Bulk local states on the black holes in AdS/CFT

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Based on the work in progress
with Tadashi Takayanagi (YITP,Kyoto)

How does the quantum gravity describe the spacetime?

- Best-understood model of the quantum gravity is AdS/CFT
- In AdS/CFT, the vacuum in CFT corresponds to the pure AdS spacetime

$$\text{Pure AdS} \Leftrightarrow |0\rangle_{\text{CFT}}$$

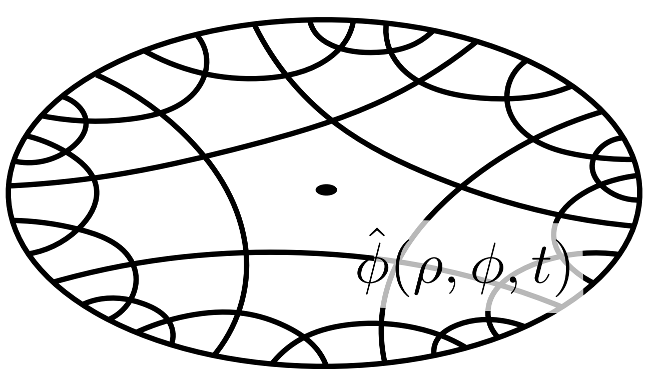
- The generic high energy states in CFT $\Leftrightarrow$ black holes in AdS

$$\text{Black holes} \Leftrightarrow |\Psi_{\text{High E}}\rangle_{\text{CFT}}$$

- In AdS/CFT, the spacetime is emergent from the Hilbert space of CFT

Can CFT reconstruct the AdS spacetime?

Can CFT describe the black hole (interior)? c.f. firewall paradox



To probe the interior of the AdS spacetime directly

→bulk local fields in AdS

How can the local fields in the AdS be represented in the CFT?

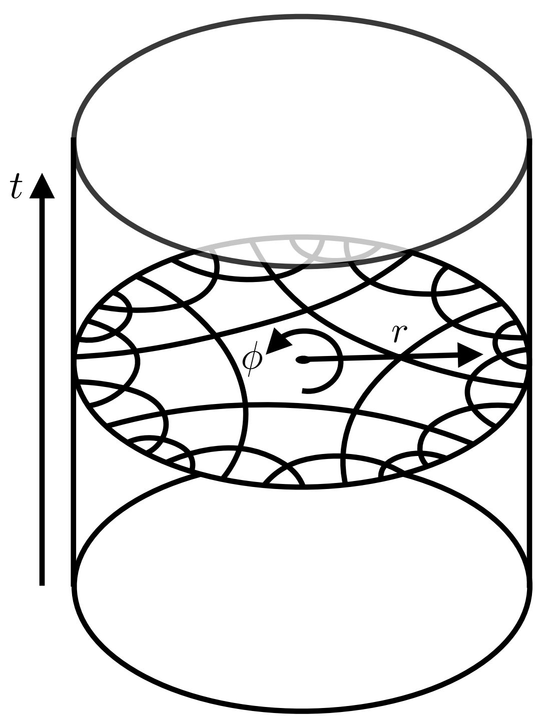
Bulk local states
in the global AdS coordinate

[Verlinde, Miyaji-Numasawa-Shiba-Takayanagi-Watanabe, Nakayama-Ooguri '15]

AdS/CFT correspondence

AdS/CFT is the correspondence between the d+1 dim. quantum gravity on AdS and large N conformal field theory on the d-dimensional boundary of AdS.

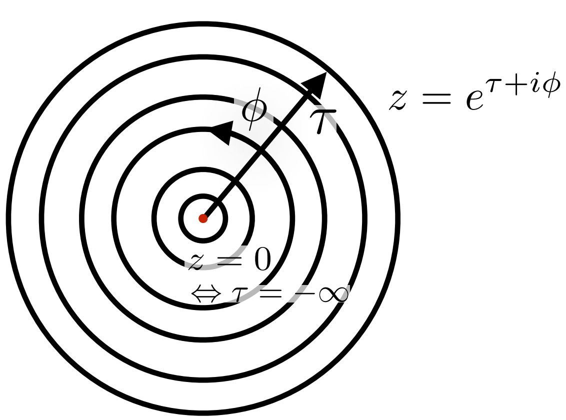
In the large N limit, the bulk gravity theory is weekly coupled (free theory).



We focus on AdS_3/CFT_2 in the leading order of 1/N.

The global coordinate of AdS is

$$ds^2 = -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\phi^2$$
$$= -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2 \qquad r = \sinh\rho$$

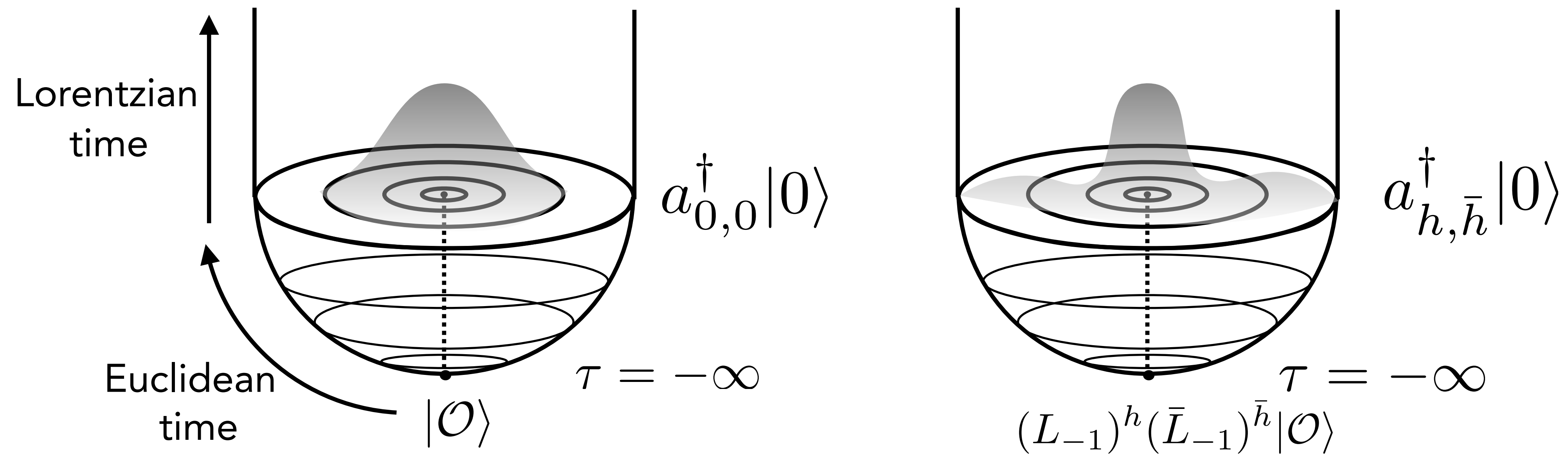


Euclidean Plane

Its boundary; $r = \infty$ is $\mathbb{S} \times \mathbb{R}$ i.e. cylinder

Its Euclidean boundary can be mapped to the euclidean plane $(z, \bar{z})$ by the conformal transformation.

One particle states in AdS



Correlators of local operators in a large N CFT factorize in the leading order of the large N expansion → "Generalized free fields"

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle = \langle \mathcal{O}_1 \mathcal{O}_2 \rangle \cdots \langle \mathcal{O}_{n-1} \mathcal{O}_n \rangle + \text{permutation}$$

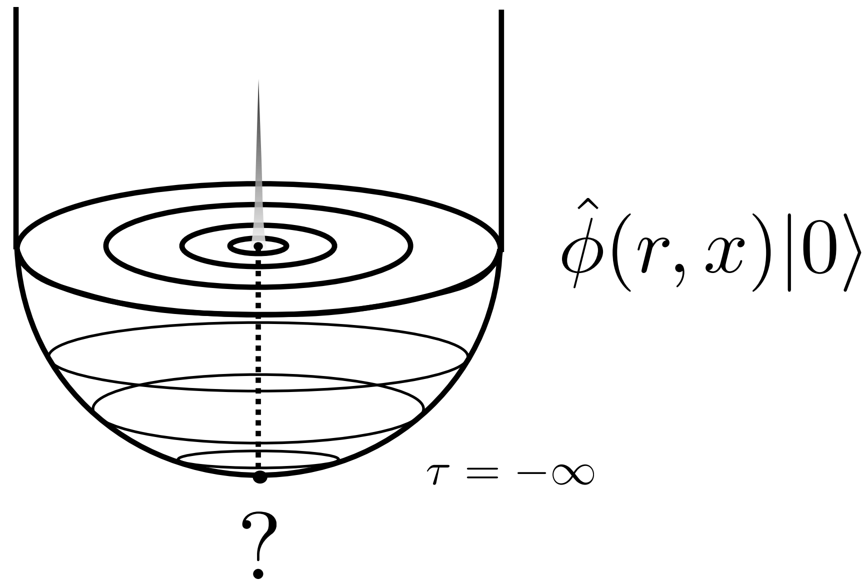
The dimension of a "multi-trace" operator $\tilde{\mathcal{O}}(x) =: \mathcal{O}_1(x) \cdots \mathcal{O}_k(x) :$

is given by $\sum_i \Delta_i + \mathcal{O}(\frac{1}{N})$ → single trace operators behave as "particles"

Single-trace operators corresponds single particle states in AdS

Primary states↔lowest energy particle state of mass $m^2 = \Delta(\Delta - 2)$

Bulk local states in AdS/CFT



In order to analyze the local physics in AdS, consider the CFT dual of states locally excited by bulk local fields.→"Bulk local states"

We focus on reconstructing free scalar fields in AdS_3

Expectation from the AdS side

Fields in AdS can be expressed as superpositions of creation operators

$$\hat{\phi}(r,\phi,t)|0\rangle \sim \sum_{h,\bar{h}} f_{h,\bar{h}}(r) e^{i(\omega t - k\phi)} a^{\dagger}_{h,\bar{h}}|0\rangle \quad \omega = \Delta + h + \bar{h}, k = h - \bar{h}$$

→We need to superpose descendants of a primary operator in the CFT side.

Symmetry

To know how to superpose descendants of a primary operator, we use the correspondence of the symmetry between AdS and CFT.

Isometry of the AdS_3 is SO(2,2);expressed in the global coordinate as

$$L_0 = i\partial_+, \quad L_\pm = e^{\pm i x^+}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_+ - \frac{1}{\sinh 2\rho}\partial_- \mp \frac{i}{2}\partial_\rho\right]$$
$$\bar{L}_0 = i\partial_-, \quad \bar{L}_\pm = e^{\pm i x^-}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_- - \frac{1}{\sinh 2\rho}\partial_+ \mp \frac{i}{2}\partial_\rho\right]$$

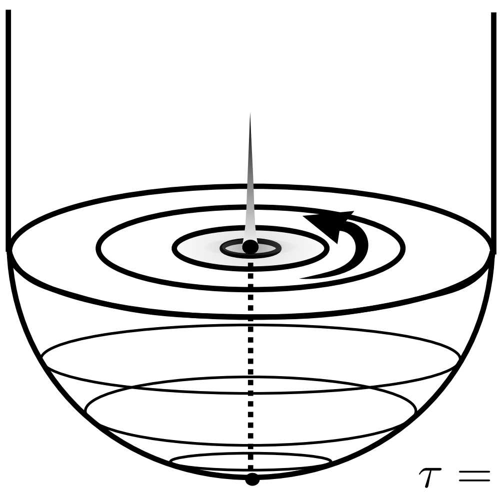
In the boundary limit, they become the generators of conformal transformations.

$$L_n = -z^{n+1}\partial_z$$
$$\bar{L}_n = -\bar{z}^{n+1}\partial_{\bar{z}} \qquad n = -1, 0, 1$$

where we wick-rotated the time and used the coordinate $z = e^{ix^+}, \bar{z} = e^{ix^-}$

Isometry of AdS⇔ (global) conformal symmetry in the CFT.

Bulk local state construction



Consider a time slice t=0.

The subgroup H of the entire conformal symmetry G=SO(2,2)=SL(2,R)×SL(2,R) which is dual to the the subgroup of the isometry which fixes a bulk point (r, x) in AdS.

The state invariant under H should be a state locally excited by $\hat{\phi}(r,x)$.

Consider the bulk local state at r=0; $|\phi\rangle \equiv \hat{\phi}(r=0,\phi,t=0)|0\rangle$. It satisfies

$$(L_0-\bar{L}_0)|\phi\rangle = (L_1+\bar{L}_{-1})|\phi\rangle = (\bar{L}_1+L_{-1})|\phi\rangle = 0$$

The solutions are expressed as Ishibashi states with respect to the SL(2,R)×SL(2,R) conformal symmetry with -1 twist.

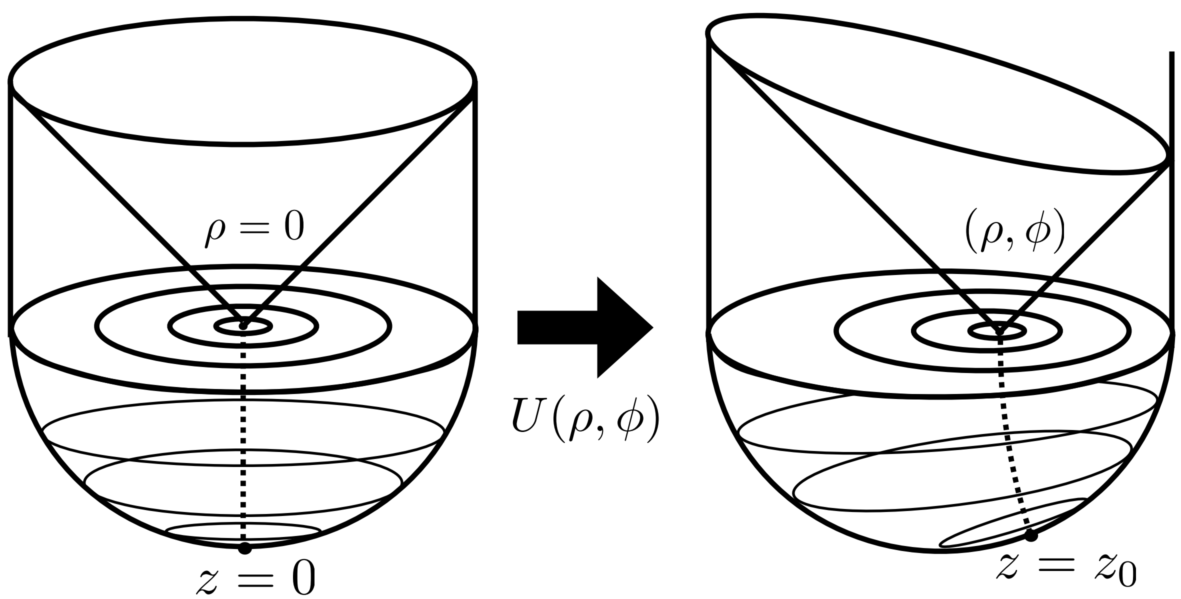
$$|\phi\rangle = \sum_k^{\infty}(-1)^k \frac{\Gamma(\Delta)}{k!\Gamma(k+\Delta)}(L_{-1})^k(\bar{L}_{-1})^k|\mathcal{O}\rangle$$

→superposition of various descendant states of a primary state.

The bulk local states at different positions⇒SL(2,R) transformations

$$|\phi(\rho,\phi,t)\rangle = e^{i(L_0+\bar{L}_0)t} e^{-\rho\left(e^{-i\phi}\frac{L_1-\bar{L}_{-1}}{2}+e^{i\phi}\frac{\bar{L}_1-L_{-1}}{2}\right)}|\phi\rangle$$

Under this conformal transformation, a boundary point $z=0$ is mapped to $z_0 = \tanh\left(\frac{\rho}{2}\right) e^{i(t+\phi)}$.Other points are mapped as the following way;



$$z' = f(z) = \frac{z+z_0}{1+e^{-2i(t+\phi)}z_0 z}$$

Define Virasoro generators around z_0

$$L_n^{z_0}\mathcal{O}(z_0) = \oint_{z_0} dz z^{n+1} f^* T(z)\mathcal{O}(z_0)$$

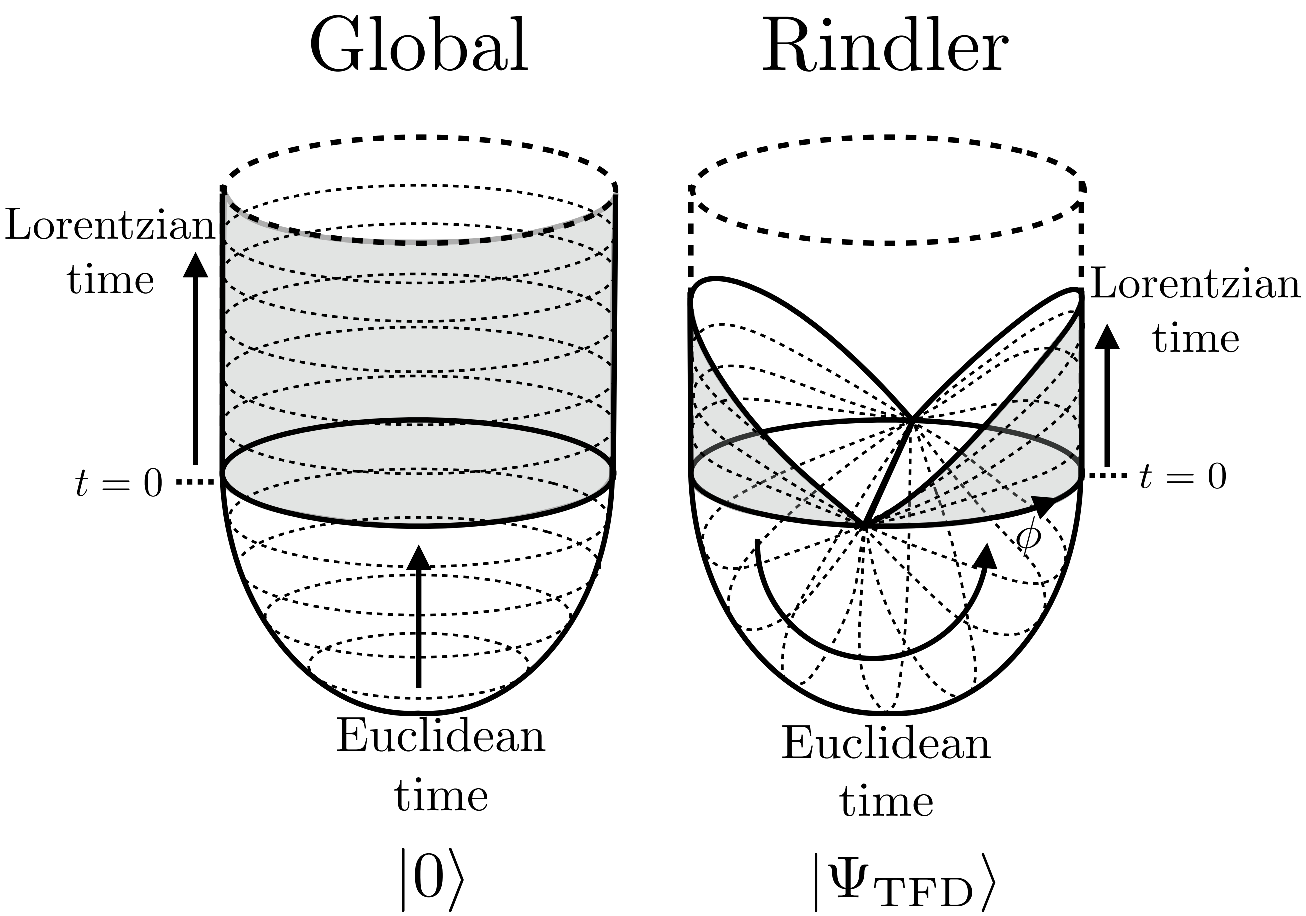
We can rewrite the bulk local states as ``Ishibashi states" around z_0

$$|\phi(\rho,\phi,t)\rangle = \sum_{k=0}^{\infty}(-1)^k \frac{\Gamma(\Delta)}{k!\Gamma(k+\Delta)}(L_{-1}^{z_0})^k(\bar{L}_{-1}^{\bar{z}_0})^k \mathcal{O}(z_0,\bar{z}_0)|0\rangle$$

The bulk coordinates are emergent as parameters of conformal transformations.

Bulk local states
on the BTZ black hole

Lessons from the Rindler AdS



Rindler AdS coordinate;

$$ds^2 = -(r^2 - R^2)dt^2 + \frac{R^2}{r^2 - R^2}dr^2 + r^2 d\phi^2$$
$$= -R^2 \sinh^2 \rho dt^2 + d\rho^2 + R\cosh^2 \rho d\phi^2$$
$$r > R\ (\rho > 0), -\infty < t < \infty, -\infty < \phi < \infty$$

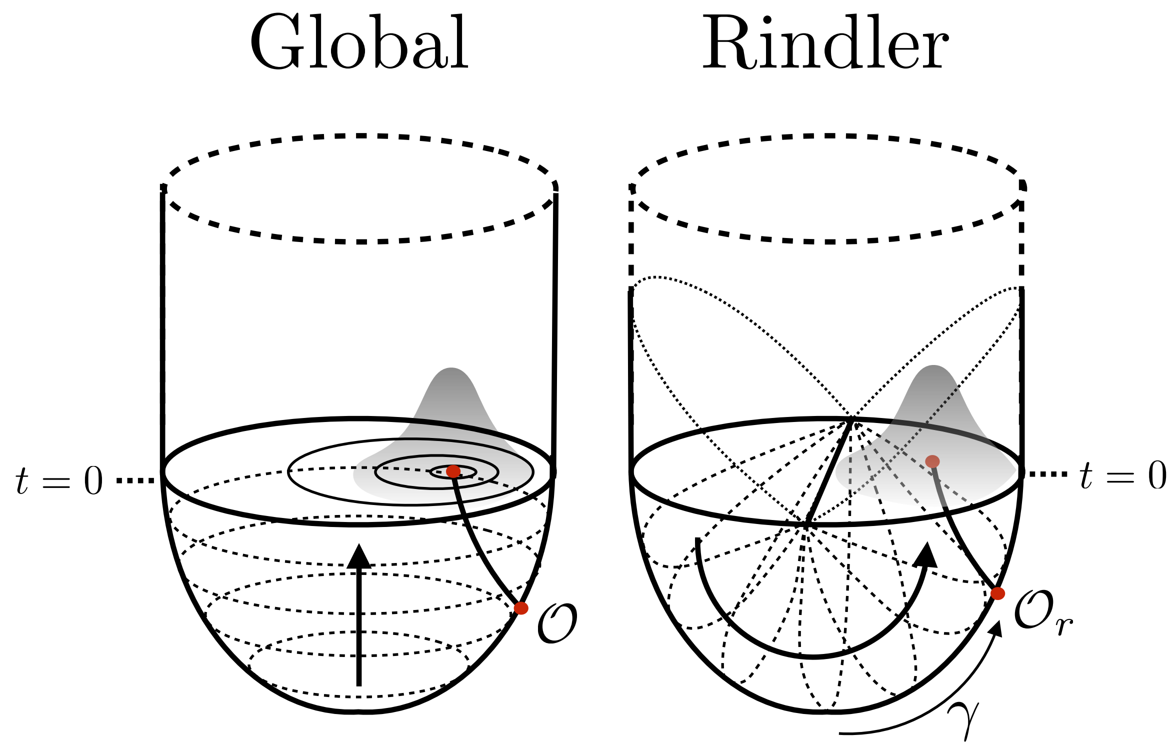
The global AdS and Rindler AdS ↪the Euclidean path integral of the dual CFT along different time slicing.

For the Rindler time slicing, it is natural to define two CFTs at t=0.

$$\mathcal{H} \simeq \mathcal{H}_l \otimes \mathcal{H}_r$$

The "vacuum state" defined at t=0 for the Rindler time slicing is written as thermofield double state.

$$|0\rangle \simeq |\Psi_{\mathrm{TFD}}\rangle = \sum_n e^{-\beta E_n/2}|E_n\rangle_l \otimes |E_n\rangle_r$$



A primary operator inserted at a boundary point makes the one particle state whose excitation is centered around the corresponding bulk point.

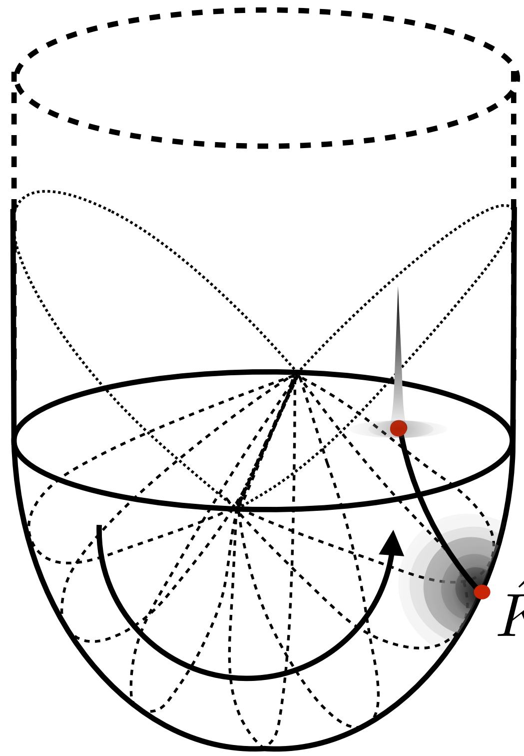
$$\mathcal{O}(z_0)|0\rangle \leftrightarrow e^{\frac{\beta}{4}(1-\gamma)H_r}\mathcal{O}_r(-\pi/2,\phi)e^{-\frac{\beta}{4}(1-\gamma)H_r}|\Psi_{\mathrm{TFD}}\rangle$$

where a primary operator inserted at $(\tau = \pi/2(\gamma-1), \phi)$.

The explicit relation between the bulk points and boundary points are

$$z_0 = \frac{\tanh\frac{\rho}{2} + i\tanh\frac{\phi+t}{2}}{1 - i\tanh\frac{\rho}{2}\tanh\frac{\phi+t}{2}} \qquad \bar{z}_0 = \frac{\tanh\frac{\rho}{2} - i\tanh\frac{\phi-t}{2}}{1 + i\tanh\frac{\rho}{2}\tanh\frac{\phi-t}{2}}$$

where (ρ, ϕ, t) are the Rindler coordinates and $(z_0, \bar{z}_0)$ is the corresponding point on the Euclidean plane.



In order to get the localize the bulk excitation, we need dress the "localizing operator" on the primary operator.

$$\hat{K}=\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma(\Delta)}{k!\Gamma(k+\Delta)}(L_{-1}^{\gamma,\phi})^k(\bar{L}_{-1}^{\gamma,\phi})^k$$

The bulk local states in the Rindler-AdS can be expressed as

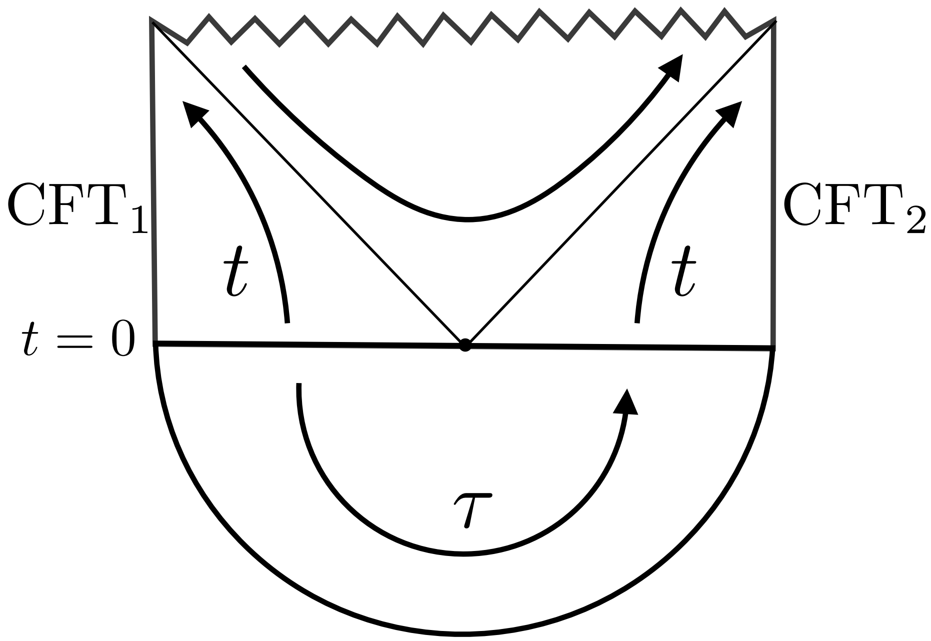
$$|\phi_r(r,\phi,t)\rangle=\sum_{k=0}^{\infty}(-1)^k\frac{\Gamma(\Delta)}{k!\Gamma(k+\Delta)}(L_{-1}^{\gamma,\phi})^k(\bar{L}_{-1}^{\gamma,\phi})^k e^{\frac{\beta}{4}(1-\gamma)H_r}\mathcal{O}_r(-\pi/2,\phi)e^{-\frac{\beta}{4}(1-\gamma)H_r}|\Psi_{\mathrm{TFD}}\rangle$$

We can also reconstruct inside the future (past) horizon by moving a primary operator in the future (past) direction of the Lorentzian time on the boundary.

We can check that two point functions of the bulk local states exactly reproduce the two point functions in the Rindler AdS.

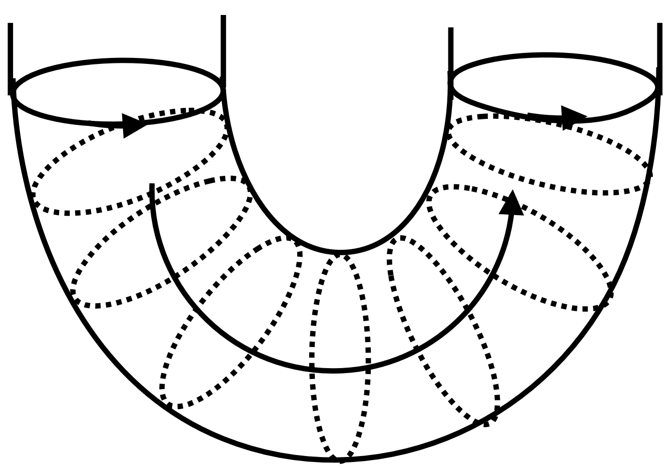
$$\langle\phi(r,\phi,t)\phi(r,\phi',t')\rangle=G^{\mathrm{Rindler}}(r,\phi,t|r,\phi',t')$$

BTZ black hole



Consider the BTZ spacetime which is dual with the thermofield double state
↪Euclidean path integral over the cylinder (half of torus) whose length is β/2.

$$|\Psi_{\text{TFD}}\rangle = \sum_n e^{-\beta E_n/2}|E_n\rangle_1 \otimes |E_n\rangle_2$$

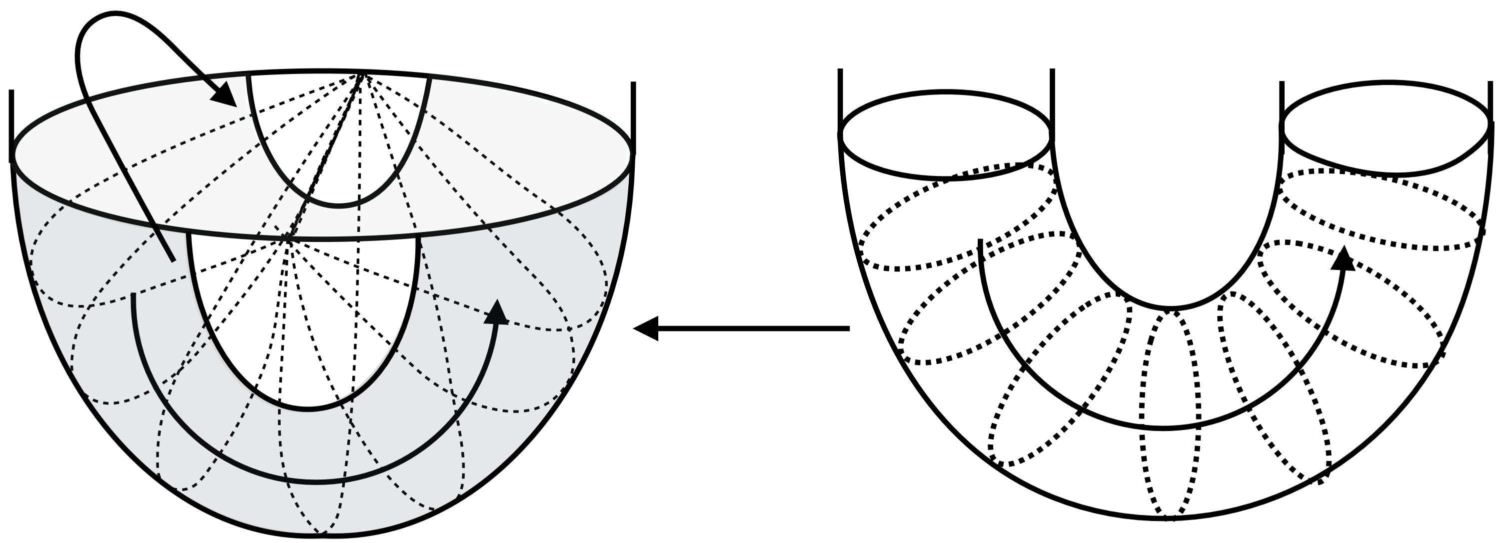


$$\begin{aligned} ds^2 &= -(r^2-R^2)dt^2 + \frac{R^2}{r^2-R^2}dr^2 + r^2 d\phi^2 \\ &= -R^2\sinh^2\rho dt^2 + d\rho^2 + R\cosh^2\rho d\phi^2 \end{aligned}$$

$$r > R\ (\rho > 0), -\infty < t < \infty, -\pi < \phi < \pi$$

Two regions are connected via a wormhole and the spacetime has the same structure as the Rindler-AdS but with the identification

$$\phi \sim \phi + 2\pi$$

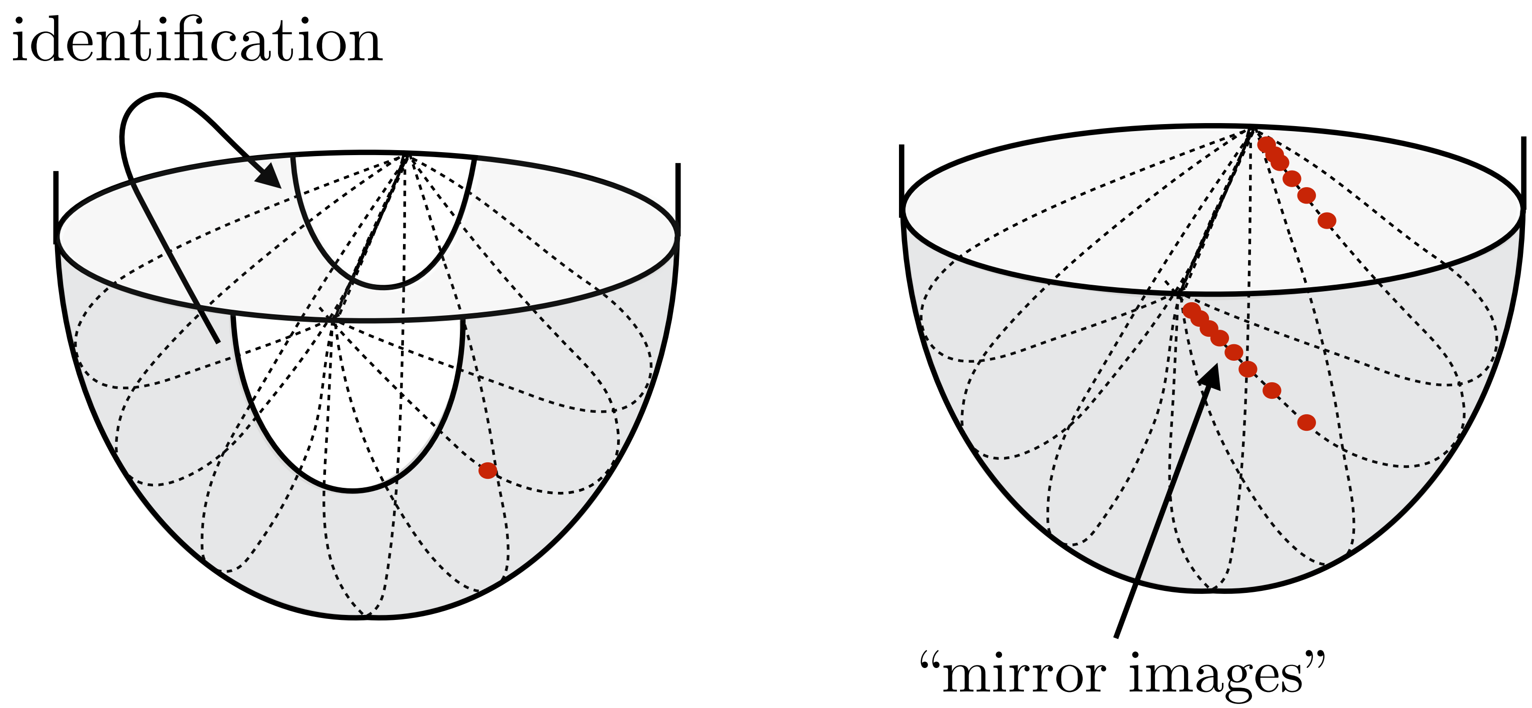


Map the boundary of the BTZ (half torus) to the boundary of the Rindler-AdS.

We define Virasoro generators and the localizing operator around a point in a similar way with the Rindler-AdS case on the cylinder (half sphere).

The bulk local states on the BTZ background can be expressed as

$$|\phi(r,\phi,t)\rangle = \sum_{k=0}^{\infty}(-1)^k \frac{\Gamma(\Delta)}{k!\Gamma(k+\Delta)}(L_{-1}^{\gamma,\phi})^k(\bar{L}_{-1}^{\gamma,\phi})^k e^{\frac{\beta}{4}(1-\gamma)H}\mathcal{O}(-\pi/2,\phi)e^{-\frac{\beta}{4}(1-\gamma)H}|\Psi_{\mathrm{TFD}}\rangle$$

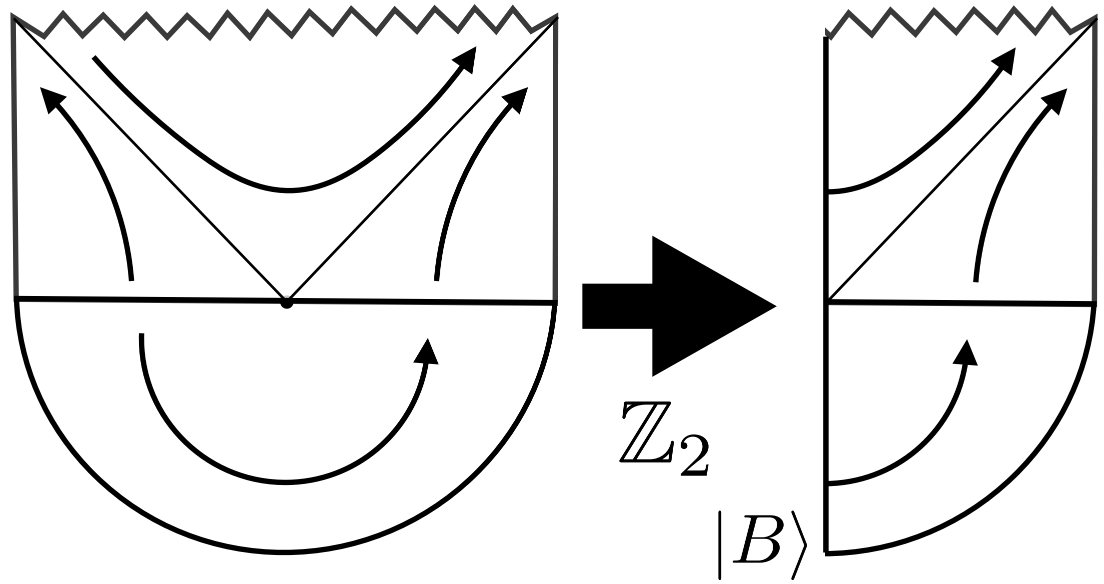


For holographic CFTs, a two point function of the bulk local states on the BTZ will be reduced to a sum of the two point functions of ``mirror” images in the Rindler-AdS.

$$G^{\mathrm{BTZ}}(r,\phi,t|r',\phi',t') = \sum_{m,n} G^{\mathrm{Rindler}}(r,\phi+2\pi m,t|r,\phi'+2\pi n,t')$$

We can see the black hole singularity as the divergences of the two point functions due to the coincidence of the mirror images.

Comments and future directions



We can make single sided black holes by the $\mathbb{Z}_2$ identification of the thermofield double states.
↪described by the boundary states.
(or cross-caps)

$$|\Psi_{\rm BH}\rangle = e^{-\beta H/4}|B\rangle$$

We can construct the bulk local states on this type of single-sided black holes in a similar way as the thermofield double case.

Bulk local states on other high energy states in the CFT?
-e.g. heavy primary states [Fitzpatrick-Kaplan-Walters '15]

$$|\Psi_{\rm BH}\rangle = |\mathcal{O}_H\rangle \quad \Delta_H \propto c$$

Our construction method seems to be state-dependent.
-e.g. Can it be applied to the shock-wave geometry? [Shenker-Stanford '14]

Our method relies on the fact that 3-dim. black holes in AdS can be made as quotients of the pure AdS. -higher dimensional black holes?

Thank you