Out-of-**T**ime-**O**rdered **C**orrelators in 2d CFT

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Holography, Quantum Entanglement and Higher Spin Gravity **Kyoto 7/02/2017**

Plan

- Introduction+Motivation
- OTOs and Quantum Chaos Bound
- Various results
- Open Questions

Motivation

- Which CFTs describe Black Holes and why?
- Necessary conditions? How can we test them?
- 2d CFTs and black holes? (n)RCFTs, Large c, W_N ,Liouville ?
- Dynamics of quantum information in 1+1 d? Universality, bounds, etc?
- What can this teach us about scrambling, quantum chaos…?

Based on: [P.C,T.Takayanagi,K.Watanabe,Y.Kusuki to appear]

[P.C,T.Numasawa,A.Veliz-Osorio'16]

[P.C,J.Simon, A.Stikonas,K.Watanabe, T.Takayanagi'15]

Black Hole Slogans:

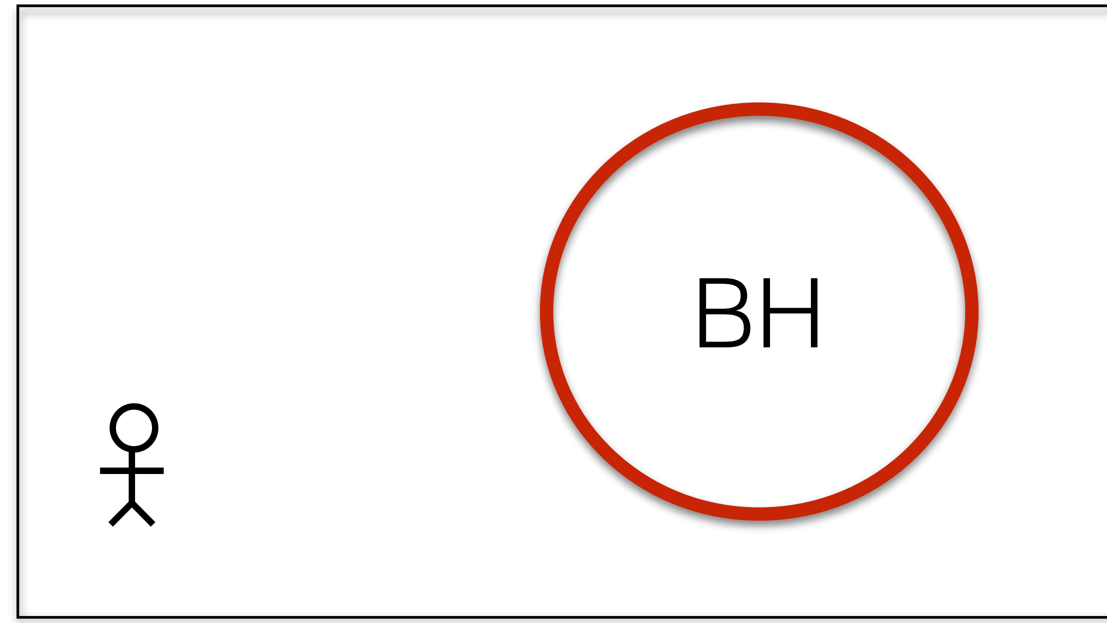
- BH are holographic (entropy) 
- BH are the fastest scramblers of information 
- BH are maximally chaotic 
- BH are (complex) quantum channels
- BH are the best quantum computers….

This Talk

AdS/CFT is our main tool to make these statements more precise

HINT: Quantum Information + Time Evolution

BH are holographic



BH entropy

$$S_{BH} = \frac{Area(Horizon)}{4G_N}$$

In 2d CFT we can relate the BH entropy to Cardy formula for the density of states

$$S_{BH} = S_C = 2\pi\sqrt{\frac{c}{6}E_L} + 2\pi\sqrt{\frac{c}{6}E_R}$$

$$c - \text{fixed}, \quad E_{L/R} \to \infty$$

Modulo the assumption that it should be valid for

$$c \to \infty, \qquad E_{L/R} \sim c$$

1) In order to describe BH, CFTs should have:

Large central Charge $c \to \infty$

Sparse Spectrum (Cardy formula valid in "extended" regime) [Hartman,Keller,Stoica,14]

$$\rho(E) = \exp[S(E)] \lesssim \exp\left[2\pi\left(E + \frac{c}{12}\right)\right] \ , \qquad E \leq \epsilon \qquad E = E_L + E_R = \Delta - \frac{c}{12}$$

In practice we can approximate correlators by the identity block.

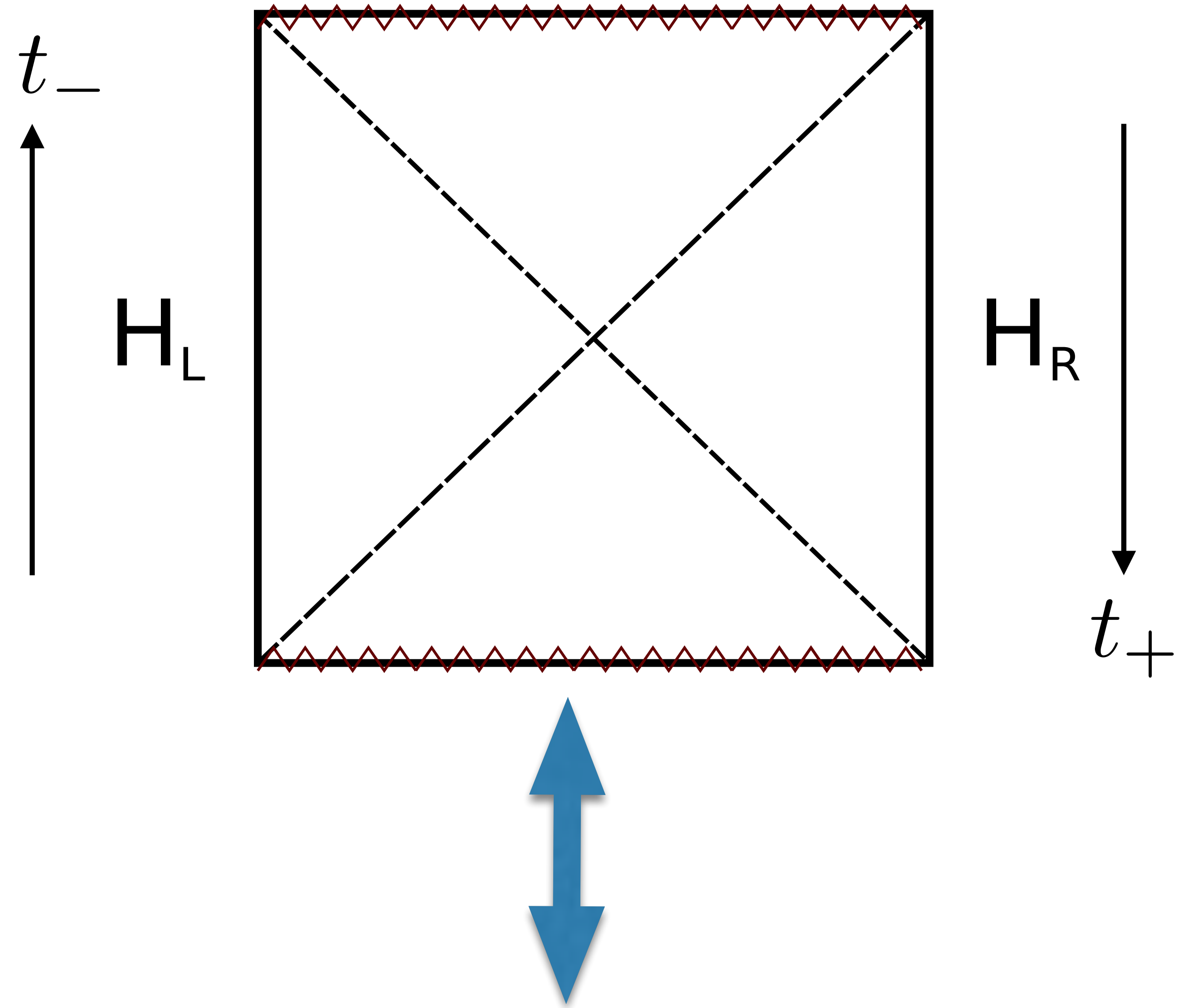
What does it imply for "physics": entanglement, CFT quenches….?

2) Black Hole Phenomenology and Chaos

AdS/CFT: BH-TFD duality

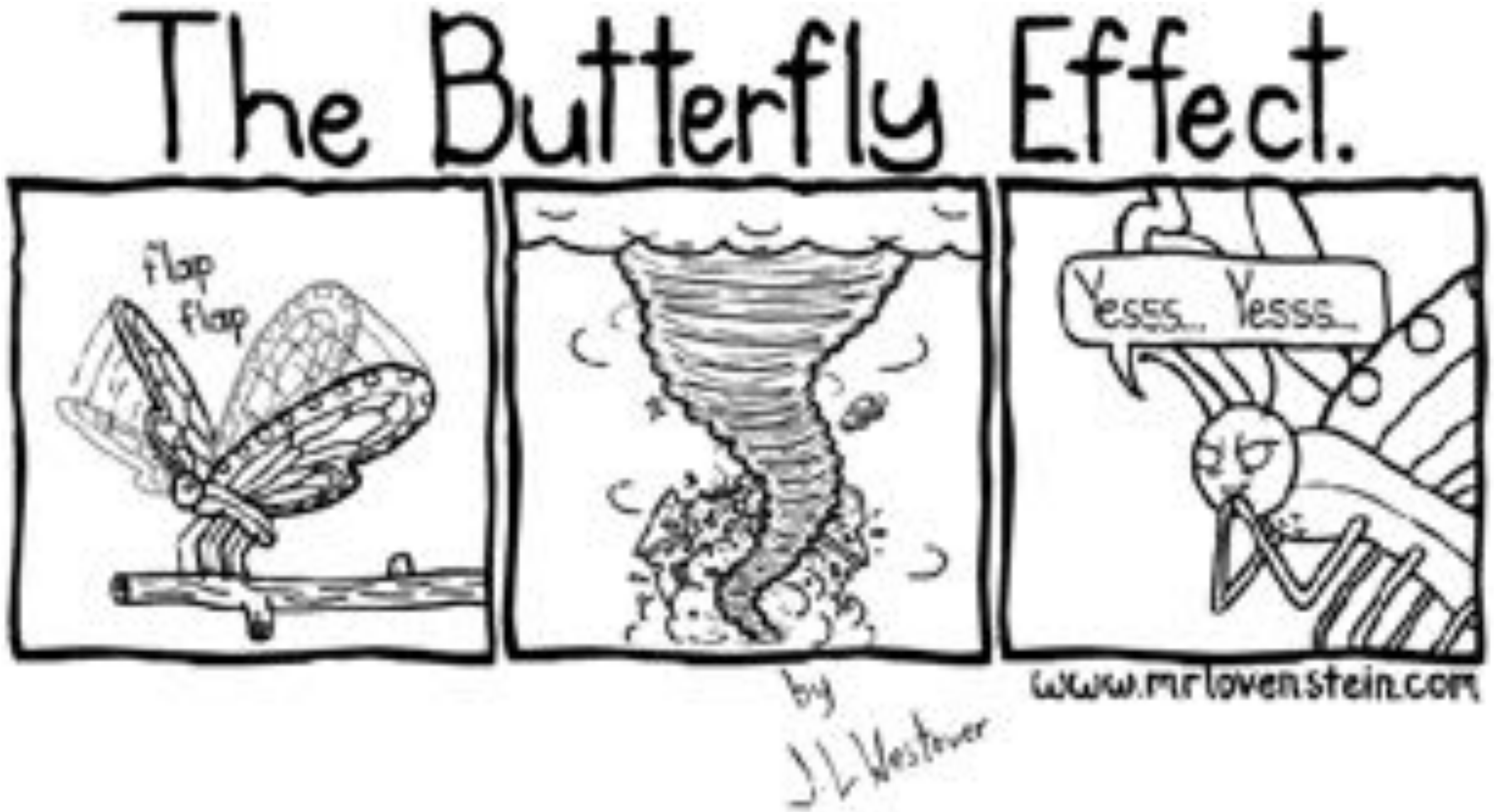
[Maldacena'01]

Eternal BH

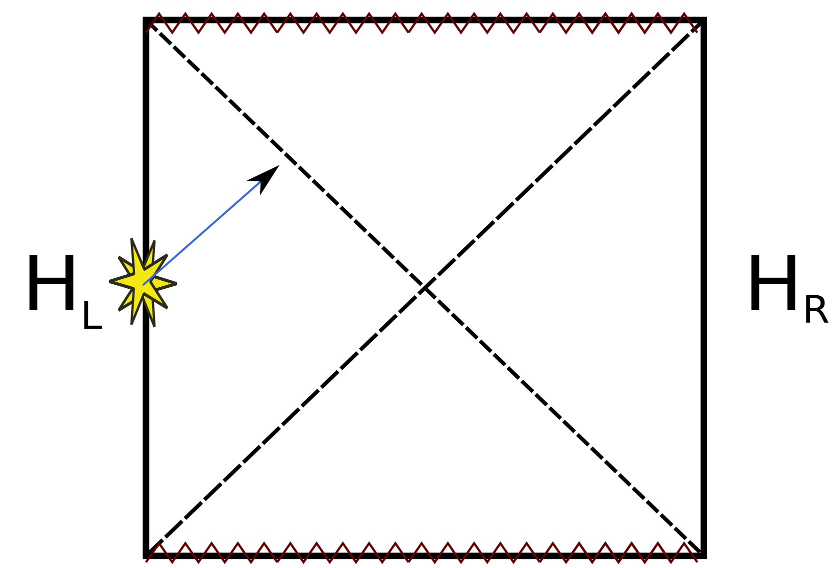


TFD

$$|\Psi_\beta\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2}E_n} |n\rangle_L |n\rangle_R$$



[Shenker,Stanford]
[Roberts,Stanford]
[+ Susskind]



$$|\psi'\rangle = e^{-iH_L t_w} O_L(x) e^{iH_L t_w} |\psi\rangle$$

$$I_{A:B} = S_A + S_B - S_{A\cup B}$$

Computed both holographically and in CFT

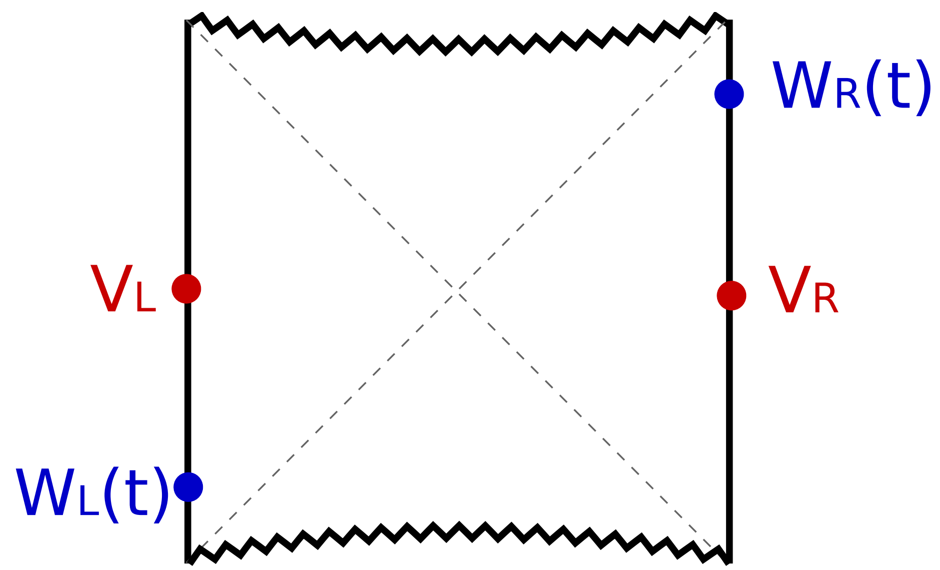
$$I_{A:B}(t_w) = 0? \qquad t_w \sim \beta \log c \sim \beta \log S$$

[Shenker,Stanford'13][PC,Simon,Stikonas,Takayanagi,Watanabe'15]

Quantum Chaos!?

<u>Holographic computations: general d</u>

[Shenker,Stanford'14]



gravitational scattering (Regge)

$$\langle VW(t)VW(t)\rangle \quad \sim G_N s \simeq G_N e^{\frac{2\pi}{\beta}t}$$

Semiclassical Chaos

SOVIET PHYSICS JETP VOLUME 28, NUMBER 6 JUNE, 1969

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

A. I. LARKIN and Yu. N. OVCHINNIKOV

$$[x(t), p(0)] \simeq -i\hbar \{x(t), p(0)\} \simeq -i\hbar \frac{\partial x(t)}{\partial x(0)} = -i\hbar e^{\lambda_L t}$$

Average square becomes O(1)

$$\langle [x(t), p(0)]^2 \rangle \simeq \hbar^2 e^{2\lambda_L t} = e^{2\lambda(t-t*)}$$

$$t^* = \frac{1}{\lambda_L} \log \frac{1}{\hbar}$$

Ehrenfest time
(decoherence!)

e.g. Inverted harmonic oscillator

Quantum Chaos in QFT

[Kitaev…to appear]

In QFT one can generalise to

$$\langle [W(t), V(0)]^2 \rangle_\beta$$

for “arbitrary” operators of the theory

$$\langle V W(t) W(t) V \rangle_\beta + \langle W(t) V V W(t) \rangle_\beta$$
$$- \langle W(t) V W(t) V \rangle_\beta - \langle V W(t) V W(t) \rangle_\beta$$

Essential information in the ratio

$$\boxed{C^\beta(t) = \frac{\langle W(t) V W(t) V \rangle_\beta}{\langle W(t) W(t) \rangle_\beta \langle V V \rangle_\beta}}$$

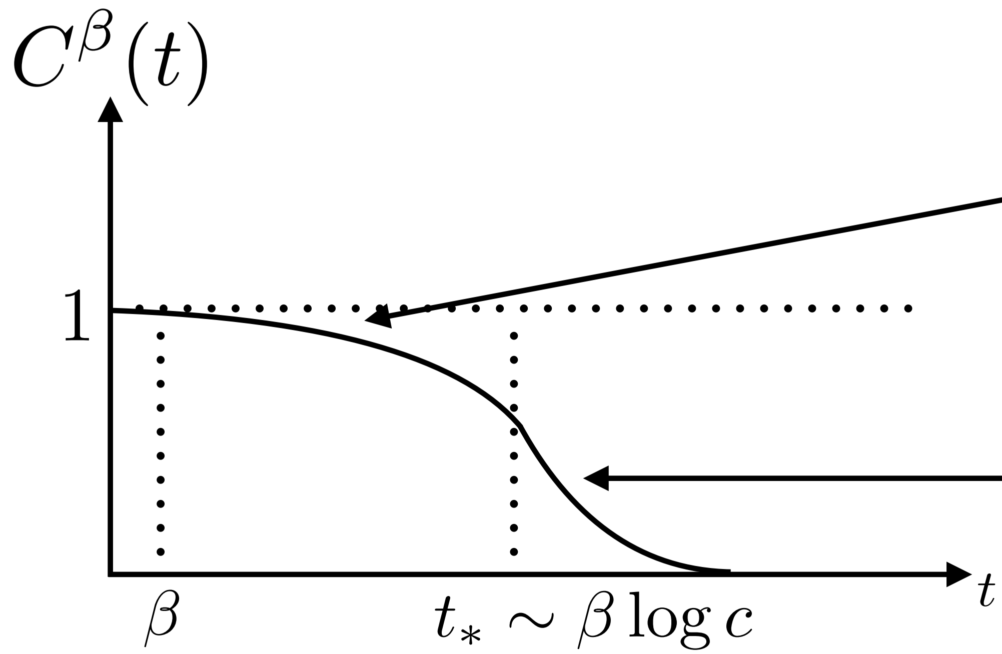
OTO-correlators

Very natural from the TFD perspective!

BH are maximally chaotic

[Maldacena,Shenker,Stanford'15]

In chaotic systems



$$C^\beta(t) \sim 1 - Ae^{\lambda_L t}$$

$$C^\beta(t) \sim e^{-a\frac{2\pi t}{\beta}}$$

Bound on chaos

$$\lambda_L \leq \frac{2\pi}{\beta}$$

BH are maximally chaotic: (=) for a system with a holographic dual (Einstein BH)

Toy models for black holes, SYK….

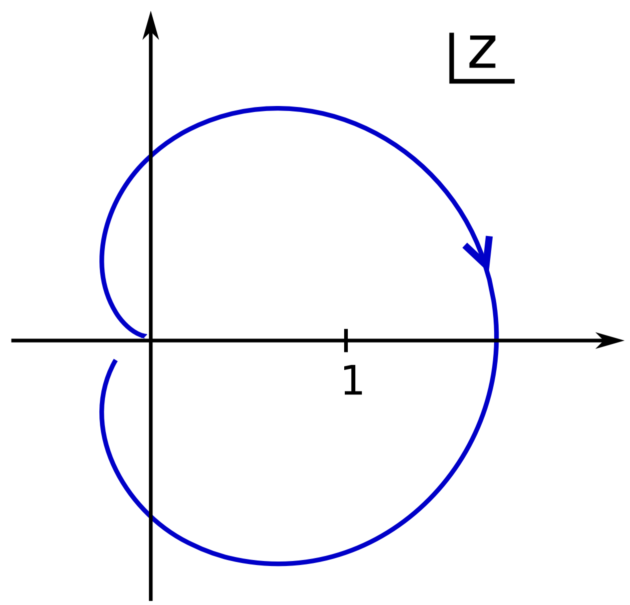
OTOs in 2d CFT

[Stanford,Roberts'15]

OTO correlators (all orderings) can be obtained from the Euclidean 4pt

$$\frac{\langle W^\dagger W V^\dagger V\rangle}{\langle W^\dagger W\rangle\langle V^\dagger V\rangle} = G(z,\bar z)$$

$$\begin{aligned}
z_1 &= e^{\frac{2\pi}{\beta}(t+i\epsilon_1)}, & \bar z_1 &= e^{-\frac{2\pi}{\beta}(t+i\epsilon_1)},\\
z_2 &= e^{\frac{2\pi}{\beta}(t+i\epsilon_2)}, & \bar z_2 &= e^{-\frac{2\pi}{\beta}(t+i\epsilon_2)},\\
z_3 &= e^{\frac{2\pi}{\beta}(x+i\epsilon_3)}, & \bar z_3 &= e^{\frac{2\pi}{\beta}(x-i\epsilon_3)},\\
z_4 &= e^{\frac{2\pi}{\beta}(x+i\epsilon_4)}, & \bar z_4 &= e^{\frac{2\pi}{\beta}(x-i\epsilon_4)}.
\end{aligned}$$



$$\epsilon_1 < \epsilon_3 < \epsilon_2 < \epsilon_4$$

$$(1-z) \to e^{-2\pi i}(1-z) \qquad z,\bar z \to 0, \qquad \bar z/z - fixed$$

OTOs in large c 2d CFT

[Stanford,Roberts'15]

[Fitzpatrick,Kaplan'16]

Identity Block at large central charge c

$$\mathcal{V}(z) \;=\; 1+\frac{2h_W h_V}{c} z^2 {}_2F_1(2,2,4,z) \;+O(1/c^2)$$

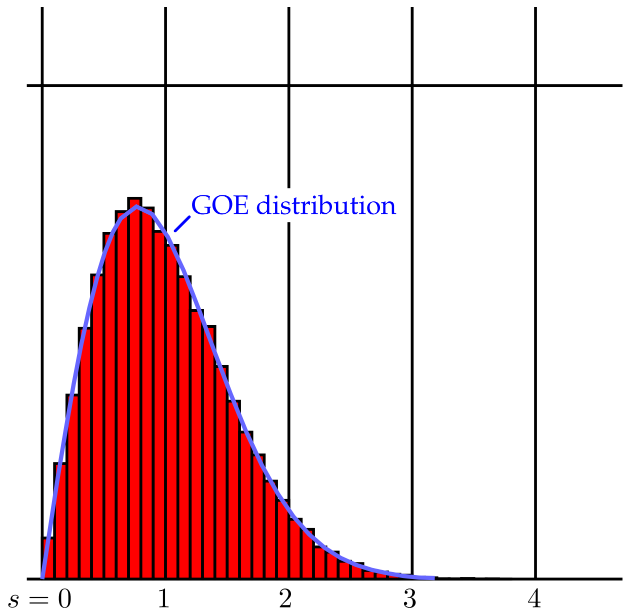
After OTO continuation

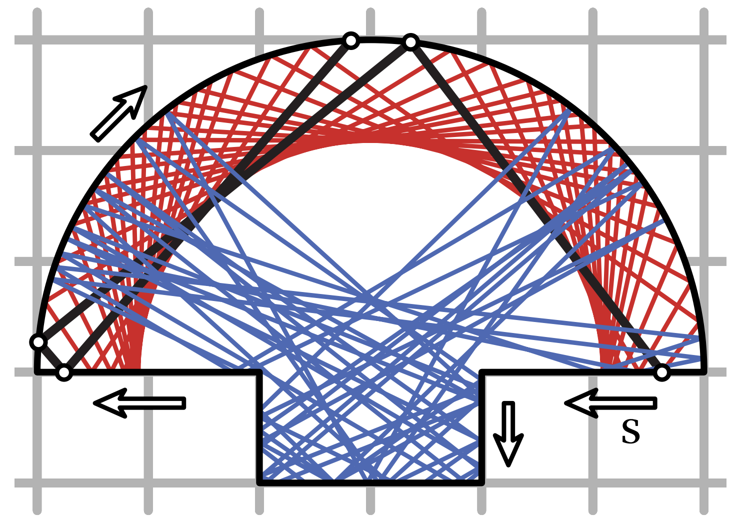
$$\mathcal{V}(z) \approx 1+\frac{48\pi i h_W h_V}{cz} = 1-\frac{12\pi h_W h_V}{c} e^{\frac{2\pi}{\beta}(t-x)} \;+O(1/c^2)$$

Large c vacuum block for HHLL captures this

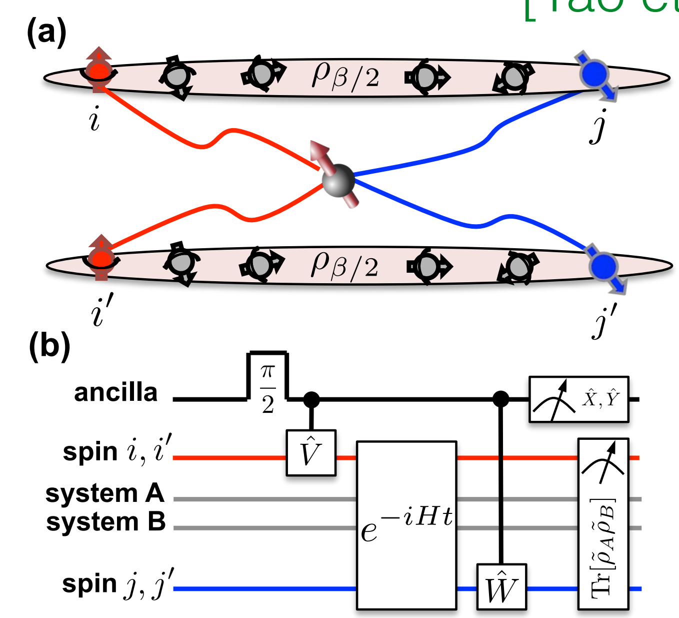
$$\left(\frac{1}{1+\frac{24\pi i h_w}{\epsilon_{12}^*\epsilon_{34}}\, e^{(2\pi/\beta)(t-t_*-x)}}\right)^{2h_v}$$

OTOs and Quantum Chaos?





[Yao et al]

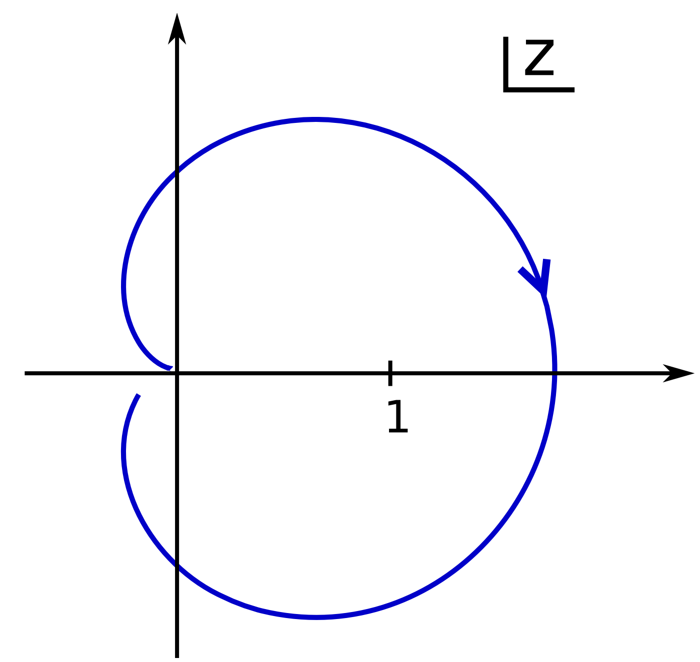


- OTOs and "Standard definitions" of Quantum Chaos
- Quantum Chaos and Non-Integrability?
- Experimental access to OTOs!

In RCFT

[P.C,Numasawa,Veliz-Osorio'16]
[Gu,Xi'16]

$$C^{\beta}_{ij}(t) \equiv \frac{\left\langle \mathcal{O}_i^\dagger(t)\mathcal{O}_j^\dagger \mathcal{O}_i(t)\mathcal{O}_j \right\rangle_\beta}{\left\langle \mathcal{O}_i^\dagger \mathcal{O}_i \right\rangle_\beta \left\langle \mathcal{O}_j^\dagger \mathcal{O}_j \right\rangle_\beta} = \mathcal{G}(z,\bar{z})$$



$$\mathcal{G}(z,\bar{z}) = \sum_p \mathcal{F}^{ii}_{jj}(p|z)\bar{\mathcal{F}}^{ii}_{jj}(p|\bar{z})$$

$$\mathcal{F}^{ii}_{jj}(p|z) \to \sum_q \mathcal{M}_{pq}\mathcal{F}^{ii}_{jj}(q|z)$$

$$\boxed{C^{\beta}_{ij}(t) \to \frac{1}{d_i d_j}\frac{S^*_{ij}}{S_{00}}}$$

$$\mathcal{M}_{00} = \frac{S^*_{ij}}{S_{00}}\frac{S_{00}}{S_{0i}}\frac{S_{00}}{S_{0j}}$$

[Bonderson, Shtengel, Slingerland '06]

"Monodromy scalar" can be measured in anyon interferometry experiments!

Large c but “rational”: SU(N)$_k$ WZW

[P.C,Numasawa,Veliz-Osorio’16]

State excited by the operator in the fundamental rep.

$$g^{\alpha}_{\beta}(x)\,|0\rangle \qquad\qquad h=\bar{h}=\frac{N^2-1}{2N(k+N)}$$

4-point correlator from K-Z equations

$$\mathcal{G}(z,\bar{z})=\sum_{i,j} I_i\bar{I}_j\sum_n X_{nn}\mathcal{F}_i^{(n)}(z)\mathcal{F}_j^{(n)}(\bar{z})$$

From the monodromy of F we can confirm our constant!

$$C^{\beta}_{ij}(t)\to\cos\left(\frac{2\pi}{k+2}\right)\cos^{-1}\left(\frac{\pi}{k+2}\right)$$

For SU(2) at level k

$\left(T^2\right)^n / \mathbb{Z}_n$

[P.C,Y.Kusuki,K.Watanabe,T.Takayanagi arXiv:17…]

OTO correlators with W,V given by twist operators as functions of the radius η

$$C_\beta(t) = |1-z|^{-4\Delta_n} F_n(z,\bar{z})$$

e.g. n=2 we can write

$$F_2(z,\bar{z}) = \frac{\Theta^2(0|T)}{f_{1/2}(z)\bar{f}_{1/2}(\bar{z})} = 2^{-4/3}|z|^{1/3}|1-z|^{1/3} Z_\eta^2(\tau,\bar{\tau})$$

modular parameter is related to the cross-ratios as

$$\tau \equiv \frac{i f_{1/2}(1-z)}{f_{1/2}(z)} \qquad\qquad f_{1/2}(z) = {}_2F_1(1/2,1/2,1,z)$$

OTO continuation is equivalent to

$$\tau \to \frac{\tau}{1+2\tau} \qquad\qquad S\bar{T}^2 S$$

$\left(T^2\right)^n / \mathbb{Z}_n$

[P.C,Y.Kusuki,K.Watanabe,T.Takayanagi arXiv:17…]

Rational radius behaves as in standard RCFTs $\eta = \dfrac{p}{q}$

n=2

$$C_\beta(t) \to \left(S\bar{T}^2 S\right)_{00}$$

for higher n, and $pq \in n\mathbb{Z}$ values sensitive to pq

$$C_\beta(t) \to \frac{n}{(2pq)^{n-1}}$$

Irrational radius $\eta \neq \dfrac{p}{q}$

$$C_\beta(t) \to \left(\frac{\beta}{t}\right)^{n-1}$$

Polynomial decay at late times

W_N

[P.C. work in progress]

We can consider the 4pt correlator of (f,1) and (1,f) in

$$\frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}} \qquad\qquad c = (N-1)\left(1 - \frac{N(N+1)}{p(p+1)}\right)$$

both operators increase entanglement entropies by the log [N]

Operators (f,1) are consider as light but (1,f) as heavy.

Their 4pt correlator

$$G_{\phi_+\phi_-}(x) = |1-x|^{-2\Delta_+}|x|^{\frac{2}{N}}\left|1+\frac{1-x}{Nx}\right|^2$$

The OTO shows no chaos.

[Perlmutter, 1602.08272]

Summary

- OTOs allow us to sharpen our constraints on Black Holes in AdS/CFT.
- OTOs are interesting new tools to explore aspects of quantum chaos
- In 2d CFTs we can compute them in various models and explore their properties
- We get a new perspective on the known CFT data from OTOs (classification of RCFTs?)
- OTOs already got their "own life" in MB physics (MBL), similarly to Entanglement Entropy.

Open Questions

- Black hole slogans and quantum chaos? Are the “necessary conditions“ independent?
- Can we interpret OTOs or extract the same information from some QI tools ? QI Metric, Fidelity, Loschmidt echo [M.Miyaji ’16]?
- CFT data and bootstrap: can we make some general arguments about OTOs, late time values, Lyapunov exponent?
- Higher-point OTOs and Jones polynomials? Is CFT a quantum computer?
- Late time physics beyond the scrambling time? Recurrences ?
- Other tools: Gutzwiller trace formula, spectral form factors…

どうもありがとう!