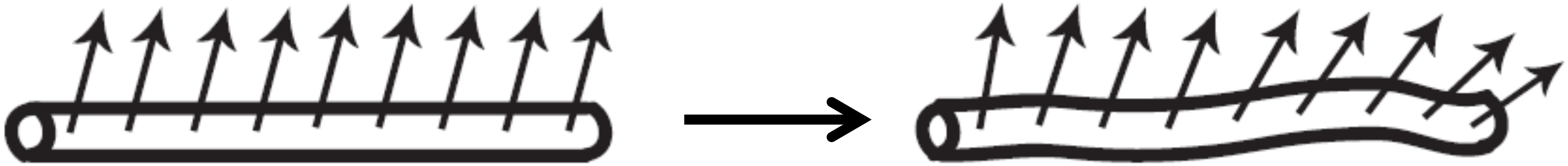
$$\underset{\uparrow}{\mathbf{C}} \times \underbrace{\frac{SU(2)_{\mathrm{C+F}}}{U(1)}} \cong \mathbf{C} \times \mathbf{C}P^1 \cong \mathbf{C} \times S^2$$

Translation
z_0

Internal
symmety

$\beta = 0, m \neq 0 << v$ No Josephson

The vortex effective theory is the $\mathbf{C}P^1$ model



"vacuum state"

fluctuation of zero modes

$$\mathcal{L}_{\mathrm{vort.eff.}} = 2\pi v^2 |\partial_\mu z_0|^2 + \frac{4\pi}{g^2}\left[\frac{\partial_\mu u^* \partial^\mu u - m^2|u|^2}{(1+|u|^2)^2}\right]$$

The monopole effective theory

$$\begin{aligned}\mathcal{L}_{\mathrm{mono.eff.}} &= \frac{4\pi}{g^2}\int_{-\infty}^{+\infty} dy \frac{e^{2my}}{(1+e^{2my})^2}[(\partial_i Y)^2 + (\partial_i \varphi)^2] \\ &= \frac{4\pi}{g^2}\frac{1}{2m}[(\partial_i Y)^2 + (\partial_i \varphi)^2].\end{aligned}$$

$\beta \neq 0 << m$

With Josephson

The vortex effective theory is the $\mathbb{C}P^1$ model

$$\mathcal{L}_{\text{vort.eff.}} = 2\pi v^2 |\partial_\mu z_0|^2 + \frac{4\pi}{g^2}\left[\frac{\partial_\mu u^* \partial^\mu u - m^2|u|^2}{(1+|u|^2)^2}\right]$$

$$-c\beta^2 D_x \qquad D_x = \frac{u+u^*}{1+|u|^2}$$

Josephoson term induced in the vortex eff. theory

$$c = \sqrt{2}\pi \int_0^\infty dr\, r(f^2 - v^2) \equiv \frac{\tilde{c}}{g^2} \qquad \tilde{c} \sim \frac{\sqrt{2}\pi}{4} \sim 1.11$$

The monopole effective theory

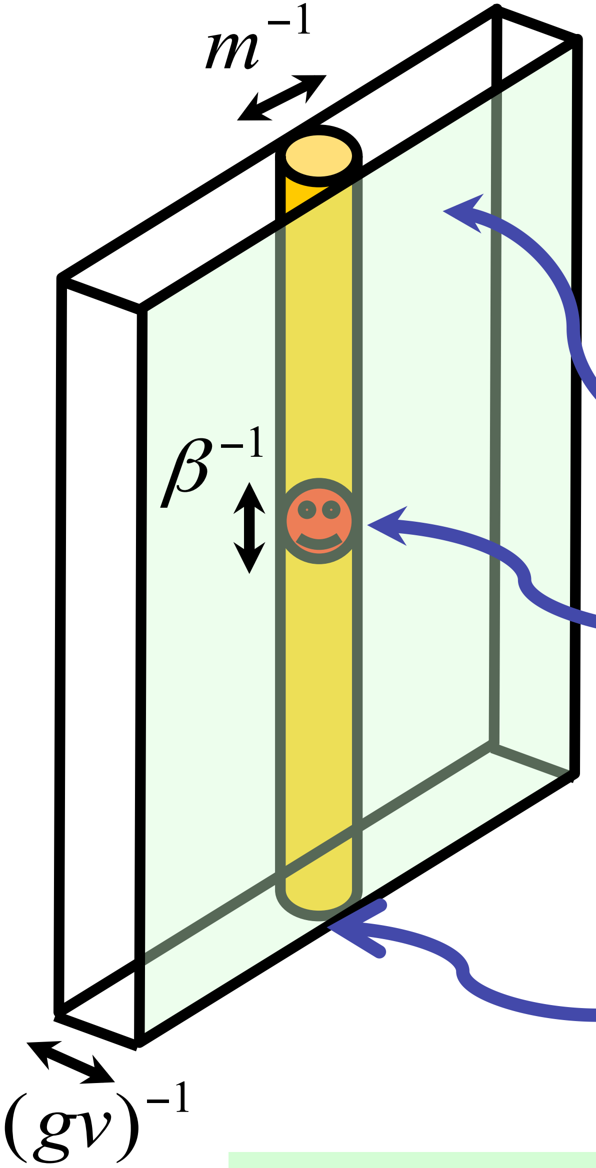
$$\mathcal{L}_{\text{mono.eff.}} = \frac{4\pi}{g^2}\frac{1}{2m}[(\partial_i Y)^2 + (\partial_i \varphi)^2] + \frac{\pi c \beta^2}{m}\cos\varphi$$

Sine-Gordon pot. induced in the monopole eff. theory

Non-Abelian

Phys.Rev.D87(2013)066008 [arXiv:1301.3268]

d=4+1 bulk



Potential

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Instantons confined by monopole strings

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$$d = 3 + 1$$

with $\beta \neq 0$

M.Kobayashi

arXiv:1304.6021

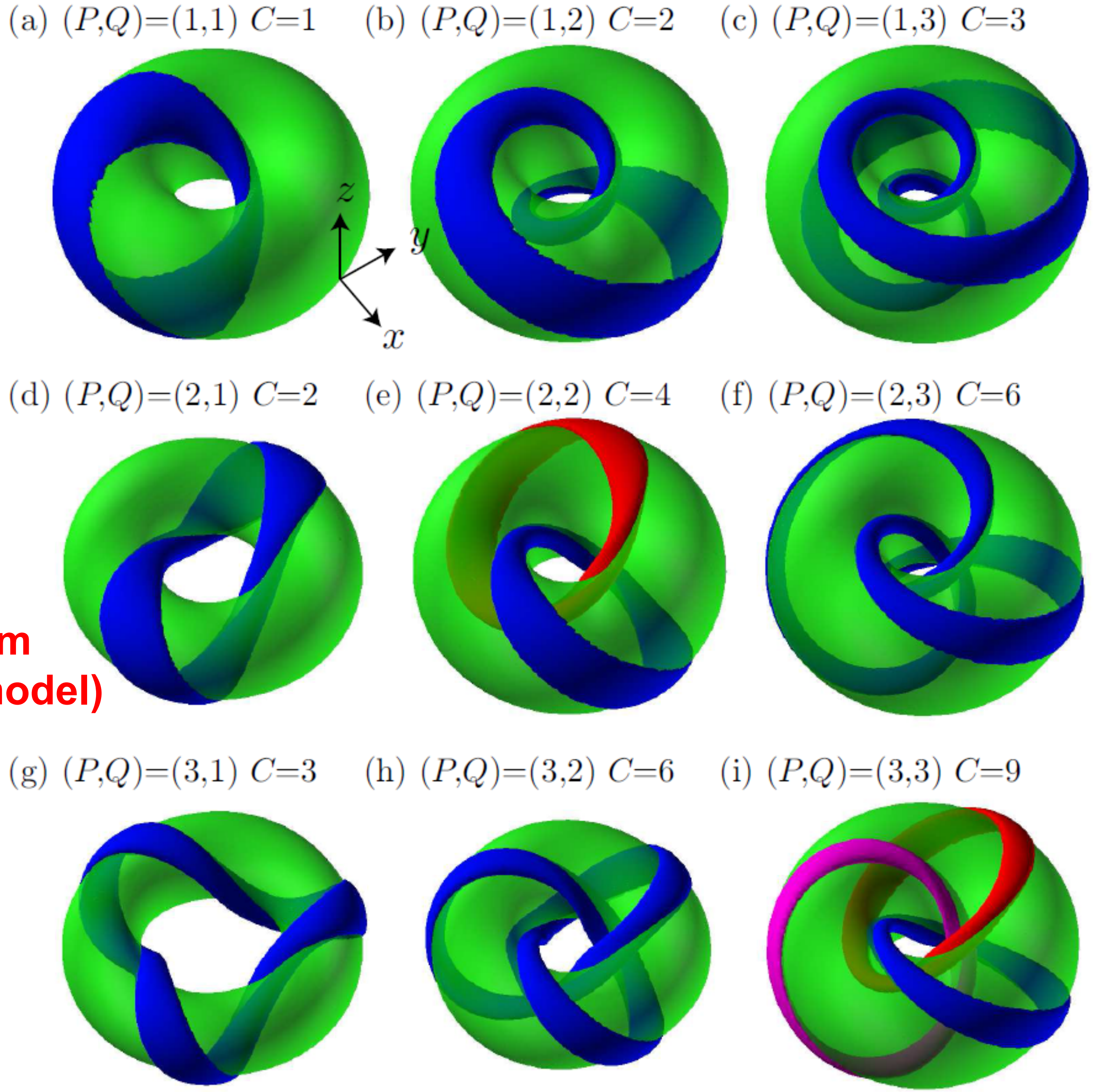
(P,Q) Torus Knots = Hopfions

4 deriv.(Skyrme) term (Faddeev-Skyrme model)

$$S^3 \rightarrow S^2$$

$$\pi_3\left(S^2\right) = \mathbf{Z}$$

256^3



(a) $(P,Q)=(1,1)$ $C=1$ (b) $(P,Q)=(1,2)$ $C=2$ (c) $(P,Q)=(1,3)$ $C=3$

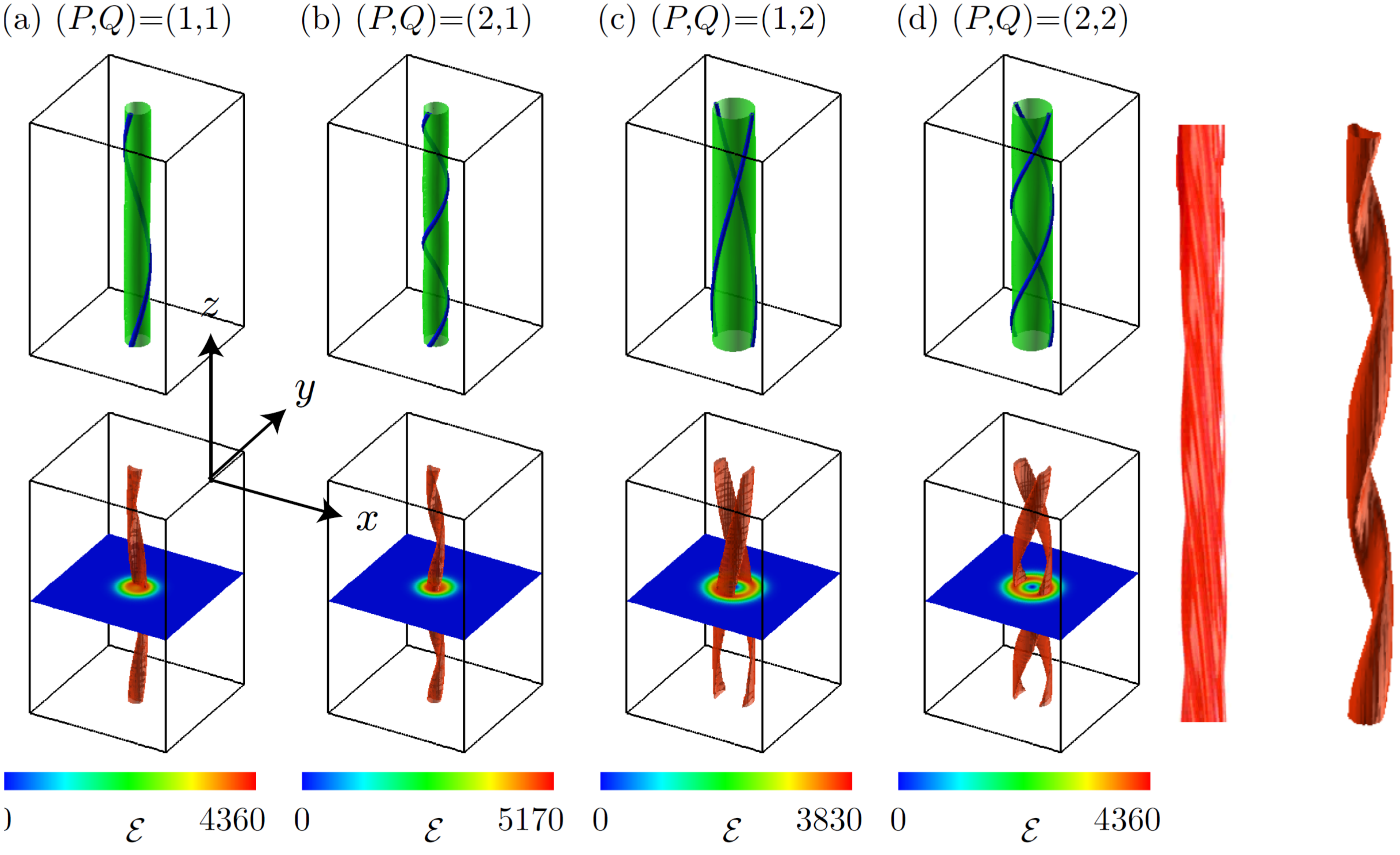
(d) $(P,Q)=(2,1)$ $C=2$ (e) $(P,Q)=(2,2)$ $C=4$ (f) $(P,Q)=(2,3)$ $C=6$

(g) $(P,Q)=(3,1)$ $C=3$ (h) $(P,Q)=(3,2)$ $C=6$ (i) $(P,Q)=(3,3)$ $C=9$

Winding Hopfion on $\mathbf{R}^{2,1} \times S^1$

M.Kobayashi & MN
NPB[arXiv:1305.7417]

Pontrjagin's homotopy $S^2 \times S^1 \rightarrow S^2$


(a) $(P,Q)=(1,1)$ (b) $(P,Q)=(2,1)$ (c) $(P,Q)=(1,2)$ (d) $(P,Q)=(2,2)$

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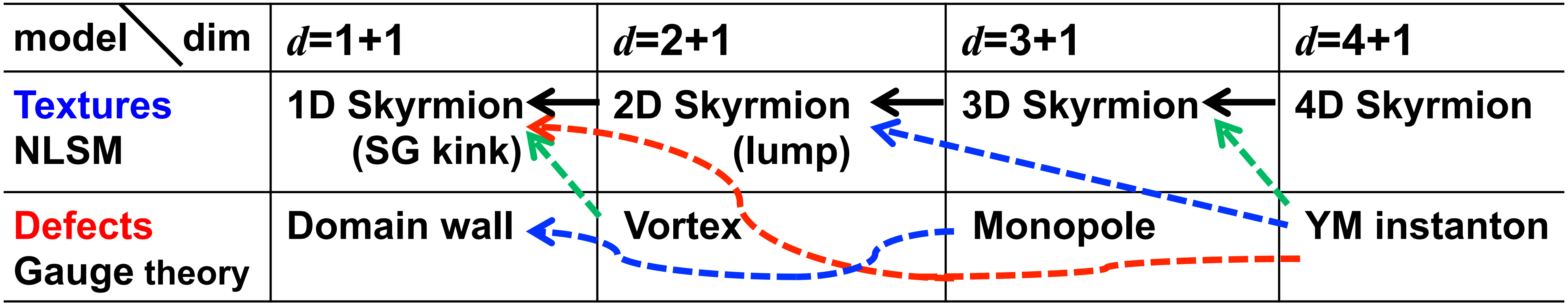
Topological Solitons and Instantons



model＼dim	d=1+1	d=2+1	d=3+1	d=4+1
Textures NLSM	1D Skyrmion (SG kink)	2D Skyrmion (lump)	3D Skyrmion	4D Skyrmion
Defects Gauge theory	Domain wall	Vortex	Monopole	YM instanton

What are relations among them?

← wall

←-- wall

←-- vortex

←-- monopole

Brane within brane (these ten years)

Hopfion = sine-Gordon kink on a curved (toroidal) domain wall

ご清聴ありがとうございました