Toric GLSM for ALE space

Masaya YATA
KEK

Based on [Tetsuji KIMURA, MY/ arXiv:1402.5580, arXiv:1406.0087]

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Introduction はじめに

It is difficult to study GLSM beyond the the topological aspect of string theory.

➡ For instance, in N=(2,2) toric GLSM, D-term constraints cannot derive a correct background geometry in the IR limit.

Challenges 課題

In order to construct the correct GLSM, it is important to find the correct F-term.

➡ As a typical example, we consider the A1-ALE space and construct the GLSM from the toric data which derives the correct background geometry and B-fields.

➡ We develop the duality transformation for GLSM including charged chiral superfields in the D-term and F-term.

Toric GLSM

$\tau = \tau_1 + \mathrm{i}\tau_2$: FI-parameter

$\Sigma = \frac{1}{\sqrt{2}}\overline{D}_+ D_- V$

The N=(2,2) Lagrangian as a GLSM

Superfields

$$\mathcal{L} = \int \mathrm{d}^4\theta \left\{ -\frac{1}{e^2}\overline{\Sigma}\Sigma + \sum_{i=1}^{k} \overline{\Phi}_i e^{2Q_i V}\Phi_i \right\} + \left\{ \sqrt{2}\int \mathrm{d}^2\tilde{\theta}(-\tau\Sigma) + (\mathrm{h.c.}) \right\}$$

Finding the SUSY vacua (Higgs branch) and taking the IR limit.

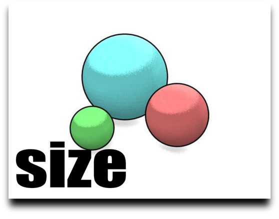
Component

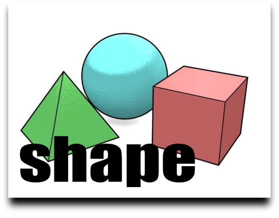
$$\mathcal{L}^{\mathrm{IR}} = -\frac{1}{2}G_{MN}\partial_m\phi^M\partial^m\phi^N + \frac{1}{2}B_{MN}\varepsilon^{mn}\partial_m\phi^M\partial_n\phi^N \quad ?$$

Higgs branch condition $\sigma = 0$ ➡ D-term constraint $\sum_{i=1}^{k} Q_i|\phi_i|^2 = \tau$ ➡ Q_i = toric charge; GLSM gives a toric CY as the background

D-term ➡ Kahler structure

F-term ➡ Complex structure





It is considered that a toric GLSM realize the target space which is specified with the toric data in the IR limit[1]. However, the correct NLSM cannot be derived from the GLSM because the toric data are embedded into only the D-term. The D-term constraint controls only the Kahler structure or the target space size. While, the F-term governs the complex structure or the shape of back ground. Thus, in order to construct the correct GLSM, we need to fix the proper F-term.

A1-ALE toric GLSM with F-term

In order to derive the correct back ground, the N=(2,2) field contents should be extended to the N=(4,4). The charged hypermultiplets {Ai ,Bi } are doublet under the SU(2)R symmetry, and the SU(2)R symmetry completely determines the F-term. The additional vector multiplet $(\tilde{V},\tilde{\Phi})$ is introduced in order to remove redundant degrees of freedom, and the charge α is arbitrary except zero.

Charge assignment for the chiral superfields in N=(4,4) theory

	(A_1, B_1)	(A_2, B_2)	(A_3, B_3)
(V, Φ)	$(+1, -1)$	$(-2, +2)$	$(+1, -1)$
$(\tilde{V}, \tilde{\Phi})$	$(0, 0)$	$(-\alpha, \alpha)$	$(0, 0)$

➡ **N=(4,4) SUSY determines F-term**

GLSM Lagrangian for A1-ALE space(superfields)

$$\mathcal{L}_{A_1} = \int \mathrm{d}^4\theta \left\{ \frac{1}{e^2}(-|\Sigma|^2 + |\Phi|^2) + \frac{1}{\tilde{e}^2}(-|\tilde{\Sigma}|^2 + |\tilde{\Phi}|^2) \right\}$$
$$+ \int \mathrm{d}^4\theta \left\{ |A_1|^2 e^{+2V} + |A_2|^2 e^{-4V-2\alpha\tilde{V}} + |A_3|^2 e^{+2V} \right\} + \int \mathrm{d}^4\theta \left\{ |B_1|^2 e^{-2V} + |B_2|^2 e^{+4V+2\alpha\tilde{V}} + |B_3|^2 e^{-2V} \right\}$$
$$+ \left\{ \sqrt{2}\int \mathrm{d}^2\theta \Big(\Phi(-A_1B_1 + 2A_2B_2 - A_3A_3 - s) + \tilde{\Phi}(\alpha A_2B_2 - \tilde{s}) \Big) + (\mathrm{h.c.}) \right\}$$
$$+ \left\{ \sqrt{2}\int \mathrm{d}^2\tilde{\theta}\Big(-t\Sigma - \tilde{t}\tilde{\Sigma} \Big) + (\mathrm{h.c.}) \right\}$$

F-term

Finding the SUSY vacua (Higgs branch) and taking the IR limit.

NLSM Lagrangian for A1-ALE space(component)

$$\mathcal{L}^{\mathrm{IR}}_{A_1} = -\frac{1}{2}\mathcal{A}^{-1}(\partial_m\rho)^2 - \frac{\rho^2}{8}\left\{ (\partial_m\vartheta)^2 + \sin^2\vartheta(\partial_m\varphi)^2 \right\} - \frac{\rho^2}{8}\mathcal{A}\left\{ (\partial_m\psi) + \cos\vartheta(\partial_m\varphi) \right\}^2$$
$$-\sqrt{2}t_2F_{01} + (\text{fermionic fields})$$

$$\mathcal{A} = 1 - \frac{\mathfrak{a}^4}{\rho^4} \qquad B_{MN} = 0$$

The NLSM represents the Eguchi-Hanson space[2]. Thus, existence of the F-term is inevitable to derive the correct geometry.

$\mathfrak{a} \le \rho \quad 0 \le \psi < 4\pi$
$0 \le \varphi < 2\pi \quad 0 \le \vartheta < \pi$

A1-ALE space (Eguchi-Hanson space) ⟵ **T-duality relation** ⟶ **Parallel NS5-branes and the B-field** $\quad \beta = -2 \quad \mathtt{V} = \varphi'$

Duality Transformation

$\mathcal{W}$: arbitrary holomorphic function
C : unconstrained complex superfields

The Rocek-Verlinde transformation[3], which converts one GLSM to the other, utilize shift symmetries of D-terms[4]. Since F-terms break the shift symmetries, it is difficult to discuss about the transformation for GLSMs with F-term. In order to do that, we rewrite the F-term to D-terms by the property of chiral superfields in the N=(4,4) vector multiplet[5].

Duality transformation of the GLSM with F-term

F-term $\sqrt{2}\int \mathrm{d}^2\theta\ \Phi\mathcal{W}\Psi$ ➡ break down the shift symmetry

F-term ➡ D-term

$$\Phi = \overline{D}_+\overline{D}_- C$$

$$\int \mathrm{d}^2\theta\alpha(-\Phi)\mathcal{W}\Psi + (\mathrm{h.c.}) = -2\alpha\int \mathrm{d}^4\theta\left\{ \Psi\mathcal{W}C + \overline{\Psi\mathcal{W}C} \right\}$$

Explicit formulation of the dualized Lagrangian

First order formalism

$$\mathcal{L} = \int \mathrm{d}^4\theta(e^{+2\alpha V + R}) + \left\{ \int \mathrm{d}^4\theta\Big(-\beta RY - \beta\mathrm{i}SY' - e^{\frac{1}{2}(R+\mathrm{i}S)}(2\sqrt{2}\alpha\mathcal{W}C + X) \Big) + (\mathrm{h.c.}) \right\}$$

Integrating out $\{Y, Y'\}$: $R = \tilde{\Psi}_1 + \overline{\tilde{\Psi}}_1, \mathrm{i}S = \tilde{\Psi}_2 - \overline{\tilde{\Psi}}_2$

Integrating out $\{X\}$: $e^{\tilde{\Psi}_1} = e^{\tilde{\Psi}_2} = \Psi$

Integrating out $\{R, Y'\}$

Absorbing X into the C

Original Lagrangian (rewrite the F-term to the D-term)

$$\mathcal{L} = \int \mathrm{d}^4\theta(|\Psi|^2 e^{+2\alpha V}) + \left\{ -2\sqrt{2}\alpha\int \mathrm{d}^4\theta\Big(\Psi\mathcal{W}C + \overline{\Psi\mathcal{W}C} \Big) \right\}$$

$\{R, S\}$: real superfields
$\{X\}$: chiral superfields
$\{Y, Y'\}$: twisted chiral superfields

Dualized Lagrangian NEW!

$$\mathcal{L}_{\mathrm{dualized}} = \int \mathrm{d}^4\theta\left\{ -2\beta(Y + \overline{Y})\log\mathcal{F} + \frac{1}{2}\mathcal{F}\mathcal{T} \right\} + \left\{ \sqrt{2}\int \mathrm{d}^2\tilde{\theta}\beta Y\Sigma + (\mathrm{h.c.}) \right\}$$

There are the explicit dualized Lagrangian and duality relation for the GLSM which includes charged chiral superfields in D-term and F-term. Next, we apply this transformation to the A1-ALE toric GLSM.

$$\mathcal{T} = -\sqrt{2}\alpha e^{-V}\left\{ e^{\frac{1}{2}(\tilde{\Psi} - \overline{\tilde{\Psi}})}\mathcal{W}C + e^{-\frac{1}{2}(\tilde{\Psi} - \overline{\tilde{\Psi}})}\overline{\mathcal{W}C} \right\}$$
$$\mathcal{F} = -\mathcal{T} + \sqrt{\mathcal{T} + 4\beta(Y + \overline{Y})}$$

Duality relation $\quad |\Psi|^2 e^{+2\alpha V} = \beta(Y + \overline{Y}) + \frac{\mathcal{T}^2}{2} - \frac{\mathcal{T}}{2}\sqrt{\mathcal{T}^2 + 4\beta(Y + \overline{Y})}$

GLSM Lagrangian for the dualized A1-ALE space(superfields)

※The A1 direction is dualized

$$\mathcal{L}_{\mathrm{dualized}\ A_1} = \int \mathrm{d}^4\theta\left\{ \frac{1}{e^2}(-|\Sigma|^2 + |\Phi|^2) + \frac{1}{\tilde{e}^2}(-|\tilde{\Sigma}|^2 + |\tilde{\Phi}|^2) \right\}$$
$$+ \int \mathrm{d}^4\theta\left\{ |A_3|^2 e^{+2V} + (|B_1|^2 + |B_3|^2)e^{-2V} + |A_2|^2 e^{-4V-2\alpha\tilde{V}} + |B_2|^2 e^{+4V+2\alpha\tilde{V}} \right\}$$
$$+ \int \mathrm{d}^4\theta\left\{ -2\beta(Y + \overline{Y})\log\mathcal{F} + \frac{1}{2}\mathcal{F}\mathcal{T} \right\} + \int \mathrm{d}^4\theta\left\{ -2\sqrt{2}\Big(A_3B_3 - 2A_2B_2 \Big)C - 2\sqrt{2}\Big(\overline{A_3B_3} - 2\overline{A_2B_2} \Big)\overline{C} \right\}$$
$$+ \left\{ \sqrt{2}\int \mathrm{d}^2\tilde{\theta}\Big(\beta Y - t \Big)\Sigma + (\mathrm{h.c.}) \right\} + \left\{ \sqrt{2}\int \mathrm{d}^2\theta\alpha\tilde{\Phi}A_2B_2 + (\mathrm{h.c.}) \right\}$$

The target space of the NLSM corresponds to the T-duality configuration of the A1-type ALE space via the Buscher rule.

Finding the SUSY vacua (Higgs branch) and taking the IR limit.

NLSM Lagrangian for the dualized A1-ALE space(component)

$$\mathcal{L}^{\mathrm{IR}}_{\mathrm{dualized}\ A_1} = -\frac{\mathcal{A}^{-1}}{2}(\partial_m\rho)^2 - \frac{\rho^2}{8}(\partial_m\vartheta)^2 - \frac{\rho^2}{4}\frac{\mathcal{A}\sin^2\vartheta}{\mathcal{A}\cos^2\vartheta + \sin^2\vartheta}(\partial_m\psi)^2$$
$$-\frac{\beta^2}{2\rho^2}\frac{1}{\mathcal{A}\cos^2\vartheta + \sin^2\vartheta}(\partial_m\mathtt{V})^2 - \frac{\beta}{2}\frac{\mathcal{A}\cos\vartheta}{\mathcal{A}\cos^2\vartheta + \sin^2\vartheta}\varepsilon^{mn}(\partial_m\mathtt{V})(\partial_n\psi)$$
$$-\sqrt{2}t_2F_{01} + (\text{fermionic fields})$$

Summary まとめ

- We explicitly obtained the A1-ALE space from N=(4,4) toric GLSM
  ➡ F-term; governing the complex structure of the geometry
- We constructed the new duality transformation for the GLSM with charged chiral superfields in F-term
  ➡ Converting F-terms to D-terms

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