Lax pairs on Yang-Baxter deformed backgrounds

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based on arXiv:1509.00173 in collaboration with T. Kameyama, J. Sakamoto and K. Yoshida

1. What are Yang-Baxter deformations?

YB deformations Klimcik (hep-th/0210095), (0802.3518)

The undeformed coset σ-model → Deformed σ-models

R-operators ↑

↓ integrable BGs

If the coset of the model is symmetric, the σ-model is integrable. Then, Yang-Baxter deformations preserve the integrability of the deformed model automatically. We can obtain some well-known backgrounds with certain r-matrices.

R is a linear operator: $\mathfrak{g} \to \mathfrak{g}$ which satisfies the Yang-Baxter equation (YBE)

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = \omega [X, Y]$$

ω=0 ⟶ homogenous YBE ★

ω≠0 ⟶ modified YBE

$$r = \sum_i a_i \wedge b_i$$

↓

$$R(X) = \sum_i (a_i\, \mathrm{STr}[b_i\, X] - b_i\, \mathrm{STr}[a_i\, X])\,, \quad (a_i\,, b_i \in \mathfrak{g})$$

This R-operator associated by tensorial r-matrix.

The Coset model

$$S = \frac{1}{2}\int d^2\sigma\ \mathrm{Tr}[A_- P_2(A_+)] = -\frac{1}{2}\int d^2\sigma\ \gamma^{\alpha\beta} g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

$$A_\alpha = g^{-1}\partial_\alpha g\,, \quad g \in G/H$$

↓ Yang-Baxter deformation with R-operator ← input

Yang-Baxter deformations of coset model Delduc, Margo and Vicedo (1308.3581)

$$S = \frac{1}{2}\int d^2\sigma\ \mathrm{Tr}[A_- P_2(J_+)] = -\frac{1}{2}\int d^2\sigma\ (\gamma^{\alpha\beta} g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu - \epsilon^{\alpha\beta} B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu)$$

$$J_\pm \equiv \frac{1}{1 \mp \eta R_g \circ P_2} A_\pm\,, \quad R_g(X) \equiv g^{-1} R(g X g^{-1}) g$$

η is the deformation parameter

output → Integrable backgrounds can be identified. ⇨

We can construct the Lax pair for this deformed system

$$\mathcal{L}_\pm = P_0(J_\pm) + \lambda^{\pm 1}\sqrt{1 + \omega\, \eta^2}\, P_2(J_\pm)$$

2. Yang-Baxter deformations of AdS$_5$ x S^5

$$\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{\mathrm{SO}(2,4)}{\mathrm{SO}(1,4)} \times \frac{\mathrm{SO}(6)}{\mathrm{SO}(5)}$$: symmetric coset ⇨

The σ-model on AdS$_5$ x S^5 is integrable. We can obtain some backgrounds from deformations with homogenous (Abelian) r-matrices.

⇦ a ω=0 type r-matrix

	resulting BG
i)	gravity duals of NCYM theory (Maldacena-Russo 1999) Matsumoto and Yoshida (1404.3657)
ii)	γ-deformed S^5 (Lunin-Maldacena 2005) Matsumoto and Yoshida (1404.1838)
iii)	Schroedinger spacetimes (Herzog-Rangamani-Ross 2008) Matsumoto and Yoshida (1504.05516)
iv)	Abelian twisted global AdS$_5$ (Dhokarh-Haque-Hashimoto 2008)

We construct the Lax pairs of these backgrounds explicitly. [arXiv: 1509.00173]

Replacement laws

When r-matrix is Abelian (including the above cases), the deformed current J can be constructed by a simple replacement law.

$$\partial x^\mu \to \partial x^\mu + f^\mu(x, \partial x)$$

⇨ Using this rule, we can easily construct the Lax pairs.

Twisted boundary condition

Frolov (hep-th/0503201), Vicedo (1504.06303)
Alday, Arutyunov and Frolov (hep-th/0512253)

For the above r-matrices, the effect of the deformation can be reinterpreted as the twisted boundary condition.

the undeformed theory + twisted boundary conditions = deformed theories

$$\tilde{g}(\tau\,, \sigma = 2\pi) = \exp\left(\frac{1}{2}\eta \int_0^{2\pi} d\sigma\ g R_g[P(J_\tau)] g^{-1}\right) \tilde{g}(\tau\,, \sigma = 0) \quad \Leftrightarrow \quad x^\mu(\sigma = 2\pi) = x^\mu(\sigma = 0) + \mathrm{const}\,, \ldots$$

3. A relation to q-deformation and future problems

A puzzle

Arutyunov, Borsato and Frolov (1507.04239)

It was confirmed that the q-deformed backgrounds fail to satisfy the SUGRA EoM.

q-deformed AdS$_5$ x S^5 — a certain r-matrix satisfies modified YBE ω≠0

→ scaling limit → the background (i) (Maldacena-Russo) ← Our deformation of AdS$_5$ x S^5 — a certain r-matrix satisfies homogenous YBE ω=0

Arutyunov, Borsato and Frolov (1312.3542)

the Lax pair from q-dfm. ⟷ the Lax pair from ω=0 type dfm.

a unitary transformation

confirmation at the level of the Lax pair

Future problem

Are the Yang-Baxter deformed backgrounds a gravity solution in general?

How about ω=0 type r-matrix? Is it always OK? or in certain cases?