- Membranes from monopole operators : Large angular momentum and M-theoretic AdS_4/CFT_3

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- based on arxiv : 1310.0016 , to appear in PTEP

  and work in progress

Quick Summary of our work

1. We studied M-theory regime of AdS_4/CFT_3.

2. We used approximation schemes which are valid for states with large angular momentum J.

3. We found AdS_4/CFT_3 works fine for large J sector at the leading order of the approx.

   In particular non-BPS states of fluctuating membranes on the AdS side are described by certain operators involving monopole operators on the CFT side.

Outline

1. Motivation for studying M-theoretic AdS_4/CFT_3

   M-theory, matrix model, AdS/CFT

2. Approximation schemes for the large J sector on the AdS side and the CFT side

3. Summary of AdS_4/CFT_3 for the large J sector

membranes in AdS $\sim$ monopole operators in CFT

1. M-theoretic AdS_4/CFT_3 is a good place to learn about M-theory, matrix model AdS/CFT.

- AdS_4/CFT_3 (Maldacena '97
Aharony-Bergman-Jafferis-Maldacena '06)

AdS side

D=11 M-theory on
$AdS_4 \times S^7/\mathbb{Z}_k$

CFT side

D=3 ABJM theory
$U(N)\times U(N)$ CS theory, level k
bifundamental matter fields

$=$

$R^6/\ell_p^6 = Nk$

(R: curvature radius, ℓ_p: Planck length)

M-theory regime

$N \gg 1, \quad k \sim O(1)$ ← this talk

cf.) IIA regime

$N \gg 1, \ k \gg 1$ 't Hooft coupling $\lambda = N/k$: fixed

$S^7/\mathbb{Z}_k \longrightarrow$ M-theory circle, radius R/k

Motivation from M-theory, matrix model

- $D=11$ M-theory has no established formulation $\left(\begin{array}{c}\text{fundamental DOF}\\ +\text{ action}\end{array}\right)$
- A good candidate : matrix model (Banks-Fischler-Shenker-Susskind '96 / de Wit-Hoppe-Nicolai '88)
  though with unsolved problems (large matrix size limit, Lorentz symm.)
- It is an important theme to further study the matrix model to establish it as a formulation of M-theory.

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- It is an important theme to further study the matrix model to establish it as a formulation of M-theory.

- Wish to combine the matrix model approach & AdS_4/CFT_3
  - ☆ Can learn about what are good observables
  - ☆ can test the matrix model & AdS_4/CFT_3 proposals simultaneously.

- Indeed, we shall show a rather direct correspondence between pp-wave matrix model (Berenstein-Maldacena-Nastase '02) and ABJM theory.

Motivation from AdS/CFT (in general)

- M-theoretic AdS_4/CFT_3 is a prime example of non-stringy AdS/CFT (~ string DOF are not fundamental)
  - ☆ membranes rather than strings
  - ☆ open/closed duality probably not essential
- 't Hooft coupling N/k not fixed
  - ⇝ focussing on planar diagrammes not allowed (in general)
- It is likely that we learn something essentially new about basic aspects of AdS/CFT

2. Good approximation schemes for large J sector on AdS side and CFT side

Difficulties about M-theoretic AdS_4/CFT_3

- **AdS side**

  no formulation of M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ for computation at the quantum level

- **CFT side**

  Coupling const. $\frac{1}{k} \sim O(1)$

  even planar approximation not applicable (in general)

Our approach

- Consider states with large orbital angular momentum J (R - charge)
- Use $1/J \ll 1$ to introduce small parameters.

☆ Essentially a WKB approx.

☆ Succesful example : Berenstein-Maldacena-Nastase '02 for AdS_5/CFT_4

☆ Our result is analogous but crucially different:

CFT operator corresponding to membranes not strings

Large J sector

AdS side

- S^7 embedded into

$X^1\ X^2\ X^3\ X^4\ X^5\ X^6\ X^7\ X^8$

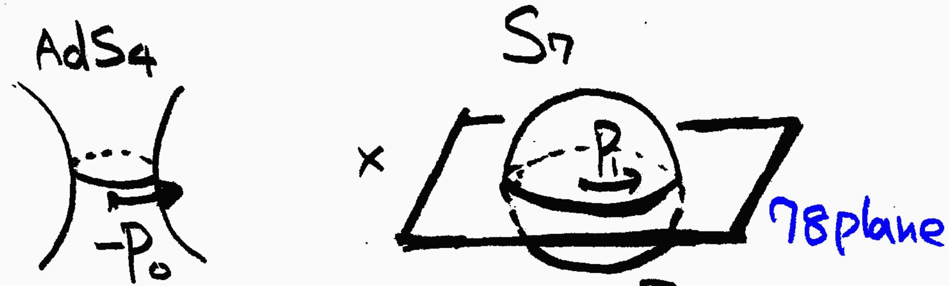
CFT side

- Complex scalars in ABJM

$\phi^1\quad \phi^2\quad \phi^3\quad \phi^4$

Large J sector

AdS side

- S^7 embedded into
  $$X^1\ X^2\ X^3\ X^4\ X^5\ X^6\ \boxed{X^7 X^8}$$
- (Angular mom. in 78 plane) $\gg 1$
- 

$$P_1 = \frac{J}{R}$$

"Light cone Hamiltonian"

$$H = -P_0 - P_1$$

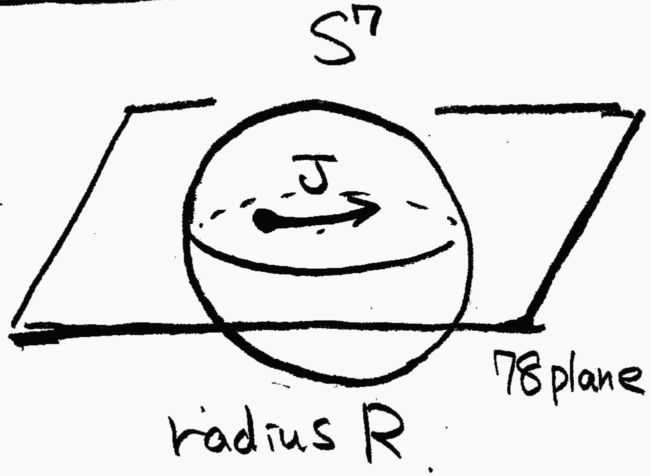
(Cf. Dobashi-Shimada-Yoneya '02)

CFT side

- Complex scalars in ABJM
  $$\phi^1\ \ \phi^2\ \ \phi^3\ \ \boxed{\phi^4}$$
- (R-charge associated with ϕ^4) $\gg 1$
- operators "containing" J numbers of ϕ^4 fields
- Δ : conformal dimension

$$H = -P_0 - P_1 \longleftrightarrow \Delta - \frac{J}{2}$$

Approx. on AdS side



- $J \gg 1 \rightsquigarrow$ obj. pushed towards equator
- if $\dfrac{(\text{size of obj.})}{R} \ll 1$

$AdS_4 \times S^7/\mathbb{Z}_k$ can be approximated by pp-wave

- pp-wave matrix model ($\ell_p = 1$)

$$H = \sum_{\alpha=1}^{9} \frac{R}{2k} P_\alpha^2 - \frac{R}{2k}\left([X,X]^2 + 2[X,Y]^2 + [Y,Y]^2\right) + \frac{k}{2R^3}\sum_{i=1}^{6} X_i^2 + \frac{2k}{R^3}\sum_{a=1}^{3} Y_a^2 + i\frac{1}{R}\epsilon^{abc} Y_a [Y_b, Y_c] + \cdots$$

$X^1, \cdots, X^6$: from S^7 ; Y_1, Y_2, Y_3 : from AdS_4 ; (matrix size) $= J/k$

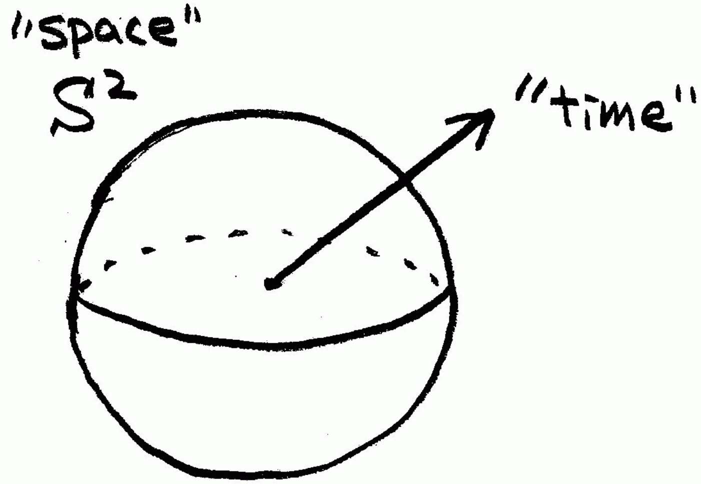
- (1-loop)/(tree) was computed. (Dasgupta-SheikhJabbari-vanRaamsdonk '02, Kim-Plefka '02)

- $N^{\frac{1}{3}} \underset{\text{loop exp.}}{\ll} J \underset{\text{pp-wave approx.}}{\ll} N^{\frac{1}{2}}$ (used $R^6 = Nk$)

Approx. on CFT side

- Use state-operator mapping

radially quantised ABJM



Approx. on CFT side

- Use state-operator mapping
- State with J R-charge

$\longrightarrow$ ϕ^4 excited J times

$\longrightarrow$ $\rho, \hat{\rho} \sim J$

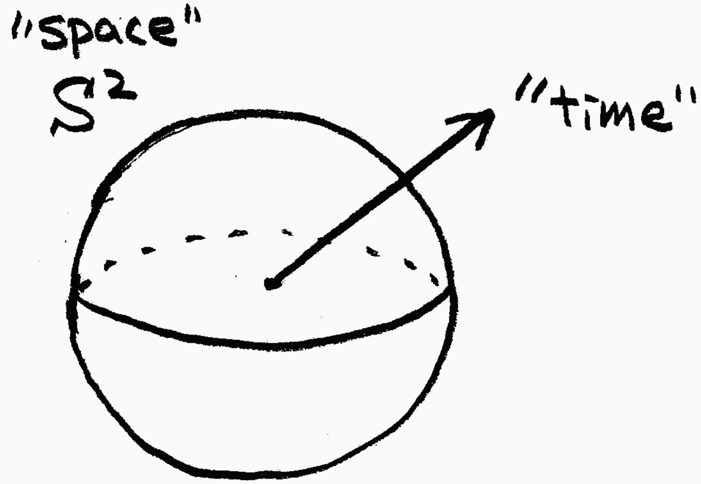
$U(N)\times U(N)$ charge density

(ϕ^4: bi-fundamental)

$\longrightarrow$ $F_{12}, \hat{F}_{12} \sim J/R$

Gauss law constr. $\frac{R}{2\pi} F_{12} = \rho, \ -\frac{R}{2\pi} \hat{F}_{12} = \hat{\rho}$

radially quantised ABJM



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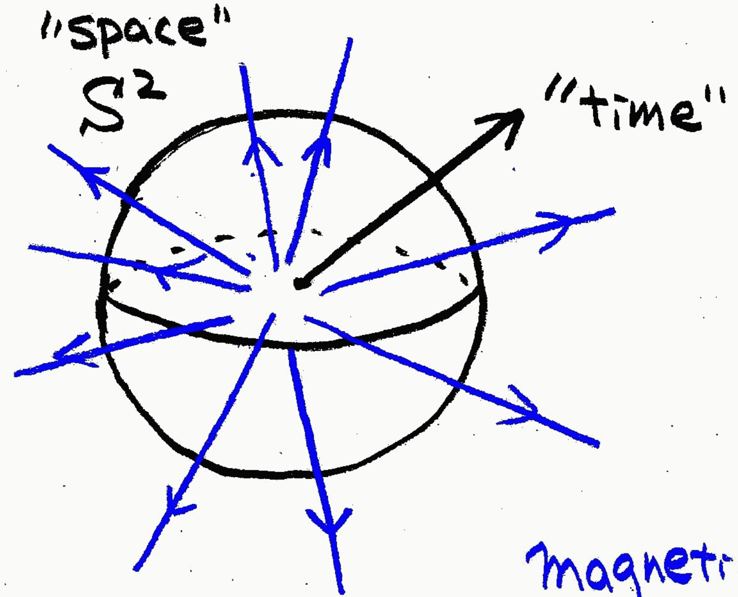

magnetic flux $\sim J/R$

☆ magnetic flux in radial quantisation = monopole operator

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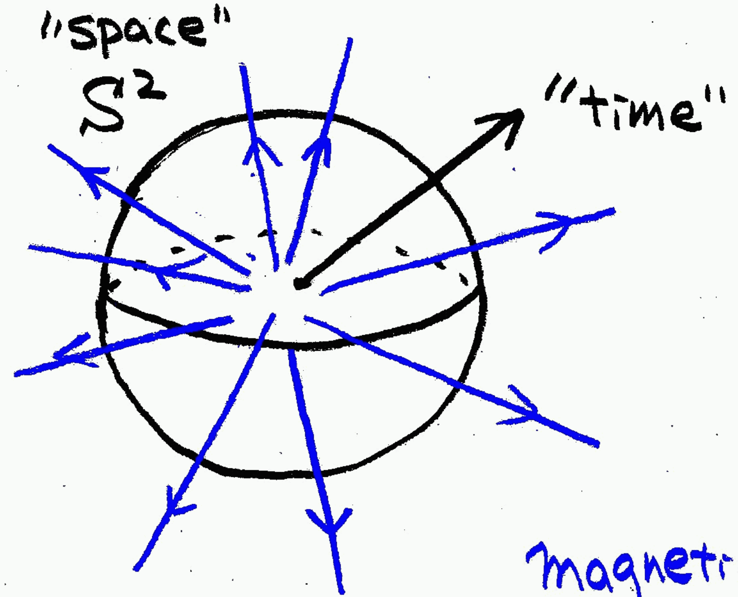
radially quantised ABJM



magnetic flux $\sim J/k$

☆ magnetic flux in radial quantisation = monopole operator

- In the presence of magnetic flux

off-diagonal DOF $\rightarrow$ heavy, mass $\sim O(J)$

diagonal DOF $\rightarrow$ light, mass $\sim O(1)$

- Integrate out heavy DOF, get effective theory for light DOF

coupling suppressed by powers of $\frac{1}{J}$ (Born-Oppenheimer approx.)

Summary of 2.

- AdS side pp-wave approx. + loop exp.

- CFT side use radial quantisation

  large R-charge $\rightarrow$ large magnetic flux

  $\rightarrow$ difference in the energy scale

  diagonal $\sim O(1)$

  off-diagonal $\sim O(J)$

  $\rightarrow$ Born-Oppenheimer approx.

- $N^{\frac{1}{3}} \ll J \ll N^{\frac{1}{2}}$

3. M-theoretic AdS_4/CFT_3 works fine at the leading order of the approximations for large J

Ground states = BPS states

non-BPS fluctuations
(excited states)

BPS states (Ground states) $k = 1$

AdS side ('02 BMN)

- classical ground state of matrix model given by reducible rep. of $SU(2)$

$$Y^a = \alpha L^a = \begin{array}{c} \\ J \end{array} \left[\begin{array}{ccc} \boxed{} & & 0 \\ & \boxed{} & \\ 0 & & \ddots \end{array} \right] \begin{array}{l} \updownarrow J_{(1)} \\ \updownarrow J_{(2)} \\ \vdots \end{array}$$

$$\underset{J}{\longleftrightarrow}$$

$(X^i = 0)$

- labelled by

$J = J_{(1)} + \cdots + J_{(n)}$

- (n) - numbers of concentric spherical membranes extended in AdS_4
- $J_{(i)}$: ang. mom of (i)-th membrane

CFT side

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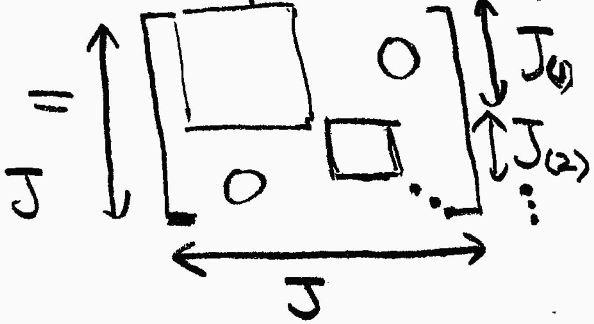

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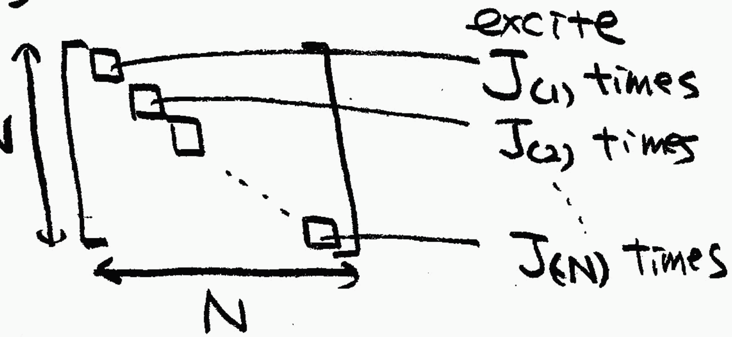
CFT side (cf. '02 Kapustin et al, '06 ABJM, '08 Kim)

- Excite diagonal elem. of Φ^4 0-mode J-times

$$\Phi^4 \sim$$



- labelled by

$$J = J_{(1)} + \cdots J_{(N)}$$

$(J_{(i)} = 0 \text{ allowed})$

- BKG gauge field : diagonal

each diagonal elem. : Dirac monopole charge $J_{(i)}$ (GNO charges)

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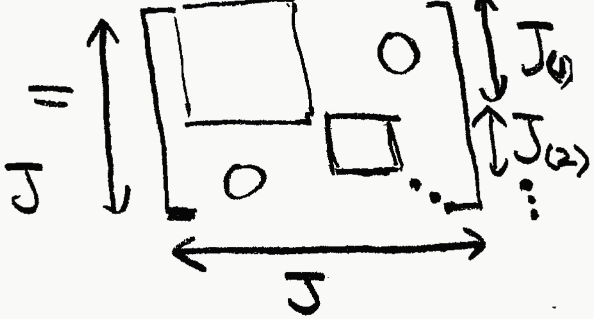

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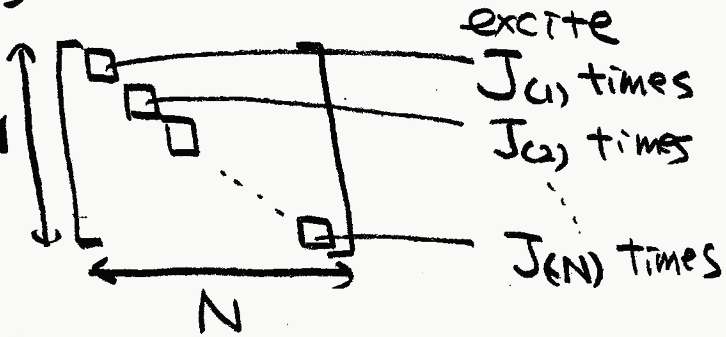
$\Phi^4 \sim N$



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matches!

(related observation '09 Simon-Sheikh Jabbari)

BPS states (Ground states) $k \neq 1$

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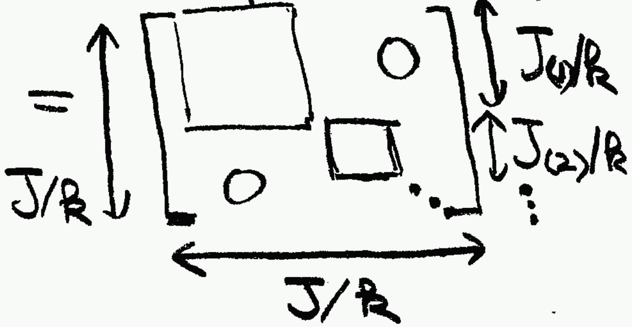

$(X^i = 0)$

- labelled by
  $J = J_{(1)} + \cdots + J_{(n)}$
- $J_{(i)}$: multiple of k, $S^7/\mathbb{Z}_k$
- (n) - numbers of concentric spherical membranes extended in AdS_4
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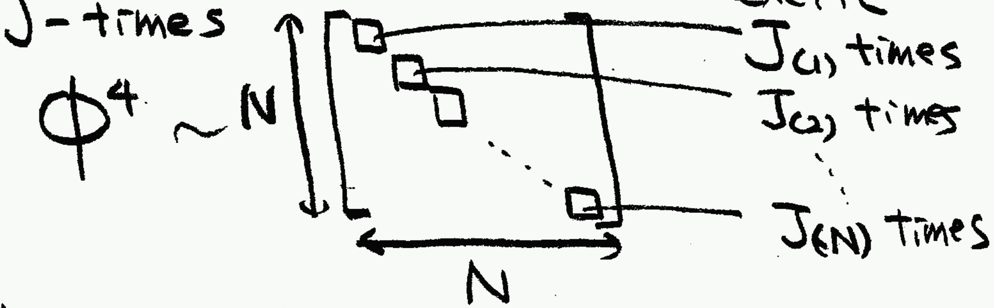

- labelled by
  $J = J_{(1)} + \cdots J_{(N)}$
  ($J_{(i)} = 0$ allowed)
- BKG gauge field : diagonal
  each diagonal elem. : Dirac monopole charge $J_{(i)}/k$
  (GNO charges)
- $J_{(i)}$: multiple of k by Dirac qtz.

matches!

(related observation '09 Simon-Sheikh Jabbari)

non-BPS fluctuation

AdS side ('02 Dasgupta - SheikhJabbari - vanRaamsdonk)

- consider single membrane case

stable S^2 - membrane extended in Y^a (AdS_4)

- fluctuation governed by

(coupled) harmonic oscillators labelled by

$Y_{\ell m}$ (+ spin) & polarisation

6 real from $X^1 \cdots X^6$ (S^7)

2 real from $Y^1 \cdots Y^3$ (AdS_4) - (gauge DOF)

16 real Fermions

- Spectrum for $X^1 \cdots X^6$

$$\omega = \sqrt{\left(\tfrac{1}{2}\right)^2 + \ell(\ell+1)}$$

$\sim X^2$ $\quad$ $\sim [X, Y]^2$

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CFT side

GNO charge $\sim \begin{bmatrix} J_0 & & \\ & \ddots & \\ & & 0 \end{bmatrix}$,

- fluctuation of the light DOF governed by ABJM on $S^2 \times \mathbb{R}$ in a carefully chosen gauge

= (coupled) harmonic oscillators labelled by $Y_{\ell m}$ (+spin) & polarisation

3 complex scalars Φ^1, Φ^2, Φ^3

Φ^4 mixed with gauge fields

4 2-component complex spinor ψ^A

- Spectrum for Φ^1, Φ^2, Φ^3

$$\omega = \sqrt{\left(\tfrac{1}{2}\right)^2 + \ell(\ell+1)}$$

mass Laplacian on S^2

non-BPS fluctuation

AdS side ('02 Dasgupta - SheikhJabbari - vanRaamsdonk)

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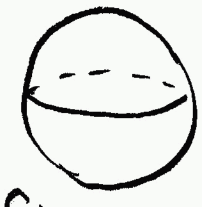

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$$\omega = \sqrt{\underset{\text{mass}}{\left(\tfrac{1}{2}\right)^2} + \underset{\text{Laplacian on } S^2}{\ell(\ell+1)}}$$

matches!

Summary and Future directions

1. M-theoretic AdS_4/CFT_3 is a good place to learn about M-theory, matrix model, AdS/CFT

- ☆ can test matrix model & AdS_4/CFT_3 simultaneously
- ☆ prime ex. of non-stringy AdS/CFT

2. Good approx. schemes for large J sector

AdS side : pp-wave approx. + loop exp., pp-wave matrix model

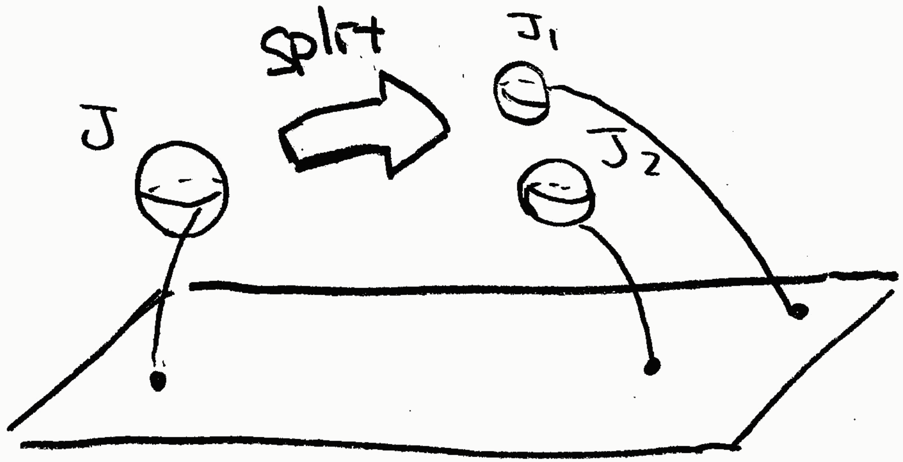
CFT side : Born-Oppenheimer approx.
radially quantised ABJM with large magnetic flux

3. AdS_4/CFT_3 works fine at the leading order approx.

BPS (ground) states	stable spherical membranes	diagonal magnetic flux GNO-charges
non-BPS fluctuation	oscillation of spherical membranes	excitation in radial quantisation

Future directions

- First subleading correction on the CFT side
  Integration out of heavy (off-diagonal) modes.
- Joining - Splitting interaction of membranes
  via instanton (kink) solution of pp-wave matrix model
  $\overset{?}{\longleftrightarrow}$ 3-pt function of monopole operators



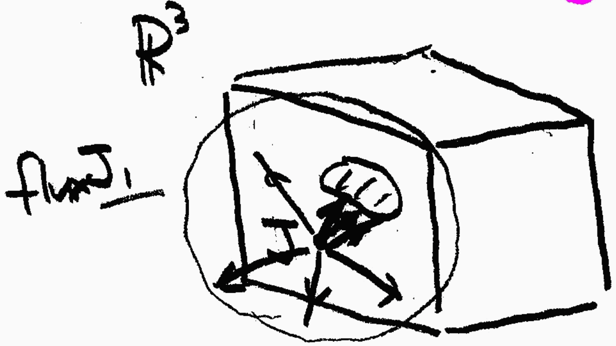
work in progress

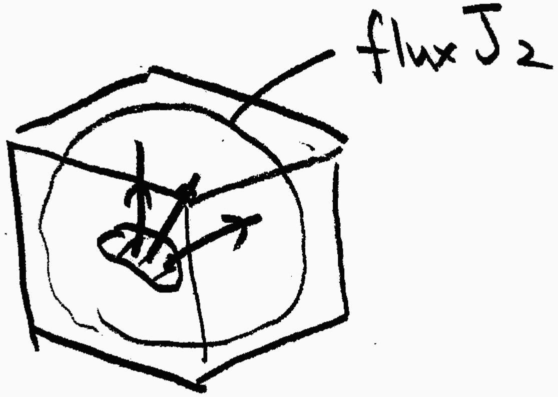
☆ Eq. governing BPS kink solution is studied ('03 Yee, Yi)

$$\dot{Y}^a = i\epsilon_{abc}[Y^b, Y^c] + Y_a$$

☆ Dimension of moduli spaces of kink connecting two vacua is known ('01 Bachas, Hoppe, Pioline) but little was known about explicit solutions.

☆ To obtain good control of the solutions, we consider large J and use continuum approx.
The problem can be mapped to 3D laplace eq. with interesting b.c.





Corresponds to splitting of a sphere with J ang mom. into two spheres with J_1, J_2

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