F-theory Family Unification:

A New Geometric Mechanism for Unparallel Three Families and Large Lepton-flavor Mixings

Shun'ya Mizoguchi KEK, Theory Center

- "*F-theory Family Unification*'': S.M., JHEP 1407 (2014) 018 arXiv:1403.7066 [hep-th]
- "*Large Lepton-flavor Mixings from E8 Kodaira Singularity: Lopsided Texture via F-theory Family Unification* ": S.M. arXiv:1407.1319 [hep-th]

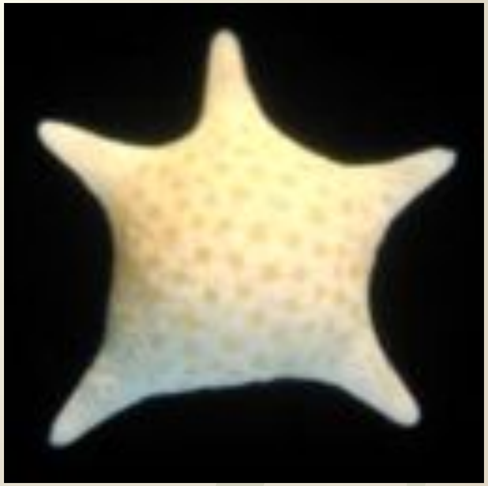
Jul.23, 2014
YITP Workshop
Strings and Fields

http://bios.sakura.ne.jp/gf/2003/starsand.html

Star Sand
（星砂）

“Star sand” is not sand

“Star sand Iriomote” by Geomr
http://en.wikipedia.org/wiki/File:Star_sand_Iriomote.jpg



Copyright (c) 1998-2010 by Gen-yu SASAKI
http://bios.sakura.ne.jp/gf/2003/starsand.html

“Tests of faraminfers”（有孔虫の外殻）：
Some kind of shells of tiny creatures

Being NEVER does not mean near-by beaches

You'll find many stands at you can find what you want

Katase-higashihama beach

How can we find star sand?

- Look into it carefully
- Find out characteristic features

$CaCO_3$

Creatures

http://bios.sakura.ne.jp/gf/2003/DSCN3448.jpg

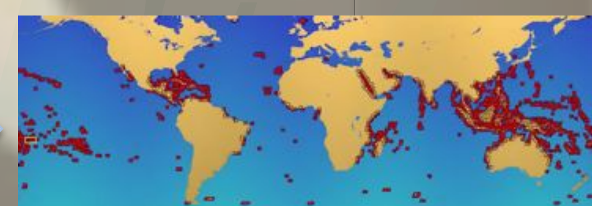
http://bios.sakura.ne.jp/gf/2003/DSCN3393.jpg

How can we find star sand?

- ◆Look into it carefully
- ◆Find out characteristic features

$CaCO_3$

→ Creatures →



Coral reefs (珊瑚礁)

http://bios.sakura.ne.jp/gf/2003/DSCN3448.jpg

http://bios.sakura.ne.jp/gf/2003/DSCN3393.jpg

Being many does not mean you can find what you want

String Landscape

© Dnally | Dreamstime Stock Photos

How can we find the Standard Model?

- Look into it carefully
- Find out characteristic features

What are characteristic features of the Standard Model?

- SU(3)xSU(2)xU(1) gauge group with a peculiar hypercharge assignment

What are characteristic features of the Standard Model?

- SU(3)xSU(2)xU(1) gauge group with a peculiar hypercharge assignment
- Three UNPARALLEL generations of quarks and leptons

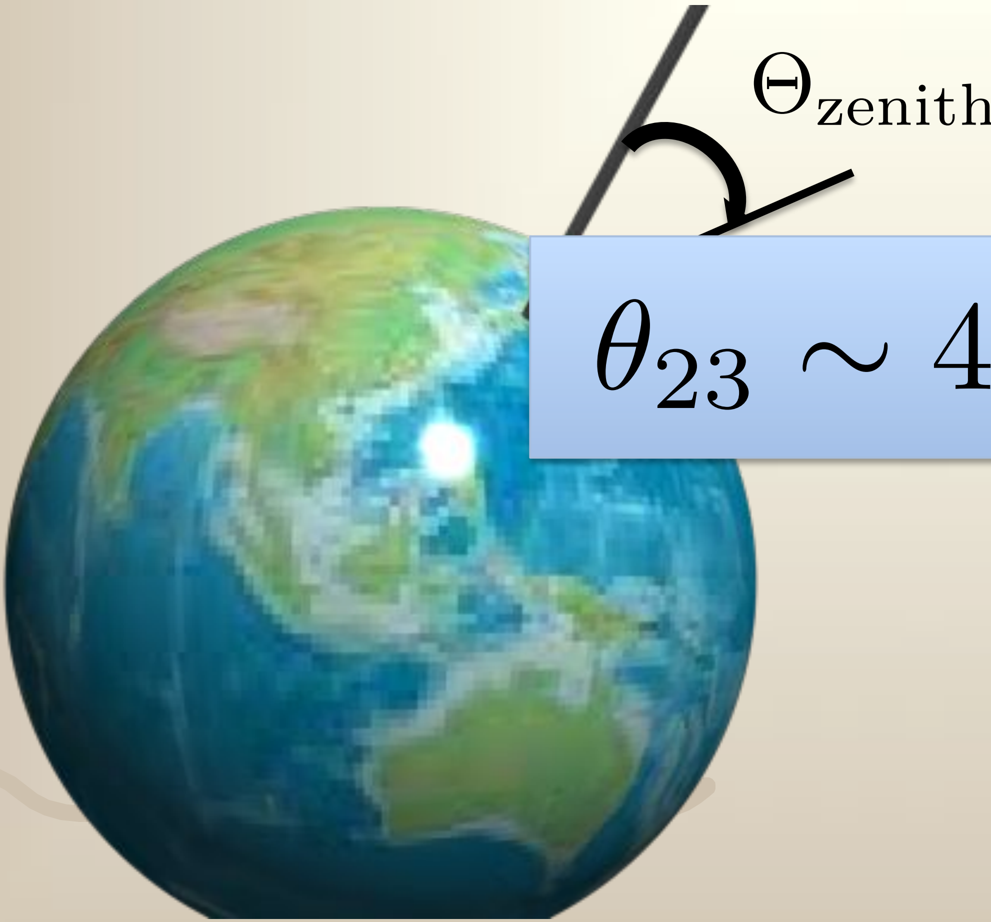
Top is heavy

$$m_{\text{top}} \sim 100,000 \times m_{\text{up}}$$

http://en.wikipedia.org/wiki/Tevatron

https://www.fnal.gov/pub/science/historical-results/

Zenith-angle dependence of the atomospheric neutrino



$$\theta_{23} \sim 45^\circ$$

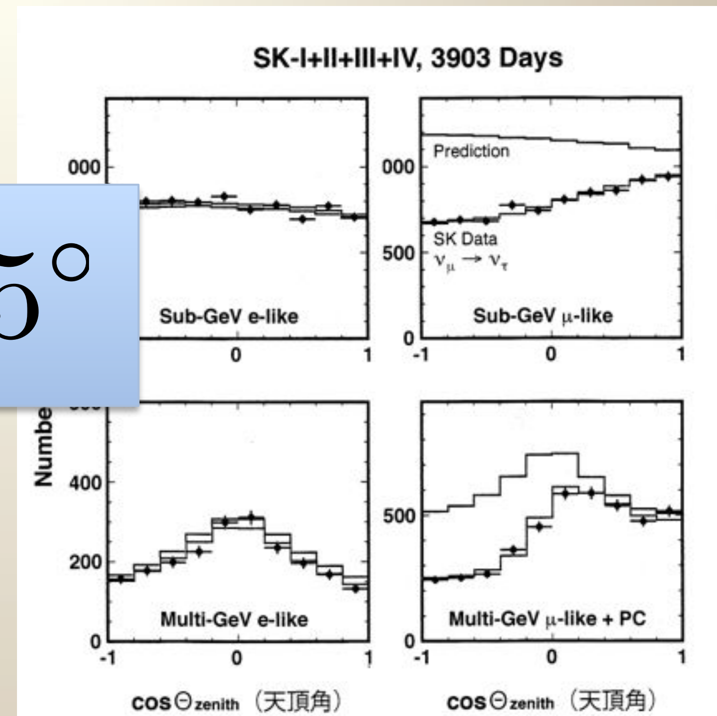
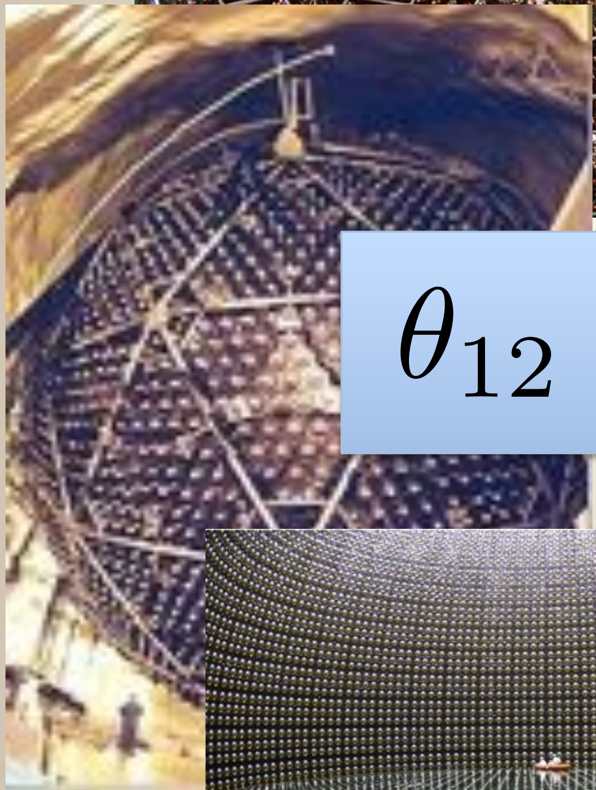


図2　Super-Kamiokande（Super-K）によって観測された大気ニュートリノの天頂角分布．図中の "Sub-GeV"（Multi-GeV）は $E \leq 1.33$ GeV（$E \geq 1.33$ GeV）のニュートリノを意味する．Super-K グループの好意による．

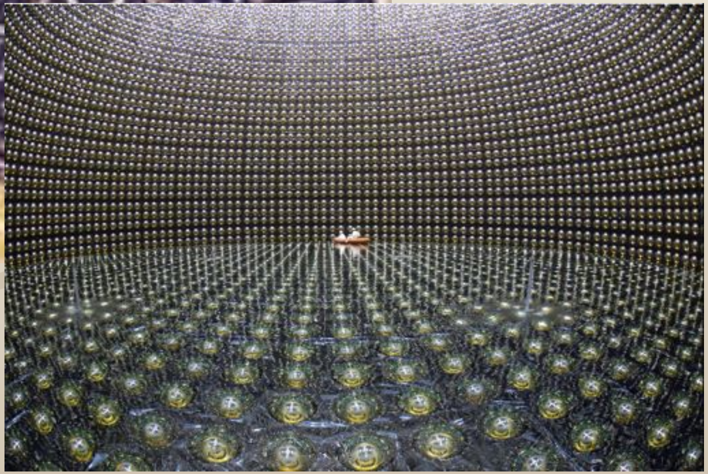
Minakata, 日本物理学会誌より

http://kamland.lbl.gov/Pictures/picgallery.html

http://neutrino.phys.ksu.edu/~dchooz/photos/



$$\theta_{12} \sim 34^\circ,\ \theta_{13} \neq 0$$



http://en.wikipedia.org/wiki/Sudbury_Neutrino_Observatory
http://www.gridpp.ac.uk/news/?p=88

http://www.quantumdiaries.org/2011/08/15/
http://hcpl.knu.ac.kr/neutrino/neutrino.html_ycyang

$$\theta_{12} \sim 34^\circ,$$

Three are not on

National Palace Museum. http://www.npm.gov.tw/en/

Standard-model-like models?

- Most (if not all) of the previous string phenomenology models REALIZE these structures by imposing artificial requirements and/or by tuning of parameters, but never EXPLAIN them
- In particular, in most cases the three generations obtained there are on EQUAL footing, and the hierarchical structure is one arranged “by hand”

What are characteristic features of the Standard Model?

- SU(3)xSU(2)xU(1) gauge group with a peculiar hypercharge assignment
- Three UNPARALLEL generations of quarks and leptons

E7

SU(5)

What are characteristic features of the Standard Model?

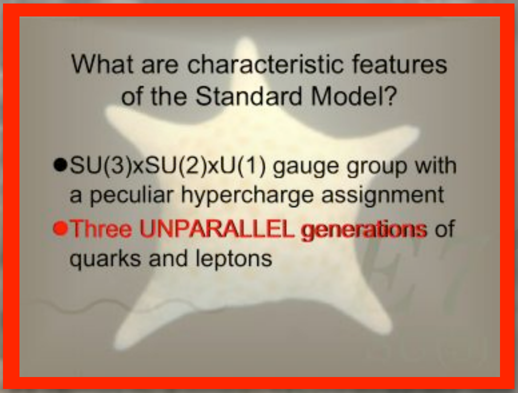
- SU(3)xSU(2)xU(1) gauge group with a peculiar hypercharge assignment
- **Three UNPARALLEL generations** of quarks and leptons

Kugo-Yangida
$E_7/(SU(5) \times U(1)^3)$
Family unification model

F-theory

How can we find the Standard Model?

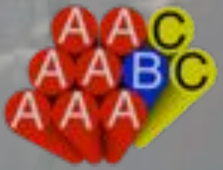
- ◆Look into it carefully
- ◆Find out characteristic features



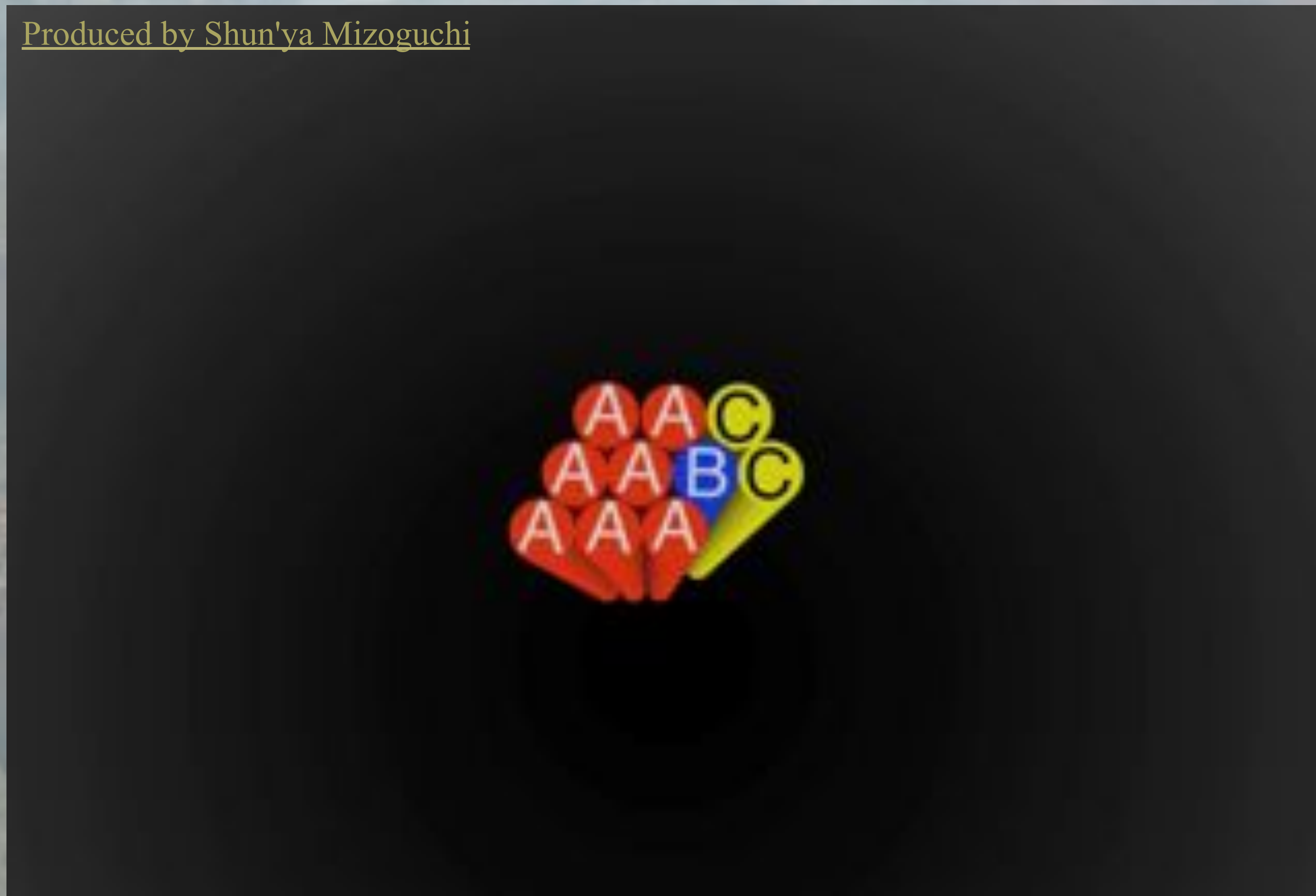
→

Kugo-Yangida
$E_7/(SU(5) \times U(1)^3)$
Family unification model

→



F-theory



Local F-theory naturally realizes Kugo-Yanagida

New geometric mechanism EXPLAINING WHY THREE generations

Plan

1. Introduction
2. Family unification
3. F-theory
4. “F-theory family unification”
5. Summary

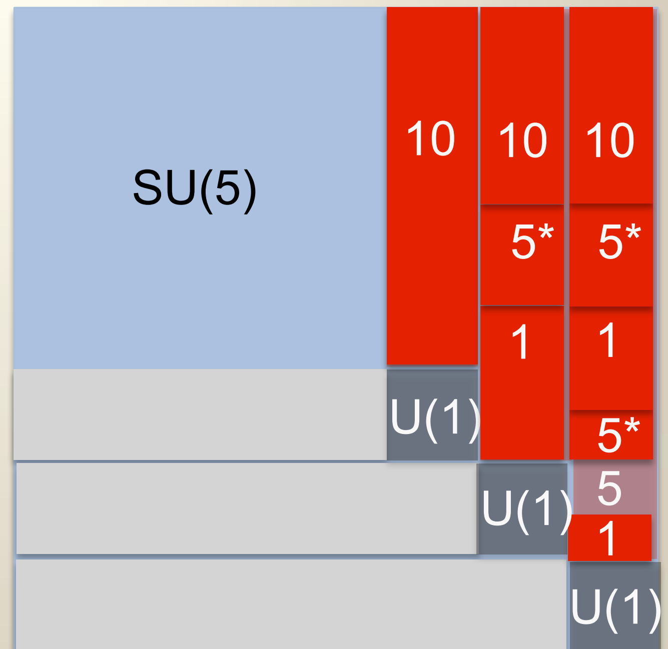
2 FAMILY UNIFICATION

Family unification

- Family unification is the idea that the quarks and leptons are the fermionic partners of the scalars of some coset supersymmetric non-linear sigma model
  Buchmuller,Peccei,Yanagida;
  Kugo,Yanagida;
  Irie,Yasui; Ong
- The importance of an “unparallel” family structure was first emphasized by Yanagida, and later by Bando, Kugo and others
  Bando,Kuramoto,Maskawa,Uehara;
  Itoh,Kugo,Kunitomo

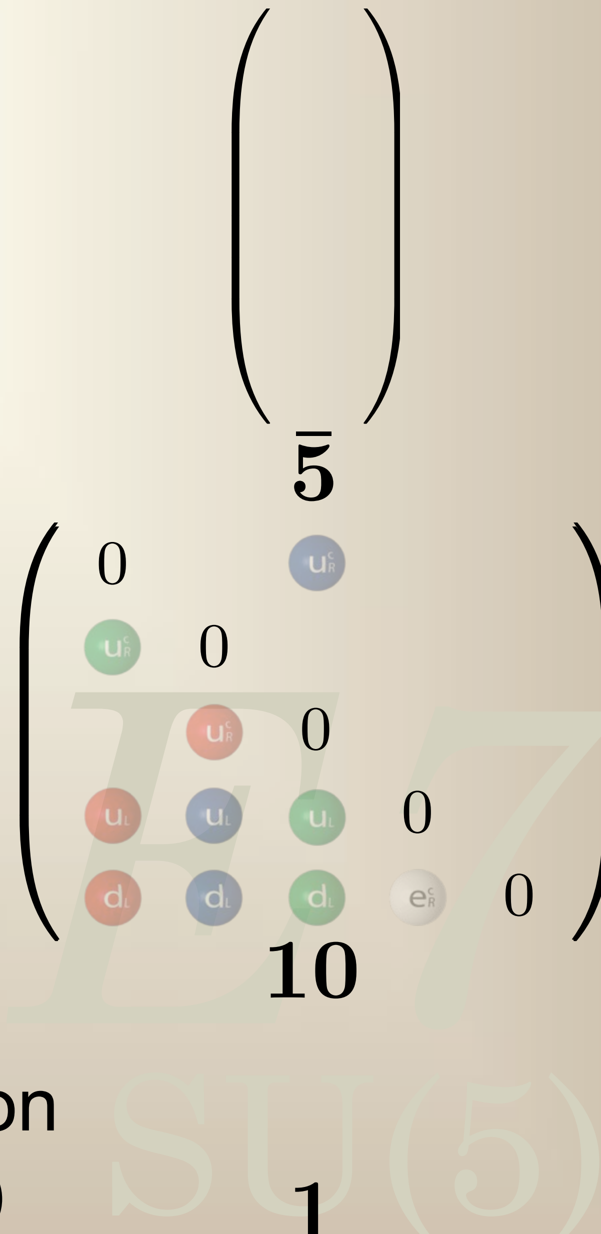
Family unification

- Remarkably, the Kugo-Yanagida model automatically realizes precisely three UNPARALLEL generations of matter fields needed for the SU(5) GUT

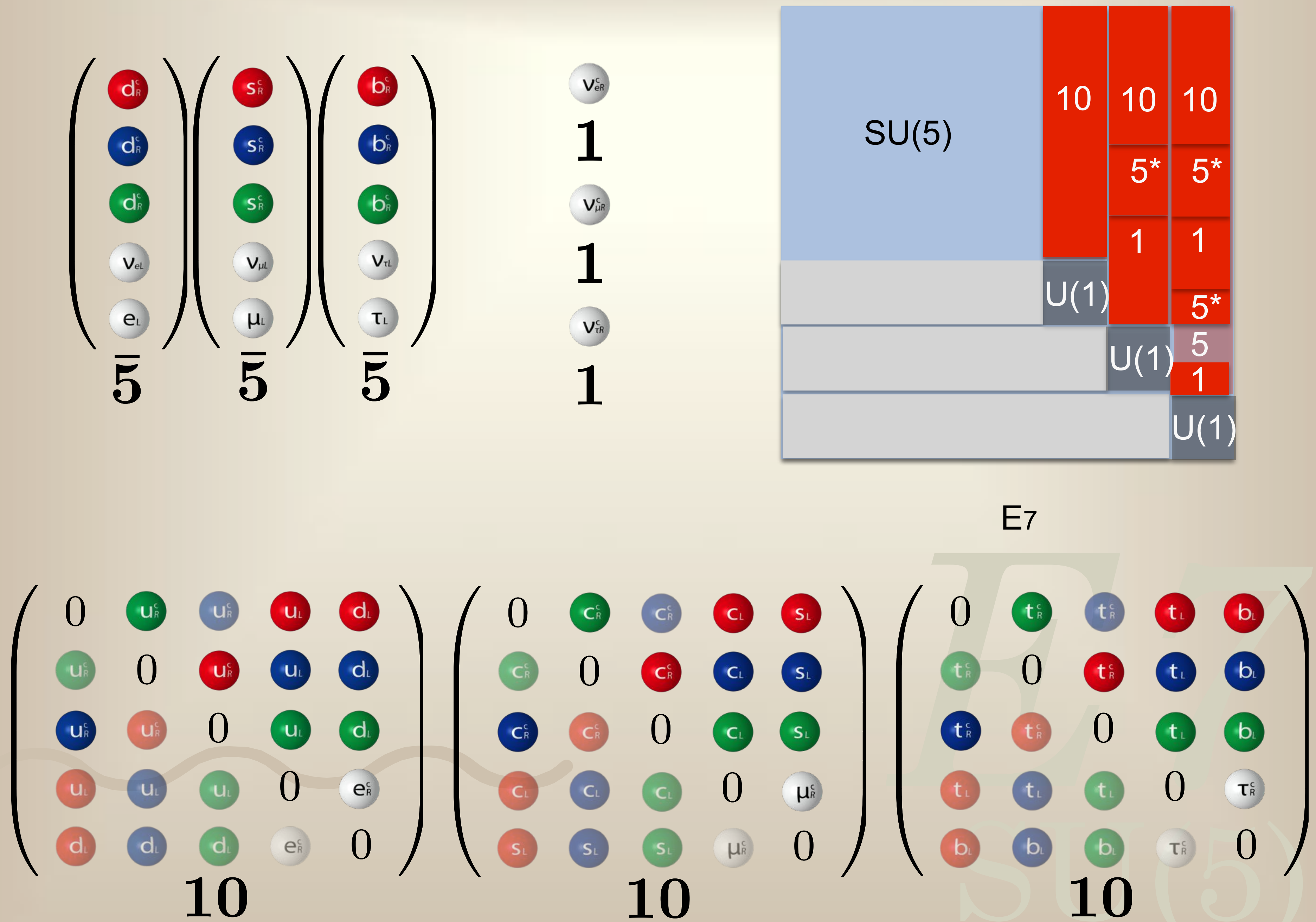


E_7

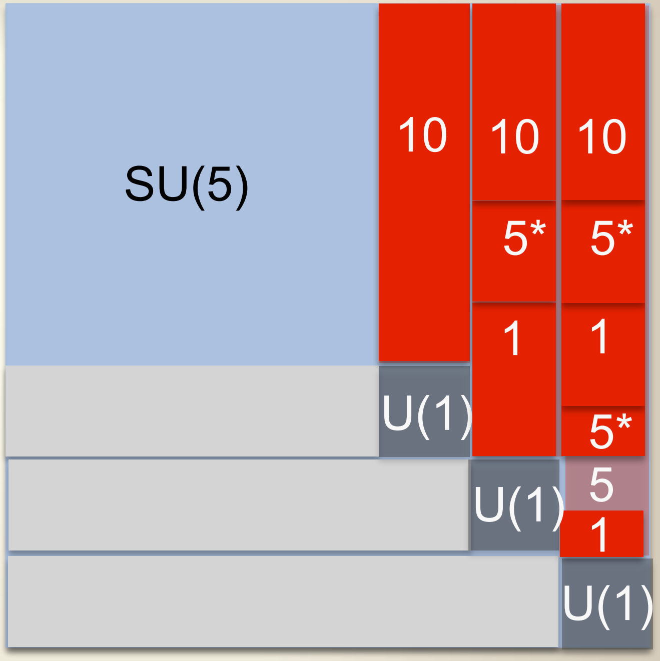
		1st generation	2nd generation	3rd generation	SU(3)	SU(2)	$U(1)_Y$
					Gauge quantum number		
Quark		u_L u_L u_L / d_L d_L d_L	c_L c_L c_L / s_L s_L s_L	t_L t_L t_L / b_L b_L b_L	3	2	1/6
		u^c_R u^c_R u^c_R	c^c_R c^c_R c^c_R	t^c_R t^c_R t^c_R	3	1	2/3
		d^c_R d^c_R d^c_R	s^c_R s^c_R s^c_R	b^c_R b^c_R b^c_R	3	1	-1/3
Lepton		ν_{eL} / e_L	$\nu_{\mu L}$ / μ_L	$\nu_{\tau L}$ / τ_L	1	2	-1/2
		ν^c_{eR}	$\nu^c_{\mu R}$	$\nu^c_{\tau R}$	1	1	-1
		e^c_R	μ^c_R	τ^c_R	1	1	0



Quarks and Leptons in one generation are grouped into $\bar{\mathbf{5}}$,**10** and **1** of SU(5)



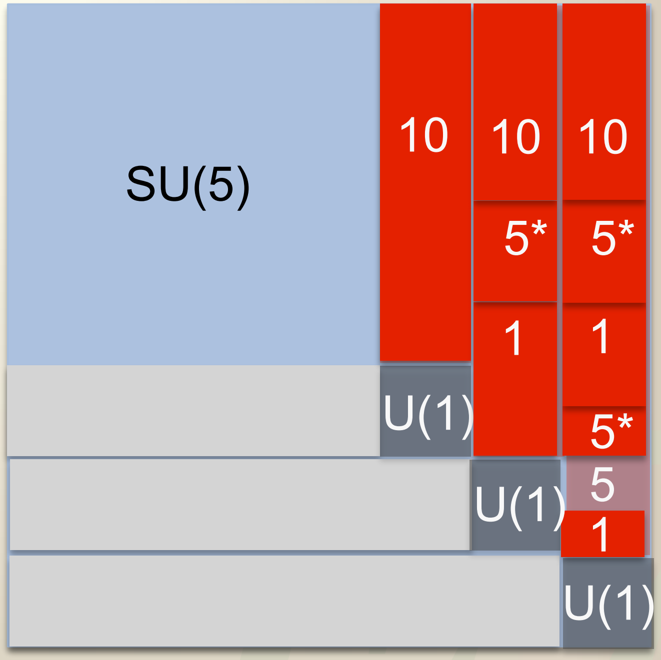
$\bar{5}$
$\bar{5}$
$\bar{5}$
1
1
1
SU(5)
10
10
10
5*
5*
1
1
5*
U(1)
U(1)
5
1
U(1)
E7
10
10
10



E7/(SU(5) × U(1)^3) Kugo-Yanagida model realizes almost minimal necessary matter content for an SU(5) GUT in an amazingly economical way

Family unification in F-THEORY

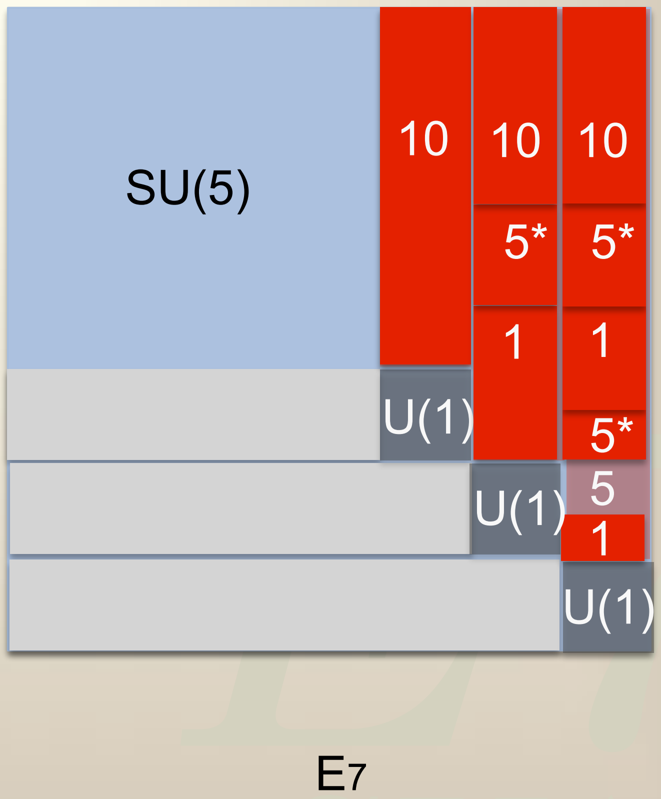
- We will show that "F-THEORY" can naturally realize such a group coset structure
- To my knowledge this is the first string-theory realization of the old idea of "family unification"



E_7

Family unification in F THEORY

- We are interested in some local geometric structure that can realize precisely three “unparallel" families
- This is because if the realization of the SM were a consequence of the global details of the entire compactification space, it would be very hard, if not impossible, to find any “reason” or “explanation” for what we observe now



3 F-THEORY

Four essential aspects of F-theory

1. Instead of considering a configuration of the IIB complex scalar $\tau = C_0 + ie^{-\phi}$, one considers a configuration of a FICTITIOUS torus whose modulus equals τ Vafa

A complex scalar τ depending only on z

http://commons.wikimedia.org/wiki/File:WorldMap-A_non-Frame.png

→ A family of tori whose shapes vary from point to point

Elliptic fibration
(「楕円」ファイブレーション)

The total space is represented as a fiber bundle whose fiber is a torus

トーラスの周期が楕円関数を使って
表せるから「楕円」という

Four essential aspects of F-theory

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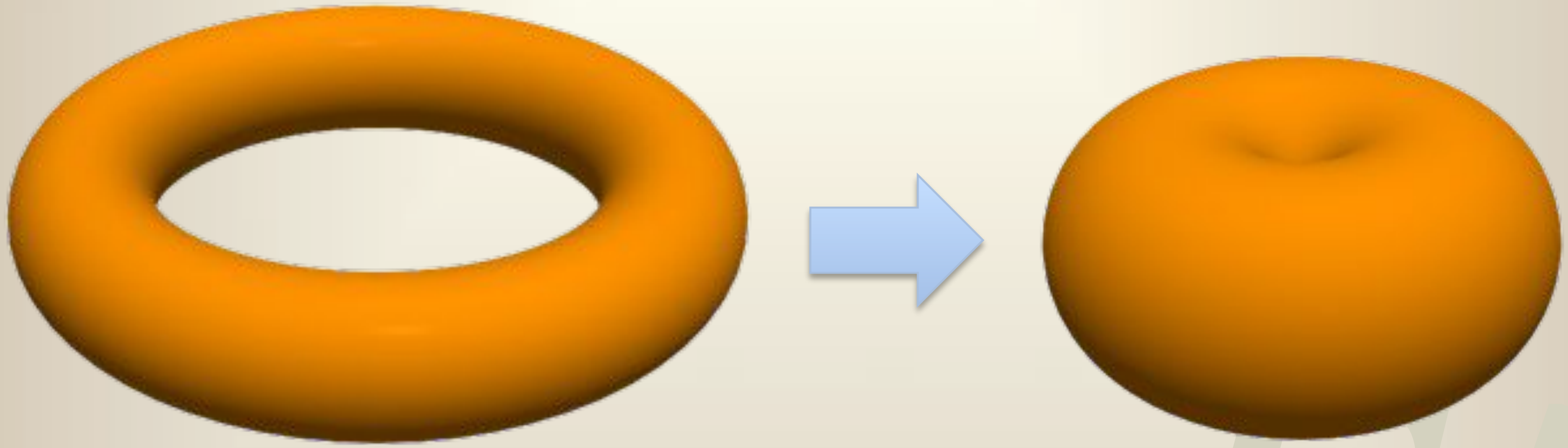
2. 7-branes are located where an elliptic (=torus) fiber degenerate and becomes singular

Degenerate torus : Singular fiber

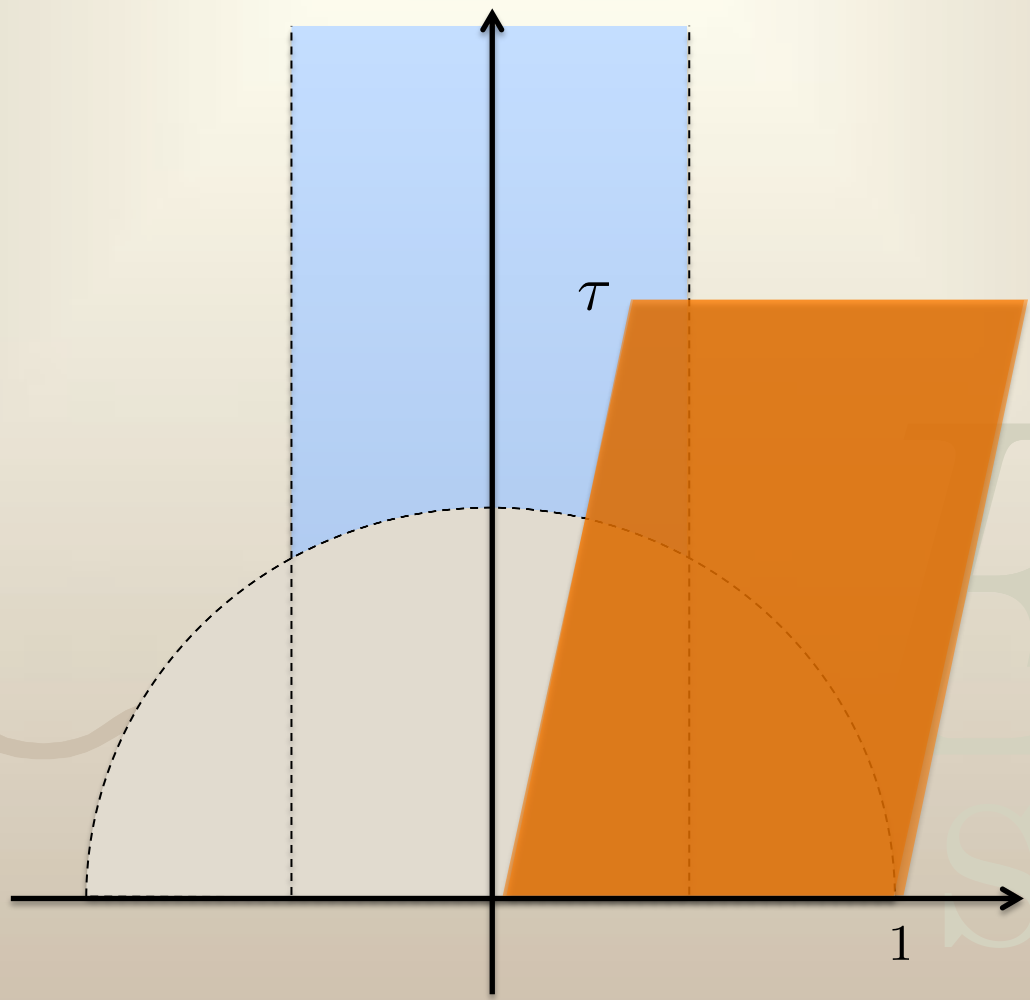


「ドーナツ」が「あんドーナツ」になる

A donut becomes “a donut hole”

http://www.fromaway.com/cooking/deep-fried-buttermilk-doughnut-holes-with-cinnamon-and-sugar

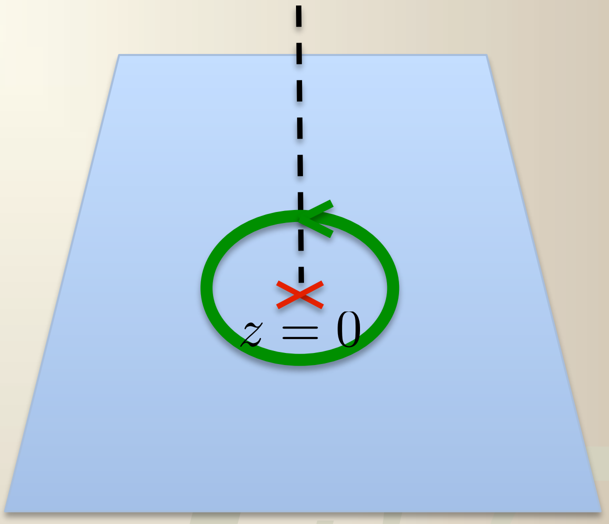
The shape of a torus: the complex structure modulus τ



D7-brane where a donut hole sits

$$\tau = \frac{1}{2\pi i} \log z, \quad f(z) = -\frac{1}{12} \log z$$

Let $z = \epsilon e^{i\theta}$



➡ $$\tau = \frac{1}{2\pi i}(\log \epsilon + i\theta)$$

➡ $$C = \mathrm{Im}\tau = \frac{\theta}{2\pi}$$

Magnetic flux:

$$\begin{aligned} \int_0^{2\pi} d\theta \partial_\theta C &= \int_0^{2\pi} d\theta \frac{1}{2\pi} \\ &= 1 \end{aligned}$$

Magnetically charged object

Four essential aspects of F-theory

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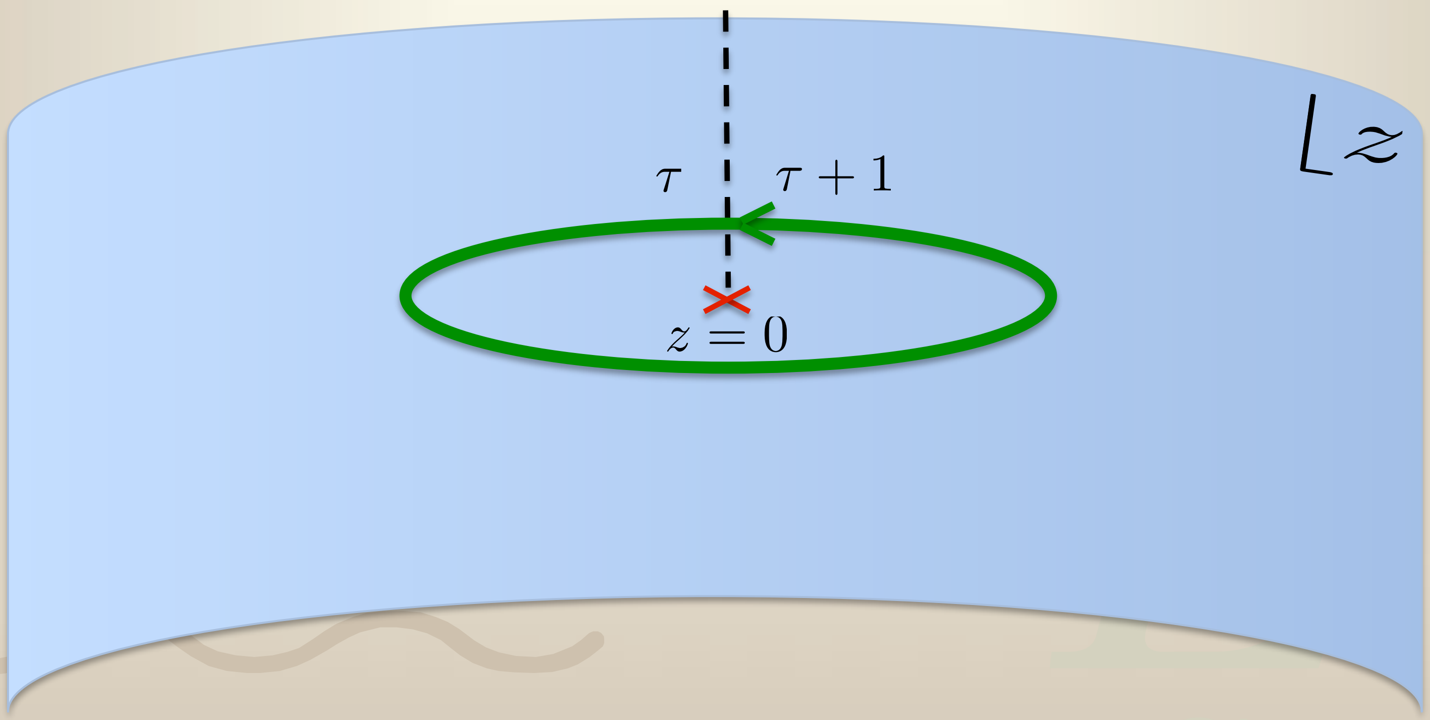
2. 7-branes are located where an elliptic (=torus) fiber degenerate and becomes singular
3. Singularities of elliptic fiberations were classified according to their types investigated by Kodaira Kodaira
4. The Kodaira singularities are described by joining/parting of 7-branes, which involves not only D-branes but general (p,q) branes DeWolfe,Hauer,Iqbal,Zwiebach

Monodromy where a donut hole sits

$$\tau = C_0 + ie^{-\phi} \quad = \frac{1}{2\pi i} \log z$$

$$z \to e^{2\pi i} z, \quad \tau \to \tau + 1$$



A,B,C: 7-branes with a different monodromy

$$K_{[p,q]} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}$$

$$A = K_{[1,0]} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \qquad B = K_{[1,-1]} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$$

$$C = K_{[1,1]} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

- All the singularity types in Kodaira's classification are described by a coalesce of A,B and C branes DeWolfe,Hauer,Iqbal,Zwiebach(1998)

Collapsible set of 7-branes are classified: Kodaira's classification

Fiber type	Singularity type	7-branes	Brane type
I_n	A_{n-1}	A^n	A_{n-1}
II	A_0	AC	H_0
III	A_1	A^2C	H_1
IV	A_2	A^3C	H_2
I_0*	D_4	A^4BC	D_4
I_n*	D_{n+4}	$A^{n+4}BC$	D_{n+4}
II*	E_8	A^7BC^2	E_8
III*	E_7	A^6BC^2	E_7
IV*	E_6	A^5BC^2	E_6

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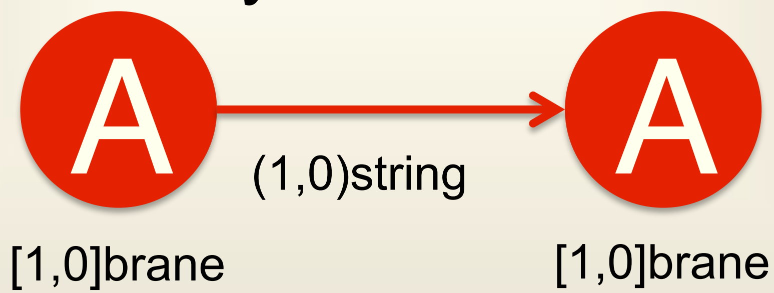
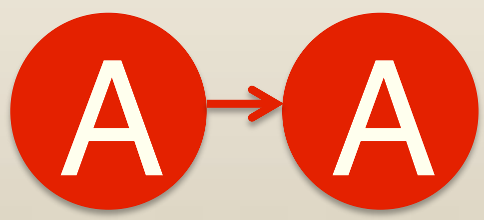
What happens if there are B- and C-branes?

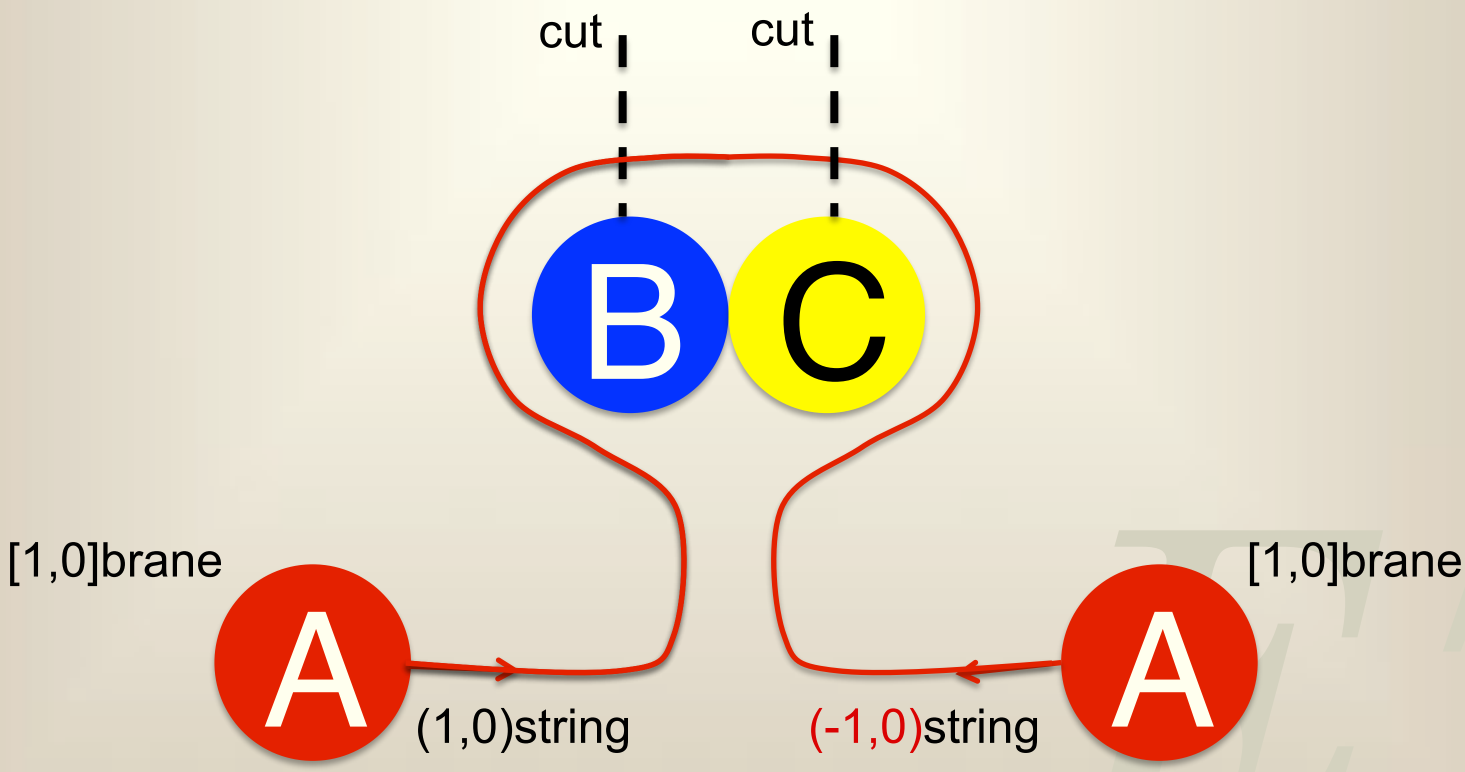
Ordinary D-brane case





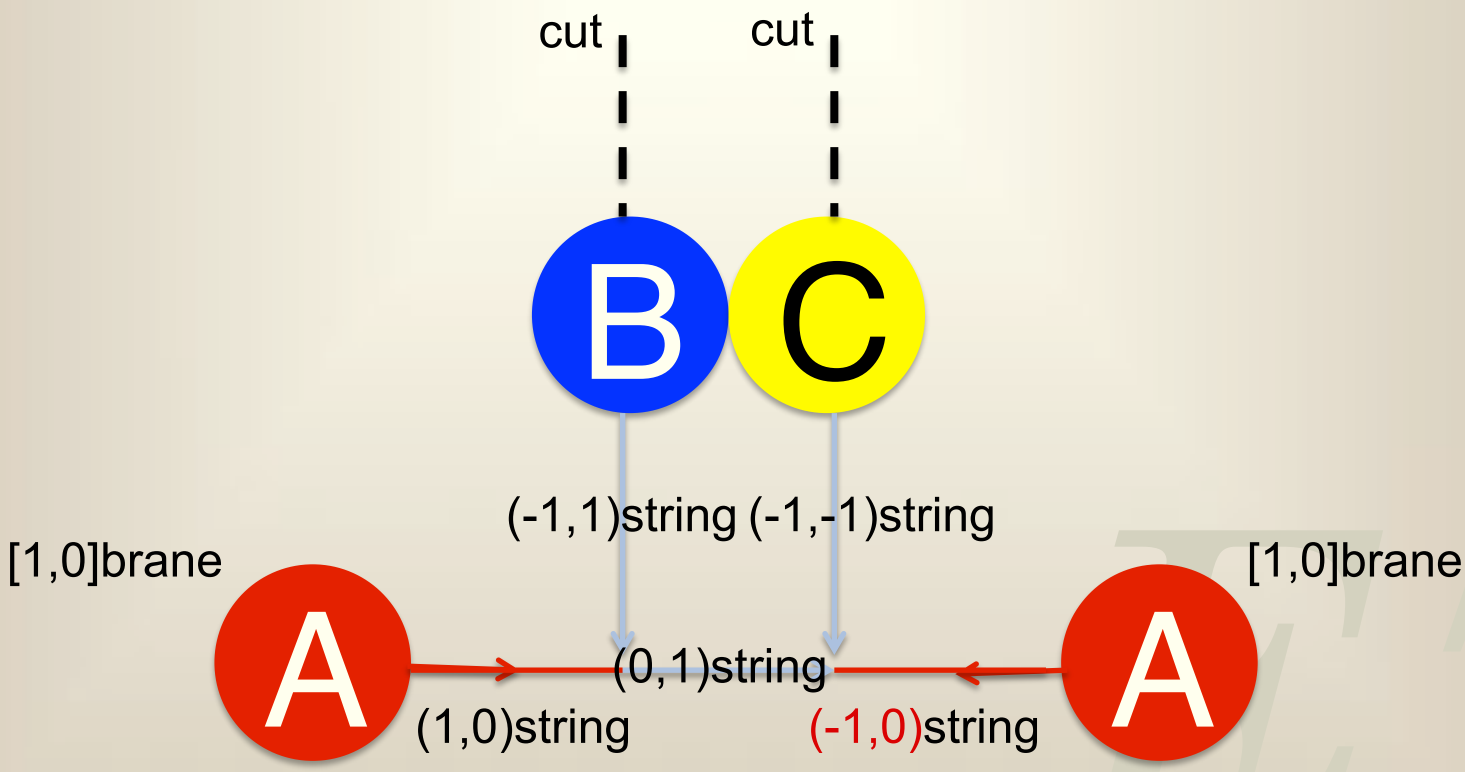
- (p,q) string can end on [p,q] branes
- Short strings between the close branes yields light “W bosons”

Monodromy around B,C branes



- (1,0) string turns into (-1,0) string ⇒ different state

String junction: (p,q) analogue of open string



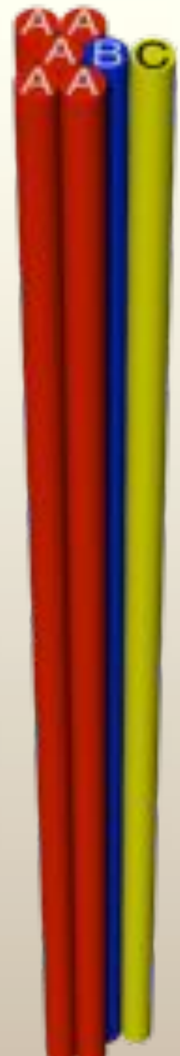
- (-1,1) and (-1,-1) strings are pulled out when the string crosses over the B and C branes

What happens if there are B- and C-branes?

- When N D-branes come on top of each other, one gets U(N) gauge symmetry Witten. In this case the relevant massless “W-bosons” are supplemented by the excitations of light open strings ending on different D-branes
- Likewise, the extra massless fields needed for the gauge symmetry enhancement to an exceptional group can be thought of as coming from the string junctions connecting the coinciding 7-branes Gaberdiel,Zwiebach

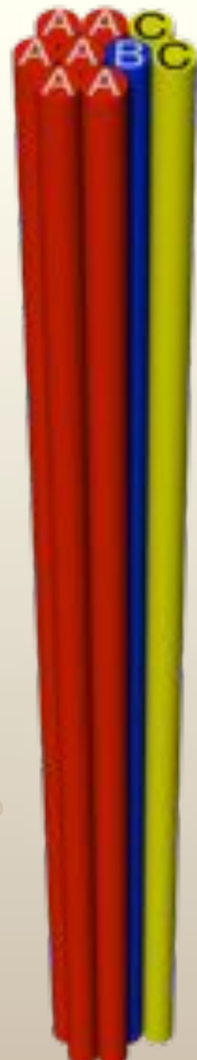
Coalesced branes and singularities: SU(5)

Coalesced branes and singularities: SO(10)



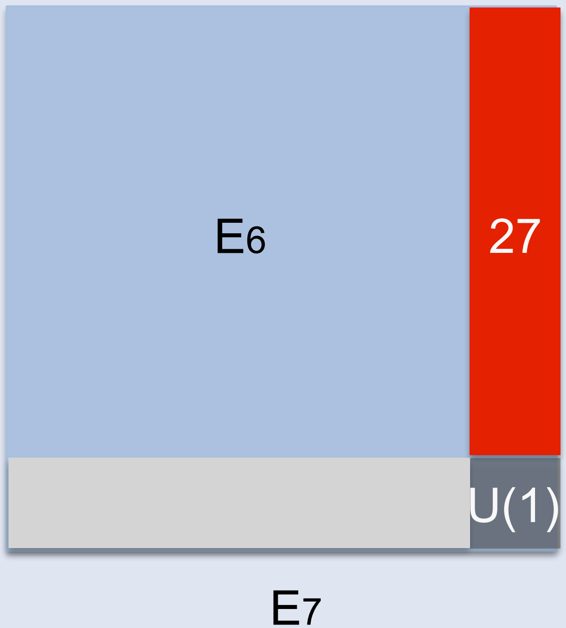
Coalesced branes and singularities: E_6

Coalesced branes and singularities: E_7



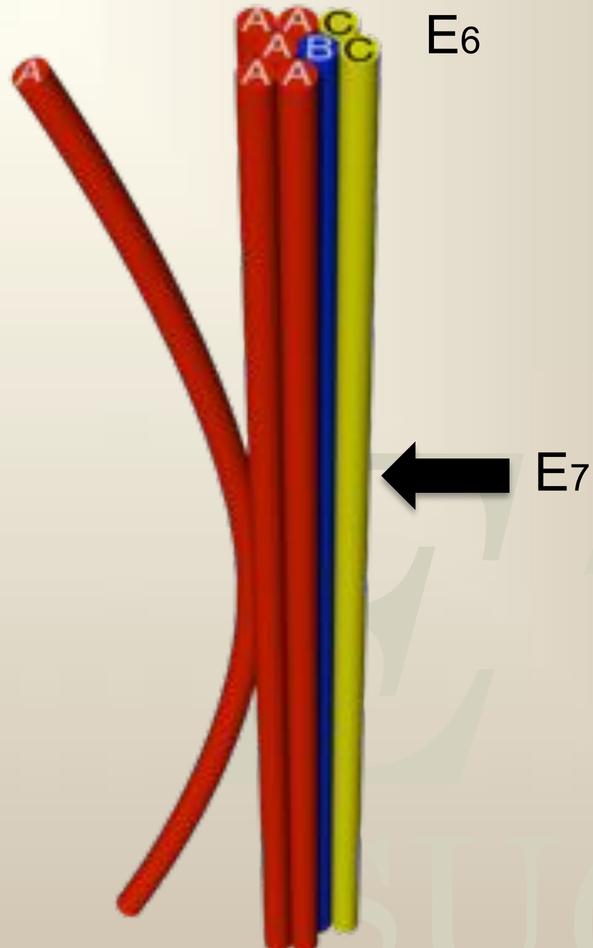
Matter from string junction”

T.Tani(谷太郎) Nucl.Phys.B602(2001)434

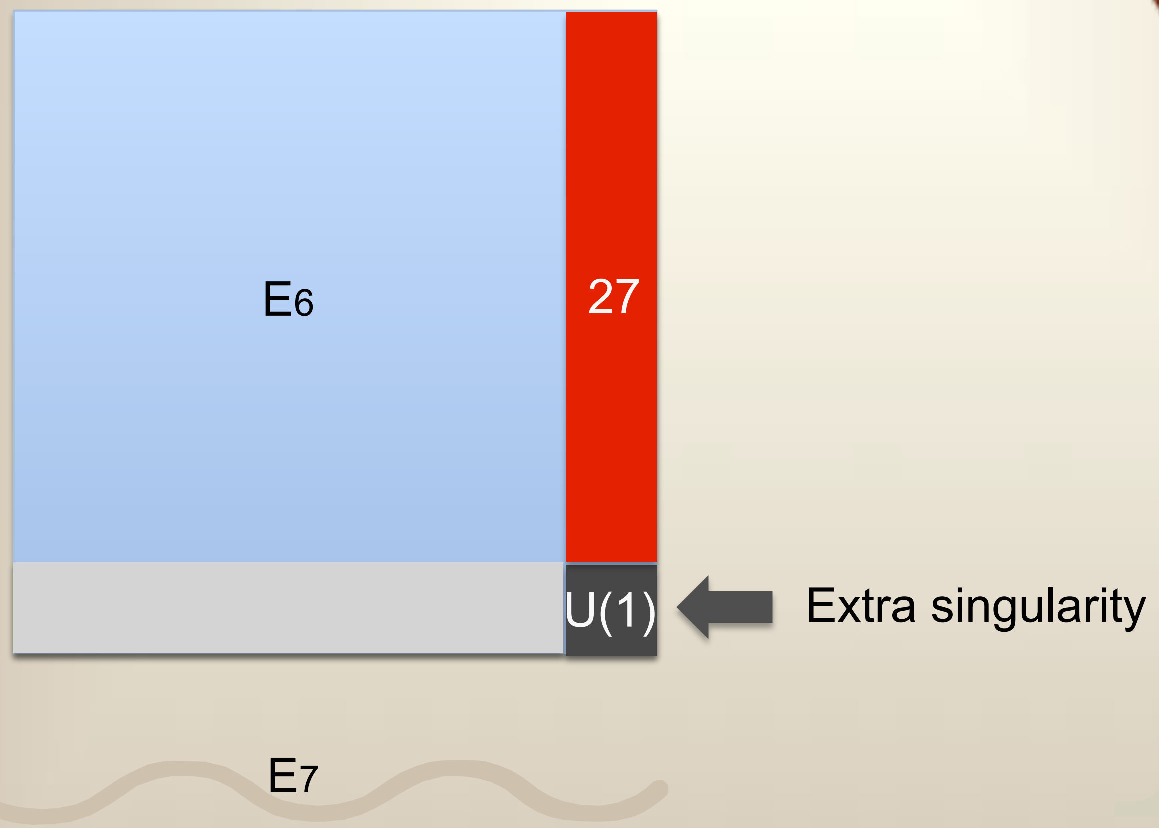


This is the same as the supersymmetric coset!!!

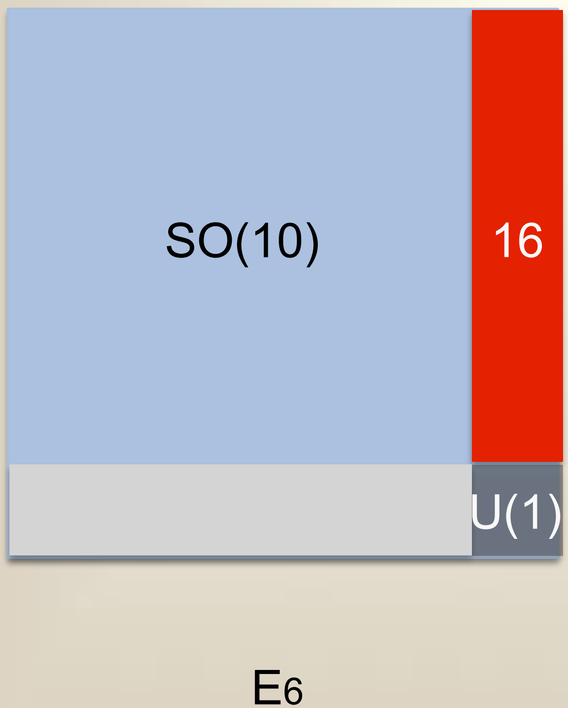
- F
- s of
  nd



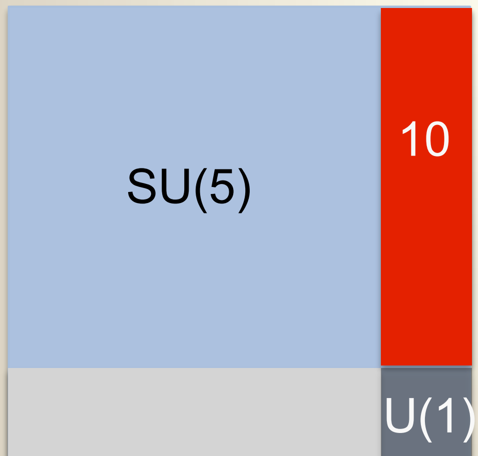
Gives a perfectly consistent picture


E6
27
U(1)
Extra singularity
E7

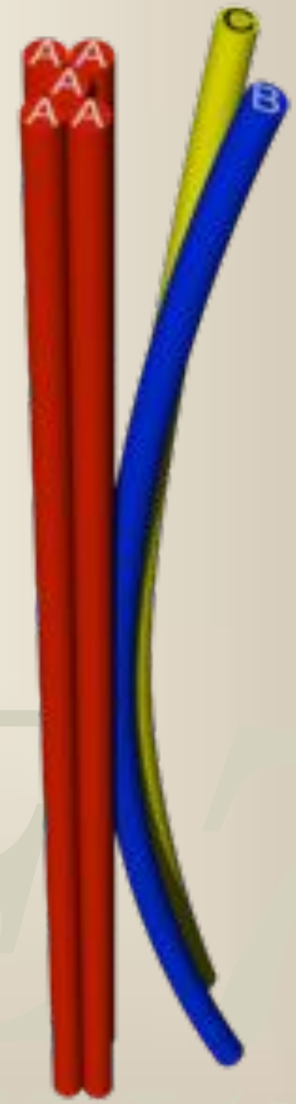

A
A
A
C
A
B
C
A
A



Extra singularity



SO(10)

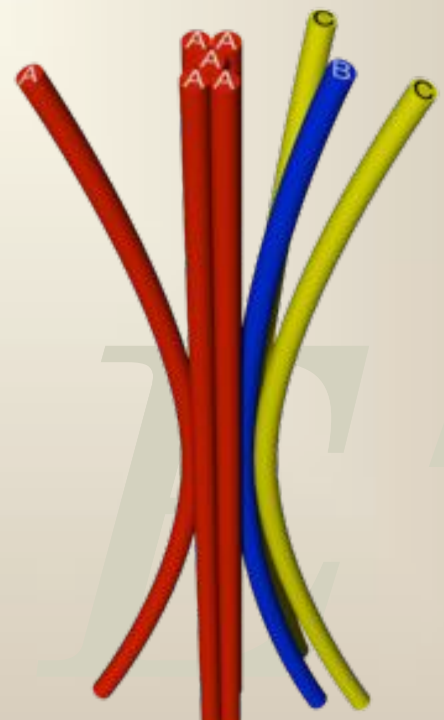


4 “F-THEORY FAMILY UNIFICATION”

Three new contributions in: “*F-theory Family Unification*”

SM, JHEP 1407(2014) 018, arXiv:1403.7066 [hep-th]

Consider a set of coalesced local 7-branes of a particular Kodaira singularity type and allow some of the branes to bend and separate from the rest, so that they meet only at an intersection point



Three new contributions in:
"*F-theory Family Unification*"

SM, JHEP 1407(2014) 018, arXiv:1403.7066 [hep-th]

I have shown that

1. The six-dimensional matter spectrum coincides (after an orbifold projection) with that of a supersymmetric coset sigma model whose target space is a corresponding homogeneous Kahler manifold

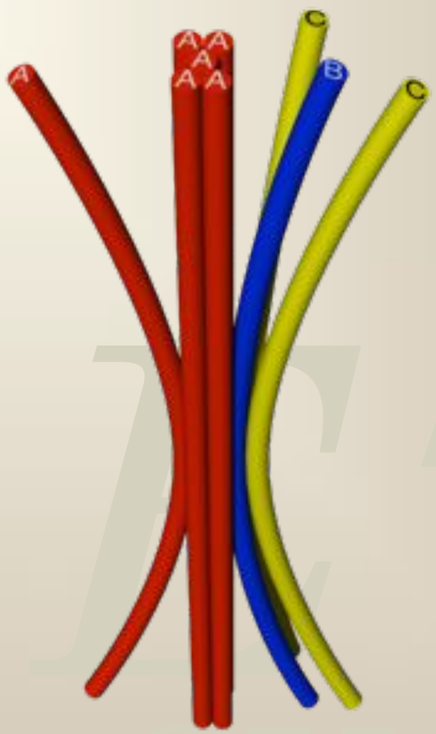


Table 2. Summary of matter fields in F-theory/heterotic duality in six dimensions. Only the cases for the split type with rank$\geq$ 2 are listed, where n is $\pm$(the number of instantons -12) in one of E_8's on the heterotic side, and r specifies how they are distributed when the commutant group is a direct product [60]. In addition to the data shown in [60], the corresponding 7-brane configurations as well as the homogeneous Kähler manifolds are also displayed.

Gauge group	Neutral hypers	Charged matter	7-branes	Homogeneous Kähler manifold
E_7	$2n+21$	$\frac{n+8}{2}\mathbf{56}$	$\mathbf{A}+\mathbf{A}^6\mathbf{BCC}$	$E_8/(E_7\times U(1))$
E_6	$3n+28$	$(n+6)\mathbf{27}$	$\mathbf{A}+\mathbf{A}^5\mathbf{BCC}$	$E_7/(E_6\times U(1))$
$SO(10)$	$4n+33$	$(n+4)\mathbf{16}$ $(n+6)\mathbf{10}$	$\mathbf{A}^5\mathbf{BC}+\mathbf{C}$ $\mathbf{A}+\mathbf{A}^5\mathbf{BC}$	$E_6/(SO(10)\times U(1))$ $SO(12)/(SO(10)\times U(1))$
$SO(8)$	$6n+44$	$(n+4)\mathbf{8}_c$ $(n+4)\mathbf{8}_s$ $(n+4)\mathbf{8}_v$	$\mathbf{A}^4\mathbf{BC}+\mathbf{C}$ $\mathbf{A}+\mathbf{A}^4\mathbf{BC}$	$E_5/(SO(8)\times U(1))$ $(=SO(10)/(SO(8)\times U(1)))$ $SO(10)/(SO(8)\times U(1))$
$SU(4)$	$8n+51$	$(4n+16)\mathbf{4}$ $(n+2)\mathbf{6}$	$\mathbf{A}^3\mathbf{BC}+\mathbf{C}$ $\mathbf{A}+\mathbf{A}^3\mathbf{BC}$	$E_4/(SO(6)\times U(1))$ $(=SU(5)/(SU(4)\times U(1)))$ $SO(8)/(SO(6)\times U(1))$
$SO(4)$	$10n+54$	$(4n+16)((\mathbf{1},\mathbf{2})$ $+(\mathbf{2},\mathbf{1}))$ $n(\mathbf{2},\mathbf{2})$	$\mathbf{A}^2\mathbf{BC}+\mathbf{C}$ $\mathbf{A}+\mathbf{A}^2\mathbf{BC}$	$E_3/(SO(4)\times U(1))$ $(=SU(3)/(SU(2)\times U(1)))$ $SO(6)/(SO(4)\times U(1))$
$SU(3)$	$12n+66$	$(6n+18)\mathbf{3}$	$\mathbf{A}+\mathbf{A}^3$	$SU(4)/(SU(3)\times U(1))$
$SO(12)$	$2n+18$	$\frac{r}{2}\mathbf{32}+\frac{n+4-r}{2}\mathbf{32}'$ $(n+8)\mathbf{12}$	$\mathbf{A}^6\mathbf{BC}+\mathbf{C}$ $\mathbf{A}+\mathbf{A}^6\mathbf{BC}$	$E_7/(SO(12)\times U(1))$ $SO(14)/(SO(12)\times U(1))$
$SU(6)$	$3n-r+21$	$\frac{r}{2}\mathbf{20}$ $(2n+16+r)\mathbf{6}$ $(n+2-r)\mathbf{15}$	$\mathbf{A}^6+\mathbf{X}_{[2,-1]}+\mathbf{C}$ $\mathbf{A}+\mathbf{A}^6$ $\mathbf{A}^6+\mathbf{B}+\mathbf{C}$	$E_6/(SU(6)\times U(1))$ $SU(7)/(SU(6)\times U(1))$ $SO(12)/(SU(6)\times U(1))$
$SU(5)$	$5n+36$	$(3n+16)\mathbf{5}$ $(n+2)\mathbf{10}$	$\mathbf{A}+\mathbf{A}^5$ $\mathbf{A}^5+\mathbf{B}+\mathbf{C}$	$SU(6)/(SU(5)\times U(1))$ $SO(10)/(SU(5)\times U(1))$

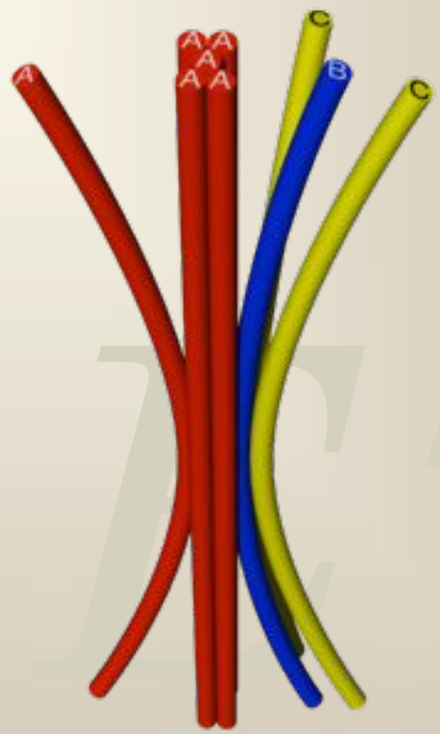
For all (the "split" cases) the patterns of gauge symmetry breaking in the F/heterotic duality investigated by Bershadsky et.al., the six-dimensional charged matter content

corresponds to a homo-geneous Kahler manifold of the relevant type

Three new contributions in:
"*F-theory Family Unification*"

SM, JHEP 1407(2014) 018, arXiv:1403.7066 [hep-th]

2. Such a brane configuration can preserve N=1 six-dimensional SUSY, at least locally



Proof of supersymmetry

$$ds_4^2 = e^{\Phi} dz d\bar{z} + e^{\Psi}(dw + \xi dz)(d\bar{w} + \bar{\xi} d\bar{z}) \qquad w = x^{\dot{6}} + i x^{\dot{7}}$$

$$\tau = \tau(z, w)$$

Φ, Ψ : real functions

ξ : complex function

Any hermitian metric can be written in this form

$$e_\mu{}^\alpha = \begin{pmatrix} e_i{}^a & 0 \\ 0 & e_{\bar{i}}{}^{\bar{a}} \end{pmatrix}$$

$$\mu = i, \bar{i}; \quad i = z, w; \quad \bar{i} = \bar{z}, \bar{w}; \quad \alpha = a, \bar{a}; \quad a = 1, 2; \quad \bar{a} = \bar{1}, \bar{2}$$

$$e_i{}^a \equiv \begin{pmatrix} e_i{}^8 + i e_i{}^9 & e_i{}^6 + i e_i{}^7 \end{pmatrix} = \begin{pmatrix} e^{\frac{\Phi}{2}} & e^{\frac{\Psi}{2}} \xi \\ 0 & e^{\frac{\Psi}{2}} \end{pmatrix},$$

$$e_{\bar{i}}{}^{\bar{a}} \equiv \begin{pmatrix} e_{\bar{i}}{}^8 - i e_{\bar{i}}{}^9 & e_{\bar{i}}{}^6 - i e_{\bar{i}}{}^7 \end{pmatrix} = \begin{pmatrix} e^{\frac{\Phi}{2}} & e^{\frac{\Psi}{2}} \bar{\xi} \\ 0 & e^{\frac{\Psi}{2}} \end{pmatrix}.$$

$$\eta_{\alpha\beta} = \begin{pmatrix} & \frac{1}{2} I \\ \frac{1}{2} I & \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$g_{\mu\nu} = e_\mu{}^\alpha \eta_{\alpha\beta} e^\beta{}_\nu, \quad ds_4 = g_{\mu\nu} dx^\mu dx^\nu$$

Choice of gamma matrices

$$\gamma^{1} \equiv \gamma^{8}+i\gamma^{9} = \begin{pmatrix} & 2 \\ 0 & \end{pmatrix} \otimes I = \begin{pmatrix} & & 2 & \\ & & & 2 \\ 0 & & & \\ & 0 & & \end{pmatrix},$$

$$\gamma^{\bar{1}} \equiv \gamma^{8}-i\gamma^{9} = \begin{pmatrix} & 0 \\ 2 & \end{pmatrix} \otimes I = \begin{pmatrix} & & 0 & \\ & & & 0 \\ 2 & & & \\ & 2 & & \end{pmatrix},$$

$$\gamma^{2} \equiv \gamma^{6}+i\gamma^{7} = \sigma_3 \otimes \begin{pmatrix} & 2 \\ 0 & \end{pmatrix} = \begin{pmatrix} & 2 & & \\ 0 & & & \\ & & & -2 \\ & & 0 & \end{pmatrix};$$

$$\gamma^{\bar{2}} \equiv \gamma^{6}-i\gamma^{7} = \sigma_3 \otimes \begin{pmatrix} & 0 \\ 2 & \end{pmatrix} = \begin{pmatrix} & 0 & & \\ 2 & & & \\ & & & 0 \\ & & -2 & \end{pmatrix}$$

Due to the holomorphic assumption we have, again,

$$P_{\bar{i}} = 0 \quad (\bar{i} = \bar{z}, \bar{w}).$$

The dilatino variation thus reads $\delta\lambda \propto P_i e_a{}^i \gamma^a \epsilon^*$

Since the leftmost columns of γ^a $(a=1,2)$ are zero, this vanishes for

$$\epsilon = \begin{pmatrix} \tilde{\epsilon} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Gravitino variation $\delta\psi_\mu$: Since the nonzero component of ϵ is only the first one, we are only concerned with the first columns of

$$
\begin{aligned}
\omega_{1\alpha\beta}\gamma^{\alpha\beta} &= \begin{pmatrix} -e^{-\frac{\Phi}{2}}(\partial_w\xi - \xi\partial_w\Phi + \partial_z\Phi) & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 2e^{-\Phi-\frac{\Psi}{2}}\left(e^{\Psi}(\bar{\xi}\partial_{\bar{w}}\xi - \partial_{\bar{z}}\xi) + e^{\Phi}\partial_{\bar{w}}\Phi\right) & * & * & * \end{pmatrix}, \\
\omega_{2\alpha\beta}\gamma^{\alpha\beta} &= \begin{pmatrix} e^{-\frac{\Psi}{2}}\left(e^{\Psi-\Phi}(\xi\partial_w\bar{\xi} - \partial_z\bar{\xi}) - \partial_w\Psi\right) & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 2e^{-\frac{\Phi}{2}}(\partial_{\bar{w}}\bar{\xi} + \bar{\xi}\partial_{\bar{w}}\Psi - \partial_{\bar{z}}\Psi) & * & * & * \end{pmatrix}, \\
\omega_{\bar{1}\alpha\beta}\gamma^{\alpha\beta} &= \begin{pmatrix} -\overline{((1,1)\text{ component of }\omega_{1\alpha\beta}\gamma^{\alpha\beta})} & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix}, \\
\omega_{\bar{2}\alpha\beta}\gamma^{\alpha\beta} &= \begin{pmatrix} -\overline{((1,1)\text{ component of }\omega_{2\alpha\beta}\gamma^{\alpha\beta})} & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix}
\end{aligned}
$$

Since the ``Bismut-like" connection contains, besides the spin connection, only Q_μ which is U(1), the gravitino variations vanish only if the off-diagonal components (of the first columns) do

SUSY transformations:
$$\delta\psi_\mu = \frac{1}{\kappa}\left(\partial_\mu - \frac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta} - \frac{i}{2}Q_\mu\right)\epsilon$$
$$\delta\lambda = \frac{i}{\kappa}P_\mu\gamma^\mu\epsilon^*$$

Since the "Bismut-like" connection contains, besides the spin connection, only Q_μ which is U(1), the gravitino variations vanish only if the off-diagonal components (of the first columns) do

$$\Rightarrow \quad e^\Psi(\xi\partial_w\bar{\xi} - \partial_z\bar{\xi}) + e^\Phi\partial_w\Phi = 0 \quad \text{and} \qquad (1)$$
$$\partial_w\xi + \xi\partial_w\Psi - \partial_z\Psi = 0 \qquad (2)$$

They are equivalent to
$$\partial_w(e^\Psi\xi\bar{\xi} + e^\Phi) = \partial_z(e^\Psi\bar{\xi}) \quad \text{and}$$
$$\partial_w(e^\Psi\xi) = \partial_z e^\Psi$$

or $$\partial_i g_{j\bar{i}} = \partial_j g_{i\bar{i}}, \quad \partial_{\bar{i}} g_{\bar{j}i} = \partial_{\bar{j}} g_{\bar{i}i}$$

$\Rightarrow$ Kähler

That the system of equations (1) (2) has a solution can be confirmed by expanding them in the coordinates and showing that the coefficients are determined order by order in this expansion

Using the solutions Φ, Ψ and ξ satisfying (1) (2)

$$\omega_{i\alpha\beta}\gamma^{\alpha\beta} = \begin{pmatrix} -\partial_i(\Phi+\Psi) & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix}, \quad \omega_{\bar{i}\alpha\beta}\gamma^{\alpha\beta} = \begin{pmatrix} +\partial_{\bar{i}}(\Phi+\Psi) & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix}$$

$$\begin{aligned} Q_i &= -\frac{i}{2}\partial_i \log(\tau - \bar{\tau}), \\ Q_{\bar{i}} &= +\frac{i}{2}\partial_{\bar{i}} \log(\tau - \bar{\tau}) \end{aligned}$$

➡ A Killing spinor exists if

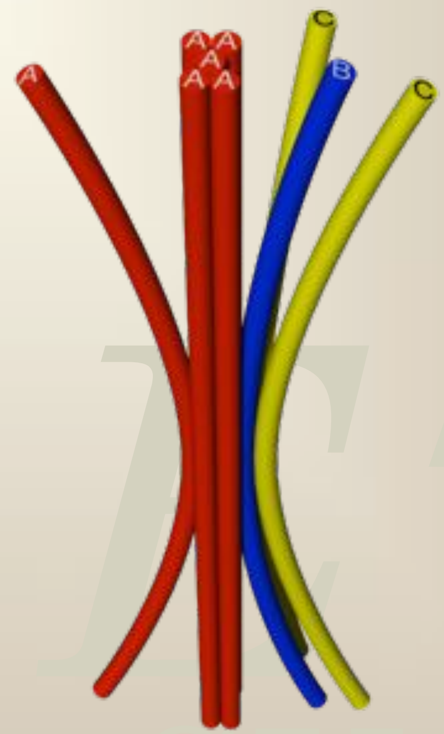
$$\Phi + \Psi = \log(\tau - \bar{\tau}) + F(z^i) + \bar{F}(\bar{z}^{\bar{i}})$$

for some holomorphic functions $\blacksquare$

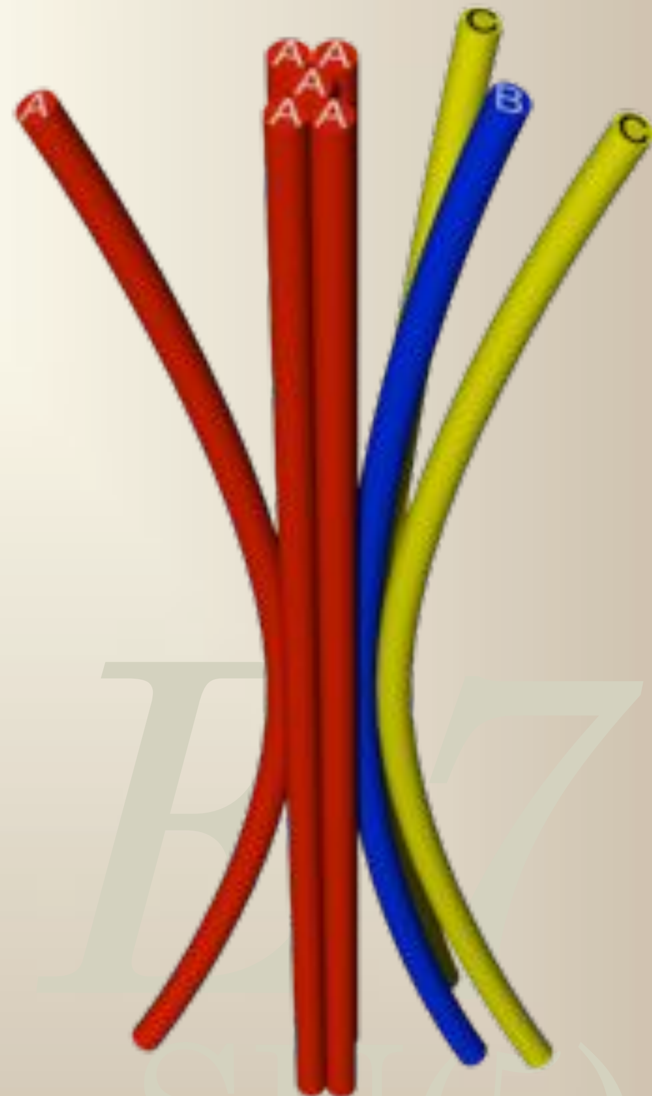
Three new contributions in: "*F-theory Family Unification*"

SM, JHEP 1407(2014) 018, arXiv:1403.7066 [hep-th]

3. If one starts from the E_7 singularity, one obtains the same chiral matter content as the E_7/ ($SU(5)xU(1)^3$) Kugo-Yanagida model yielding precisely three generations with an UNPARALLEL family structure!!!



Kugo-Yanagida in F-theory



To get a 4D theory, we still need to compactify on T^2 and take an orbifold

“*Large Lepton-flavor Mixings from E8 Kodaira Singularity*” SM, arXiv:1407.1319[hep-th]

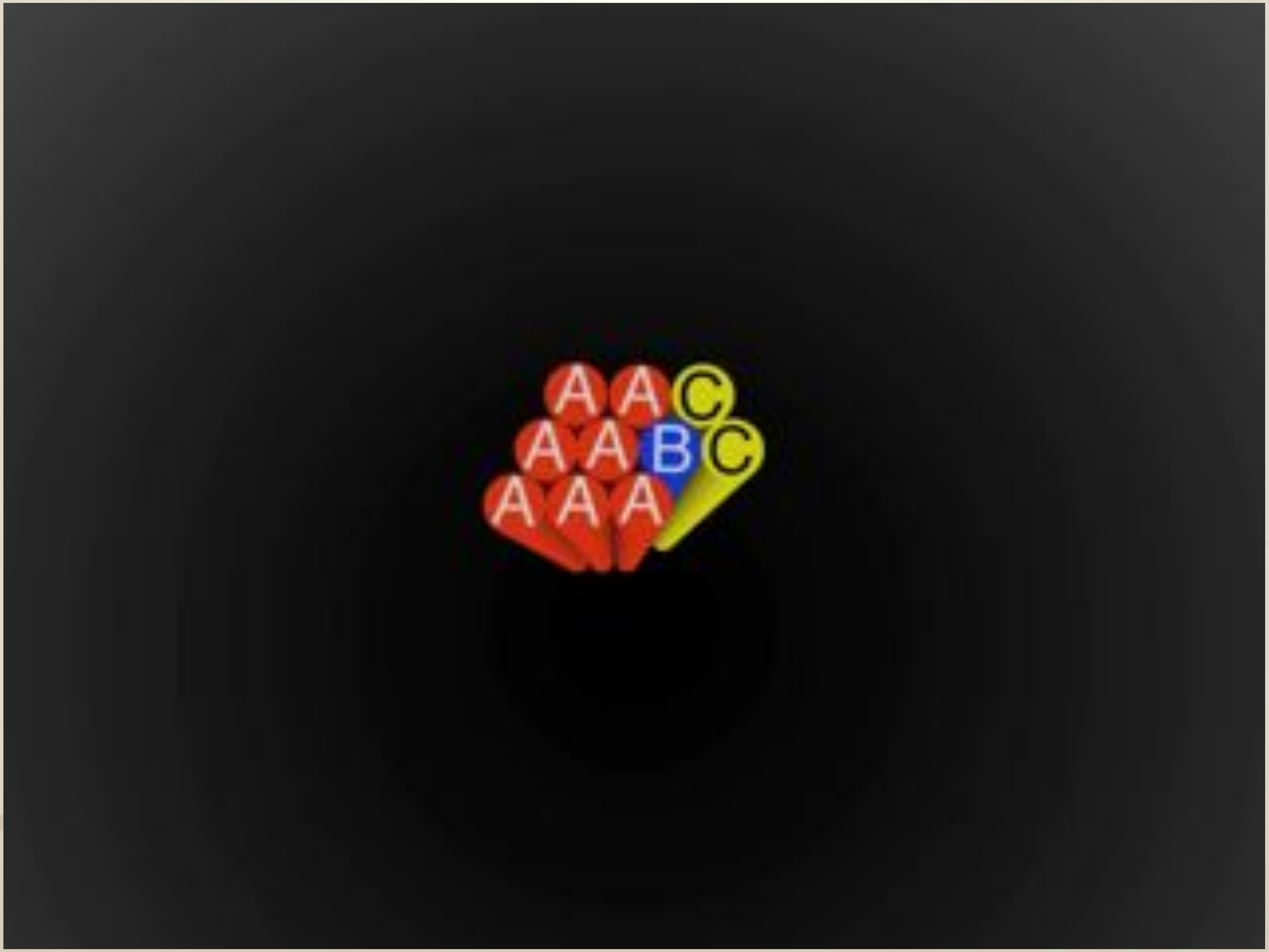
Sato-Yanagida’s scenario
using the Frogatt-Nielsen mechanism

Assume THREE PAIRS $s_i, \bar{s}_i (i = 0, 1, 2)$ of singlet scalar fields with particular U(1) charges in the $E_7/(SU(5)xU(1)^3)$ model

- Hierarchical Yukawa couplings for both the quark and lepton sectors, qualitatively in agreement
- Large lepton / small quark mixing angles

provided that $\frac{\langle s_1 \rangle}{\langle s_0 \rangle} \sim 0.05, \quad \frac{\langle s_2 \rangle}{\langle s_1 \rangle} \sim 0.05$ and $\tan\theta \sim 1$

Sato-Yanagida's scenario is naturally realized in F-theory family unification!



Necessary Frogatt-Nielsen fields naturally arise if we consider the branching of E8 singularity!

5 SUMMARY

Summary

- We have shown that a certain local 7-brane system in F theory can realize, already at the level of six dimensions, the same quantum numbers as that of the SUSY nonlinear sigma model considered in family unification
- If half of the spectrum is projected out, then it becomes precisely what we observe in nature
- We considered a set of coalesced local 7-branes of a particular singularity type and allowed some of the branes to bend and separate from the rest
- The massless spectrum was studied by investigating string junctions near the intersection and shown to be the same as the sigma model