An Approach to the information problem in a Self-consistent Model of the Black Hole Evaporation

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(with H. Kawai)

(Based on my PhD thesis and
H. Kawai, Y. Matsuo, and Y. Yokokura,
International Journal of Modern Physics A, Volume 28, 1350050 (2013).
The last part is a work in progress.)

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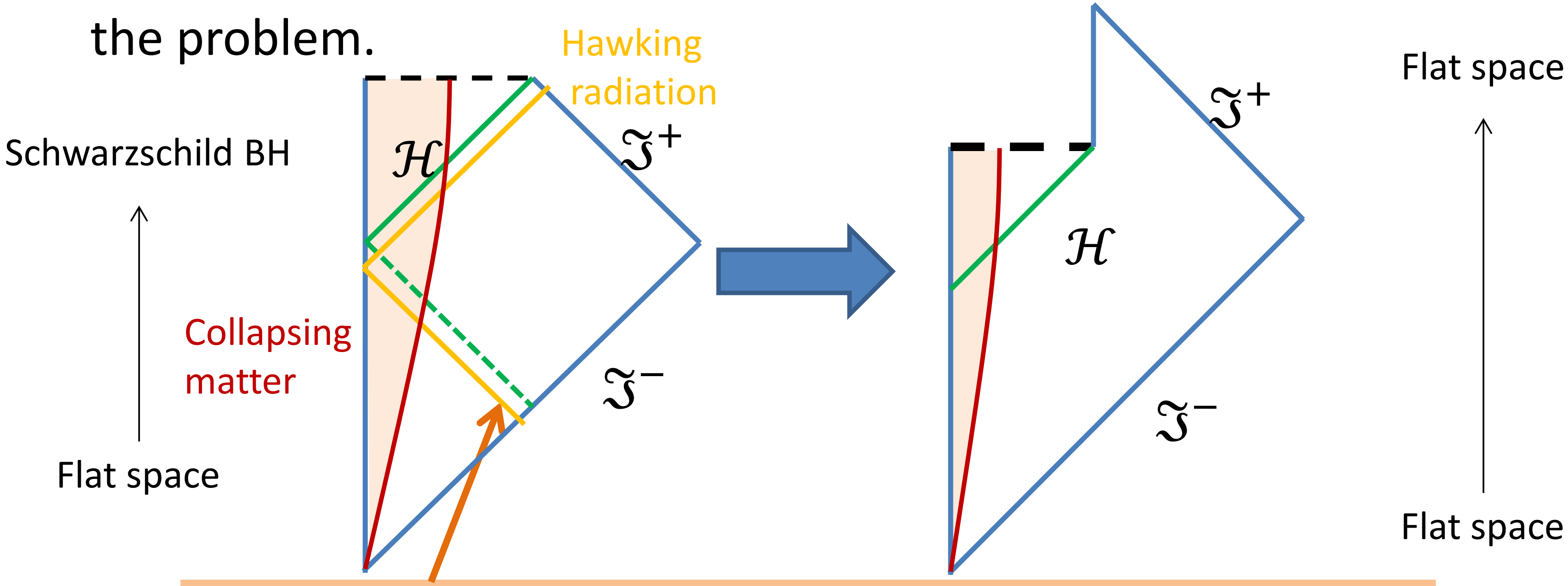
Usual approach to Information puzzle

Usually people take the following approach in the information problem:

(1) Assume formation of a BH by a collapsing process.

(2) Use the vacuum static BH solution to derive the Hawking radiation.

(3) Consider a naïve time evolution after the formation and try to solve the problem.



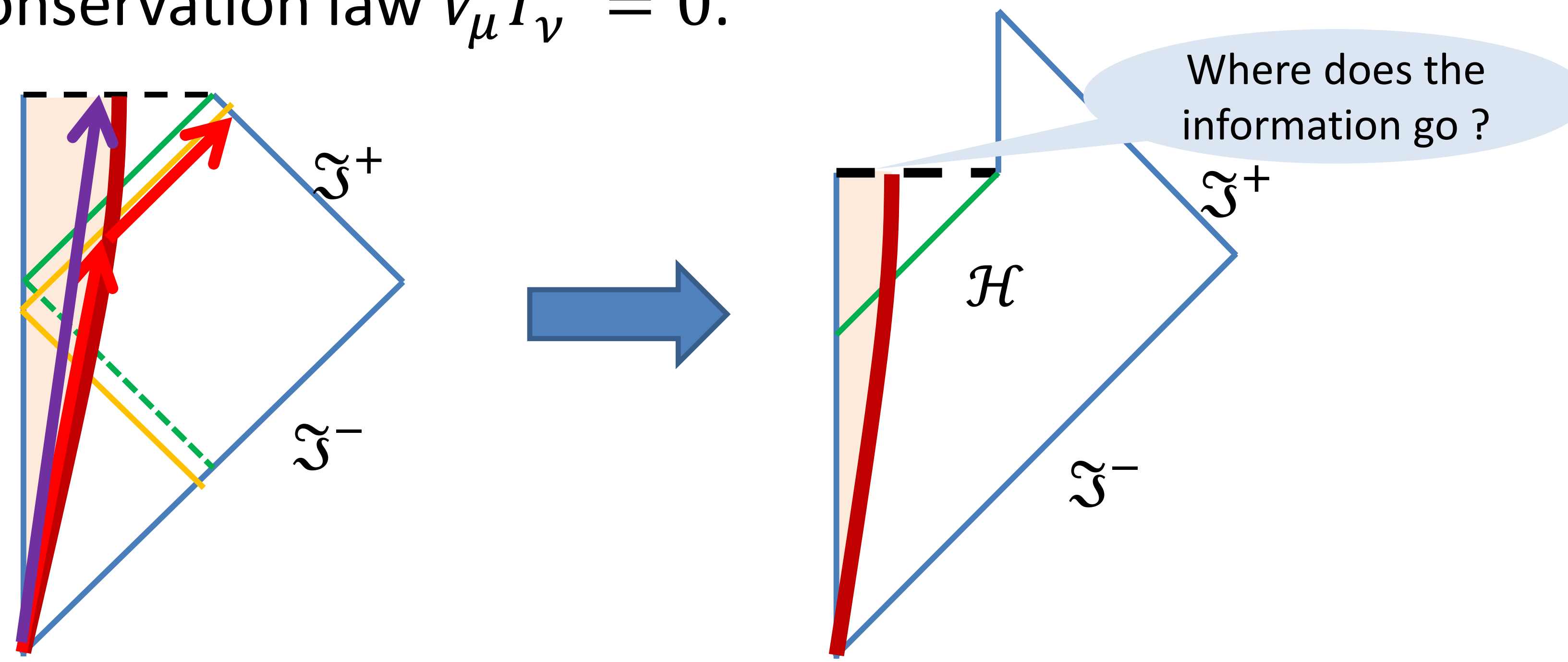
Remark: Vacuum quantum fields on $\mathfrak{I}^-$ become the Hawking radiation through this time-dependent spacetime.

The origin of the information loss

- Information = quantum state of the collapsing matter

⇒Its flow is described by the matter field ϕ_i.

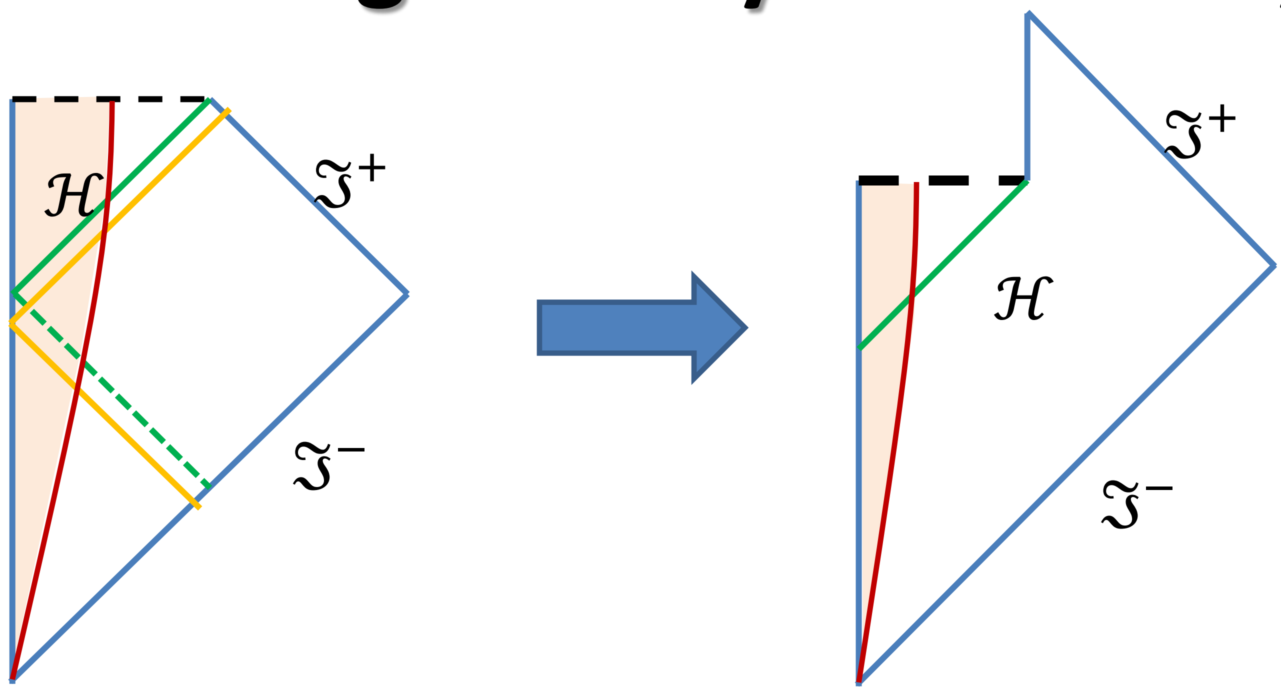
- Energy flow $T_{\mu\nu}$ is determined by the local conservation law $\nabla_\mu T^\mu_\nu = 0$.



⇒Information flow does not follow energy flow.

⇒Information will be lost!

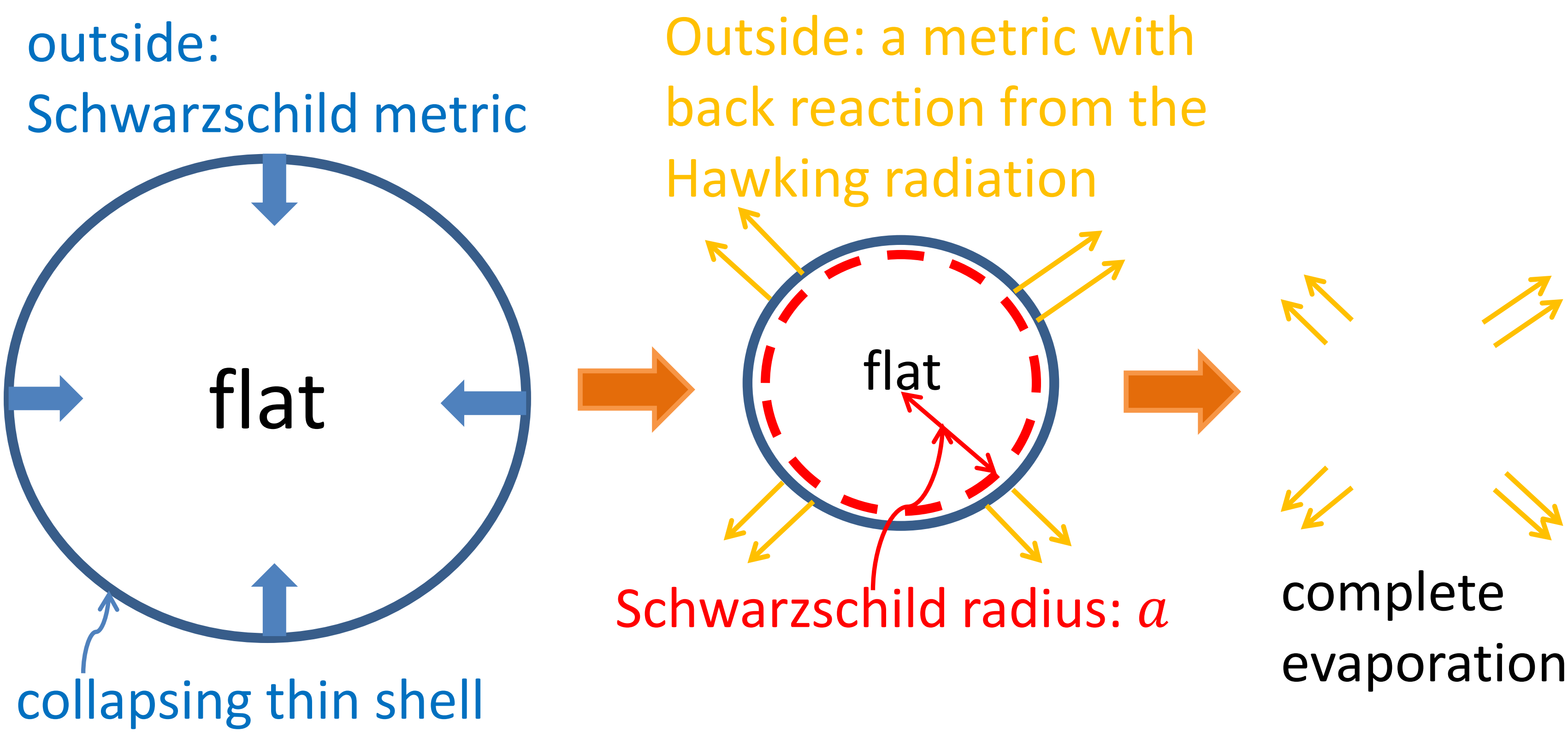
Question:
Does this geometry occur really?



<u>*Our motivation*</u>

Rather, before considering the information problem, we should solve time evolution of the evaporation more correctly to determine the geometry.

A Simple minded viewpoint of the outside observer



Question: Is this story true?

⇒ Yes, under some conditions.

Our approach: self-consistent eqs.

future goal:
Understand time evolution of
the spacetime and information

Solve matter and geometry in a fully
quantum-mechanic manner
⇒Too difficult!

Solve the semi-classical equations
in a self-consistent way

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

$$\nabla^2 \hat{\phi} = 0$$

⇒still difficult!

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Today's talk

This will be taken away later.

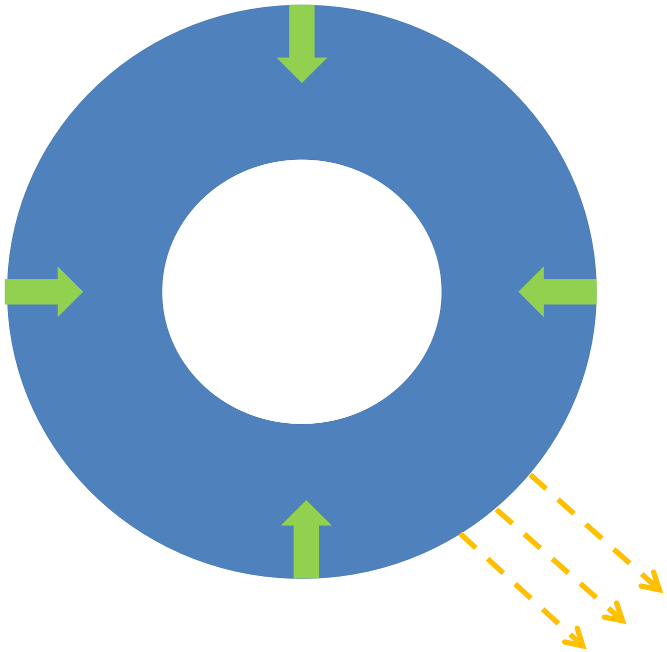
Use some approximations:

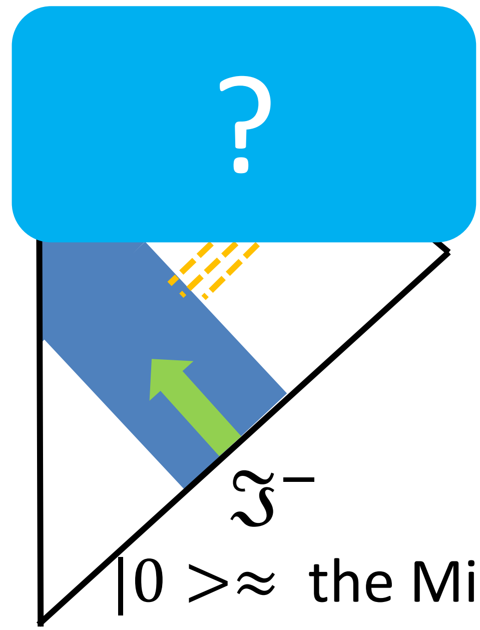
a) Eikonal approximation ⇒solve the wave eq
b) Only s-wave ⇒only single eq is sufficient
c) Large degrees of freedom: $N \gg 1$ ⇒keep $g_{\mu\nu}$ classical

1: A self-consistent model of the BH evaporation

Physical Situation

A continuously-distributed and spherical null matter





$\mathfrak{I}^-$

$|0> \approx$ the Minkowski vacuum

Hawking radiation

described by massless scalar fields: ϕ_i

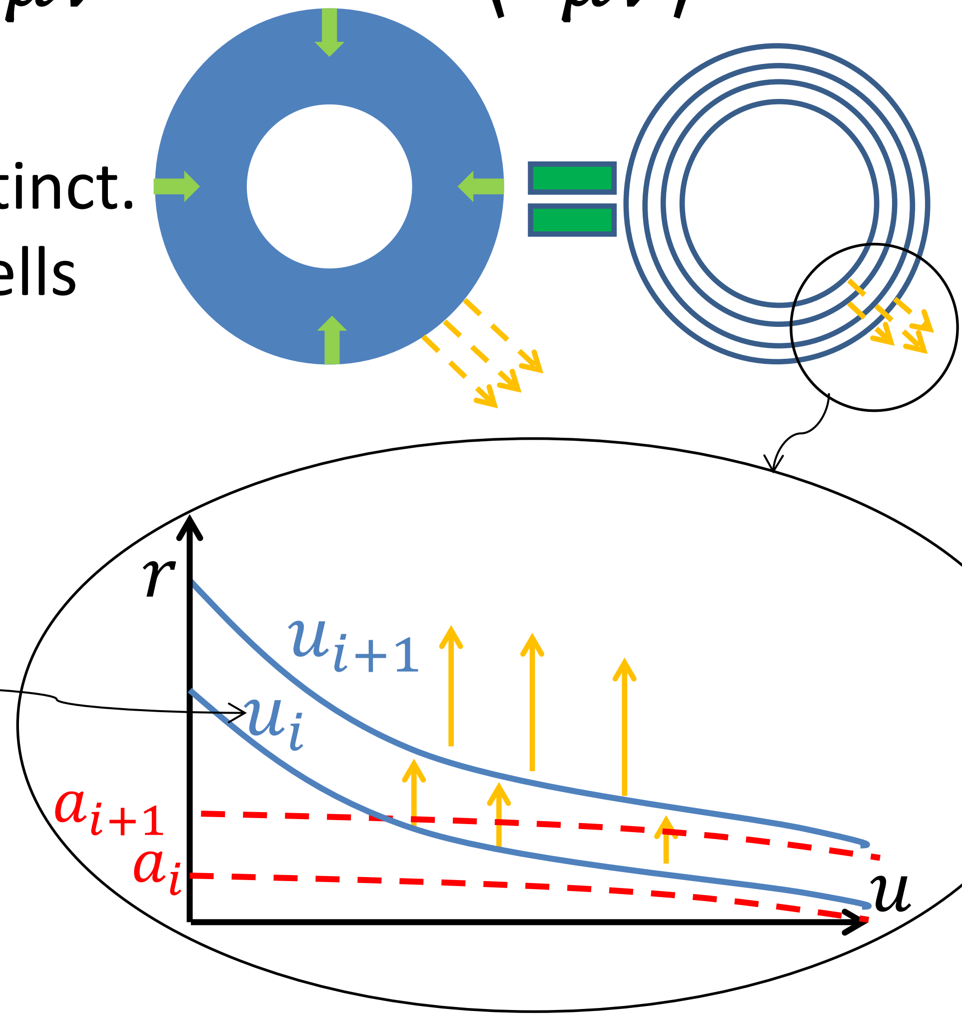
How to solve $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$

Spherical symmetry

⇒The inside and outside are distinct.

⇒Continuous matter = many shells

⇒we can focus on a single shell



s-wave and massless

⇒outgoing Vaidya metric

$$ds_i{}^2 = -\left(1 - \frac{a_i(u_i)}{r}\right) du_i{}^2$$
$$-2du_i dr + r^2 d\Omega^2$$

⇒a single unknown function

$$a(u,r) \equiv 2Gm(u,r)$$

⇒we just have to solve

$$\dot{m} = -4\pi r^2 \langle T_{\alpha\beta} u^\alpha k^\beta \rangle \equiv -J$$

⇒determine the geometry

We can evaluate this by using eikonal approximation and point-splitting regularization.

the evaporating solution

- The self-consistent solution for the inside:

$$ds^2 = -e^{-\frac{24\pi}{Nl_p{}^2}[a_{out}(u)^2 - r^2]}\left[\frac{Nl_p{}^2}{48\pi r^2} e^{-\frac{24\pi}{Nl_p{}^2}[a_{out}(u)^2 - r^2]} du + 2dr\right] du + r^2 d\Omega^2,$$

Remarks:

a) The horizon (or trapped region) does not appear.

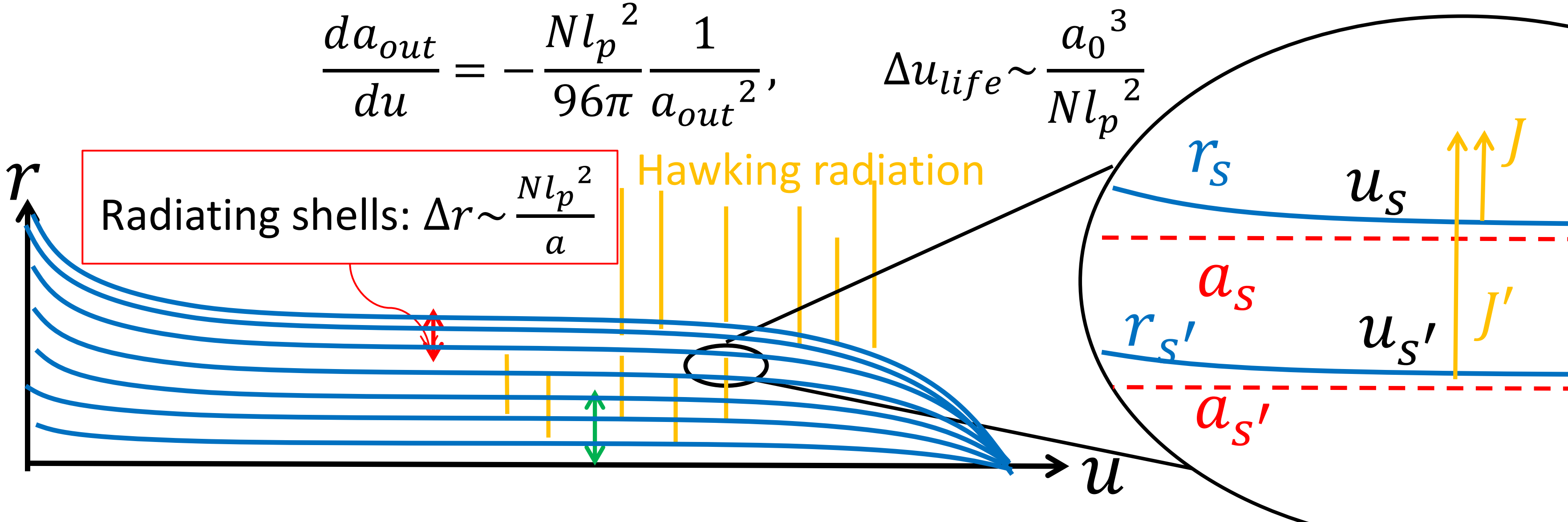
b) The classical limit $\hbar \to 0$ can not be taken ($\because$ self-consistent and non-perturbative)

- Each shell emits the Hawking radiation (following the Planck distribution) with

$$T_H(u,r) = \frac{\hbar}{4\pi a(u,r)}$$

- The total mass decreases as usual:

$$\frac{da_{out}}{du} = -\frac{Nl_p{}^2}{96\pi}\frac{1}{a_{out}{}^2}, \qquad \Delta u_{life} \sim \frac{a_0{}^3}{Nl_p{}^2}$$

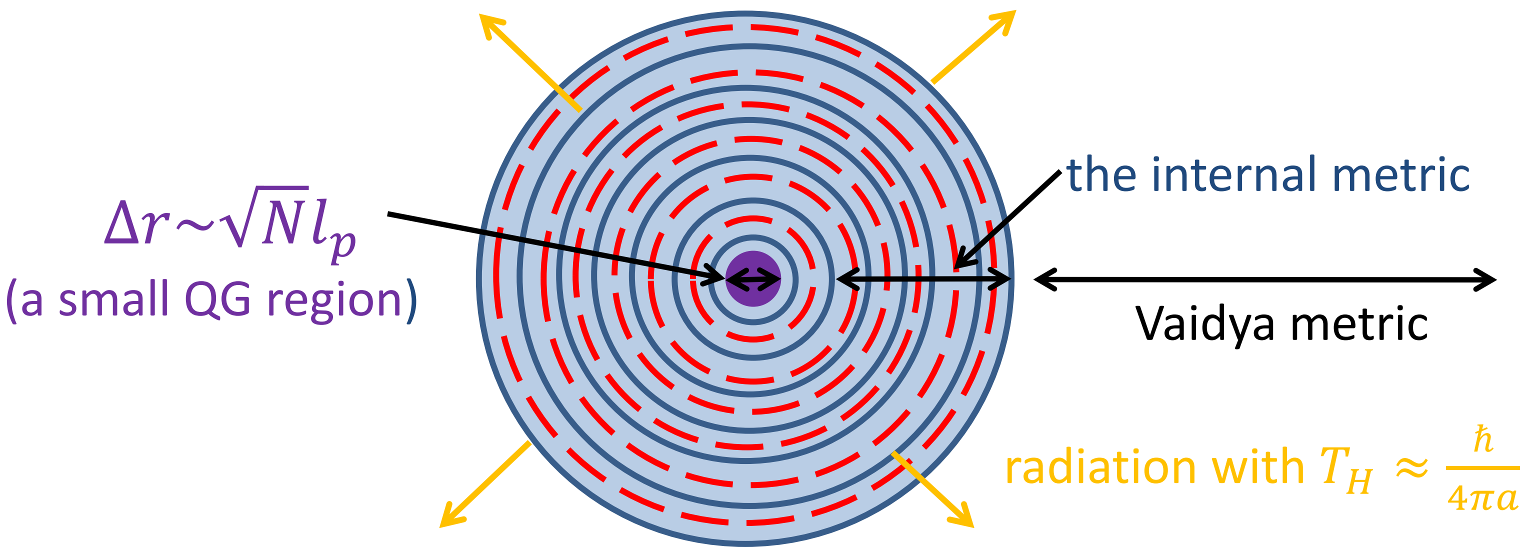


Large N effect: No large singularity

- This metric does not have a large curvature compared with ${l_p}^{-2}$ in the region $r \gg \sqrt{N} l_p$ if N is sufficiently large (but finite), $N \gg 100$:

$$R, \sqrt{R_{\alpha\beta}R^{\alpha\beta}}, \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} \sim \frac{100}{N{l_p}^2}$$

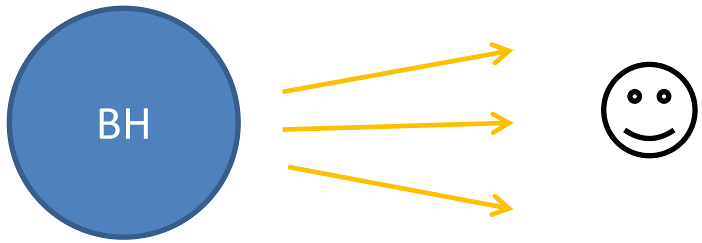
⇒This black hole can evaporate without horizon or large singularity, as if one peels off an onion.



2 BH entropy

Hawking's idea of BH entropy: What is BH entropy?

- Gather up the radiation in the distance.



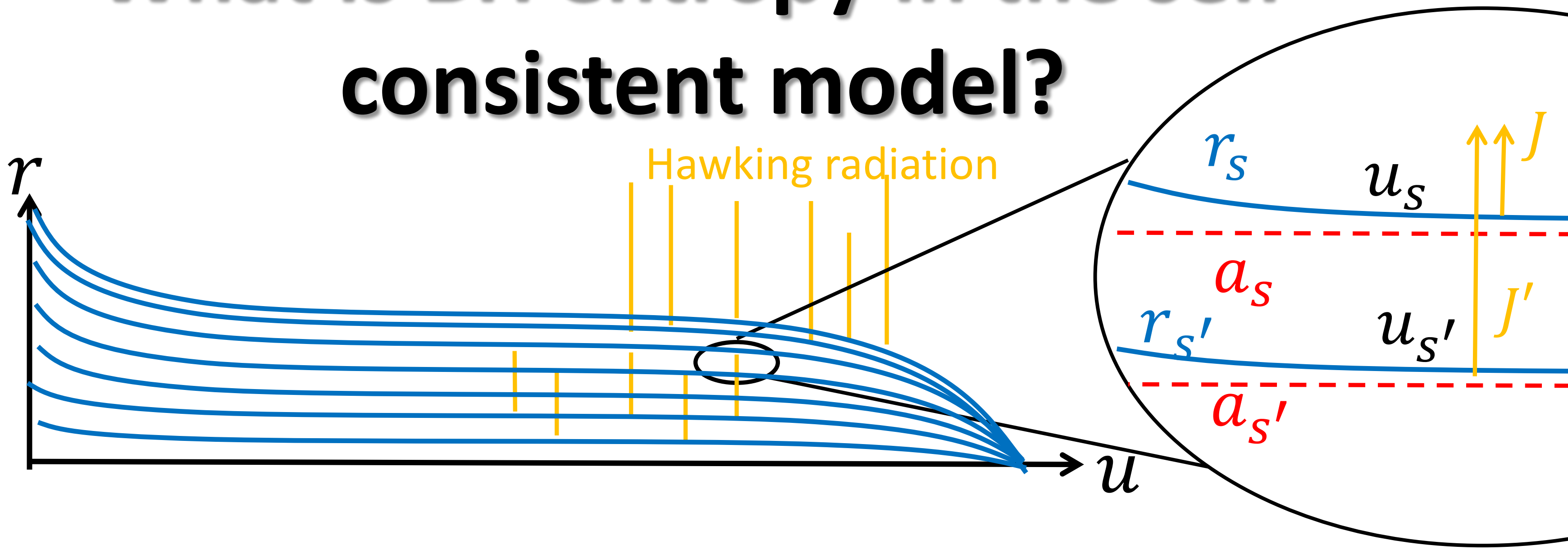
- If the evaporation process is adiabatic, then

BH entropy = entropy of the radiation:

$$S_{BH} = \int \frac{dM}{T_H} = \frac{A}{4l_p^2}$$

Clausius relation (or 1st law)

What is BH entropy in the self-consistent model?



BH entropy

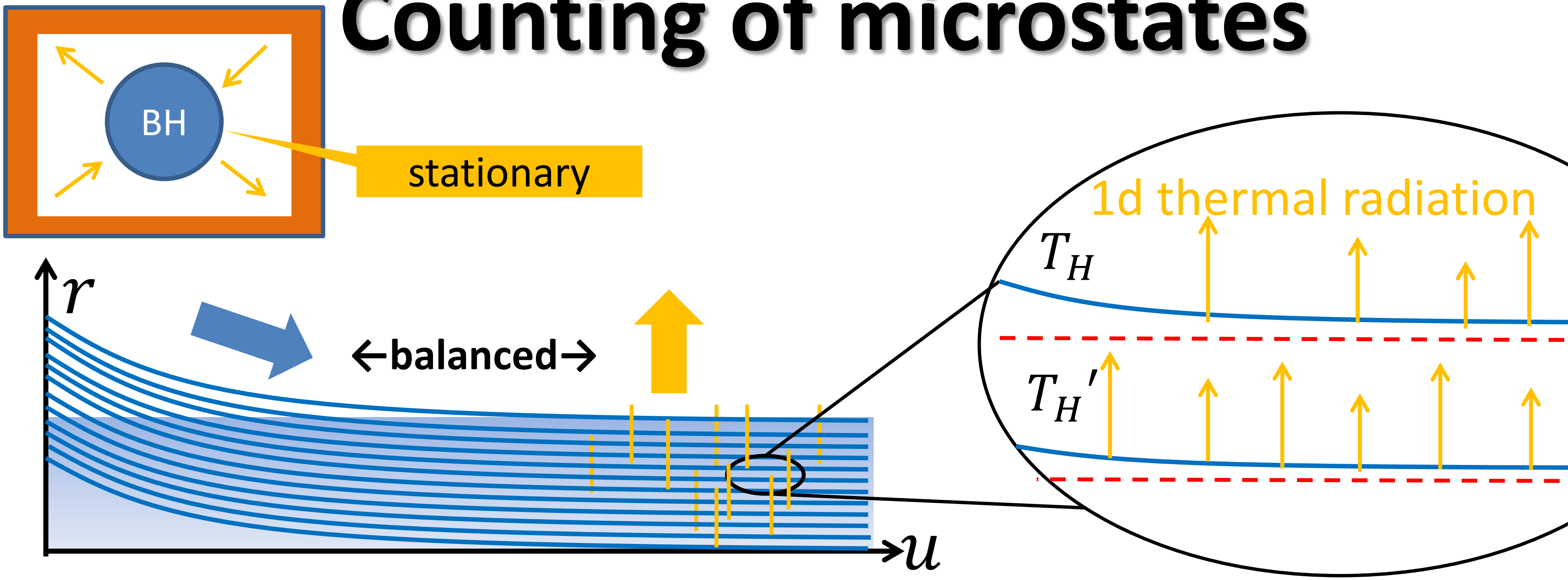
= entropy of the radiation from each shell

(= entropy of the collapsing matter)

if the information is conserved.
⇒entropy problem = information problem

Let's count their microstates!

Counting of microstates



Consider a BH in the heat bath:

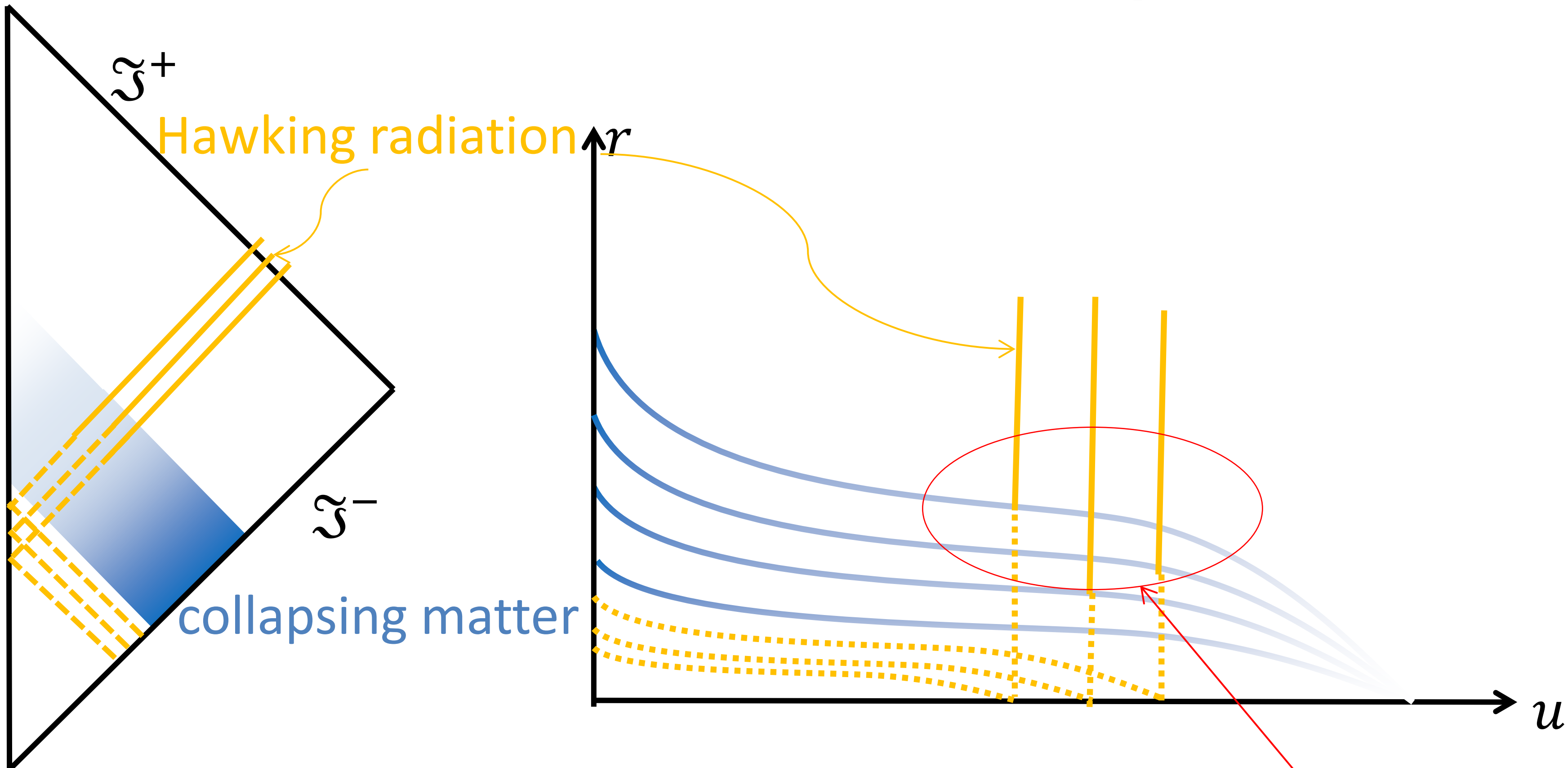
$$ds^2 = -\frac{N{l_p}^2}{48\pi r^2} e^{-\frac{48\pi}{N{l_p}^2}[a^2-r^2]} dt^2 + \frac{48\pi r^2}{N{l_p}^2} dr^2 + r^2 d\Omega^2$$

- 1d thermal radiations & techniques of statistical mechanics
  ⇒counting the microstates of the radiations
  ⇒the black hole entropy.

$$S_{BH} = \int_0^a dr\sqrt{g_{rr}}\, s = \frac{A}{4{l_p}^2}$$

3 the Information problem

How about the information problem?



- The matter seems to keep falling.

⇒Information loss?

⇒However, the eikonal approximation will be broken at $O(1)$ in this region. What happens there?

Summary

- Construct a self-consistent model which describes a BH from formation to evaporation including the back reaction from the Hawking radiation, under three assumptions.
- Obtain an asymptotic solution representing the inside of the hole, which emits the Hawking radiation and evaporates completely without forming large horizon or singularity.
- Reproduce the entropy area law by counting microstates inside the hole.
- Discuss the information problem by analyzing local energy conservation in the field-theoretic manner.

Thank you very much!