Breaking of higher spin gauge symmetry from dual large *N* CFTs

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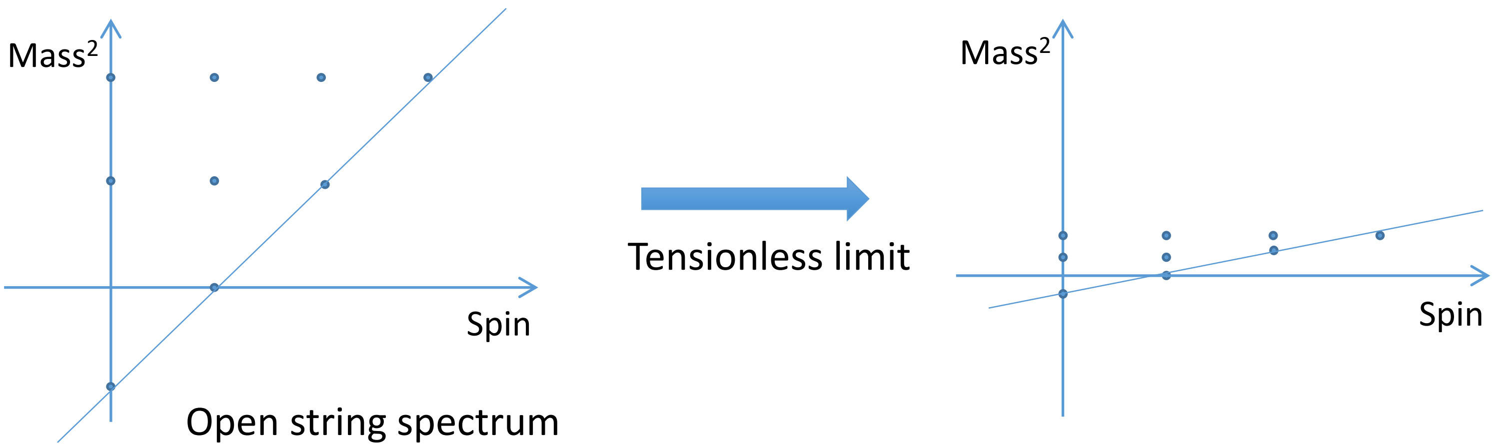
Workshop ``Holography, Quantum Entanglement and Higher Spin Gravity''

Strings from higher spin fields

- Superstring theory includes a lot of higher spin excitations, but they are too heavy to observe
- Higher spin gauge symmetry is expected to appear at the tensionless limit
- Superstring theory may be described by higher spin gauge theory with broken symmetry [Gross '88]



Symmetry breaking from dual CFT

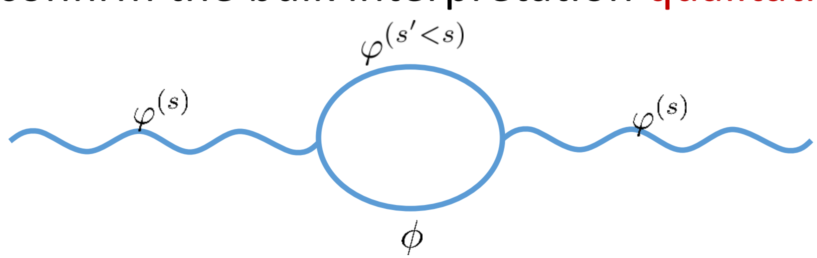
- Klebanov-Polyakov duality

4d Vasiliev (Neumann b.c.) ⟷ 3d O(N) free boson

[Witten '01] $\quad\quad\quad\quad\quad\quad$ $\frac{f}{2}(\phi^i\phi_i)^2$

4d Vasiliev (Dirichlet b.c.) ⟷ 3d critical O(N) model

- CFT implies bulk symmetry broken due to [Girardello-Porrati-Zaffaroni '02]
  - One-loop effects
  - Change of boundary condition for bulk scalar
- We would like to confirm the bulk interpretation qualitatively



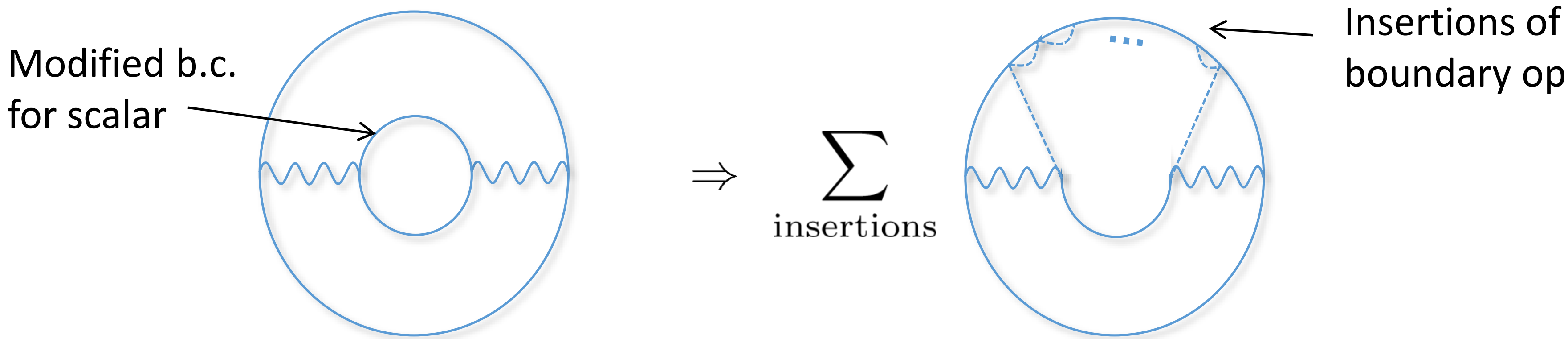
A summary of our results

- Compute anomalous dimensions of higher spin currents in conformal perturbation theory [YH '16] (c.f. [Lang-Ruhl '93])

$$\Delta_s = s + 1 + \frac{16(s-2)}{3\pi^2 N(2s-1)} + \mathcal{O}(\tfrac{1}{N^2})$$

$$\Rightarrow M_s^2 = \Delta_s(\Delta_s - 3) - (s+1)(s-2) = \frac{16(s-2)}{3\pi^2 N} + \mathcal{O}(\tfrac{1}{N^2})$$

- From bulk Witten diagrams to boundary conformal perturbation theory



- Generalize to other examples with higher spin duals [YH-Wada '16,'17]

Plan of talk

- Introduction
- Anomalous dimensions in 3d critical O(N) model
- Dual higher spin interpretation
- Limitation of the method & generalizations
- Conclusion

3d O(N) vector model

- The system with N free real bosons $\phi_i(x)\ (i=1,2,\ldots,N)$ has conserved currents with even spin s

$$J_{\mu_1\cdots\mu_s}(x) \propto \phi^i(x)\partial_{\mu_1}\cdots\partial_{\mu_s}\phi_i(x)+\cdots$$

$$\Rightarrow J_s^{\epsilon}(x) \equiv J_{\mu_1\cdots\mu_s}(x)\epsilon^{\mu_1}\cdots\epsilon^{\mu_s}\ \ (\epsilon\cdot\epsilon=0)$$

- We treat the relevant deformation

$$\Delta S = \frac{f}{2}\int d^3x\,\mathcal{O}(x)\mathcal{O}(x),\ \ \mathcal{O}=\phi_i\phi^i$$

in the conformal perturbation theory

$$\left\langle \prod_{i=1}^{n}\Phi_i(x_i)\right\rangle_f = \frac{\left\langle \prod_{i=1}^{n}\Phi_i(x_i)e^{-\Delta S}\right\rangle_0}{\left\langle e^{-\Delta S}\right\rangle_0}$$

Evaluated in the free theory

Scalar 2pt. function

- Deformed scalar 2pt. function

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_f = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_0 - \frac{f}{2}\int d^3x_3 \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_3)\rangle_0 + \cdots$$

- Dominant contribution for large *N* is

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_f = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_0 - f\int d^3x_3 \langle \mathcal{O}(x_1)\mathcal{O}(x_3)\rangle_0 \langle \mathcal{O}(x_2)\mathcal{O}(x_3)\rangle_0 + \cdots$$

  due to the large *N* factorization $\langle \mathcal{O}^n\rangle_0/(\sqrt{\langle \mathcal{O}^2\rangle\rangle_0})^n \propto N^{1-n/2}$

- Evaluated around fixed point $f \sim \infty$
  - Scaling dimension changes as $\Delta = 1 \Rightarrow \Delta = 2$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_0 = \frac{2N}{|x_{12}|^2}$$

$$\Rightarrow \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_f \sim \frac{1}{f^2}\frac{1}{4\pi^4 N}\frac{1}{|x_{12}|^4} \equiv \frac{1}{f^2}G(x_{12})$$

Current 2pt. function

- Anomalous dimension $\tau_s \equiv \Delta_s - s - 1$ can be read off from log|*x*| term in current 2pt. function

$$\langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_{f\to\infty} = N_s(1+\delta_s)\frac{P_s(x_{12})}{|x_{12}|^{2s+2+2\tau_s}} + \mathcal{O}(N^{-1})$$
$$= N_s\frac{P_s(x_{12})}{|x_{12}|^{2s+2}}(1+\delta_s - 2\tau_s \log|x_{12}|) + \mathcal{O}(N^{-1})$$
$$\left(N_s = \frac{N(2s)!}{(s!)^2},\ P_s(x_{12}) = \frac{(2\epsilon\cdot x_{12})^{2s}}{|x_{12}|^{2s}}\right)$$

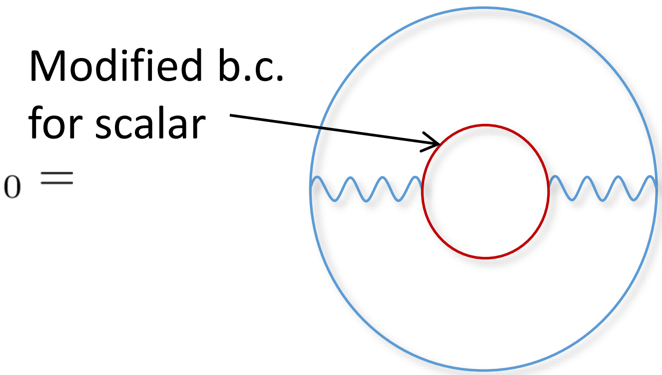
- The expression of current 2pt. function for large *f*, *N*

$$\langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_f = \langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_0 + I_1 + I_2 + \mathcal{O}(1/f, 1/N)$$
$$I_1 = \frac{1}{2}\int d^3x_3 d^3x_4 \langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0 G(x_{34})$$
$$I_2 = \frac{1}{2}\int d^3x_3 d^3x_4 d^3x_5 d^3x_6 \langle J_s^\epsilon(x_1)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0 \langle J_s^\epsilon(x_2)\mathcal{O}(x_5)\mathcal{O}(x_6)\rangle_0 G(x_{35}) G(x_{46})$$

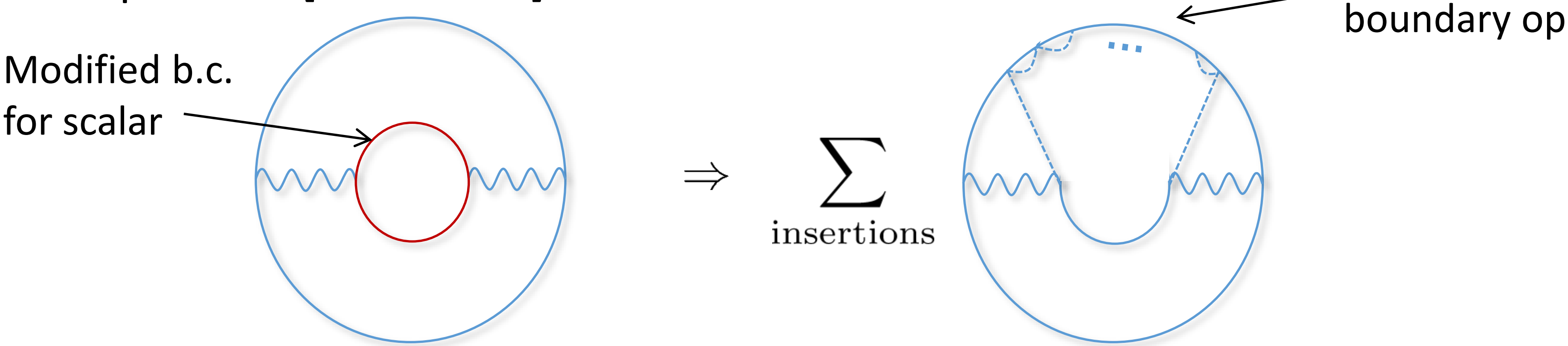
One-loop Witten diagram

- Correction to current 2 pt. function from one-loop Witten diagram

$$\langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_{f\to\infty} - \langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_0 =$$



- Modifying bulk scalar propagator corresponds to inserting boundary operators [Witten ’01]



Leading order corrections

- Current 2 pt. function to the order f^2, $1/N$

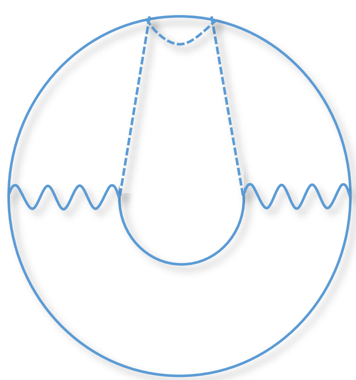
$$\langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_f = \langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\rangle_0 + \tilde{I}_1 + \tilde{I}_2 + \mathcal{O}(f^2, 1/N)$$

$$\tilde{I}_1 = \frac{f^2}{2}\int d^3x_3 d^3x_4 \langle J_s^\epsilon(x_1) J_s^\epsilon(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0 \langle \mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0$$

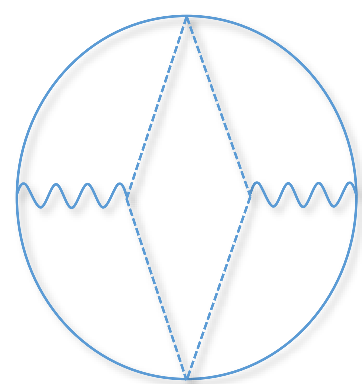
$$\tilde{I}_2 = \frac{f^2}{2}\int d^3x_3 d^3x_4 \langle J_s^\epsilon(x_1)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0 \langle J_s^\epsilon(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0$$

- Corresponding Witten diagrams

$\tilde{I}_1 \Leftrightarrow$

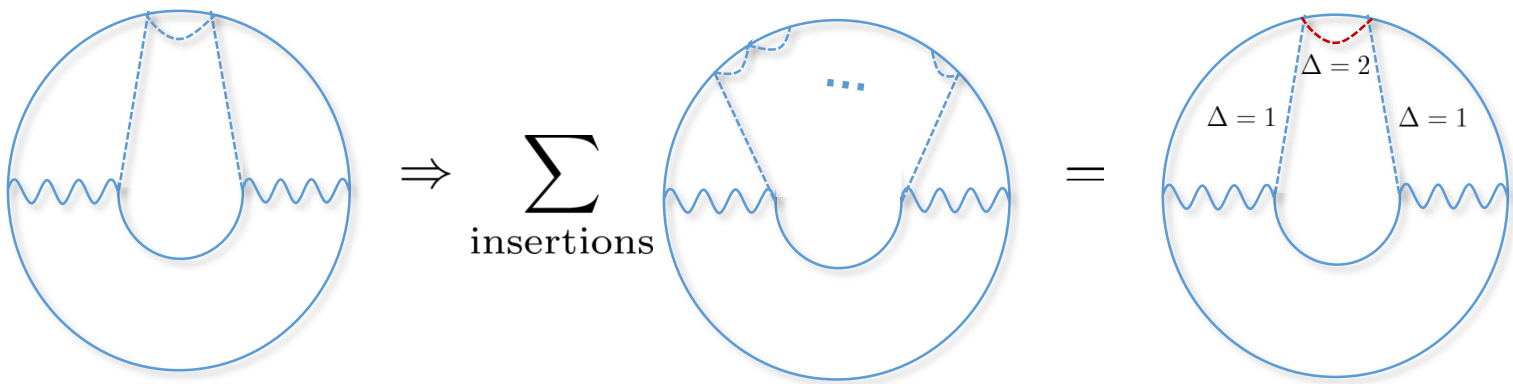


$\tilde{I}_2 \Leftrightarrow$

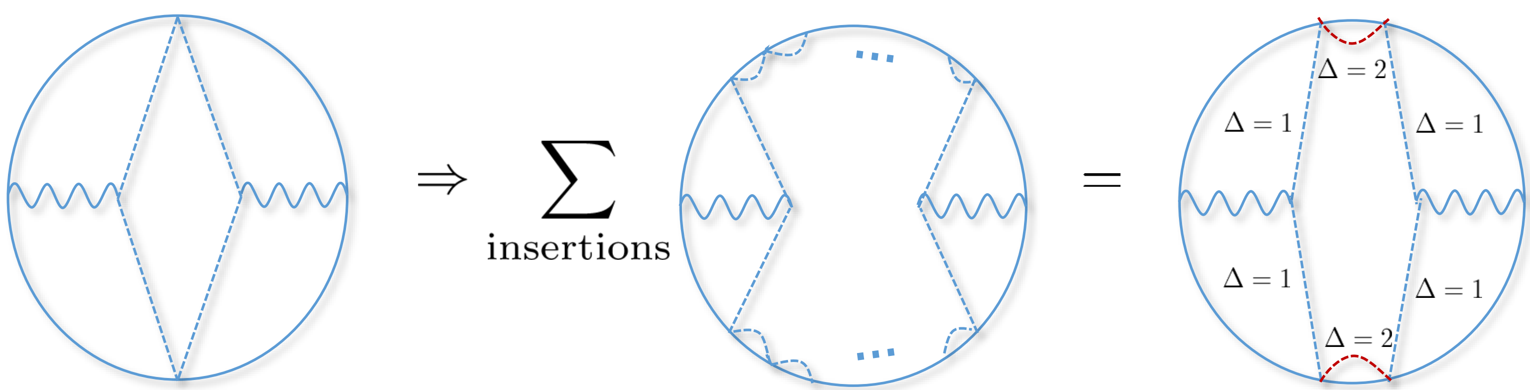


Higher order corrections

- Corrections to $\tilde{I}_1 \Rightarrow I_1$

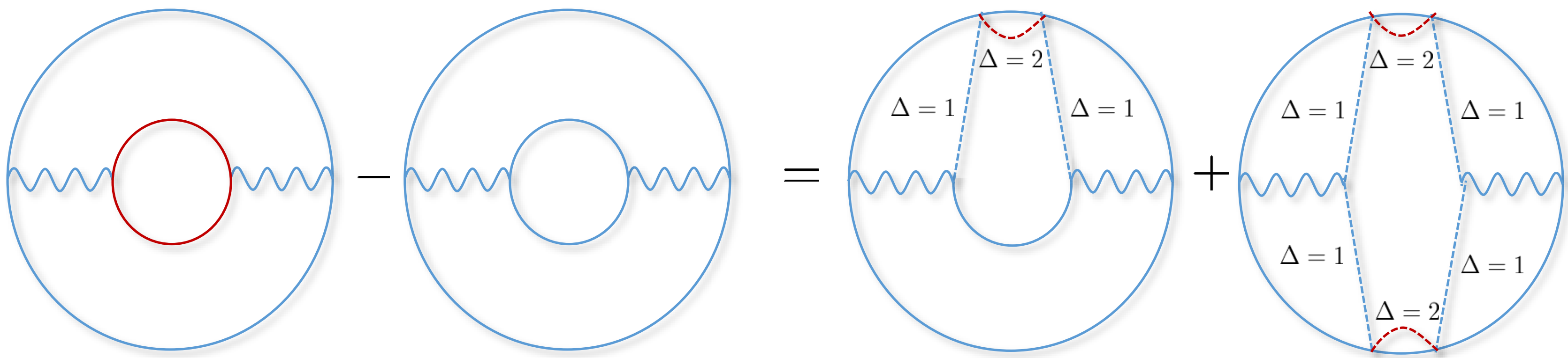


- Corrections to $\tilde{I}_2 \Rightarrow I_2$

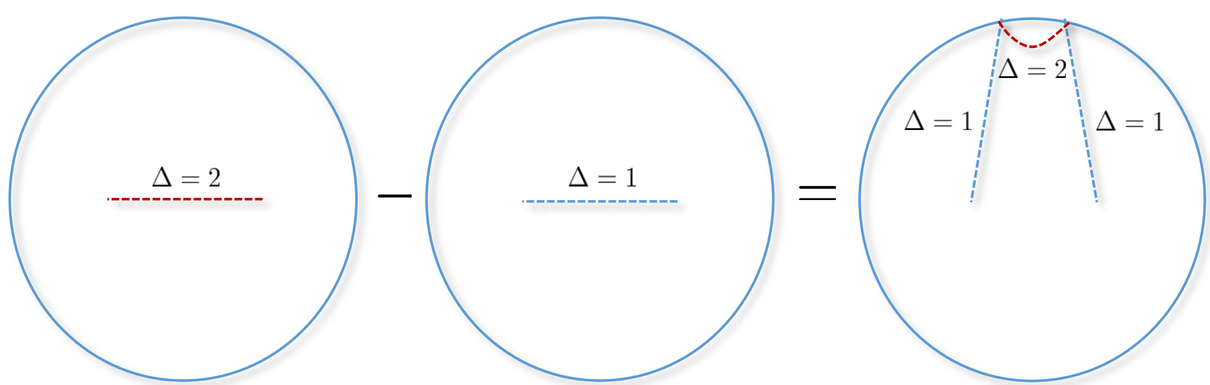


Cutting the loops

- The one-loop Witten diagram can be evaluated from only the products of tree Witten diagrams



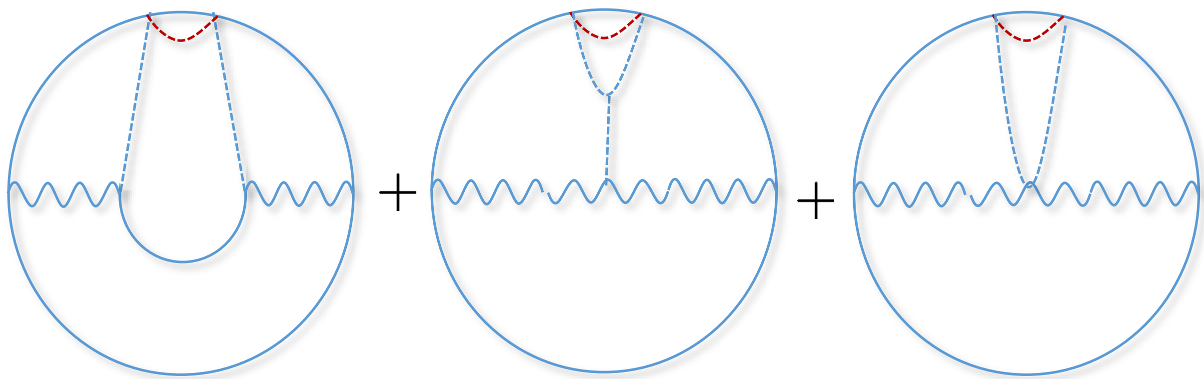
- Crucial identity was proposed in [Hartman-Rastelli ’06, Giombi-Yin ’11]



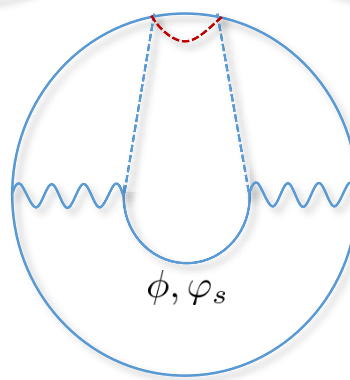
Limitation of the method

- Not every bulk information can be obtained from the dual CFT
- CFT 4pt. corresponds to the sum of Witten diagrams (c.f.[Giombi-Yin ’11])

$$I_1 = \frac{1}{2}\int d^3x_3 d^3x_4 \langle J_s^\epsilon(x_1) J_s^\epsilon(x_2) \mathcal{O}(x_3)\mathcal{O}(x_4)\rangle_0 G(x_{34})$$

$\Leftrightarrow$ 

- We cannot identity what are the bulk intermediate states



Other examples with higher spin duals

- We have extended the analysis to systems such that [YH-Wada ’16, ’17]
  - O(N) vector model in d dimensions (reproduces [Lang-Ruhl ’93])
  - Gross-Neven model in d dimensions (reproduces [Muta-Popovic ’77])
  - 3d supersymmetric U(N) model (marginal deformation ⇔ Turning on string tension, new results)
- We want to examine Chern-Simons matter theory as in [Aharony-Gur-Ari-Yacoby, Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin ’11]
  - Anomalous dimensions of higher spin currents were obtained recently in different methods [Giombi-Gurucharan-Kirilin-Prakash-Skvortsov ’16]
  - ABJ triality suggests closer relations to superstring theory [Chang-Minwalla-Sharma-Yin ’12]

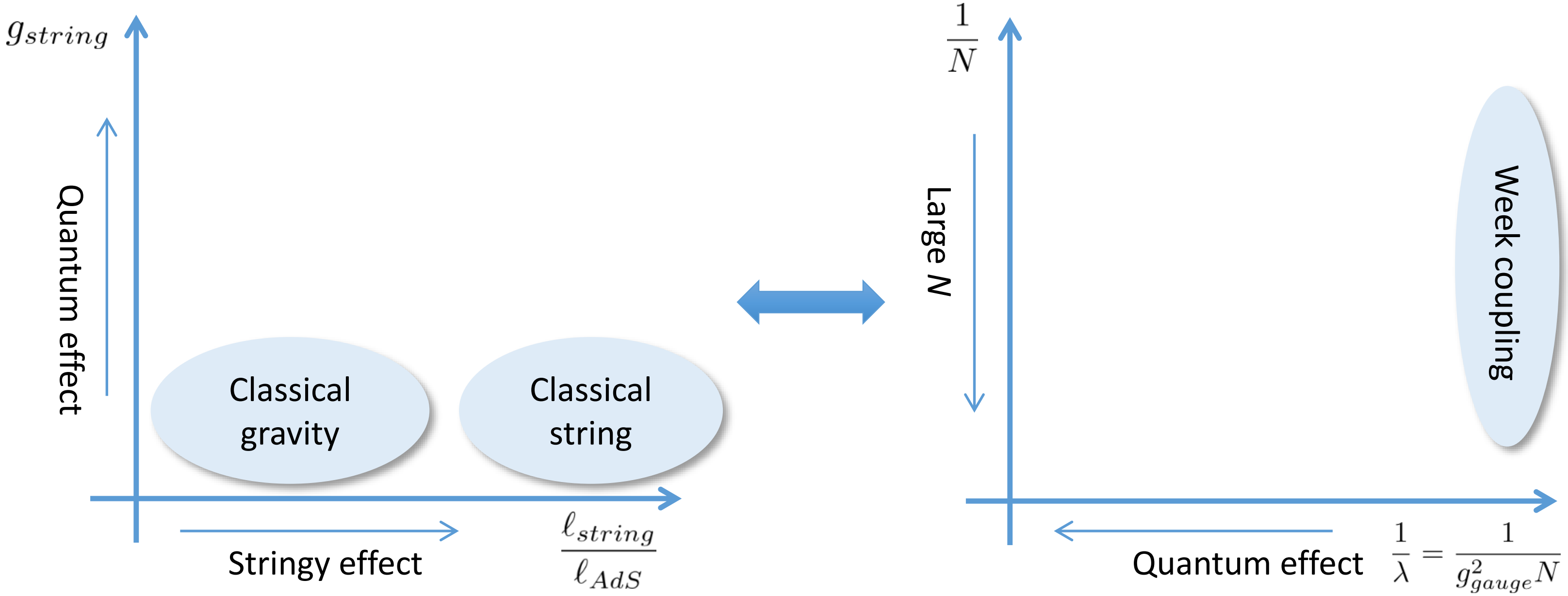
Conclusion

- Symmetry breaking for Klebanov-Polyakov duality
  - Anomalous dimensions are reproduced in conformal perturbation theory
  - CFT computation can be interpreted in terms of bulk Witten diagrams
  - Analysis has been generalized to other examples with higher spin duals
- There are many open problems
  - Understand the bulk one-loop computation in more details
  - Extend the analysis to the case with coupling to Chern-Simons gauge fields
  - Compute correlation functions in conformal perturbation theory and interpret the computation in the dual bulk theory

Back ups

AdS/CFT map

- Superstrings on AdS5xS5
- 4d U(N) gauge theory



- Tensionless limit of superstring theory (higher spin gauge theory) corresponds to weakly coupled gauge theory

Short v.s. long multiplet

[Girardello-Porrati-Zaffaroni '02]

- Fields on AdS4 are classified by representations of so(4,1)
  - Δ : scaling dimension, s : spin $(\Delta \geq s+1)$
- Shortening of representation

$$\lim_{\Delta \to s+1} D(\Delta, s) \to D(s+1, s) \oplus D(s+2, s-1)$$

Short representation: $\partial \cdot J^{(s)} = 0$

- Goldstone modes from bound states with $(S, n) = (s - s' - 1, 0)$

$$D(s'+1, s') \otimes D(2,0) = \bigoplus_{S=0}^{\infty} \bigoplus_{n=0}^{\infty} D(s'+S+n+3, s'+S)$$

  - A remark: spin 2 currents are kept conserved