基研研究会「場の理論と弦理論」(2013年8月21日)

Gauged Linear Sigma Model for Exotic Five-brane

木村 哲士

立教大学理学部 物理学科・数理物理学研究センター

arXiv:1304.4061 (accepted by Nucl. Phys. B)

arXiv:1305.4439 (accepted by JHEP)

佐々木伸氏との共同研究

良く知る物体	string dualities → in D-dimensions	馴染みの薄い物体
D-branes, NS-branes		exotic branes

特 徴 :

✓ 漸近的振る舞いや質量に赤外発散が入る

(co-dimension ≤ 2)

✓ 計量などが一価関数にならない

(非自明なモノドロミー)

5^2_2-brane

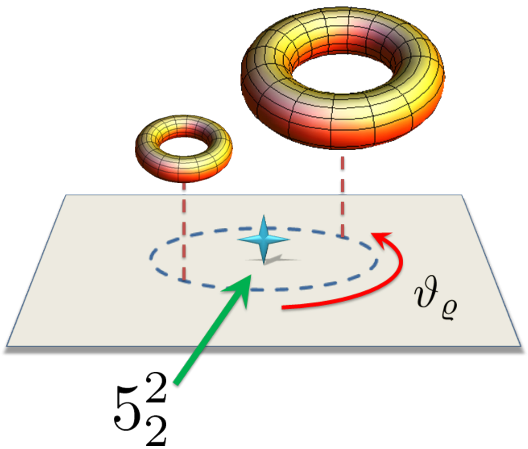
NS5-brane	0	1	2	3	4	5	6	7	8	9

T-dual along x^9

KK-monopole	0	1	2	3	4	5	6	7	8	$\widetilde{9}$

T-dual along x^8

5^2_2-brane	0	1	2	3	4	5	6	7	$\widetilde{8}$	$\widetilde{9}$



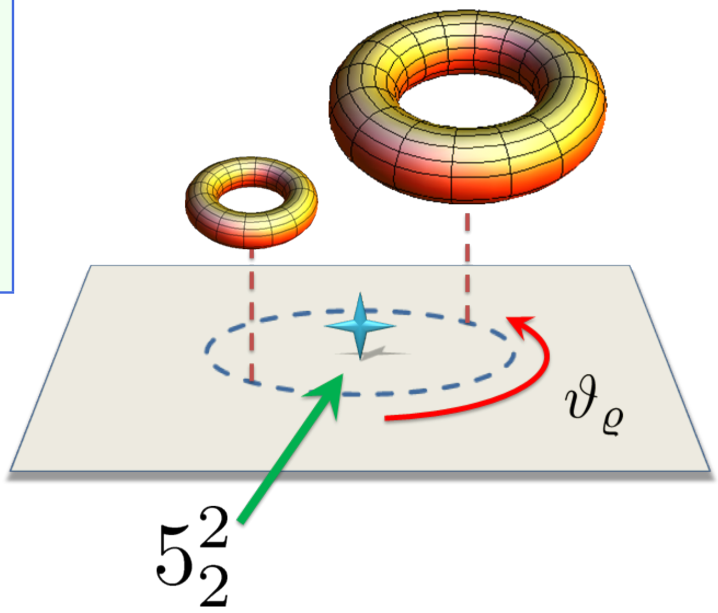
$$ds^2 = dx^2_{012345} + H\big[d\varrho^2 + \varrho^2 d\vartheta_\varrho^2\big] + \frac{H}{K}\big[(d\widetilde{x}^8)^2 + (d\widetilde{x}^9)^2\big]$$

$$B_{89} = -\frac{\sigma\,\vartheta_\varrho}{K}, \quad e^{2\Phi} = \frac{H}{K}, \quad K = H^2 + (\sigma\vartheta_\varrho)^2$$

$$H = h + \sigma\,\log\left(\frac{\mu}{\varrho}\right)$$

$$\vartheta_\varrho = 0 \quad : \quad G_{\widetilde{8}\widetilde{8}} = G_{\widetilde{9}\widetilde{9}} = \frac{1}{H}$$

$$\vartheta_\varrho = 2\pi \quad : \quad G_{\widetilde{8}\widetilde{8}} = G_{\widetilde{9}\widetilde{9}} = \frac{H}{H^2 + (2\pi\sigma)^2}$$



$$ds^2 = dx^2_{012345} + H\big[d\varrho^2 + \varrho^2 d\vartheta_\varrho^2\big] + \frac{H}{K}\big[(d\widetilde{x}^8)^2 + (d\widetilde{x}^9)^2\big]$$

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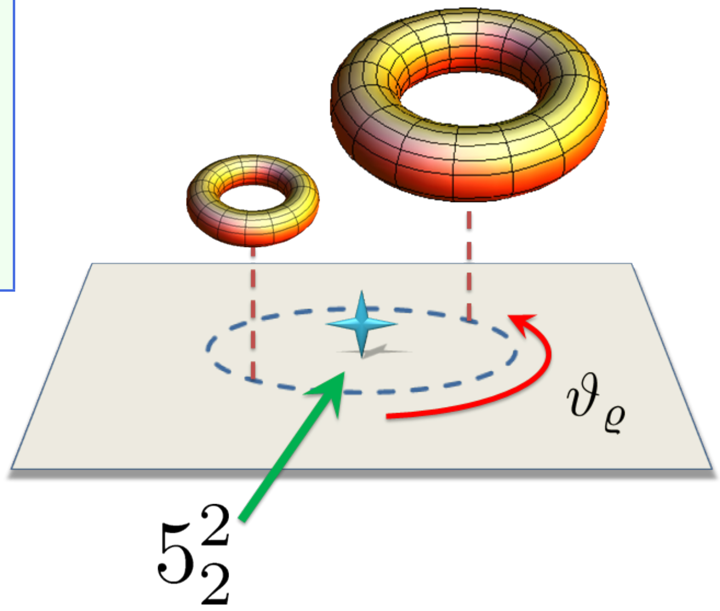


Locally geometric, しかし Globally non-geometric (non–single-valued metric)

フラックスコンパクト化に於ける T-fold の具体例

J. de Boer and M. Shigemori arXiv:1209.6056

C. Hull hep-th/0406102

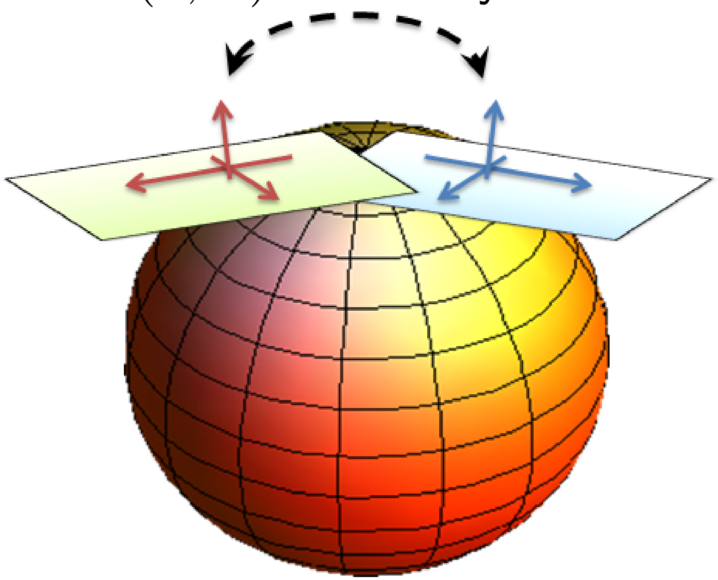
Non-geometric structure という概念の出現

構造群 $=$ 一般座標変換群 $(GL(d,\mathbb{R}))$ $\subset$ 双対変換群 $(O(d,d)$, U-双対変換, etc.)

↑

弦理論の双対性に起因

$GL(d,\mathbb{R}) \subset$ duality transf.



Generalized Geometry
Doubled Geometry

5^2_2-brane (T-fold) はこの概念の具現化

エキゾチックブレーンが見せる新しい時空構造

monodrofolds

これまでは主に、点粒子をプローブとした超重力理論での記述であった

弦をプローブとした Worldsheet 理論 で解析する

(弦理論特有のエキゾチックブレーンは弦理論で理解すべし)

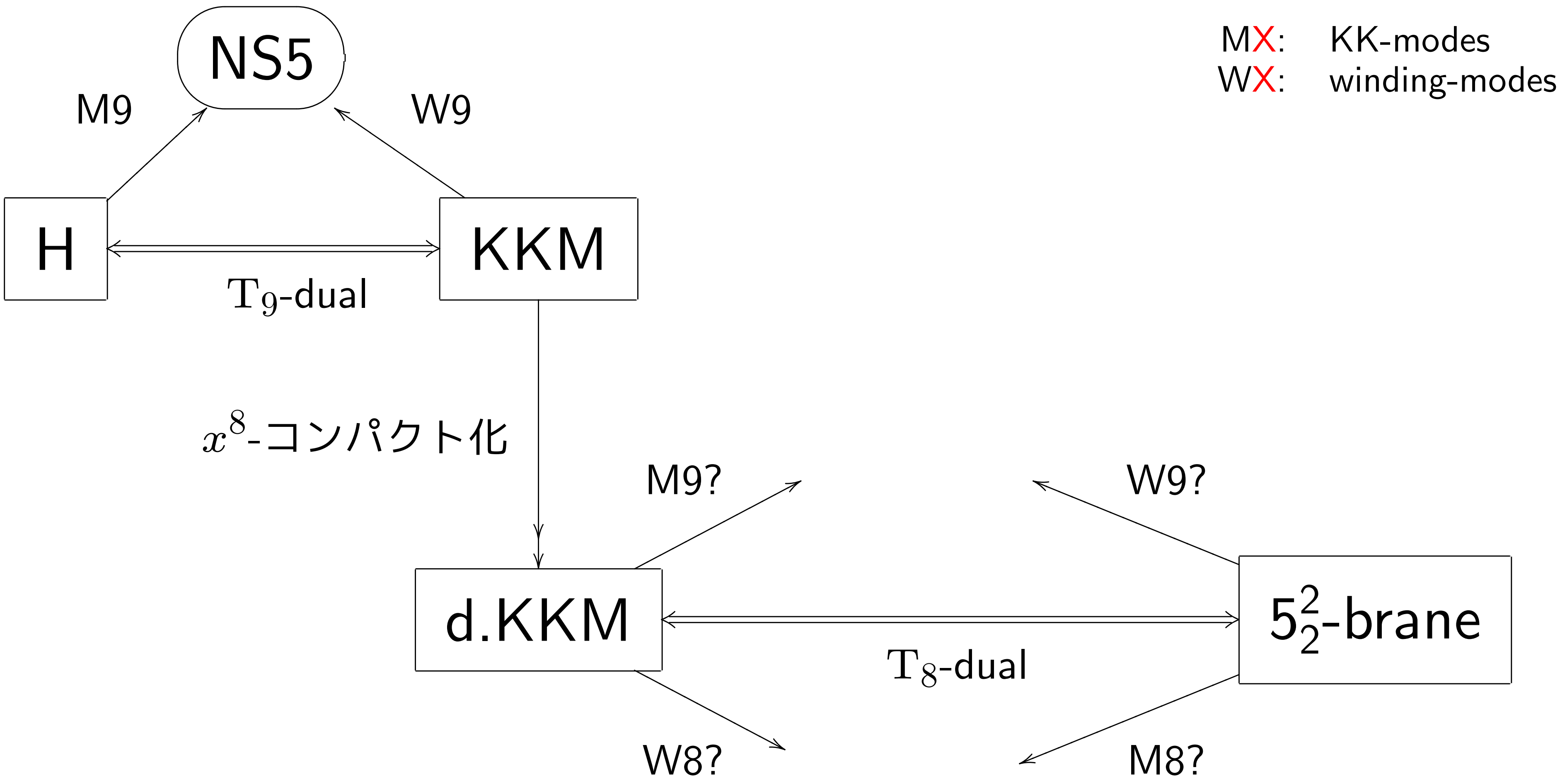
Worldsheet 理論

- ☞ nonlinear sigma model (NLSM)
- ☞ conformal field theory (CFT)

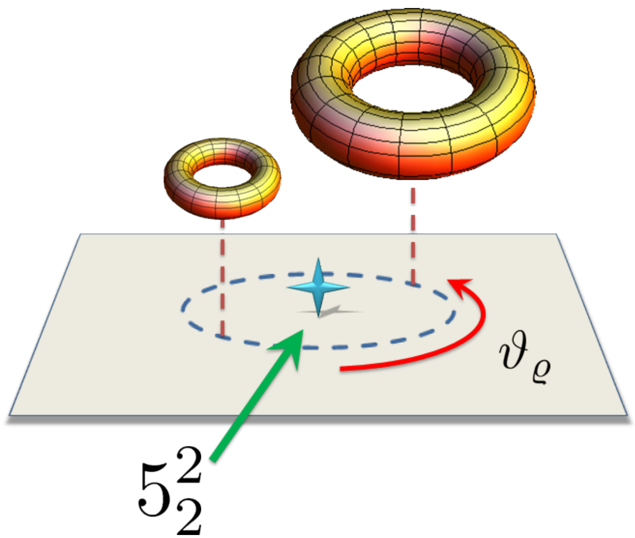
- ☞ gauged linear sigma model (GLSM)

GLSM (UltraViolet) $\Longleftrightarrow$ NLSM/CFT (InfraRed)

$\mathcal{N} = (4,4)$ GLSM で見える(と期待できる)物理



弦をプローブとして 5^2_2-brane を記述する



T-duality : NS5 → KKM → 5^2_2-brane

technique 1

F-terms ありでの Roček-Verlinde formula

↓

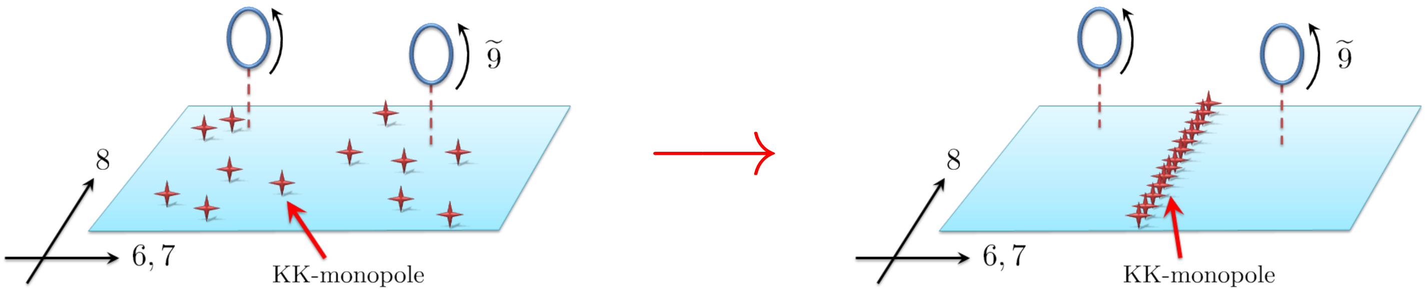
F-terms を D-terms に書き換える

technique 2

2 つめの isometry

↓

KKM を無限個並べる



$$\begin{aligned}\mathscr{L}_{\mathsf{H}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta} \Theta + \overline{\Psi} \Psi \Big) \\ & + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\mathsf{h.c.}) \Big\} \\ & + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, (t_a - \Theta) \Sigma_a + (\mathsf{h.c.}) \Big\}\end{aligned}$$

GLSM for H-monopoles (NS5)

D. Tong hep-th/0204186

$$\mathscr{L}_{\rm H} = \sum_{a=1}^{k} \int d^4\theta \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, e^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, e^{-2V_a} \widetilde{Q}_a \Big\} + \int d^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big)$$
$$+ \sum_{a=1}^{k} \Big\{ \sqrt{2} \int d^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\}$$
$$+ \sum_{a=1}^{k} \Big\{ \sqrt{2} \int d^2\widetilde{\theta} \, (t_a - \Theta) \Sigma_a + (\text{h.c.}) \Big\}$$

GLSM for H-monopoles (NS5)

D. Tong hep-th/0204186

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		役割
neutral HM	chiral $\Psi = \frac{1}{\sqrt{2}}(X^6 + i\, X^8) + \ldots$	twisted chiral $\Theta = \frac{1}{\sqrt{2}}(X^7 + i\, X^9) + \ldots$	時空座標
VMs	twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}} \overline{D}_+ D_- V_a$	chiral Φ_a	isometry のゲージ化
charged HMs	chiral Q_a $(+)$	chiral $\widetilde{Q}_a$ $(-)$	時空を曲げる
FI parameters	$s_a = s_{6,a} + i\, s_{8,a}$	$t_a = t_{7,a} + i\, t_{9,a}$	five-branes の位置

$$\begin{aligned}\mathscr{L}_{\mathsf{H}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta} \Theta + \overline{\Psi} \Psi \Big) \\ & + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\mathsf{h.c.}) \Big\} \\ & + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, (t_a - \Theta) \Sigma_a + (\mathsf{h.c.}) \Big\}\end{aligned}$$

GLSM for H-monopoles (NS5)

D. Tong hep-th/0204186

Ψ と Θ について duality (Roček-Verlinde) 変換

$$
\begin{aligned}
\mathscr{L}_{\text{new}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int \mathrm{d}^4\theta \Big\{ - \frac{g^2}{2} \Big(\Xi + \overline{\Xi} - \sqrt{2} \sum_{a=1}^{k} (C_a + \overline{C}_a) \Big)^2 \Big\} \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big\} \\
& - \sqrt{2} \int \mathrm{d}^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_{a=1}^{k} (C_a - \overline{C}_a) - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (X^9 A_{n,a})
\end{aligned}
$$

GLSM for exotic 5^2_2-brane : **NEW!**

$$
\begin{aligned}
\mathscr{L}_{\text{new}} \;=\;& \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int \mathrm{d}^4\theta \, \Big\{ - \frac{g^2}{2} \Big(\Xi + \overline{\Xi} - \sqrt{2} \sum_{a=1}^{k} (C_a + \overline{C}_a) \Big)^2 \Big\} \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big\} \\
& - \sqrt{2} \int \mathrm{d}^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_{a=1}^{k} (C_a - \overline{C}_a) - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (X^9 A_{n,a})
\end{aligned}
$$

$$
\begin{aligned}
- \frac{1}{g^2} (\Theta + \overline{\Theta}) \;&=\; (\Gamma + \overline{\Gamma}) + 2 \sum_a V_a \\
- \frac{1}{g^2} (\mathring{\Psi} + \overline{\mathring{\Psi}}) \;&=\; (\Xi + \overline{\Xi}) - \sqrt{2} \sum_a (C_a + \overline{C}_a)
\end{aligned}
$$

$\Sigma_a = \frac{1}{\sqrt{2}} \overline{D}_+ D_- V_a$ そして $\Phi_a = \overline{D}_+ \overline{D}_- C_a$

$$
\begin{aligned}
\mathscr{L}_{\text{new}} \ &= \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int \mathrm{d}^4\theta \, \Big\{ - \frac{g^2}{2} \Big(\Xi + \overline{\Xi} - \sqrt{2} \sum_{a=1}^{k} (C_a + \overline{C}_a) \Big)^2 \Big\} \\
&\quad + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big\} \\
&\quad - \sqrt{2} \int \mathrm{d}^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_{a=1}^{k} (C_a - \overline{C}_a) - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (X^9 A_{n,a})
\end{aligned}
$$

$\int \mathrm{d}^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_{a} (C_a - \overline{C}_a)$ は一見 isometry を壊しそうだが、必要な存在！

もしも $\int \mathrm{d}^4\theta\,(\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_a (C_a - \overline{C}_a)$ がなかったら $\cdots$

- ☹ IR極限では chiral model になってしまう [conflict w/ $\mathcal{N} = (4,4)$ SUSY]
- ☹ 時空計量が一価関数になってしまう [conflict w/ non-trivial monodromy]
- ☹ 時空上の B-場が登場しなくなってしまう [conflict w/ original NS5-brane]

この項が存在するので

T-duality変換を実行する前の座標場 X^8 がまだ系に存在する

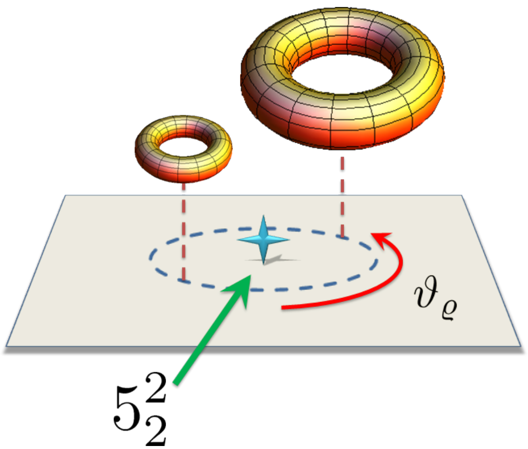
↓

積分して追い出す (可能)

一価関数でない時空上の (G, B) が導出される

Steps to NLSM for 5^2_2-brane geometry details superfields

1. SUSY 真空を探す $\mathscr{L}^{\text{potential}} = 0$
2. charged HMs $(Q_a, \widetilde{Q}_a)$ の拘束条件を解く
3. IR 極限下で VM (V_a, Φ_a) を積分
4. ★ T-dual 座標場 X^8 を積分



$$(X^6, X^7; Y^8, Y^9)$$

Step 1.,2.,3. :

$$\begin{aligned}\mathscr{L} \;=\;& -\frac{1}{2}H\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] - \frac{1}{2H}(\partial_m Y^9)^2 \\ & -\frac{(\Omega_8)^2}{2H}(\partial_m X^8)^2 + \frac{\Omega_8}{H}(\partial_m Y^9)(\partial^m X^8) \\ & -\frac{(\Omega_6)^2}{2H}(\partial_m X^6)^2 - \frac{\Omega_6\Omega_8}{H}(\partial_m X^6)(\partial^m X^8) + \frac{\Omega_6}{H}(\partial_m Y^9)(\partial^m X^6) \\ & -\sqrt{2}\,\varepsilon^{mn}\sum_a \partial_m((X^9 - t_{9,a})A_{n,a}) + \varepsilon^{mn}(\partial_m X^8)(\partial_n Y^8)\end{aligned}$$

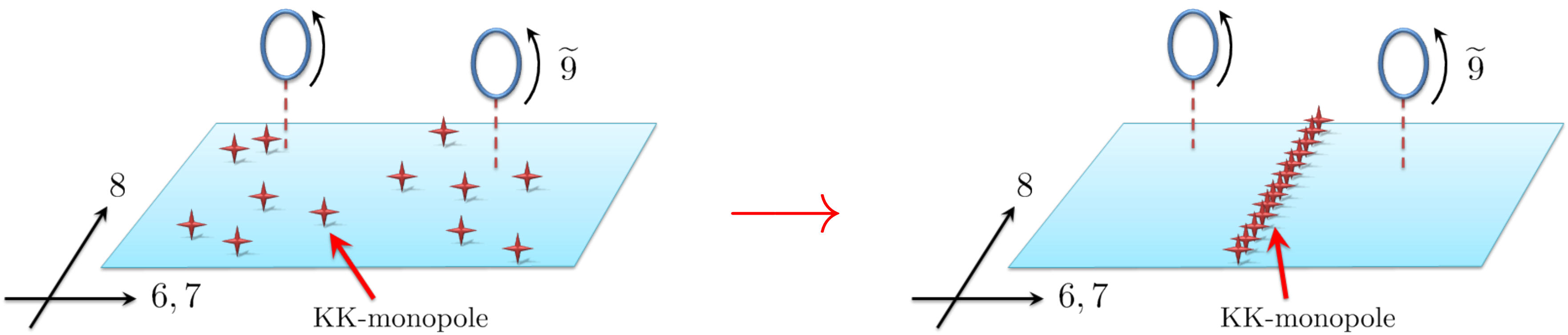
$$H \;=\; \frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a}, \qquad \Omega_6 \;=\; \sum_a \frac{X^8 - s_{8,a}}{\sqrt{2}\,R_a(R_a + (X^7 - t_{7,a}))}, \quad \Omega_8 \;=\; -\sum_a \frac{X^6 - s_{6,a}}{\sqrt{2}\,R_a(R_a + (X^7 - t_{7,a}))}$$

$$A_{m,a} = -\frac{1}{2R_a H}\big(\partial_m Y^9 - \Omega_i\,\partial_m X^i\big) - \frac{1}{\sqrt{2}}\Omega_{i,a}\,\partial_m X^i$$

Step 4. : $s_{6,a} = 0 = t_{7,a} = t_{9,a}$ and $s_{8,a} = 2\pi \mathcal{R}_8 \, a$ with $\boxed{k \to \infty}$

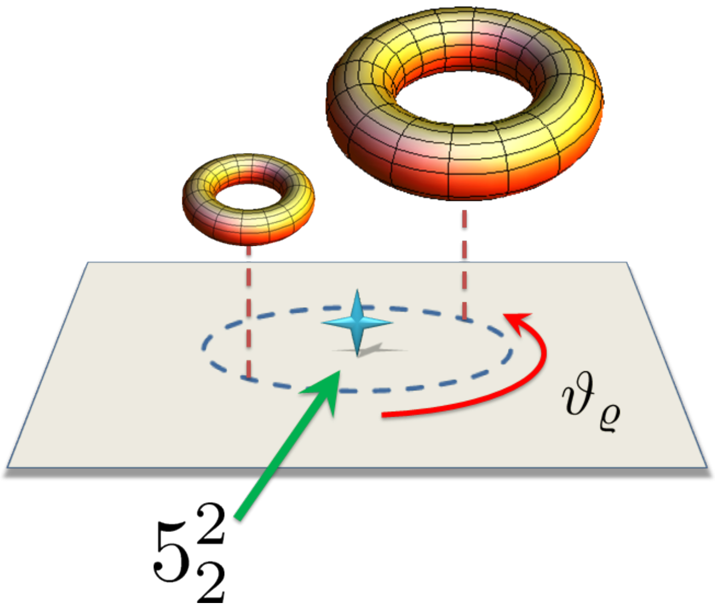
main

$$\longrightarrow \begin{cases} H \to h_0 + \sigma \log(\mu/\varrho) & : \text{ co-dim. 2} \qquad \varrho^2 = (X^6)^2 + (X^7)^2 \\ \Omega_6 \to 0 & : \text{ isometry along } X^8 \\ \Omega_8 \to \sigma \arctan(X^7/X^6) \equiv \sigma \, \vartheta_\varrho & : \text{ “non–single-valued” metric} \end{cases}$$

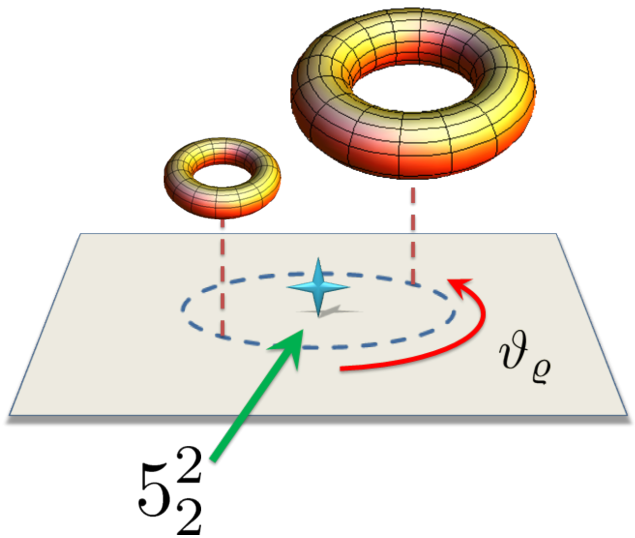


X^8 を積分 : $\partial_m X^8 = \dfrac{H}{K}\Big[\dfrac{\sigma \vartheta_\varrho}{H}(\partial_m Y^9) + \varepsilon_{mn}(\partial^n Y^8)\Big]$ $\qquad K = H^2 + (\sigma \vartheta_\varrho)^2$

$$
\begin{aligned}
\mathscr{L}_{\text{NLSM}} \;&=\; -\frac{1}{2} H \Big[(\partial_m \varrho)^2 + \varrho^2 (\partial_m \vartheta_\varrho)^2 \Big] - \frac{1}{2} \frac{H}{K} \Big[(\partial_m Y^8)^2 + (\partial_m Y^9)^2 \Big] \\
&\quad - \frac{\sigma \vartheta_\varrho}{K} \, \varepsilon^{mn} (\partial_m Y^8)(\partial_n Y^9) - \sqrt{2}\, \varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a})
\end{aligned}
$$



まとめ

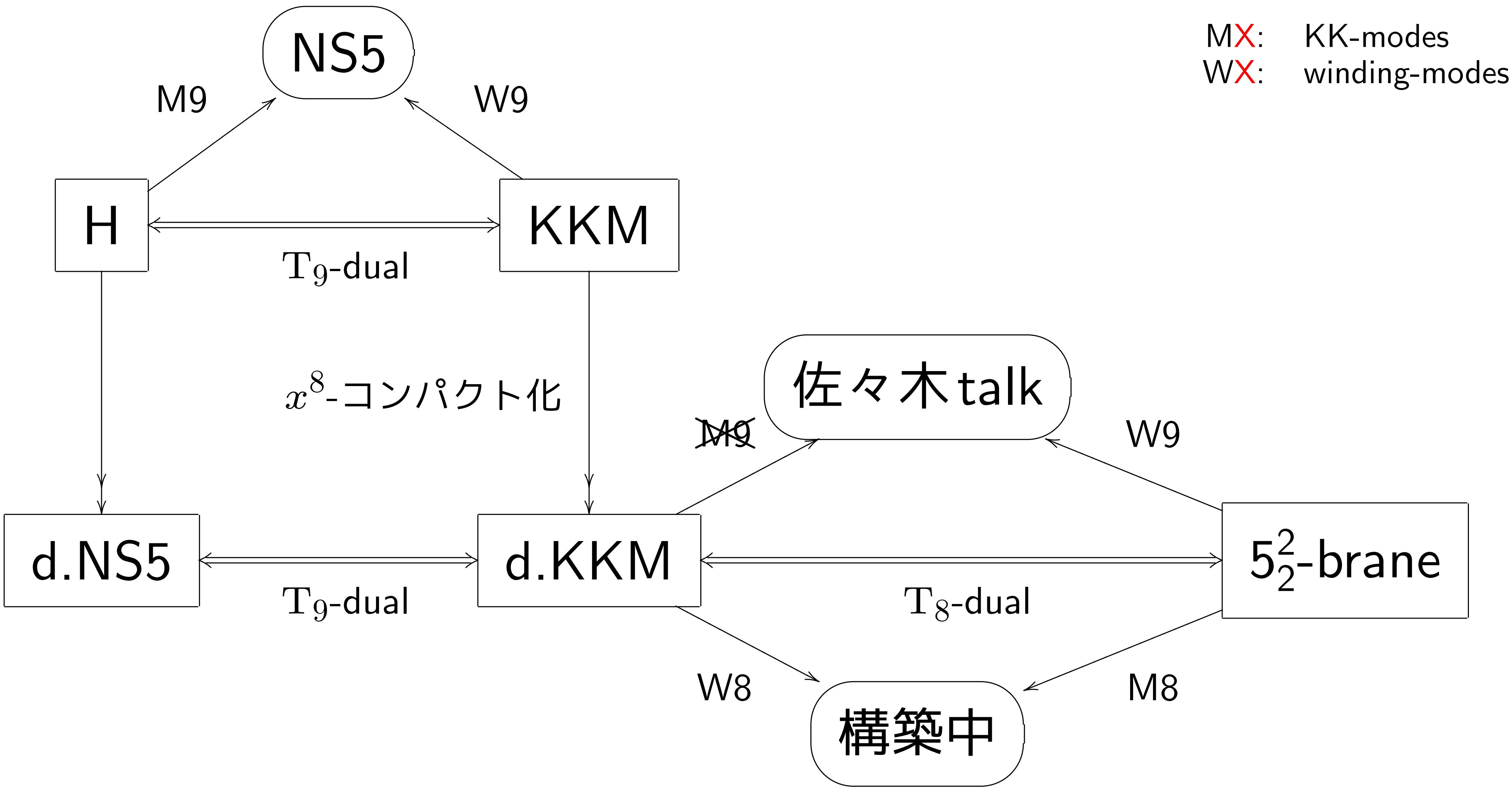


We established GLSM for exotic five-brane!

- ✓ [co-dim. 2] [2 つの isometry] [一価関数でない計量] を実現
- ✓ この GLSM の最初の応用：量子補正 $\longleftarrow$ 佐々木氏の講演

この $\mathcal{N} = (4,4)$ GLSM はまだまだ多くの情報を持つだろう ($\mathcal{N} = (2,2)$ GLSM の様に) GLSM

- ✓ さらなる量子補正 (次ページ)
- ✓ defect NS5-brane と 5^2_2-brane の束縛状態 de Boer and Shigemori
- ✓ modular invariance, dipole description ? T. Kikuchi, T. Okada and Y. Sakatani
- ✓ より本質的な GLSM がある (!?)


NS5
M9
W9
H
KKM
T9-dual
x8-コンパクト化
佐々木talk
M9
W9
d.NS5
T9-dual
d.KKM
T8-dual
5 2 2-brane
W8
M8
構築中
MX: KK-modes
WX: winding-modes

Thanks

Appendix

10D string theory $=$ D-dim spacetime $\otimes$ compact space $\mathcal{M}_d$ main

$\mathcal{M}_d$	geometry associated with G_{mn}	Conventional geometry (manifold) $O(d)$ global symmetry [Calabi-Yau, etc]	ordinary compactifications
	geometry associated with G_{mn}, B_{mn}	Generalized geometry $O(d,d;\mathbb{Z})$ T-duality symmetry [T-fold]	flux compactifications
	geometry associated with G_{mn}, $\tau = C_{(0)} + \mathrm{i}\,\mathrm{e}^{-\Phi}$	Generalized geometry $SL(2,\mathbb{Z})$ S-duality symmetry [S-fold]	F-theory
	geometry associated with G_{mn}, B_{mn}, Φ, $C_{(p)}$	Generalized geometry $E_{d+1(d+1)}(\mathbb{Z})$ U-duality symmetry [U-fold]	compactifications with non-abelian gauge

M-theory on $S^1(R_s)$	mass/tension ($l_s \equiv 1$)	type IIA
longitudinal M2	1	F1
transverse M2	$\frac{1}{g_s}$	D2
longitudinal M5	$\frac{1}{g_s}$	D4
transverse M5	$\frac{1}{g_s^2}$	NS5
longitudinal KK6	$\frac{R_{TN}^2}{g_s^2}$	KK5
KK6 with $R_{TN} = R_s$	$\frac{1}{g_s}$	D6
transverse KK6	$\frac{R_{TN}^2}{g_s^3}$	6_3^1

0	1	2	3	4	5	6	7	8	9	M
✓	✓	✓	✓	✓	✓	✓	S^1	$\mathbb{R}^3$		
KK6 $\to 6_3^1$							Taub-NUT			

$$b_n^c \; : \; M = \frac{(R_1 \cdots R_c)^2}{g_s^n}$$

for review: N. Obers and B. Pioline hep-th/9809039

$$b^c_n$$

b : spatial dimensions

c : # of isometry directions

n : mass of brane $\sim g_s^{-n}$

NS5-brane : 5_2-brane

KK-monopole : 5^1_2-brane

more : 5^2_2-brane , etc

an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

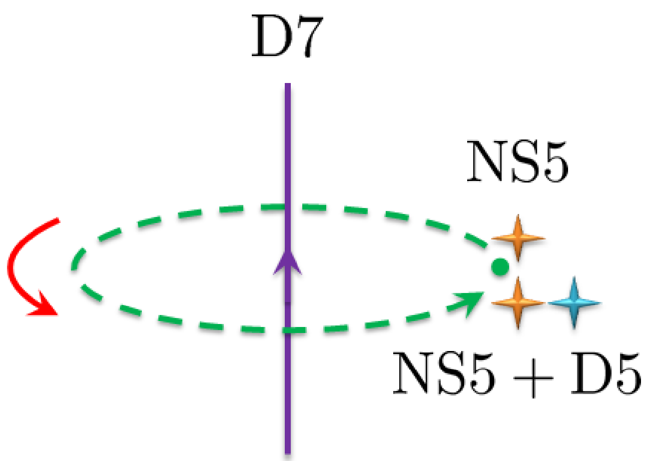
A very rough sketch on D7-brane (co-dim. 2 in 10D)

Moving around a D7-brane induces an $SL(2,\mathbb{Z})$ monodromy charge q

$$q : (C_2, B_2) \to (C_2 + B_2, B_2)$$

Since co-dim. 2, brane charges are not conserved but

$$(0, \text{NS5}) \to (\text{D5}, \text{NS5})$$

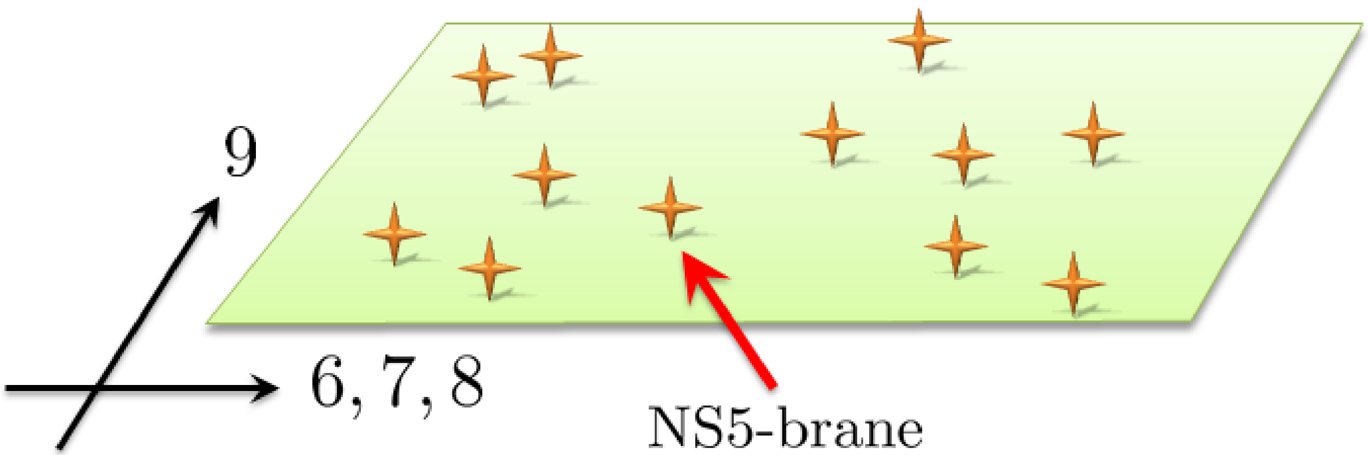


an instructive discussion : J. de Boer and M. Shigemori arXiv:1209.6056

NS5-branes

- co-dim. 4 ($\mathbb{R}^4$, $\vec{x} \in \mathbb{R}^4$)
- $ds^2 = dx^2_{012345} + H(x)\Big[(dx^6)^2 + (dx^7)^2 + (dx^8)^2 + (dx^9)^2\Big]$
- $H(x) = 1 + \sum_p \frac{Q}{|\vec{x} - \vec{x}_p|^2}\,, \qquad H_{mnp} = \varepsilon_{mnp}{}^q\,\partial_q \log H(x)\,, \qquad \Phi = \frac{1}{2}\log H(x)$

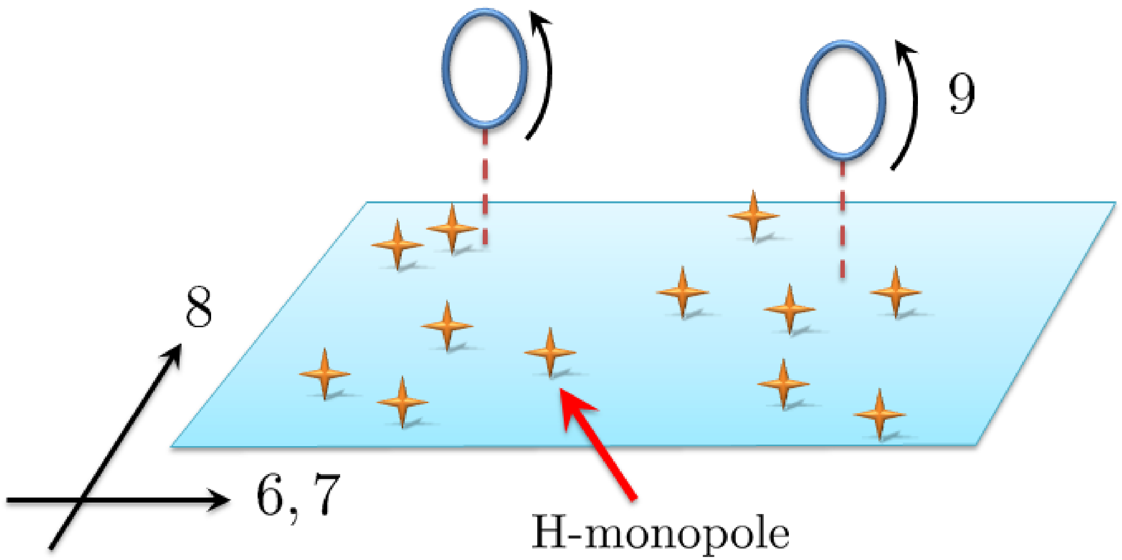
$dx^2_{012345} = -dt^2 + (dx^1)^2 + \ldots + (dx^5)^2$



NS5-branes (smeared), or H-monopoles

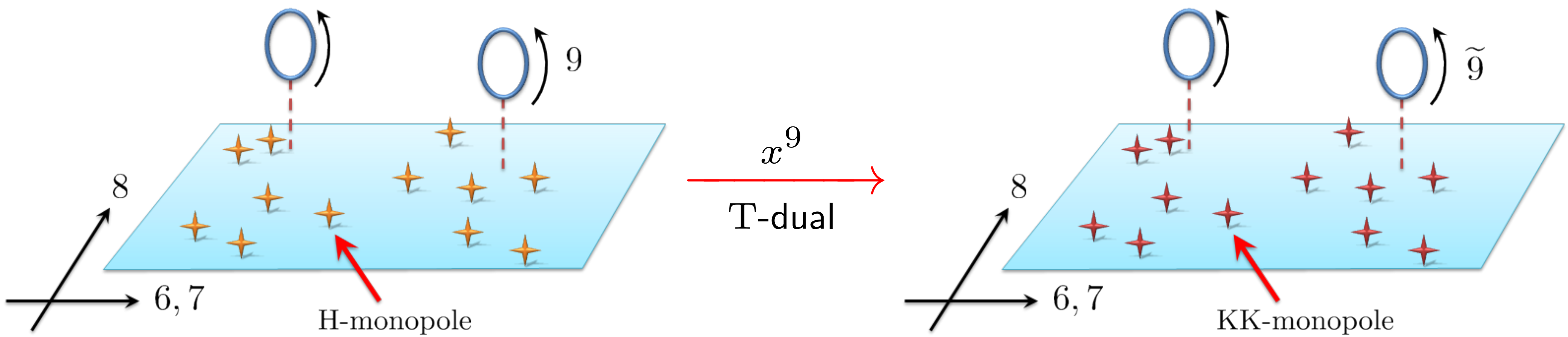
- co-dim. 3 ($\mathbb{R}^3 \times S^1\,, \quad \vec{r} \in \mathbb{R}^3$)
- $\mathrm{d}s^2 \;=\; \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2 + (\mathrm{d}x^9)^2\Big]$ NLSM
- $H(x) = 1 + \sum_p \frac{Q'}{|\vec{r} - \vec{r}_p|}\,, \qquad H_{mnp} = \varepsilon_{mnp}{}^q\,\partial_q\,\log H(x)\,, \qquad \Phi = \frac{1}{2}\log H(x)$

$\mathrm{d}x^2_{012345} = -\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + \ldots + (\mathrm{d}x^5)^2$



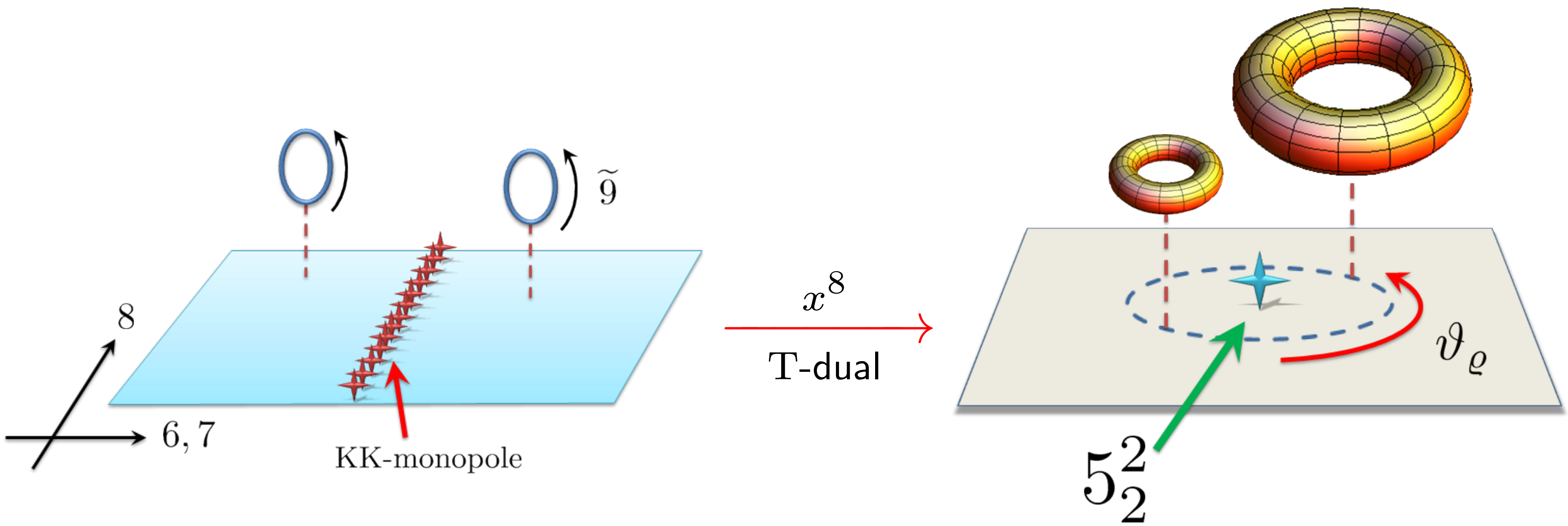
KK-monopoles

- co-dim. 3 ($\mathbb{R}^3 \times \widetilde{S}^1$: Taub-NUT space, $\vec{r} \in \mathbb{R}^3$)
- $\mathrm{d}s^2 \ = \ \mathrm{d}x^2_{012345} + H(x)\Big[(\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2\Big] + \dfrac{1}{H(x)}\big(\mathrm{d}\widetilde{x}^9 + \omega\big)^2$ NLSM
- $H(x) = 1 + \sum_p \dfrac{Q'}{|\vec{r} - \vec{r}_p|}, \qquad H_{mnp} = 0 = \Phi$



5^2_2-brane

- co-dim. 2 ($\mathbb{R}^2 \times T^2$)
- $H(x) = h + \sigma \log\left(\frac{\mu}{\varrho}\right)$, $(\varrho, \vartheta_\varrho) \in \mathbb{R}^2$



- T-duality transformation is represented as

(free) string	sign flip (parity) in right-mover	momentum $\leftrightarrow$ winding
spacetime	Buscher rule	$(G_{mn}, B_{mn}) \to (G'_{mn}, B'_{mn})$
SUSY sigma model	Roček-Verlinde formula	chiral $\leftrightarrow$ twisted chiral

Gauged Linear Sigma Model (GLSM) はとても強力！

$\mathcal{N} = (2,2)$ 理論の場合：

✓ CY/LG (geometry/CFT) 対応 [phases]

$\leftarrow$ E. Witten hep-th/9301042

✓ ミラー対称性 [T-duality]

$\leftarrow$ K. Hori and C. Vafa hep-th/0002222

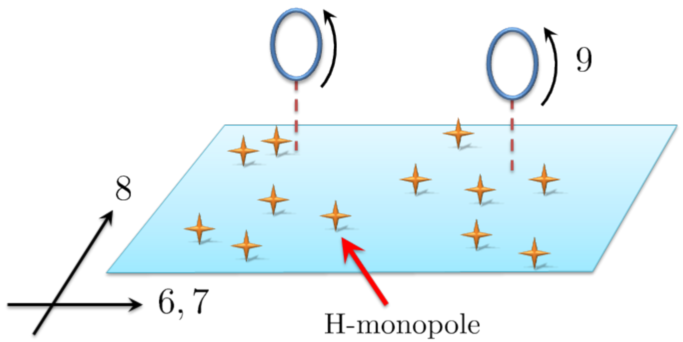
✓ 量子 Kähler moduli 空間 [instantons]

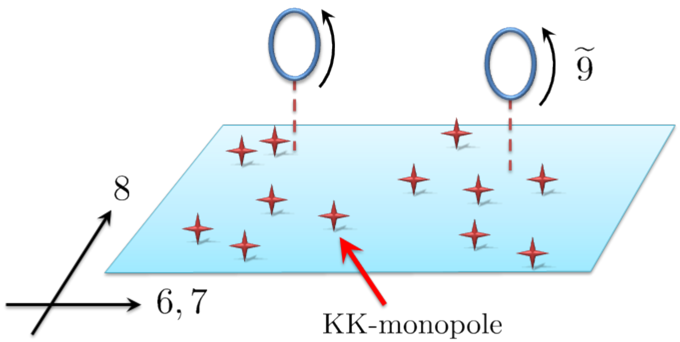
$\leftarrow$ N. Doroud et al arXiv:1206.2606; H. Jockers et al arXiv:1208.6244

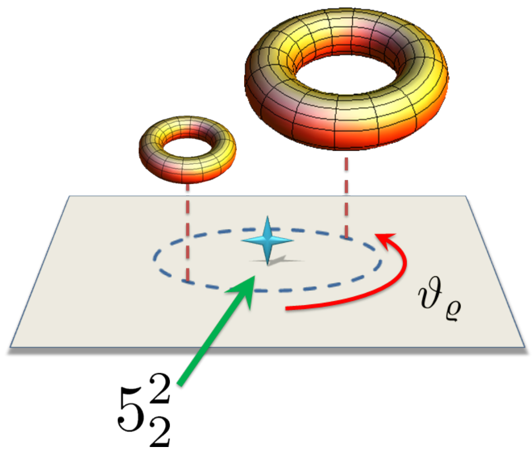
- 弦をプローブとした NS5-branes や KK-monopoles について：

$\mathcal{N} = (4, 4)$ GLSM の IR 極限で実現

D. Tong hep-th/0204186; J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097



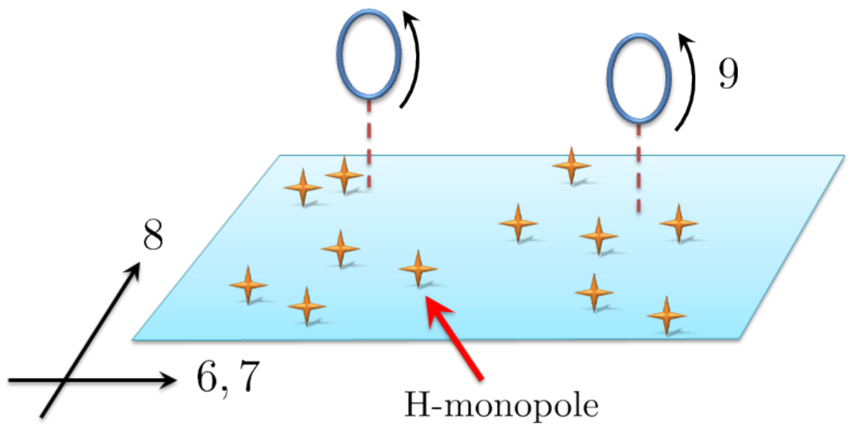




5^2_2-brane の記述には

1. 調和関数が対数関数 (co-dim. 2)
2. 2つめの isometry の存在
3. 時空計量が一価関数ではない

LESSON 1 : GLSM for H-monopoles



- $\mathcal{N} = (4,4)$ for H-monopoles by $\mathcal{N} = (2,2)$ language :

superfields

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		役割
neutral HM	chiral $\Psi = \frac{1}{\sqrt{2}}(X^6 + \mathrm{i}\, X^8) + \ldots$	twisted chiral $\Theta = \frac{1}{\sqrt{2}}(X^7 + \mathrm{i}\, X^9) + \ldots$	時空座標
VM	twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}}\overline{D}_+ D_- V_a$	chiral Φ_a	isometry のゲージ化
charged HMs	chiral Q_a $(+)$	chiral $\widetilde{Q}_a$ $(-)$	時空を曲げる
FI parameters	$s_a = s_{6,a} + \mathrm{i}\, s_{8,a}$	$t_a = t_{7,a} + \mathrm{i}\, t_{9,a}$	five-branes の位置

$$
\begin{aligned}
\mathscr{L}_{\mathrm{H}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \Big(- \overline{\Theta}\Theta + \overline{\Psi}\Psi \Big) \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi)\Phi_a \Big) + (\mathrm{h.c.}) \Big\} \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, (t_a - \Theta)\Sigma_a + (\mathrm{h.c.}) \Big\}
\end{aligned}
$$

Bosonic Lagrangian after integrating-out auxiliary fields :

$$\mathscr{L}_{\mathsf{H}}^{\mathsf{kin}} = \sum_a \frac{1}{e_a^2}\Big[\frac{1}{2}(F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2\Big] - \sum_a \Big[|D_m q_a|^2 + |D_m \widetilde{q}_a|^2\Big]$$
$$-\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 + (\partial_m X^9)^2\Big] - \sqrt{2}\sum_a (X^9 - t_{9,a})\, F_{01,a}$$

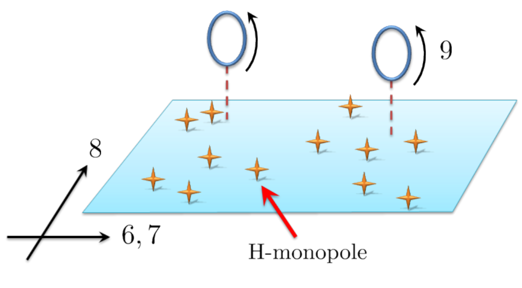
$$\mathscr{L}_{\mathsf{H}}^{\mathsf{pot}} = -2\sum_a \big(|\sigma_a|^2 + |\phi_a|^2\big)\big(|q_a|^2 + |\widetilde{q}_a|^2 + g^2\big)$$
$$-\sum_a \frac{e_a^2}{2}\Big(|q_a|^2 - |\widetilde{q}_a|^2 - \sqrt{2}(X^7 - t_{7,a})\Big)^2 - \sum_a e_a^2\Big|\sqrt{2}\, q_a \widetilde{q}_a - (X^6 - s_{6,a}) - \mathrm{i}(X^8 - s_{8,a})\Big|^2$$

Steps to NLSM for H-monopoles

geometry | details

1. SUSY vacua $\mathscr{L}^{\mathsf{pot}} = 0$
2. solve constraints on $(q_a, \widetilde{q}_a)$
3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$\Longrightarrow$



$(X^6, X^7; X^8, X^9)$

1. SUSY vacua

main

$$\sigma_a = 0 = \phi_a \,, \qquad |q_a|^2 - |\widetilde{q}_a|^2 = \sqrt{2}(X^7 - t_{7,a}) \,, \qquad \sqrt{2}\, q_a \widetilde{q}_a = (X^6 - s_{6,a}) + \mathrm{i}(X^8 - s_{8,a})$$

2. solve constraints on $(q_a, \widetilde{q}_a)$

$$q_a = -\frac{\mathrm{i}}{2^{1/4}}\,\mathrm{e}^{-\mathrm{i}\alpha_a}\sqrt{R_a + (X^7 - t_{7,a})} \,, \qquad \widetilde{q}_a = \frac{\mathrm{i}}{2^{1/4}}\,\mathrm{e}^{+\mathrm{i}\alpha_a}\frac{(X^6 - s_{6,a}) + \mathrm{i}(X^8 - s_{8,a})}{\sqrt{R_a + (X^7 - t_{7,a})}}$$

$$|D_m q_a|^2 + |D_m \widetilde{q}_a|^2 = \frac{1}{2\sqrt{2}R_a}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2\Big] + \sqrt{2}\,R_a\Big(\partial_m \alpha_a - A_{m,a} - \frac{1}{\sqrt{2}}\Omega_{i,a}\,\partial_m X^i\Big)^2$$

$$R_a = \sqrt{(X^6 - s_{6,a})^2 + (X^7 - t_{7,a})^2 + (X^8 - s_{8,a})^2}$$

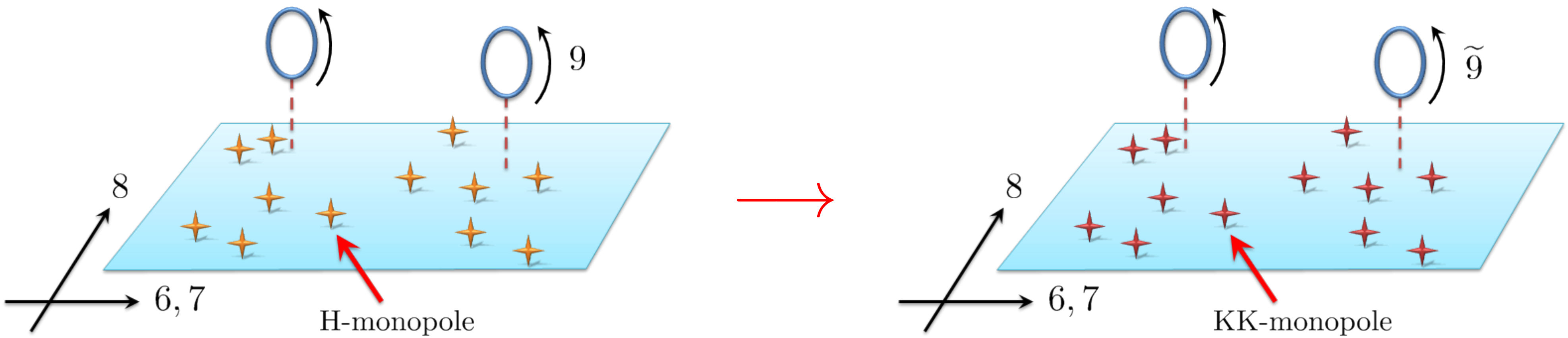
$$\Omega_{i,a}\,\partial_m X^i = \frac{-(X^6 - s_{6,a})\partial_m X^8 + (X^8 - s_{8,a})\partial_m X^6}{\sqrt{2}\,R_a(R_a + (X^7 - t_{7,a}))}$$

3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$$A_{m,a} = -\frac{1}{\sqrt{2}}\Omega_{i,a}\,\partial_m X^i - \frac{1}{2R_a}\,\varepsilon_{mn}\,\partial^n X^9 \,, \qquad \Omega_i = \sum_a \Omega_{i,a}$$

$$\Longrightarrow \quad \mathscr{L}_{\mathrm{H}}^{\mathrm{NLSM}} = -\frac{1}{2}\Big(\frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2}R_a}\Big)\Big[(\partial_m \vec{X})^2 + (\partial_m X^9)^2\Big] + \varepsilon^{mn}\,\Omega_i\,\partial_m X^i\,\partial_n X^9$$

LESSON 2 : T-duality



$\Theta \to \Gamma$:

$$\begin{aligned}
\mathscr{L}_{\mathsf{H}} \ni \mathscr{L}_{\Theta} &= \int \mathrm{d}^4\theta \Big(-\frac{1}{g^2}\overline{\Theta}\Theta \Big) + \sum_a \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta}\,(-\Theta)\Sigma_a + (\mathsf{h.c.}) \Big\} \\
&= \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}(\Theta + \overline{\Theta})^2 - 2(\Theta + \overline{\Theta}) \sum_a V_a \Big\} - \sqrt{2}\,\varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a})
\end{aligned}$$

$$\downarrow$$

$$\mathscr{L}_{B\Gamma} = \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2} B^2 - 2B \sum_a V_a - B(\Gamma + \overline{\Gamma}) \Big\} - \sqrt{2}\,\varepsilon^{mn} \sum_a \partial_m (X^9 A_{n,a})$$

real $\overline{B} = B$; chiral $\overline{D}_{\pm}\Gamma = 0$

$\Theta \to \Gamma$:

$$\mathscr{L}_{B\Gamma} \;=\; \int \mathrm{d}^4\theta \Big\{ -\frac{1}{2g^2}B^2 - 2B\sum_a V_a - (\Gamma + \overline{\Gamma})B \Big\} - \sqrt{2}\,\varepsilon^{mn}\sum_a \partial_m(X^9 A_{n,a})$$

Integrating out Γ, $\overline{\Gamma}$: $\to$ GLSM for H-monopoles $\qquad B = \Theta + \overline{\Theta}$

or, Integrating out B : $\to$ GLSM for KK-monopoles $\qquad \frac{1}{g^2}B = -(\Gamma + \overline{\Gamma}) - 2\sum_a V_a$

duality relation : $\qquad \Theta = \frac{1}{\sqrt{2}}(X^7 + \mathrm{i}X^9) + \ldots, \quad \Gamma = \frac{1}{\sqrt{2}}(Y^7 + \mathrm{i}Y^9) + \ldots$

$$\Theta + \overline{\Theta} \;=\; -g^2(\Gamma + \overline{\Gamma}) - 2\,g^2\sum_a V_a \quad \to \quad \boxed{\begin{aligned} X^7 &= -g^2\,Y^7 \\ \pm(\partial_0 \pm \partial_1)X^9 &= -g^2(D_0 \pm D_1)Y^9 \\ D_m Y^9 &= \partial_m Y^9 + \sqrt{2}\sum_a A_{m,a} \end{aligned}}$$

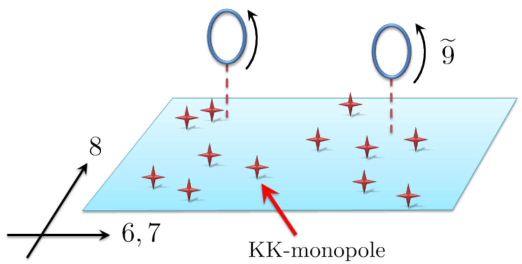
$$
\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \;=\; & \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(-\overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} \\
& + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int \mathrm{d}^4\theta \, \frac{1}{g^2} \, \overline{\Psi}\Psi \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\
& + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big\} - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (X^9 A_{n,a})
\end{aligned}
$$

Steps to NLSM for KK-monopoles

geometry | details | superfields

1. SUSY vacua $\mathscr{L}^{\text{pot}} = 0$
2. solve constraints on $(q_a, \widetilde{q}_a)$
3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$\Longrightarrow$



$(X^6, X^7; X^8, Y^9)$

1. SUSY vacua

main

$$\sigma_a = 0 = \phi_a \, , \qquad |q_a|^2 - |\widetilde{q}_a|^2 = \sqrt{2}(X^7 - t_{7,a}) \, , \qquad \sqrt{2}\, q_a \widetilde{q}_a = (X^6 - s_{6,a}) + \mathrm{i}(X^8 - s_{8,a})$$

2. solve constraints on $(q_a, \widetilde{q}_a)$

$$q_a = -\frac{\mathrm{i}}{2^{1/4}} \mathrm{e}^{-\mathrm{i}\alpha_a} \sqrt{R_a + (X^7 - t_{7,a})} \, , \qquad \widetilde{q}_a = \frac{\mathrm{i}}{2^{1/4}} \mathrm{e}^{+\mathrm{i}\alpha_a} \frac{(X^6 - s_{6,a}) + \mathrm{i}(X^8 - s_{8,a})}{\sqrt{R_a + (X^7 - t_{7,a})}}$$

$$|D_m q_a|^2 + |D_m \widetilde{q}_a|^2 = \frac{1}{2\sqrt{2} R_a} \Big[(\partial_m X^6)^2 + (\partial_m X^7)^2 + (\partial_m X^8)^2 \Big] + \sqrt{2}\, R_a \Big(\partial_m \alpha_a - A_{m,a} - \frac{1}{\sqrt{2}} \Omega_{i,a}\, \partial_m X^i \Big)^2$$

$$R_a = \sqrt{(X^6 - s_{6,a})^2 + (X^7 - t_{7,a})^2 + (X^8 - s_{8,a})^2}$$

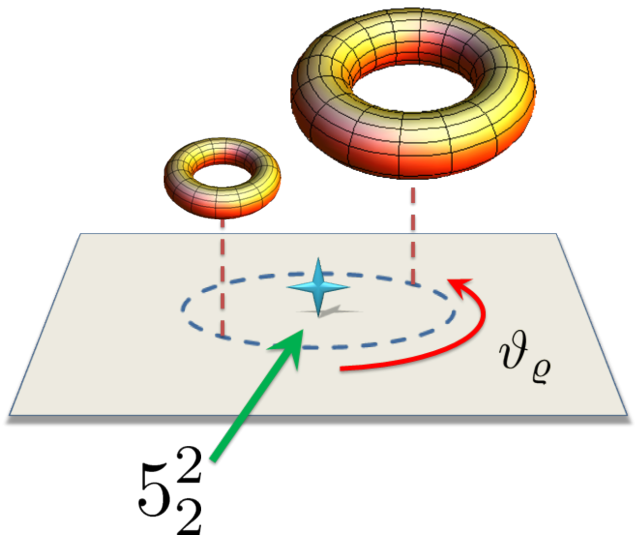
$$\Omega_{i,a}\, \partial_m X^i = \frac{-(X^6 - s_{6,a}) \partial_m X^8 + (X^8 - s_{8,a}) \partial_m X^6}{\sqrt{2}\, R_a (R_a + (X^7 - t_{7,a}))}$$

3. IR limit $e_a \to \infty$, and integrate out $A_{m,a}$

$$A_{m,a} = -\frac{1}{2 R_a H} \big(\partial_m Y^9 - \Omega_i\, \partial_m X^i \big) - \frac{1}{\sqrt{2}} \Omega_{i,a}\, \partial_m X^i \, , \qquad \Omega_i = \sum_a \Omega_{i,a} \, , \qquad H = \frac{1}{g^2} + \sum_a \frac{1}{\sqrt{2} R_a}$$

$$\mathscr{L}_{\mathrm{KK}}^{\mathrm{NLSM}} = -\frac{1}{2} H (\partial_m \vec{X})^2 - \frac{1}{2} H^{-1} (\partial_m Y^9 - \Omega_i\, \partial_m X^i)^2 - \sqrt{2}\, \varepsilon^{mn} \sum_a \partial_m ((X^9 - t_{9,a}) \mathring{A}_{n,a})$$

MAIN : 5^2_2-brane



$\Psi \to \Xi$:

$$\begin{aligned}
\mathscr{L}_{\mathsf{KK}} \ni \mathscr{L}_{\Psi} &= \int \mathrm{d}^4\theta \, \frac{1}{g^2} \overline{\Psi} \Psi + \sum_a \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, (-\Psi) \Phi_a + (\text{h.c.}) \Big\} \\
&= \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} (\Psi + \overline{\Psi})^2 - \sqrt{2} (\Psi + \overline{\Psi}) \sum_a (C_a + \overline{C}_a) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\Psi - \overline{\Psi})^2 - \sqrt{2} (\Psi - \overline{\Psi}) \sum_a (C_a - \overline{C}_a) \Big\}
\end{aligned}$$

$$\downarrow$$

$$\begin{aligned}
\mathscr{L}_{RSX\Xi} &= \int \mathrm{d}^4\theta \, \Big\{ \frac{a}{g^2} R^2 - \sqrt{2} R \sum_a (C_a + \overline{C}_a) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - \sqrt{2} (\mathrm{i}S) \sum_a (C_a - \overline{C}_a) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}
\end{aligned}$$

$\overline{R} = R, \ \overline{S} = S, \ \overline{D}_+ \Xi_{1,2} = 0 = D_- \Xi_{1,2}, \ \overline{D}_\pm X = 0, \ \Phi_a = \overline{D}_+ \overline{D}_- C_a$

$\Psi \to \Xi$:

$$\widetilde{\mathscr{L}} = \int \mathrm{d}^4\theta \Big\{ \frac{a}{g^2} R^2 - \sqrt{2} R \sum_a (C_a + \overline{C}_a) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\}$$
$$+ \int \mathrm{d}^4\theta \Big\{ \frac{2a-1}{2g^2} (\mathrm{i}S)^2 - \sqrt{2} (\mathrm{i}S) \sum_a (C_a - \overline{C}_a) + \mathrm{i}S(\Xi_2 - \overline{\Xi}_2) + \mathrm{i}S(X - \overline{X}) \Big\}$$

Integrating out Ξ_1, Ξ_2, X : $\to$ GLSM for KK-monopoles

or, Integrating out R, Ξ_2 : $\to$ new GLSM

$$\frac{2a}{g^2} R = -(\Xi_1 + \overline{\Xi}_1) + \sqrt{2} \sum_a (C_a + \overline{C}_a)$$

duality relation at $a = \frac{1}{2}$:

$$\Psi = \frac{1}{\sqrt{2}} (X^6 + \mathrm{i}\, X^8) + \ldots$$

$$\Psi + \overline{\Psi} = -g^2 (\Xi_1 + \overline{\Xi}_1) + \sqrt{2}\, g^2 \sum_a (C_a + \overline{C}_a)$$

$X^6 \sim$ real part of Ξ

$\partial X^8 \sim \partial$(imaginary part of Ξ) + "gauge" fields in C_a

$$\begin{aligned}
\mathscr{L}_{\text{new}} \ &= \sum_{a=1}^{k} \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} \Big(- \overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \overline{Q}_a \, \mathrm{e}^{2V_a} Q_a + \overline{\widetilde{Q}}_a \, \mathrm{e}^{-2V_a} \widetilde{Q}_a \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \frac{g^2}{2} \Big(\Gamma + \overline{\Gamma} + 2 \sum_{a=1}^{k} V_a \Big)^2 + \int \mathrm{d}^4\theta \, \Big\{ - \frac{g^2}{2} \Big(\Xi + \overline{\Xi} - \sqrt{2} \sum_{a=1}^{k} (C_a + \overline{C}_a) \Big)^2 \Big\} \\
&\quad - \sqrt{2} \int \mathrm{d}^4\theta \, (\mathring{\Psi} - \overline{\mathring{\Psi}}) \sum_{a=1}^{k} (C_a - \overline{C}_a) \\
&\quad + \sum_{a=1}^{k} \Big\{ \sqrt{2} \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + \sqrt{2} \int \mathrm{d}^2\widetilde{\theta} \, t_a \Sigma_a + (\text{h.c.}) \Big\} - \sqrt{2} \, \varepsilon^{mn} \sum_{a=1}^{k} \partial_m (X^9 A_{n,a})
\end{aligned}$$

$\mathcal{N} = (4,4)$	$\mathcal{N} = (2,2)$		役割
neutral HM	twisted chiral $\Xi = \frac{1}{\sqrt{2}}(X^6 + \mathrm{i}\, Y^8) + \ldots$	chiral $\Gamma = \frac{1}{\sqrt{2}}(X^7 + \mathrm{i}\, Y^9) + \ldots$	時空座標
VMs	twisted chiral $\Sigma_a = \frac{1}{\sqrt{2}} \overline{D}_+ D_- V_a$	chiral $\Phi_a = \overline{D}_+ \overline{D}_- C_a$	isometry のゲージ化
charged HMs	chiral Q_a $(+)$	chiral $\widetilde{Q}_a$ $(-)$	時空を曲げる
FI parameters	$s_a = s_{6,a} + \mathrm{i}\, s_{8,a}$	$t_a = t_{7,a} + \mathrm{i}\, t_{9,a}$	five-branes の位置

$$\mathscr{L}^{\text{kin}}_{\text{new}} = \sum_a \frac{1}{e_a^2}\Big[\frac{1}{2}(F_{01,a})^2 - |\partial_m \sigma_a|^2 - |\partial_m \phi_a|^2\Big] - \sum_a \Big[|D_m q_a|^2 + |D_m \widetilde{q}_a|^2\Big]$$
$$-\frac{1}{2g^2}\Big[(\partial_m X^6)^2 + (\partial_m X^7)^2\Big] - \frac{g^2}{2}\Big[(\partial_m Y^8)^2 + (D_m Y^9)^2\Big] - \sqrt{2}\sum_a (X^9 - t_{9,a})\, F_{01,a}$$

$$\mathscr{L}^{\text{pot}}_{\text{new}} = -2g^2 \sum_{a,b}(\sigma_a \overline{\sigma}_b + 4 M_{c,a}\overline{M}_{c,b}) - 2\sum_a \big(|\sigma_a|^2 + 4|M_{c,a}|^2\big)\big(|q_a|^2 + |\widetilde{q}_a|^2\big)$$
$$-\sum_a \frac{e_a^2}{2}\Big(|q_a|^2 - |\widetilde{q}_a|^2 - \sqrt{2}(X^7 - t_{7,a})\Big)^2 - \sum_a e_a^2 \Big|\sqrt{2}\, q_a \widetilde{q}_a - (X^6 - s_{6,a}) - \mathrm{i}(X^8 - s_{8,a})\Big|^2$$
$$+ \frac{g^2}{2}\sum_{a,b}(A_{c=,a} + \overline{A}_{c=,a})(B_{c+\!\!+,b} + \overline{B}_{c+\!\!+,b})$$

$$(\partial_0 + \partial_1) X^8 = -g^2 (\partial_0 + \partial_1) Y^8 + g^2 \sum_a (B_{c+\!\!+,a} + \overline{B}_{c+\!\!+,a})$$
$$(\partial_0 - \partial_1) X^8 = +g^2 (\partial_0 - \partial_1) Y^8 + g^2 \sum_a (A_{c=,a} + \overline{A}_{c=,a})$$

$$+\frac{g^2}{2}\sum_{a,b}(A_{c=,a} + \overline{A}_{c=,a})(B_{c+\!\!+,b} + \overline{B}_{c+\!\!+,b}) = -\frac{1}{2g^2}(\partial_m X^8)^2 + \frac{g^2}{2}(\partial_m Y^8)^2 + \varepsilon^{mn}(\partial_m X^8)(\partial_n Y^8)$$

GLSM is a powerful tool, also in this stage :

Worldsheet instantons in NLSM can be captured by soliton (vortex) solutions in gauge theory

Take the configuration : $\phi_a = 0 = \sigma_a$ with $g^2 \to 0$ and finite e_a^2

$$\mathscr{L}_{\mathsf{E}} = \sum_{a=1}^{k} \Big[\frac{1}{2e_a^2} (F_{12,a})^2 + |D_m q_a|^2 + \frac{e_a^2}{2} \big(|q_a|^2 - \sqrt{2}\, \zeta_a \big)^2 + \mathrm{i} \sqrt{2}\, X^9\, F_{12,a} \Big]$$

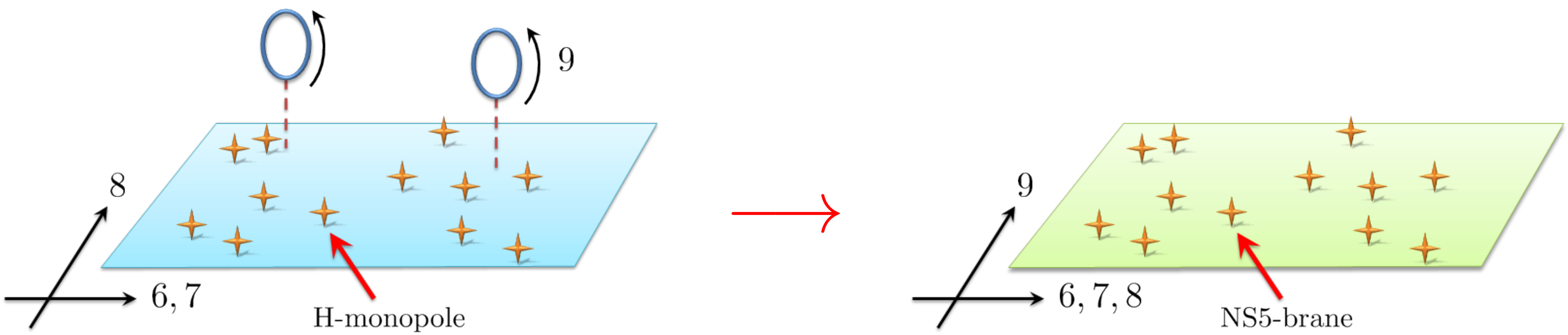
$$F_{12,a} = \mp e_a^2 \big(|q_a|^2 - \sqrt{2}\, \zeta_a \big) \, , \qquad 0 = (D_1 \pm \mathrm{i} D_2) q_a$$

Abrikosov-Nielsen-Olesen vortex eq.

then, $S_{\mathsf{E}} = \frac{1}{2\pi} \int \mathrm{d}^2 x\, \mathscr{L}_{\mathsf{E}} = \sqrt{2} \sum_{a=1}^{k} \Big(\zeta_a |n_a| - \mathrm{i}\, X^9\, n_a \Big)$ $\quad n_a = -\frac{1}{2\pi} \int \mathrm{d}^2 x\, F_{12,a}$

Worldsheet instanton corrections to GLSM for H-monopoles :

unfolding effect on compactified circle x^9



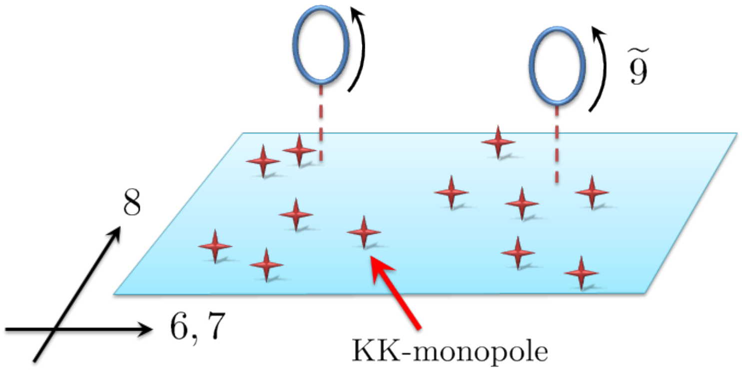
$$H = \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2} R_a} \to \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2} R_a} \sum_{n_a=1}^{\infty} \mathrm{e}^{-n_a R_a} \Big[\mathrm{e}^{+\mathrm{i}\, n_a X^9} + \mathrm{e}^{-\mathrm{i}\, n_a X^9} \Big]$$

$$= \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2} R_a} \frac{\sinh(R_a)}{\cosh(R_a) - \cos(X^9)}$$

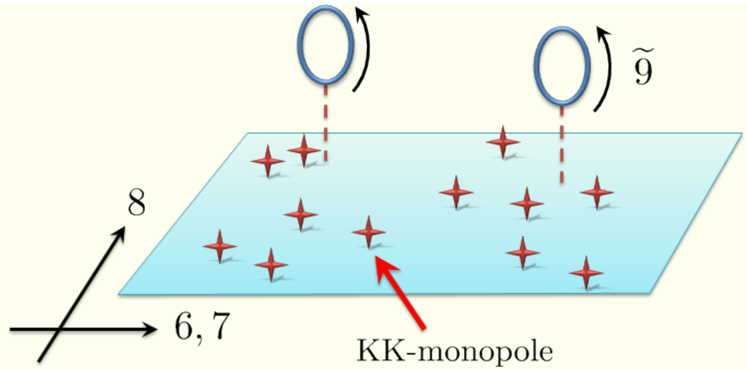
D. Tong hep-th/0204186

Worldsheet instanton corrections to GLSM for KK-monopoles :

string winding modes along x^9



$$H \to \frac{1}{g^2} + \sum_{a=1}^{k} \frac{1}{\sqrt{2}R_a} \frac{\sinh(R_a)}{\cosh(R_a) - \cos(X^9)}$$



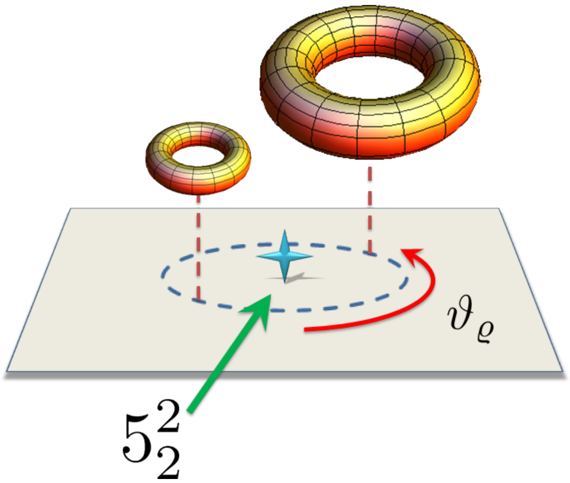
+

string winding along
dual circle x^9

J. Harvey and S. Jensen hep-th/0507204; K. Okuyama hep-th/0508097

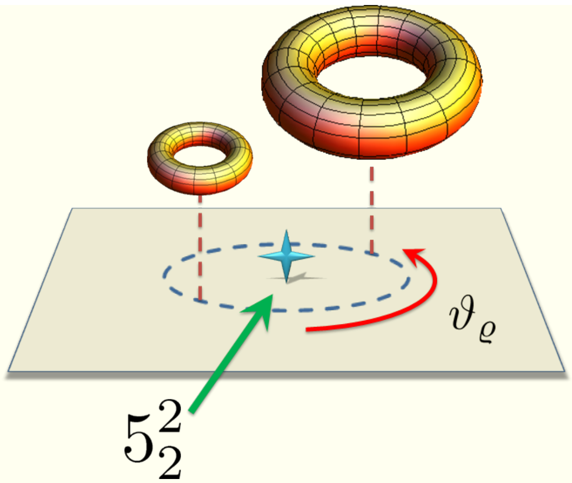
Worldsheet instanton corrections to GLSM for 5^2_2-brane :

string winding modes along x^9



$$H \quad \rightarrow \quad h_0 + \sigma \log(\mu/\varrho) + \sigma \sum_{n \neq 0} \exp(\mathrm{i}\, n\, X^9) K_0(|n|\varrho)$$

$\longrightarrow$



$+$

string winding along dual circle x^9

S. Sasaki and TK arXiv:1305.4439

$$
\begin{aligned}
\Psi &= \frac{1}{\sqrt{2}}(X^6 + \mathrm{i} X^8) + \mathrm{i}\sqrt{2}\,\theta^+ \chi_+ + \mathrm{i}\sqrt{2}\,\theta^- \chi_- + 2\mathrm{i}\,\theta^+\theta^- G + \ldots \\
\Xi &= \frac{1}{\sqrt{2}}(Y^6 + \mathrm{i}\, Y^8) + \mathrm{i}\sqrt{2}\,\theta^+ \overline{\xi}_+ + \mathrm{i}\sqrt{2}\,\overline{\theta}^- \xi_- + 2\mathrm{i}\,\theta^+\overline{\theta}^- G_{\Xi} + \ldots \\
\Theta &= \frac{1}{\sqrt{2}}(X^7 + \mathrm{i} X^9) + \mathrm{i}\sqrt{2}\,\theta^+ \overline{\widetilde{\chi}}_+ + \mathrm{i}\sqrt{2}\,\overline{\theta}^- \widetilde{\chi}_- + 2\mathrm{i}\,\theta^+\overline{\theta}^- \widetilde{G} + \ldots \\
\Gamma &= \frac{1}{\sqrt{2}}(Y^7 + \mathrm{i}\, Y^9) + \mathrm{i}\sqrt{2}\,\theta^+ \zeta_+ + \mathrm{i}\sqrt{2}\,\theta^- \zeta_- + 2\mathrm{i}\,\theta^+\theta^- G_{\Gamma} + \ldots \\
Q_a &= q_a + \mathrm{i}\sqrt{2}\,\theta^+ \psi_{+,a} + \mathrm{i}\sqrt{2}\,\theta^- \psi_{-,a} + 2\mathrm{i}\,\theta^+\theta^- F_a + \ldots \\
\widetilde{Q}_a &= \widetilde{q}_a + \mathrm{i}\sqrt{2}\,\theta^+ \widetilde{\psi}_{+,a} + \mathrm{i}\sqrt{2}\,\theta^- \widetilde{\psi}_{-,a} + 2\mathrm{i}\,\theta^+\theta^- \widetilde{F}_a + \ldots \\
V_a &= \theta^+\overline{\theta}^+(A_{0,a} + A_{1,a}) + \theta^-\overline{\theta}^-(A_{0,a} - A_{1,a}) - \sqrt{2}\,\theta^-\overline{\theta}^+\sigma_a - \sqrt{2}\,\theta^+\overline{\theta}^-\overline{\sigma}_a \\
&\quad -2\mathrm{i}\,\theta^+\theta^-\big(\overline{\theta}^+\overline{\lambda}_{+,a} + \overline{\theta}^-\overline{\lambda}_{-,a}\big) + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^-\big(\theta^+\lambda_{+,a} + \theta^-\lambda_{-,a}\big) - 2\,\theta^+\theta^-\overline{\theta}^+\overline{\theta}^- D_{V,a} \\
\Phi_a &= \phi_a + \mathrm{i}\sqrt{2}\,\theta^+ \widetilde{\lambda}_{+,a} + \mathrm{i}\sqrt{2}\,\theta^- \widetilde{\lambda}_{-,a} + 2\mathrm{i}\,\theta^+\theta^- D_{\Phi,a} + \ldots \qquad = \overline{D}_+\overline{D}_- C_a \\
C_a &= \phi_{c,a} + \mathrm{i}\sqrt{2}\,\theta^+\psi_{c+,a} + \mathrm{i}\sqrt{2}\,\theta^-\psi_{c-,a} + \mathrm{i}\sqrt{2}\,\overline{\theta}^+\chi_{c+,a} + \mathrm{i}\sqrt{2}\,\overline{\theta}^-\chi_{c-,a} \\
&\quad +2\mathrm{i}\,\theta^+\theta^- F_{c,a} + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^- M_{c,a} + 2\mathrm{i}\,\theta^+\overline{\theta}^- G_{c,a} + 2\mathrm{i}\,\overline{\theta}^+\theta^- N_{c,a} + \theta^-\overline{\theta}^- A_{c=,a} + \theta^+\overline{\theta}^+ B_{c\#,a} \\
&\quad -2\mathrm{i}\,\theta^+\theta^-\overline{\theta}^+\zeta_{c+,a} - 2\mathrm{i}\,\theta^+\theta^-\overline{\theta}^-\zeta_{c-,a} + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^-\theta^+\lambda_{c+,a} + 2\mathrm{i}\,\overline{\theta}^+\overline{\theta}^-\theta^-\lambda_{c-,a} - 2\theta^+\theta^-\overline{\theta}^+\overline{\theta}^- D_{c,a}
\end{aligned}
$$

with $\overline{D}_\pm = -\dfrac{\partial}{\partial\overline{\theta}^\pm} + \mathrm{i}\,\theta^\pm(\partial_0 \pm \partial_1)$, $D_\pm = \dfrac{\partial}{\partial\theta^\pm} - \mathrm{i}\,\overline{\theta}^\pm(\partial_0 \pm \partial_1)$

H KK 522