Phase Diagram of a Holographic Superconductor Model with s-wave and d-wave

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[M.N arXiv:1403.6070]

① Abstract

Motivation

Understanding of a phase structure about different superconducting states from holographic superconductor

Set up

4-dim AdS black hole + U(1) gauge field + scalar field + tensor field + coupling η

Plan

derive EOMs from the gravity model

↓

study the solution's property

↓

make a phase diagram

Result

We get a rich phase structure.

② Gravity model

$$S=\int d^4x\sqrt{-g}\Big[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\mathcal{L}_{\rm s}+\mathcal{L}_{\rm d}-\eta|\psi|^2|\Phi_{\mu\nu}|^2\Big]$$

$$\mathcal{L}_{\rm s}=-m_{\rm s}^2|\psi|^2-|D_\mu\psi|^2$$

$$\mathcal{L}_{\rm d}=-|D_\rho\Phi_{\mu\nu}|^2+2|D_\mu\Phi^{\mu\nu}|^2+|D_\mu\Phi|^2-[(D_\mu\Phi^{\mu\nu})^*D_\nu\Phi+{\rm c.c.}]$$
$$-m_{\rm d}^2(|\Phi_{\mu\nu}|^2-|\Phi|^2)+2R_{\mu\nu\rho\lambda}\Phi^{*\mu\rho}\Phi^{\nu\lambda}-\frac{1}{4}R|\Phi|^2-ie_{\rm d}F_{\mu\nu}\Phi^{*\mu\lambda}\Phi_\lambda^\nu$$

[F. Benini, et al. , 2010]

$$ds^2=\frac{1}{z^2}(-f(z)dt^2+dx^2+dy^2+\frac{dz^2}{f(z)}) \qquad f(z)=1-\Big(\frac{z}{z_0}\Big)^3$$

Hawking temperature $T=\frac{3}{4\pi z_0}$

- We introduce ψ and $\Phi_{\mu\nu}$ as order parameters.
- We consider direct coupling η only.
- probe limit (Matter fields don't change the metric.)
- We introduce different electric charge [P. Basu, et al. , 2010] for a rich phase structure.

$$m_{\rm s}^2=-2 \qquad m_{\rm d}^2=0$$
$$e_{\rm s}=1 \qquad e_{\rm d}=1.95$$

③ Equations of motion

ansatz $\psi=\psi(z),\quad \Phi_{xy}=\Phi_{yx}=\frac{\varphi(z)}{2z^2},\quad A_t=\phi(z)$

s-wave $\psi''+\Big(\frac{f'}{f}-\frac{2}{z}\Big)\psi'+\frac{\phi^2}{f^2}\psi+\frac{2}{z^2f}\psi-\frac{\eta\varphi^2}{2z^2f}\psi=0$

d-wave $\varphi''+\Big(\frac{f'}{f}-\frac{2}{z}\Big)\varphi'+\frac{(1.95\phi)^2}{f^2}\varphi-\frac{\eta\psi^2}{z^2f}\varphi=0$

A_t $\phi''-\frac{2\psi^2}{z^2f}\phi-\frac{(1.95\varphi)^2}{z^2f}\phi=0$

EOMs have four types of solutions.

- the normal conducting solution $\psi=0,\quad \varphi=0$
- the s-wave single solution $\psi\neq0,\quad \varphi=0$
- the d-wave single solution $\psi=0,\quad \varphi\neq0$
- the s+d coexistent solution $\psi\neq0,\quad \varphi\neq0$

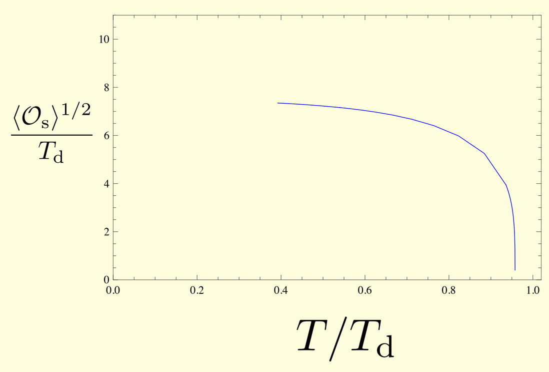
boundary conditions

$\psi\to\langle\mathcal{O}_{\rm s}\rangle z^2 \ (z\to0) \qquad \varphi\to\langle\mathcal{O}_{\rm d}\rangle z^3 \ (z\to0) \qquad \phi\to0 \ (z\to z_0)$

④ Solutions of EOMs

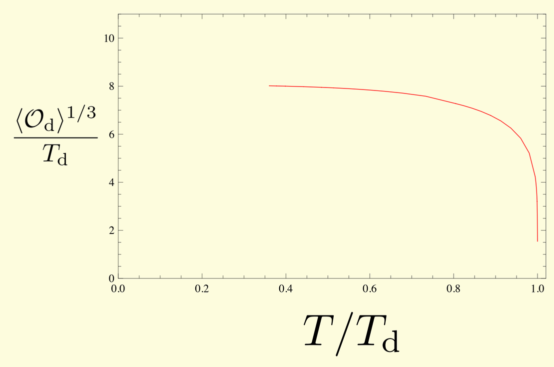
the s-wave single solution

$\psi\to\langle\mathcal{O}_{\rm s}\rangle z^2 \quad (z\to0)$



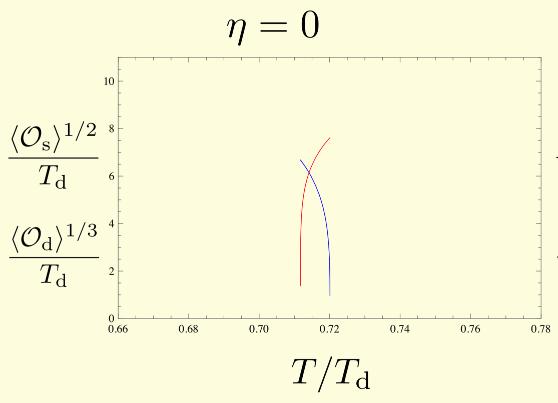
the d-wave single solution

$\varphi\to\langle\mathcal{O}_{\rm d}\rangle z^3 \quad (z\to0)$

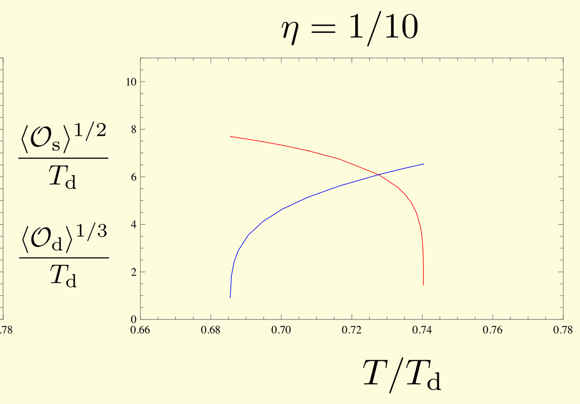


the s+d coexistent solution

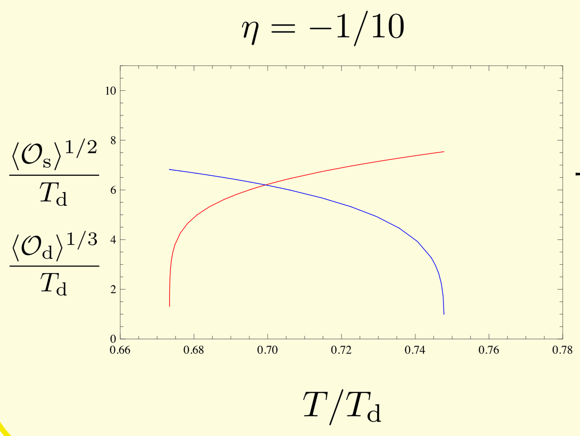
$\eta=0$



$\eta=1/10$



$\eta=-1/10$



If we change η, the s+d coexistent solution's property changes.

⑤ Free energy density

To make a phase diagram, we calculate free energy density of each solution. In holographic superconductor, free energy density is related to a classical Euclid action $\mathcal{S}_{\rm onshell}$. The solution which free energy density is minimum is favored.

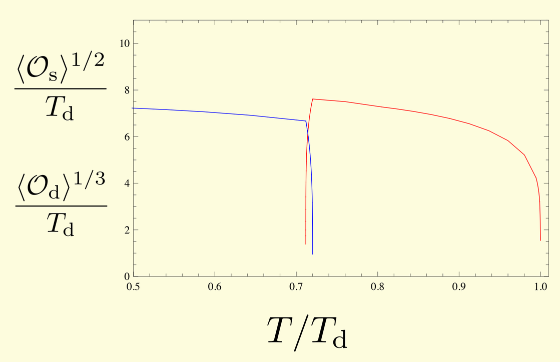
free energy density

$$\frac{S_{\rm onshell}}{\beta V_2}=-\frac{\mu\rho}{2}+\int\frac{\phi^2\psi^2}{z^2f}dz+\int\frac{(1.95)^2\phi^2\varphi^2}{2z^2f}dz-\int\eta\frac{\psi^2\varphi^2}{2z^4}dz$$

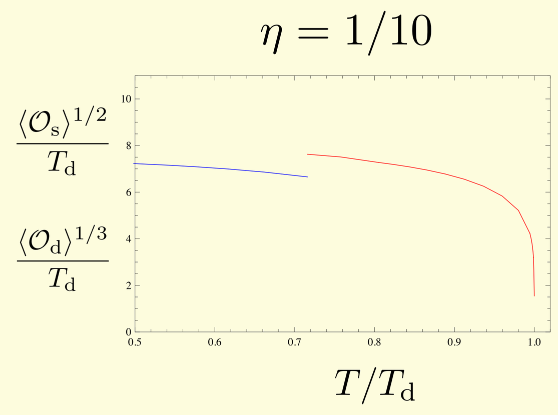
$\phi\to\mu-\rho z \quad (z\to0) \qquad \beta=\int dt \quad V_2=\int dxdy$

favored solutions

$\eta=0$

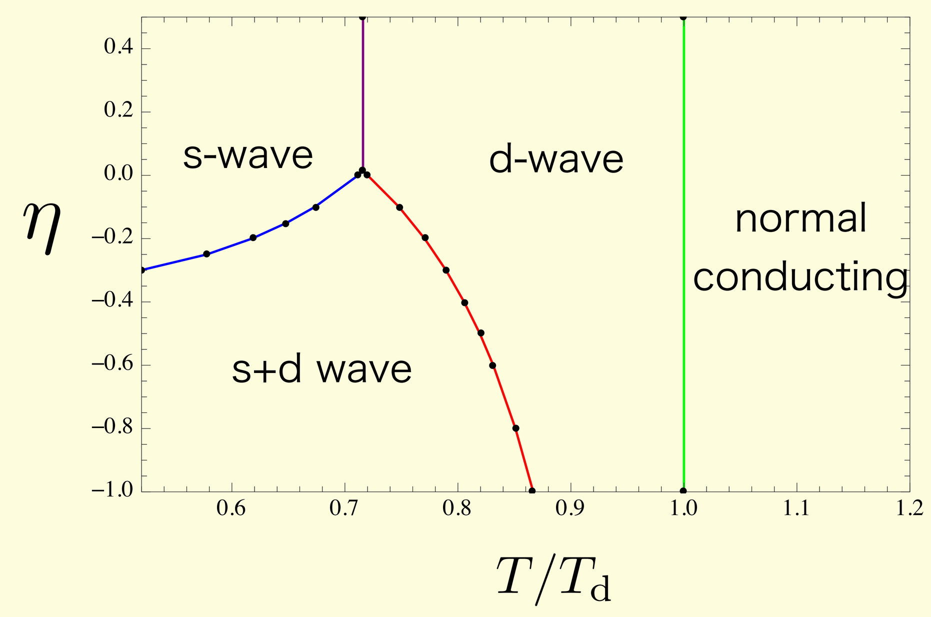


$\eta=1/10$

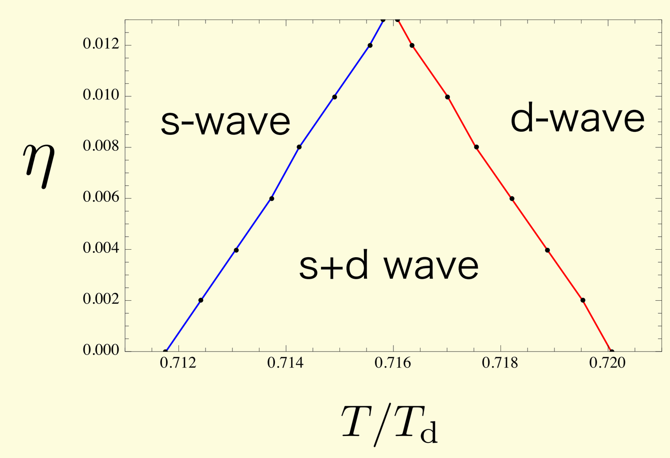


If η is enough large, the s+d coexistent solution is not favored.

⑥ Phase diagram



near $\eta=0$



If we choose specific parameters, we can get a rich phase diagram including a triple point and four phases.

⑦ Future work

- back reaction
- other coupling
- other parameters
- other ansatz

A Holographic Superconductor Model with Josephson Coupling

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[Work in Progress]

① Abstract :

Consider a classical solutions of a holographic model with Josephson coupling.

Result : If we choose specific parameters, the number of the solutions increases in three scalar model.

② Two scalar model

$$S=\int d^4x\sqrt{-g}\Big[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-|D_\mu\varphi_1|-|D_\mu\varphi_2| \\ -m_1^2|\varphi_1|^2-m_2^2|\varphi_2|^2-\epsilon_{12}(\varphi_1^*\varphi_2+\varphi_1\varphi_2^*)\Big]$$

[Wen-Yu Wen, et al. , 2013]

- Introduce two complex scalar fields φ_i and nonzero Josephson coupling $\epsilon_{12}\neq 0$
- probe limit (4-dim AdS black hole)
- Same electric charge for gauge invariance
- Consider a classical solution $\varphi_1\neq 0,\ \ \varphi_2\neq 0$ and introduce real scalar fields $\psi_i>0$ and phases θ_i as

$$\varphi_i=\psi_i e^{i\theta_i}$$

- Because the action doesn't include $|\varphi_i|^4$, we can use the diagonalization.

③ Equations of motion

ansatz $\psi_i=\psi_i(z),\ A_t=A_t(z),\ \theta_i=\mathrm{const}$

$\epsilon'_{12}\equiv\epsilon_{12}\cos(\theta_1-\theta_2)$

A^ν	$\nabla_\mu F^{\mu\nu}-2\psi_1^2A^\nu-2\psi_2^2A^\nu=0$
ψ_1	$\nabla_\mu\nabla^\mu\psi_1-A_\mu A^\mu\psi_1-m_1^2\psi_1-\epsilon'_{12}\psi_2=0$
ψ_2	$\nabla_\mu\nabla^\mu\psi_2-A_\mu A^\mu\psi_2-m_2^2\psi_2-\epsilon'_{12}\psi_1=0$
θ_1	$2\epsilon_{12}\psi_1\psi_2\sin(\theta_1-\theta_2)=0$
θ_2	$2\epsilon_{12}\psi_1\psi_2\sin(\theta_2-\theta_1)=0$

To simplify EOMs, diagonalize $\begin{pmatrix} m_1^2 & \epsilon'_{12} \\ \epsilon'_{12} & m_2^2\end{pmatrix}$.

eigenvalues

$$\lambda_1=\frac{m_1^2+m_2^2-\sqrt{(m_1^2-m_2^2)^2+4\epsilon'^2_{12}}}{2}$$

$$\lambda_2=\frac{m_1^2+m_2^2+\sqrt{(m_1^2-m_2^2)^2+4\epsilon'^2_{12}}}{2}$$

eigenvectors

$$\psi'_1=\frac{-\epsilon'_{12}\psi_1+(m_1^2-\lambda_1)\psi_2}{\sqrt{\epsilon'^2_{12}+(m_1^2-\lambda_1)^2}}$$

$$\psi'_2=\frac{(m_1^2-\lambda_1)\psi_1+\epsilon'_{12}\psi_2}{\sqrt{\epsilon'^2_{12}+(m_1^2-\lambda_1)^2}}$$

④ Solutions of EOMs

- $\theta_1-\theta_2=0$ or π because $2\epsilon_{12}\psi_1\psi_2\sin(\theta_1-\theta_2)=0$.
- EOMs of ψ'_i

$$\nabla_\mu\nabla^\mu\psi'_i-A_\mu A^\mu\psi'_i-\lambda_i\psi'_i=0$$

There is no solution of $\psi'_1\neq 0,\ \ \psi'_2\neq 0$ because $\lambda_1\neq\lambda_2$. [P. Basu, et al. , 2010]

- $\epsilon'_{12}>0\to\psi'_1=0,\ \ \psi'_2\neq 0$ is the solution.
  $\epsilon'_{12}<0\to\psi'_1\neq 0,\ \ \psi'_2=0$ is the solution.

- Free energy of the solution of $\epsilon'_{12}<0$ is smaller than that of $\epsilon'_{12}>0$ because $\lambda_1<\lambda_2$.
- If $|\epsilon_{12}|$ is large, λ_1 is smaller than the value of BF bound.

⑤ Three scalar model

$$S=\int d^4x\sqrt{-g}\Big[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-|D_\mu\varphi_1|-|D_\mu\varphi_2|-|D_\mu\varphi_3| \\ -m_1^2|\varphi_1|^2-m_2^2|\varphi_2|^2-m_3^2|\varphi_3|^2 \\ -\epsilon_{12}(\varphi_1^*\varphi_2+\varphi_1\varphi_2^*) \\ -\epsilon_{23}(\varphi_2^*\varphi_3+\varphi_2\varphi_3^*)-\epsilon_{31}(\varphi_3^*\varphi_1+\varphi_3\varphi_1^*)\Big]$$

EOMs

A^ν	$\nabla_\mu F^{\mu\nu}-2\psi_1^2A^\nu-2\psi_2^2A^\nu-2\psi_3^2A^\nu=0$
ψ_1	$\nabla_\mu\nabla^\mu\psi_1-A_\mu A^\mu\psi_1-m_1^2\psi_1-\epsilon'_{12}\psi_2-\epsilon'_{31}\psi_3=0$
ψ_2	$\nabla_\mu\nabla^\mu\psi_2-A_\mu A^\mu\psi_2-m_2^2\psi_2-\epsilon'_{23}\psi_3-\epsilon'_{12}\psi_1=0$
ψ_3	$\nabla_\mu\nabla^\mu\psi_3-A_\mu A^\mu\psi_3-m_3^2\psi_3-\epsilon'_{31}\psi_1-\epsilon'_{23}\psi_2=0$
θ_1	$2\epsilon_{12}\psi_1\psi_2\sin(\theta_1-\theta_2)+2\epsilon_{31}\psi_1\psi_3\sin(\theta_1-\theta_3)=0$
θ_2	$2\epsilon_{12}\psi_2\psi_1\sin(\theta_2-\theta_1)+2\epsilon_{23}\psi_2\psi_3\sin(\theta_2-\theta_3)=0$
θ_3	$2\epsilon_{31}\psi_3\psi_1\sin(\theta_3-\theta_1)+2\epsilon_{23}\psi_3\psi_2\sin(\theta_3-\theta_2)=0$

If $\epsilon_{12}\neq 0,\ \ \epsilon_{23}\neq 0,\ \ \epsilon_{31}\neq 0$, there is some possibility that a solution of $\sin(\theta_1-\theta_2)\neq 0,\ \ \sin(\theta_2-\theta_3)\neq 0,\ \ \sin(\theta_3-\theta_1)\neq 0$ exists.

⑥ A condition for the solution to exist

- From EOMs of θ_i ,

$$\psi_2=\frac{\epsilon_{31}\sin(\theta_3-\theta_1)}{\epsilon_{23}\sin(\theta_2-\theta_3)}\psi_1,$$

$$\psi_3=\frac{\epsilon_{12}\sin(\theta_1-\theta_2)}{\epsilon_{23}\sin(\theta_2-\theta_3)}\psi_1.$$

- Substituting them to EOMs of ψ_i,

$$(\nabla_\mu\nabla^\mu-A_\mu A^\mu)\psi_1+(-m_1^2+\frac{\epsilon_{12}\epsilon_{31}}{\epsilon_{23}})\psi_1=0,$$

$$(\nabla_\mu\nabla^\mu-A_\mu A^\mu)\psi_1+(-m_2^2+\frac{\epsilon_{23}\epsilon_{12}}{\epsilon_{31}})\psi_1=0,$$

$$(\nabla_\mu\nabla^\mu-A_\mu A^\mu)\psi_1+(-m_3^2+\frac{\epsilon_{31}\epsilon_{23}}{\epsilon_{12}})\psi_1=0.$$

θ_i dependence is cancelled!

- If $m_1^2-\frac{\epsilon_{12}\epsilon_{31}}{\epsilon_{23}}=m_2^2-\frac{\epsilon_{23}\epsilon_{12}}{\epsilon_{31}}=m_3^2-\frac{\epsilon_{31}\epsilon_{23}}{\epsilon_{12}}$, the solution of $\sin(\theta_1-\theta_2)\neq 0,\ \ \sin(\theta_2-\theta_3)\neq 0,\ \ \sin(\theta_3-\theta_1)\neq 0$ exists.

⑦ Why does the solution exist?

If $m_1^2-\frac{\epsilon_{12}\epsilon_{31}}{\epsilon_{23}}=m_2^2-\frac{\epsilon_{23}\epsilon_{12}}{\epsilon_{31}}=m_3^2-\frac{\epsilon_{31}\epsilon_{23}}{\epsilon_{12}}$,

$\begin{pmatrix} m_1^2 & \epsilon_{12} & \epsilon_{31} \\ \epsilon_{12} & m_2^2 & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{23} & m_3^2\end{pmatrix}$ has degenerate eigenvalues.

eigenvalues

$$(m_1^2-\frac{\epsilon_{12}\epsilon_{31}}{\epsilon_{23}},m_1^2-\frac{\epsilon_{12}\epsilon_{31}}{\epsilon_{23}},m_1^2+\frac{\epsilon_{23}\epsilon_{12}}{\epsilon_{31}}+\frac{\epsilon_{31}\epsilon_{23}}{\epsilon_{12}})$$

The solutions of non-degenerate eigenvalues → Two constraints of other eigenvectors

The solutions of degenerate eigenvalues → One constraint of other eigenvector

Because the number of constraints decreases, the number of the solutions increases.