Boundary states as Holographic duals of Trivial spacetimes

Masamichi Miyaji[1] with S. Ryu[2], T. Takayanagi[1], and X. Wen[2]

1. Yukawa Institute for Theoretical Physics, 2. University of Illinois

JHEP05(2015)152

Abstract

Recent studies in AdS/CFT show that quantum entanglement of CFT plays essential role in dual spacetimes. It was shown that connectivity of dual spacetime corresponds to quantum entanglement of subsystems[1], and linearized Einstein equation of dual spacetime corresponds to first law of entanglement of CFT[2][3]. In our study, **we show conformal boundary states have no short range entanglement. This means these states are dual to trivial, small spacetimes.**

With this result in hands we can use conformal boundary state as final state of Multi-scale entanglement renormalization anzatz(MERA), whose networks are conjectured[4] to describe bulk anti de Sitter spacetimes in AdS/CFT correspondence.

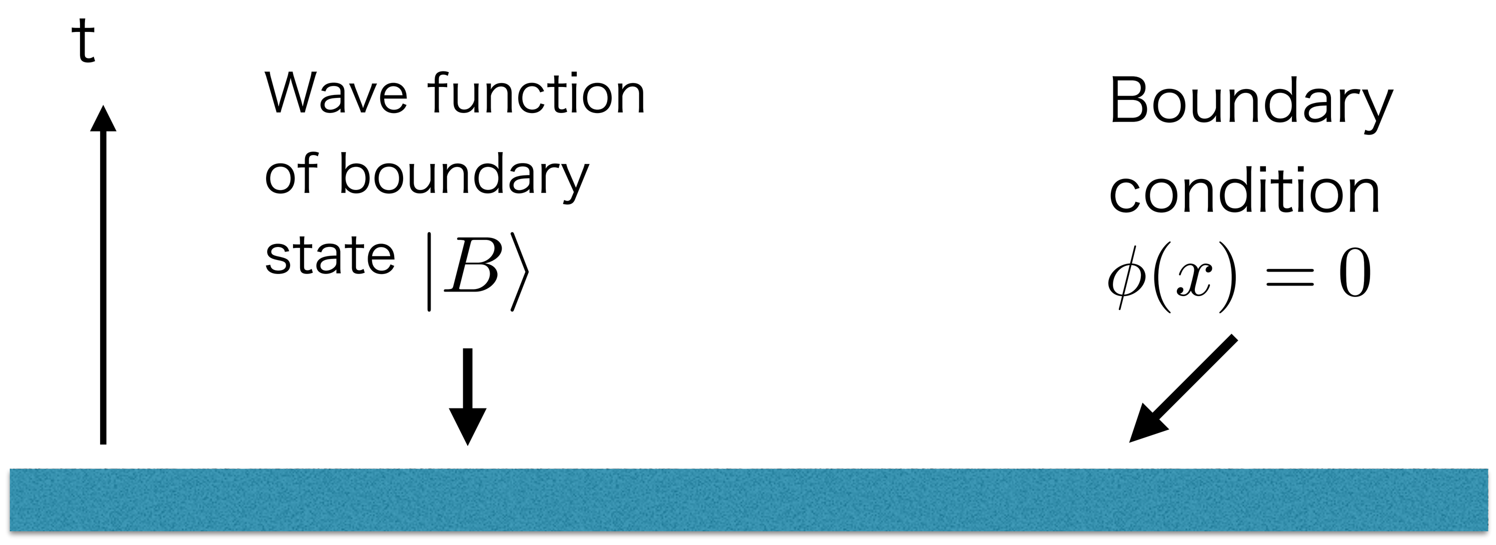
CFT boundary states

In QFT, we call states that are constrained by boundary conditions of local fields as boundary states. For example, Dirichlet boundary state is defined by

$$\hat{\phi}(x)|B\rangle = 0$$

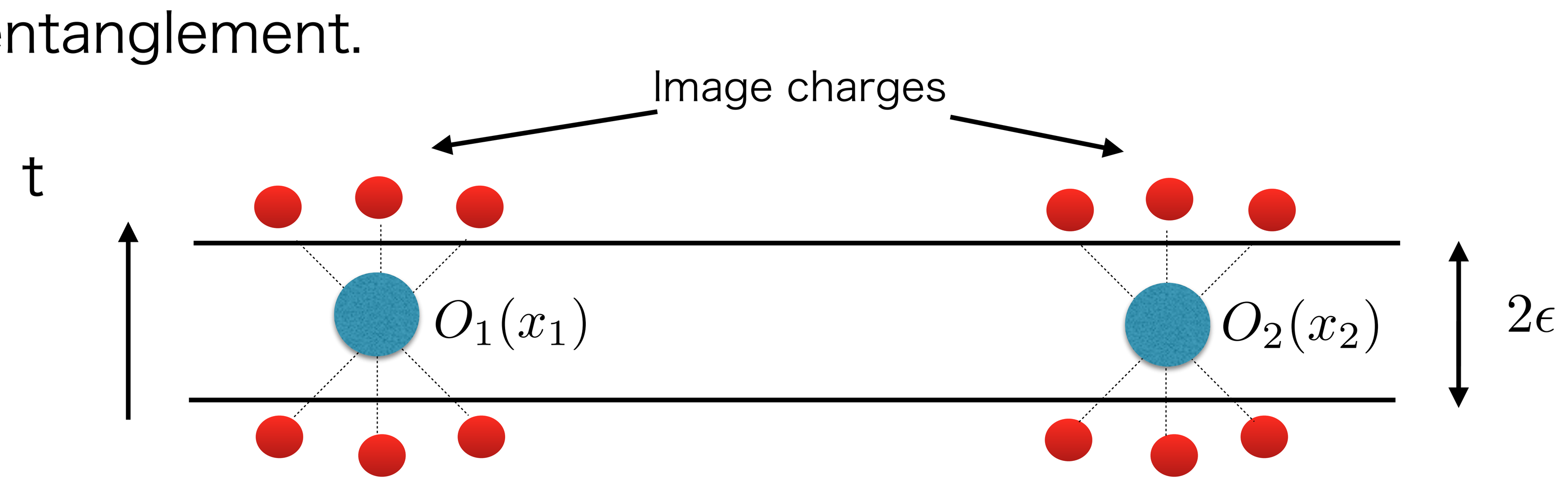
for scalars.

When those constraints are conformally invariant, we call CFT boundary states.



The absence of entanglement in conformal boundary states

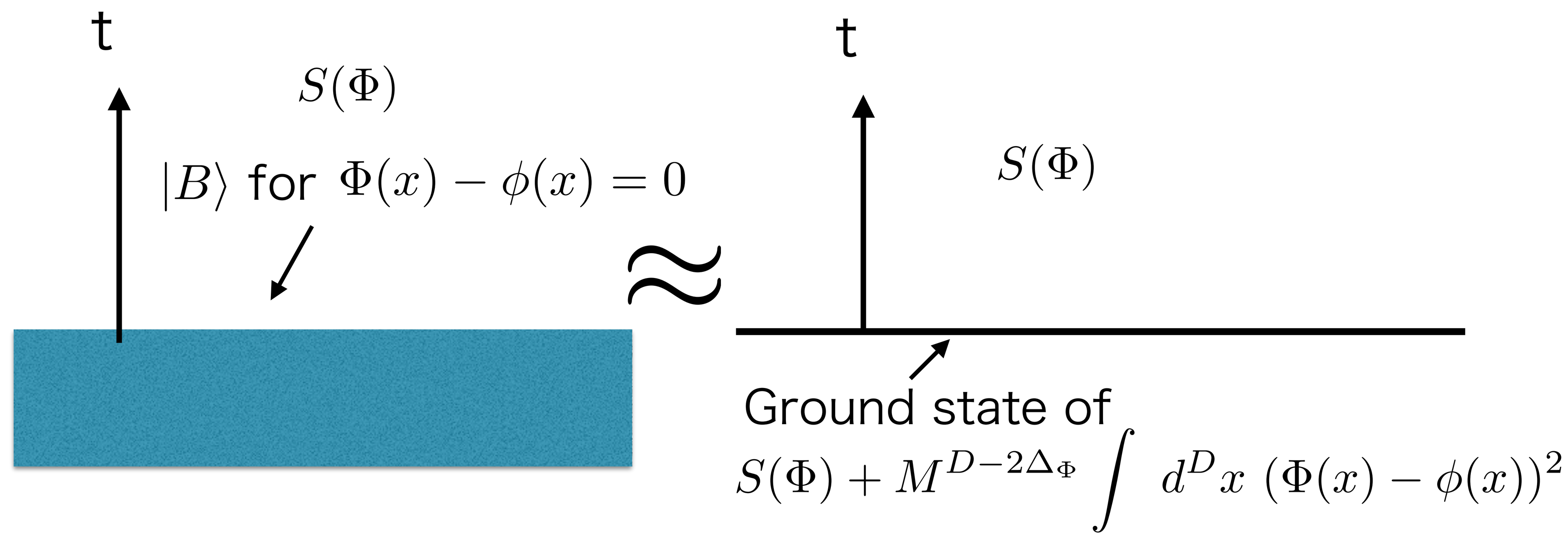
By method of images, n-point functions on boundary state factorizes. This shows boundary states have no entanglement.



$$\langle B|e^{-\epsilon H}O_1(x_1)O_2(x_2)...e^{-\epsilon H}|B\rangle$$

$$\approx \langle B|e^{-\epsilon H}O_1(x_1)e^{-\epsilon H}|B\rangle\langle B|e^{-\epsilon H}O_2(x_2)e^{-\epsilon H}|B\rangle...$$

Or in path integral, boundary state is realized as a ground state of Hamiltonian deformed by large mass term. This also implies boundary states have no entanglement.



In addition, we can explicitly confirm that Dirichlet and Neumann boundary states of 2d free Dirac fermion have no entanglement.

$$S_A = \frac{1}{3}log\frac{4\epsilon}{\pi a}$$

From these argument, we can see boundary state indeed has no divergent entanglement entropy for **general CFT**.

Implication for Entanglement renormalization

MERA is an efficient representation of CFT ground states[5][6]. MERA tensor networks starts from IR state (or top tensor) whose degrees of freedom are only topological. By coarse graining and adding quantum entanglement to that state, we obtain true ground state of the CFT. IR states of MERA therefore have no entanglement coming from dynamical degrees of freedom. Therefore, we can use conformal boundary states as IR state of MERA.

Conclusion

- Conformal boundary states have no short range entanglement. They only have topological entanglement. So they can be used as IR states of MERA.
- Conformal boundary states are dual to small, trivial spacetimes.

Future problems

- Identify intermediate states of MERA for arbitrary CFT, in particular, large N CFT.
- Study quantum gravity effects?
- More on tensor networks and quantum gravity….

[1]Raamsdonk (2010)
[2]Nozaki, Numasawa, Takayanagi (2013)
[3]Lashkari, McDermott, Raamsdonk(2013)
[4]B. Swingle (2009)
[5]G. Vidal (2006)
[6]G. Evenbly, G. Vidal (2007)
[7]J. Haegeman,T. J.Osborne, H. Verschelde, and F. Verstraete (2011)