Thermal Vacuum State for Multiple Closed Superstrings in the Framework of Thermo Field Dynamics

arXiv:1511.xxxxx

Hokkaido Univ.

Kenji Hotta

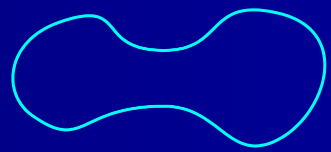
1. Introduction

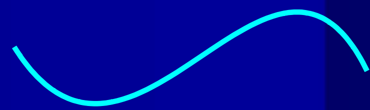
- Hagedorn Temperature $\mathcal{T}_H$ (type II)

  maximum temperature for perturbative strings

  A single energetic string captures most of the energy.

$$\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'}$$

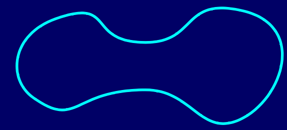




$Z(\beta) \rightarrow \infty$ for $\beta < \beta_H$

The finite temperature systems of strings have been mainly investigated in Matsubara formalism.

- **Hagedorn Transition of Closed Strings**



$Z(\beta) \rightarrow \infty$ for $T > T_H$ (Matsubara Method)

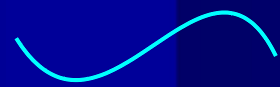
winding tachyon in the Euclidean time direction

Hagedorn Transition (Sathiapalan, Kogan, Atick-Witten)

A phase transition takes place due to the condensation of tachyon fields. (stable minimum?)

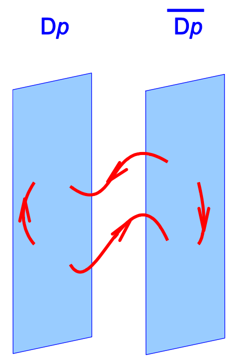
- **Brane-antibrane Pair Creation Transition**

Dp-$\overline{Dp}$ Pairs are unstable at zero temperature



finite temperature system of Dp-$\overline{Dp}$ Pairs

→ $D9$-$\overline{D9}$ pairs become stable near the Hagedorn temperature.



Hotta

Relation between two phase transitions?

we have conjectured that

$D9$-$\overline{D9}$ Pairs are created by the Hagedorn transition of closed strings.

These works are based on Matsubara Method.

One of the method to investigate finite temperature system is thermo field dynamics (TFD).

↓

finite temperature system of Dp-$\overline{Dp}$ based on TFD

↓

finite temperature system of closed superstring based on TFD?

■ Thermo Field Dynamics (TFD) Takahashi-Umezawa

statistical average

$$\langle A \rangle = Z^{-1}(\beta) \sum_n \langle n|\hat{A}|n\rangle e^{-\beta E_n}$$

We can represent it as

$$\langle A \rangle = \left\langle 0(\beta) \middle| \hat{A} \middle| 0(\beta) \right\rangle$$

by introducing a fictitious copy of the system.

$$|0(\beta)\rangle = Z^{-\frac{1}{2}}(\beta) \sum_n e^{-\frac{\beta E_n}{2}} |n\rangle \otimes |\tilde{n}\rangle$$ thermal vacuum state

The fictitious state is interpreted as `hole' state.

Hawking-Unruh effect can be described by TFD.

Unruh Effect in bosonic open string theory

Hata-Oda-Yahikozawa

➡ closed string which can propagate bulk spacetime

finite temperature system of closed superstring

based on TFD?

Contents

2. Second Quantized Closed Superstring

- Light-Cone Gauge SFT cf) Kaku-Kikkawa, Hua

We consider free string case.

action

$$I_0 = \int_0^\infty dp^+ \int \mathcal{D}^8 X \left[\Phi^*_{p^+}(X,\psi) \left\{ i\frac{\partial}{\partial X^+} - \hat{p}^- \right\} \Phi_{p^+}(X,\psi) \right]$$

$$\hat{p}^- = \int_0^{2\pi p^+} d\sigma \sum_{I=2}^{9} \left\{ -\frac{\pi}{2}\frac{\delta^2}{\delta\left\{X^I(\sigma)\right\}^2} + \frac{1}{2\pi}\left(\frac{\partial X^I(\sigma)}{\partial\sigma}\right)^2 + 2i\left(\psi_L^I(\sigma)\frac{\partial}{\partial\sigma}\frac{\delta}{\delta\psi_L^I(\sigma)} - \psi_R^I(\sigma)\frac{\partial}{\partial\sigma}\frac{\delta}{\delta\psi_R^I(\sigma)}\right)\right\}$$

eq. of motion

$$i\frac{\partial}{\partial X^+}\Phi_{p^+}(X,\psi) = \hat{p}^-\Phi_{p^+}(X,\psi)$$

$$\Phi_{NSNS}, \Phi_{RR}, \Phi_{NSR}, \Phi_{RNS}$$

We show only the NS-NS sector case.

mode expansion

$$X^I(\sigma) = x^I + \sum_{l=1}^{\infty} \left\{ \frac{x_l^I}{\sqrt{l}} \cos\left(\frac{l\sigma}{p^+}\right) + \frac{y_l^I}{\sqrt{l}} \sin\left(\frac{l\sigma}{p^+}\right) \right\}$$

$$\psi_L^I = \sum_{r \in Z+\frac{1}{2}} \psi_r^I \exp\left(-\frac{ir\sigma}{p^+}\right)$$

$$\psi_R^I = \sum_{r \in Z+\frac{1}{2}} \overline{\psi}_r^I \exp\left(\frac{ir\sigma}{p^+}\right)$$

$$\hat{p}^- = \frac{1}{2p^+}\left\{-\sum_{I=2}^{9} \frac{\partial^2}{\partial\left(x^I\right)^2} + \frac{2}{\alpha'}\left(\hat{B} + \hat{F} - 1\right)\right\}$$

$$\hat{B} = \sum_{l=1}^{\infty}\sum_{I=2}^{9} \frac{l}{2}\left\{-\frac{\partial^2}{\partial\left(x_n^I\right)^2} - \frac{\partial^2}{\partial\left(y_n^I\right)^2} + \left(x_n^I\right)^2 + \left(y_n^I\right)^2 - 2\right\}$$

$$\hat{F} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=2}^{9} r\left(\psi_r^I \frac{\partial}{\partial \psi_r^I} + \overline{\psi}_r^I \frac{\partial}{\partial \overline{\psi}_r^I}\right)$$

solution

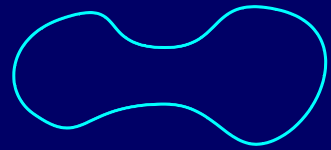
$$\Phi_{NSNS,\alpha} = \exp\left(i\boldsymbol{p}\cdot\boldsymbol{x} - ip_\alpha^- x^+\right)\overline{\phi}_{B,\alpha_1}\phi_{B,\alpha_2}\overline{\phi}_{NS,\alpha_3}\phi_{NS,\alpha_4}$$

$$\phi_{B,\alpha} = \prod_{l=1}^{\infty}\prod_{I=2}^{9} \frac{l^{\frac{1}{4}}}{2^{\frac{n}{2}}\pi^{\frac{1}{4}}\sqrt{(n_l^I)!}} H_{n_l^I}(x_l^I) \exp\left(-\frac{1}{2}\, {x_l^I}^2\right)$$

$$\phi_{NS,\alpha} = \prod_{r=\frac{1}{2}}^{\infty}\prod_{I=2}^{9} \left(\psi_r^I\right)^{n_r^I}$$

$$\alpha = \left\{p^+, \boldsymbol{p}, N_B, N_{NS}, \overline{N}_B, \overline{N}_{NS}\right\}$$

Mass Spectrum



$$M_{NSNS}{}^2 = \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_{NS} - 1\right)$$

$$M_{RR}{}^2 = \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_R\right)$$

space time boson

$$M_{NSR}{}^2 = \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_R - \frac{1}{2}\right)$$

$$M_{RNS}{}^2 = \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_{NS} - \frac{1}{2}\right)$$

space time fermion

$$N_B = \sum_{l=1}^{\infty}\sum_{I=2}^{9} l n_l^I, \quad N_{NS} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=2}^{9} r n_r^I, \quad N_R = \sum_{m=1}^{\infty}\sum_{I=2}^{9} m n_m^I$$

We show only the NS-NS mode case.

GSO projection

left-moving modes : $\frac{1}{2}(1+G)$

right-moving modes : $\frac{1}{2}\left(1+\overline{G}\right)$

$$G = -(-1)^{\sum_{r=\frac{1}{2}}^{\infty}\sum_{I=2}^{9} n_r^I}$$

level-matching condition

$$N_B + N_{NS} - \overline{N}_B - \overline{N}_{NS} = 0$$

$$\delta_{n,n'} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \ \exp\left[2\pi i \tau_1 \left(n - n'\right)\right]$$

second quantization

$$\Phi_{NSNS} = \sum_{\alpha} \left(A^{\dagger}_{NSNS,\alpha} \Phi^{*}_{NSNS,\alpha} + \Phi_{NSNS,\alpha} A_{NSNS,\alpha} \right)$$

commutation relation

$$\left[A_{NSNS,\alpha}, A^{\dagger}_{NSNS,\alpha'} \right] = \delta_{\alpha,\alpha'}$$

$$\int_0^{\infty} dp^{+} \int D^{16}z \; \Phi^{*}_{NSNS} \; \hat{p}^{-} \Phi_{NSNS} = \sum_{\alpha} \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{2p^{+}} \; A^{\dagger}_{NSNS,\alpha} A_{NSNS,\alpha}$$

$$D^{16}z = d^8x \prod_{l=1}^{\infty} d^8x_l d^8y_l \prod_{r=\frac{1}{2}}^{\infty} d\psi_r d\bar{\psi}_r$$

Hamiltonian

$$H_{NSNS} = \frac{1}{\sqrt{2}} \sum_{\alpha} \left(p^{+} + \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{2p^{+}} \right) \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} A^{\dagger}_{NSNS,\alpha} A_{NSNS,\alpha}$$

level-matching condition

$$\mathcal{P}_{NSNS,\alpha} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \;\; \exp\left[2\pi i \tau_1 \left(N_B + N_{NS} - \bar{N}_B - \bar{N}_{NS} \right) \right]$$

GSO projection

$$P_{NSNS,\alpha} = \frac{1}{4} (1 + G_{n_r}) \left(1 + \bar{G}_{\bar{n}_r} \right)$$

$$G = -(-1)^{\sum_{r=\frac{1}{2}}^{\infty} \sum_{I=2}^{9} n_r^I}$$

3. Thermal Vacuum State and Free Energy

- Thermal Vacuum State for NS-NS Strings

generator of Bogoliubov tr.

$$G_{NSNS} = i\sum_{\alpha} \theta_{NSNS,\alpha}\left(A^{\dagger}_{NSNS,\alpha}\tilde{A}^{\dagger}_{NSNS,\alpha} - \tilde{A}_{NSNS,\alpha}A_{NSNS,\alpha}\right)$$

$$\alpha = \{p^{+}, \boldsymbol{p}, N_B, N_{NS}, \overline{N}_B, \overline{N}_{NS}\}$$

thermal vacuum state for multiple strings

$$\begin{aligned}|0_{NSNS}(\theta)\rangle &\equiv e^{-iG_{NSNS}}|0\rangle\rangle \\ &= \exp\left[\sum_{\alpha} \theta_{NSNS,\alpha}\left(A^{\dagger}_{NSNS,\alpha}\tilde{A}^{\dagger}_{NSNS,\alpha} - \tilde{A}_{NSNS,\alpha}A_{NSNS,\alpha}\right)\right]|0\rangle\rangle \\ &= \prod_{\alpha}\left\{\frac{1}{\cosh(\theta_{NSNS,\alpha})}\exp\left[\tanh(\theta_{NSNS,\alpha})A^{\dagger}_{NSNS,\alpha}\tilde{A}^{\dagger}_{NSNS,\alpha}\right]\right\}|0\rangle\rangle\end{aligned}$$

$$A_{NSNS,\alpha}|0\rangle\rangle = \tilde{A}_{NSNS,\alpha}|0\rangle\rangle = 0$$

Free Energy for Multiple NS-NS String

$$F_{NSNS}(\theta) = \left\langle 0_{NSNS}(\theta) \middle| \left(H_{NSNS} - \frac{1}{\beta} K_{NSNS} \right) \middle| 0_{NSNS}(\theta) \right\rangle$$

Hamiltonian

$$H_{NSNS} = \frac{1}{\sqrt{2}} \sum_{\alpha} \left(p^+ + \frac{|\boldsymbol{p}|^2 + M_{NSNS}^2}{2p^+} \right) \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} A_{NSNS,\alpha}^{\dagger} A_{NSNS,\alpha}$$

entropy

$$\begin{aligned} K_{NSNS} \;=\; -\sum_{\alpha} \Big(& A_{NSNS,\alpha}^{\dagger} A_{NSNS,\alpha} \ln \sinh^2 \theta_{NSNS,\alpha} \\ & - A_{NSNS,\alpha} A_{NSNS,\alpha}^{\dagger} \ln \cosh^2 \theta_{NSNS,\alpha} \Big) \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} \end{aligned}$$

$$\begin{aligned} F_{NSNS}(\theta) \;=\; & \sum_{\alpha} \mathcal{P}_{NSNS,\alpha} P_{NSNS,\alpha} \\ & \times \left\{ \sinh^2 \theta_{NSNS,\alpha} \left(E_{NSNS,\alpha} + \frac{1}{\beta} \ln \tanh^2 \theta_{NSNS,\alpha} \right) \right. \\ & \left. - \frac{1}{\beta} \ln \cosh^2 \theta_{NSNS,\alpha} \right\} \end{aligned}$$

Relation between β and θ .

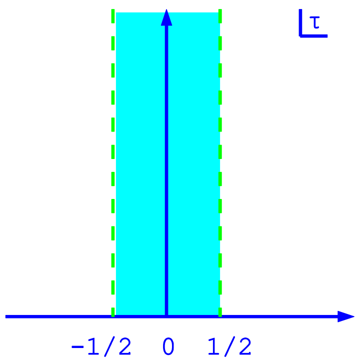
$$\frac{\partial}{\partial \theta_{NSNS,\alpha}} F_{NSNS}(\theta) = 0$$

$$\tanh \theta_{NSNS,\alpha} = \exp\left(-\frac{\beta E_{NSNS,\alpha}}{2}\right)$$

$$F_{NSNS}(\beta) = -\sum_{\alpha}\sum_{w=1}^{\infty}\frac{1}{w\beta}\mathcal{P}_{NSNS,\alpha}P_{NSNS,\alpha}\exp\left(-w\beta E_{NSNS,\alpha}\right)$$

$$\sum_{\alpha} \to \sum_{n_l,\bar{n}_l,n_r,\bar{n}_r}\frac{\sqrt{2}\ v_9}{(2\pi)^9}\int_0^{\infty}dp^+\int_{-\infty}^{\infty}d^8p \qquad \tau_2 \equiv \frac{2\sqrt{2}\ \pi\beta}{{\beta_H}^2 p^+} \qquad \tau \equiv \tau_1 + i\tau_2 \qquad \beta_H = 2\pi\sqrt{2\alpha'}$$

$$F_{NSNS}(\beta) \;=\; -\frac{8(2\pi)^8 v_9}{\beta_H^{10}}\int_S \frac{d^2\tau}{\tau_2^6}\frac{1}{|\vartheta_1{}'(0|\tau)|^8} \times \left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4\right)(0|\tau)\left\{\sum_{w=1}^{\infty}\exp\left(-\frac{2\pi w^2\beta^2}{\beta_H^2\tau_2}\right)\right\}$$



domain of integration S

Free Energy for Multiple Strings

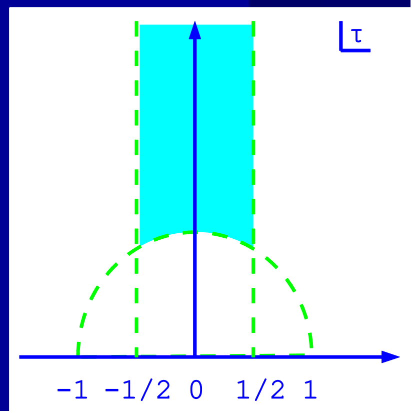
Summing over the free energy for all sectors, we obtain

$$F(\beta) = F_{NSNS}(\beta) + F_{RR}(\beta) + F_{NSR}(\beta) + F_{RNS}(\beta)$$

$$\begin{aligned} F(\beta) \;=\; & -\frac{8(2\pi)^8 v_9}{{\beta_H}^{10}} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^{\infty} d\tau_2 \frac{1}{\tau_2^6} \frac{1}{|{\vartheta_1}'(0|\tau)|^8} \\ & \times \Bigg[\left\{ \left(\vartheta_3^4 - \vartheta_4^4\right)\left(\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4\right) + \vartheta_2^4 \bar{\vartheta}_2^4 \right\} (0|\tau) \;\sum_{w=1}^{\infty} \exp\left(-\frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2} \right) \\ & - \left\{ \left(\vartheta_3^4 - \vartheta_4^4\right) \bar{\vartheta}_2^4 + \vartheta_2^4 \left(\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4\right) \right\} (0|\tau) \;\sum_{w=1}^{\infty} (-1)^w \exp\left(-\frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2} \right) \Bigg] \end{aligned}$$

This equals to the free energy in the S-representation based on Matsubara formalism.

We can transform this to F-representation or Dual-representation by using modular transformation.



4. Conclusion and Discussion

- Closed Superstring Gas in TFD

  We computed thermal vacuum state and free energy
  for multiple closed superstring based on TFD.

  The free energy for multiple strings agrees with that
  based on the Matsubara formalism.

- Hawking-Unruh Effect

  closed strings in curved spacetime

  Unruh Effect in bosonic open string theory
  Hata-Oda-Yahikozawa

  ➡ covariant closed superstrings in TFD?

  black hole firewall Almheiri-Marolf-Polchinski-Sully

  Planck solid model Hotta

Half-formed D9-$\overline{\text{D9}}$ Pairs

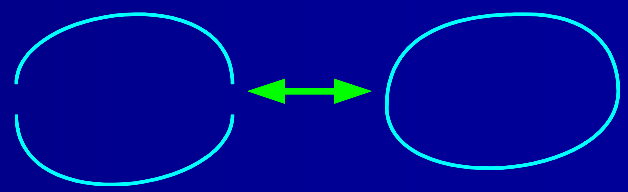
D9-$\overline{\text{D9}}$ pairs are half-formed before the Hagedorn transition.

The vacuum moves from closed string vacuum towards open string vacuum.

↓

Closed strings in the bulk and open strings on D9-$\overline{\text{D9}}$ pairs are in thermal equillibrium.

D9-$\overline{\text{D9}}$ pairs and open strings are annihilated and closed strings are created and vice versa.



Do D9-$\overline{\text{D9}}$ pairs and open strings play a role of ‘hole’ for closed strings?

D9-$\overline{\text{D9}}$ pairs as thermal states?

cf) Cantcheff The thermal vacuum state for open bosonic string is reminiscent of the D-brane boundary state of a closed string.