Spontaneous Compactification of Bimetric Theory

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Nahomi Kan (Gifu National College of Technology),
Takuya Maki (Japan Women's College of Physical Education)
Kiyoshi Shiraishi (Yamaguchi University)

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In a six-dimensional model of bimetric theory,
massless and massive gravitons
emerge with a spontaneous
compactification to four dimensions.

Why massive gravity?

- How does gravity behave at a cosmological scale?
- In the very early universe, is the evolution of scale factors different from the present behavior?
- As an exercise for covariant approach to quantum gravity

Problems in massive gravity

- **vDVZ discontinuity** (van Dam & Veltman; Zakharov, 1970)

  the limit $m \to 0$ is different from GR

- **Vainshtein Mechanism** (1972 PLB39)

  nonlinear effects at strong gravity is important(!?)

- **Boulware-Deser Ghost** (1972 PRD6)

A ghost degree of freedom appears in nonlinear massive gravity, because it is impossible to make a scalar constraint without derivatives (from a single metric).

Bigravity (Bimetric gravity)

massless and massive gravitons

Ghost-free Non-linear massive gravity

(one metric is non-dynamical)

dRGT (de Rham, Gabadadze, Tolley) 2010

Hassan, Rosen 2011

Ghost-free Non-linear Bigravity

Hassan, Rosen, Schmidt-May, 2012

(very sorry to many other authors for omitting their work...)

mass term $\sim\sqrt{-g}\sum_n C_n\,\delta^{\mu_1\mu_2\cdots\mu_n}_{\nu_1\nu_2\cdots\nu_n} K^{\nu_1}_{\mu_1} K^{\nu_2}_{\mu_2}\cdots K^{\nu_n}_{\mu_n}$

two metrics $g_{\mu\nu},\quad f_{\mu\nu}$

$$K^{\nu}_{\mu}=\sqrt{g^{\nu\rho}f_{\rho\mu}}$$

The generalized Kronecher delta

$$\delta^{\mu_1\mu_2\cdots\mu_p}_{\nu_1\nu_2\cdots\nu_p}\equiv\begin{vmatrix}\delta^{\mu_1}_{\nu_1} & \delta^{\mu_1}_{\nu_2} & \cdots & \delta^{\mu_1}_{\nu_p}\\ \delta^{\mu_2}_{\nu_1} & \delta^{\mu_2}_{\nu_2} & \cdots & \delta^{\mu_2}_{\nu_p}\\ \vdots & \vdots & \ddots & \vdots\\ \delta^{\mu_p}_{\nu_1} & \delta^{\mu_p}_{\nu_2} & \cdots & \delta^{\mu_p}_{\nu_p}\end{vmatrix}$$

vierbein massive gravity

$$e_g{}^A_M \eta_{AB} e_g{}^B_N = g_{MN}\,, \qquad e_f{}^A_M \eta_{AB} e_f{}^B_N = f_{MN}$$

mass term $\sim \sum_n c_n \, \delta^{M_1 M_2 \ldots M_n}_{A_1 A_2 \ldots A_D} \, e_g{}^{A_1}_{M_1} \cdots e_f{}^{A_D}_{M_D}$ (n e_g's, D-n e_f's)

According to Alexandrov (GRG46(2014)1639), vierbein bigravity is also Ghost free.

Six-dimensional Model of BimetricTheory

In models of Bigravity, Massive gravity

mass of massive graviton <--'new' scale, by hand(?)

we wish to consider connections to some other scales and/or mechanisms.

We need mixing part in the action of bimetric theory.

gauge field mixing (Holdom, 1986)

$$\mathcal{L} = -\frac{1}{4}\left(F_{1\mu\nu}F_1{}^{\mu\nu} + F_{2\mu\nu}F_2{}^{\mu\nu} + 2\alpha F_{1\mu\nu}F_2{}^{\mu\nu}\right)$$

(this is also used in recent models of dark matter) idea: Consider flux mixing and metric mixing at the same time=at the stage of compactification of extra space!

The six-dimensional model:

$$S = S_g[g, F_g] + S_f[f, F_f] + S_{int}[g, f, F_g, F_f]$$

$$S_g = \int d^6x\sqrt{-g}\left[\frac{1}{2\kappa_g^2}R_g - \frac{1}{4}g^{MK}g^{NL}F_{g\,MN}F_{g\,KL} - \Lambda_g\right]$$

$$S_f = \int d^6x\sqrt{-f}\left[\frac{1}{2\kappa_f^2}R_f - \frac{1}{4}f^{MK}f^{NL}F_{f\,MN}F_{f\,KL} - \Lambda_f\right]$$

$$S_{int} = -\frac{\alpha}{96}\int d^6x\,\epsilon^{MNRSTL}\epsilon_{ABCDEF}e_{g\,M}^{\;A}e_{f\,N}^{\;B}e_{g\,R}^{\;C}e_{f\,S}^{\;D}F_{g\,TL}F_{f\,JK}e_g^{\;EJ}e_f^{\;FK}$$

α : **a dimensionless parameter**

Compactification

extra space $= S^2$

$$g_{mn}dx^m dx^n = a^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$f_{mn}dx^m dx^n = b^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

(two metrics of two radii in general)

curvatures:

$$R_{g\,mn} = \frac{1}{a^2} g_{mn}\,, \qquad R_{f\,mn} = \frac{1}{b^2} f_{mn}$$

fluxes:

$$F_g = dA_g = -\frac{n_g}{2ea^2} a\, d\theta \wedge a \sin\theta d\varphi\,, \qquad F_f = dA_f = -\frac{n_f}{2eb^2} b\, d\theta \wedge b \sin\theta d\varphi$$

Compactification of Six-dimensional Einstein-Maxwell theory as in

Randjbar-Daemi, Salam, Strathdee NPB214(1983)

a static solution is given as a minimum point of the effective potential for scales of extra dimensions
with fine-tuning to obtain flat four-dimensional spacetime

$$V(a,b) = a^2\left(-\frac{1}{\kappa_g^2 a^2}+\frac{n_g^2}{8e^2a^4}+\Lambda_g\right)+b^2\left(-\frac{1}{\kappa_f^2 b^2}+\frac{n_f^2}{8e^2b^4}+\Lambda_f\right)$$
$$+2\alpha ab\left(\frac{n_g n_f}{8e^2a^2b^2}\right).$$

or, if we set $x \equiv b/a, \quad y \equiv ab,$

$$V(x,y) = -\frac{1}{\kappa_g^2}-\frac{1}{\kappa_f^2}+\frac{1}{8e^2 y}\left(n_g^2 x+\frac{n_f^2}{x}+2\alpha n_g n_f\right)+y\left(\frac{\Lambda_g}{x}+\Lambda_f x\right)$$

V is minimized at $y = y_0$:

$$y_0 = \frac{1}{2\sqrt{2}e}\sqrt{\left(n_g^2 x + \frac{n_f^2}{x} + 2\alpha n_g n_f\right)\left(\frac{\Lambda_g}{x} + \Lambda_f x\right)^{-1}}$$

$$V(x, y_0) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{1}{\sqrt{2}e}\sqrt{\left(n_g^2 x + \frac{n_f^2}{x} + 2\alpha n_g n_f\right)\left(\frac{\Lambda_g}{x} + \Lambda_f x\right)}$$

the parameter tuning must be done as the above takes a minimum value at $x = x_0$

masses of gravitons

The four-dimensional effective action for gravitons after the compactification

$$
\begin{aligned}
S^{(4)} \;=\; & 4\pi \int d^4x \left\{ \sqrt{-g^{(4)}} \left[\frac{a^2}{2\kappa_g^2} R_g^{(4)} + \frac{1}{\kappa_g^2} - \frac{n_g^2}{8e^2a^2} - \Lambda_g a^2 \right] \right. \\
& + \sqrt{-f^{(4)}} \left[\frac{b^2}{2\kappa_g^2} R_f^{(4)} + \frac{1}{\kappa_f^2} - \frac{n_f^2}{8e^2b^2} - \Lambda_f b^2 \right] \\
& \left. - \frac{\alpha}{12} \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} {e_g}^a_\mu {e_f}^b_\nu {e_g}^c_\rho {e_f}^d_\sigma \frac{n_g n_f}{8e^2ab} \right\} ,
\end{aligned}
$$

In the weak field limit: $e_g = \eta + \frac{1}{2} h_g,\; e_f = \eta + \frac{1}{2} h_f$,

we find $\sqrt{-g^{(4)}} = \det e_g = 1 + \frac{1}{2}[h_g] + \frac{1}{8}[h_g]^2 - \frac{1}{8}[h_g^2] + O(h^3)$

and
$$\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}e_{g\mu}^{\ a}e_{f\nu}^{\ b}e_{g\rho}^{\ c}e_{f\sigma}^{\ d}$$
$$= 1+\frac{1}{4}([h_g]+[h_f])+\frac{1}{48}([h_g]^2+4[h_g][h_f]+[h_f]^2)$$
$$-\frac{1}{8}([h_g^2]+4[h_gh_f]+[h_f^2])+O(h^3)\,.$$

Now we get the Lagrangian for two gravitons h_g, h_f.

To eliminate the linear term in h_g, h_f, we should choose the parameters as

$$\frac{1}{\kappa_g^2}=\frac{n_g^2x_0}{8e^2y_0}+\frac{\alpha n_gn_f}{8e^2y_0}+\Lambda_g\frac{y_0}{x_0}\,,$$
$$\frac{1}{\kappa_f^2}=\frac{n_f^2}{8e^2x_0y_0}+\frac{\alpha n_gn_f}{8e^2y_0}+\Lambda_fx_0y_0$$

These come from Two constraints from variations of two lapse functions.

Then, the Lagrangian for two gravitons reads:

$$-\frac{1}{2}\partial_\rho H_{0\mu\nu}\partial^\rho H_0{}^{\mu\nu} + \partial_\rho H_0{}^\rho{}_\mu \partial_\nu H_0{}^{\nu\mu} - \partial_\mu H_0{}^{\mu\nu}\partial_\nu H_0 + \frac{1}{2}\partial_\rho H_0 \partial^\rho H_0$$

$$-\frac{1}{2}\partial_\rho H_{1\mu\nu}\partial^\rho H_1{}^{\mu\nu} + \partial_\lambda H_1{}^\lambda{}_\mu \partial_\nu H_1{}^{\nu\mu} - \partial_\mu H_1{}^{\mu\nu}\partial_\nu H_1 + \frac{1}{2}\partial_\rho H_1 \partial^\rho H_1$$

$$+\frac{1}{2}\frac{\alpha n_g n_f}{12 e^2 y_0^2}\left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0}\right)\left(H_1^2 - H_{1\,\mu\nu}^{\;2}\right) ,$$

where $H_0 \equiv \dfrac{\frac{\kappa_f}{\kappa_g x_0} h_g + \frac{\kappa_g x_0}{\kappa_f} h_f}{\sqrt{\frac{4}{y_0}\left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0}\right)}}$, $\quad H_1 \equiv \dfrac{h_g - h_f}{\sqrt{\frac{4}{y_0}\left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0}\right)}}$

massless massive

A simple case:

- $\Lambda_g = \Lambda_f \equiv \Lambda,\ \kappa_g = \kappa_f \equiv \kappa$ and $n_g = n_f \equiv n$

mass-square of massive graviton: $m^2 = \dfrac{4\sqrt{2\Lambda}\alpha e}{3n(1+\alpha)^{3/2}}$

(tuning: $\frac{1}{\kappa^2} = \frac{n\sqrt{\Lambda/2}}{e}\sqrt{1+\alpha}$ etc.)

Since $(m^2/\frac{2}{a^2}) = \frac{\alpha}{3(1+\alpha)}$,

mass of massive graviton is less than that of a KK mode

generalization to multigravity -->spectra with hierarchy

Summary and outlook

In a Six-dimensional Bimetric model,
massless and massive gravitons appear
at the spontaneous compactification.

- Cosmology (many constraints!)
- Spectrum of spin2, spin1, spin0
- Tuning-free theory? (extension of models of Salam-Sezgin-Nishino-Maeda?)