Workshop “Holography, Quantum Entanglement and Higher Spin Gravity”
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BMS Supertranslations and Gravitational Memory

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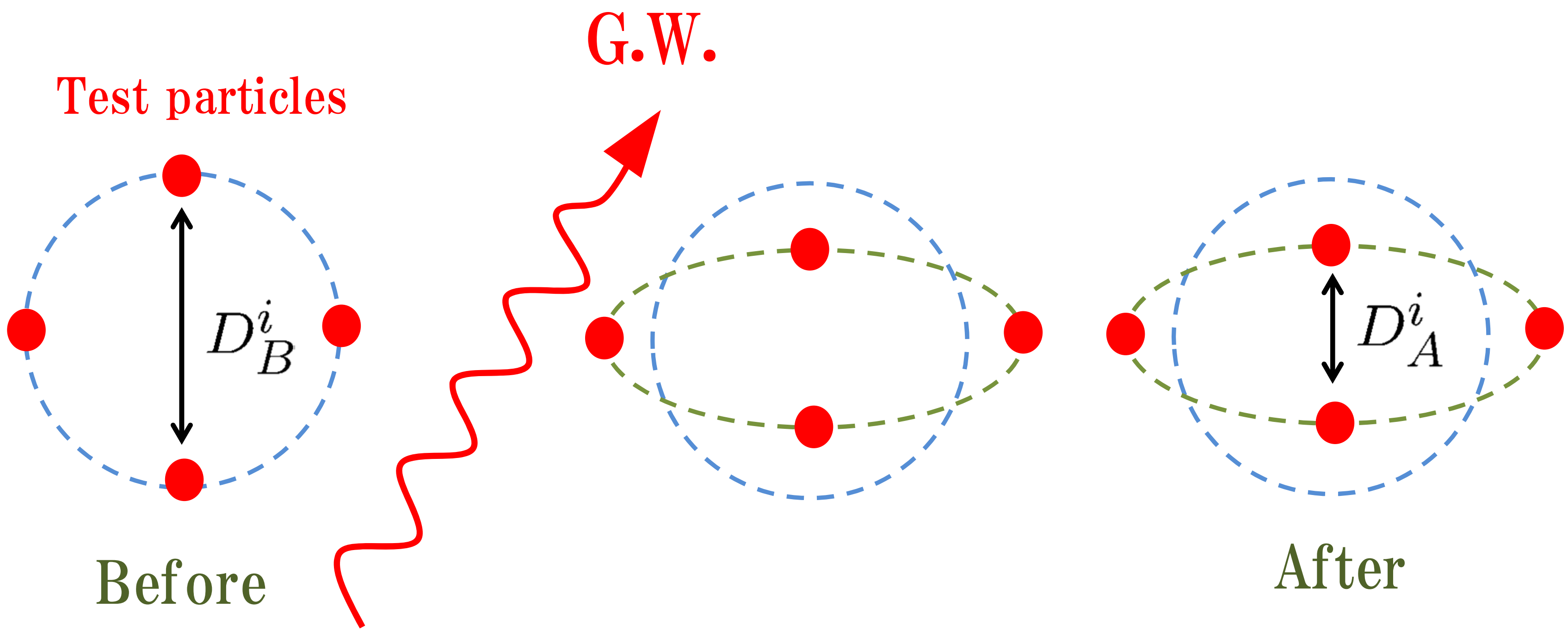
1702.00095 D. Garfinkle and A. Tolish

Memory Effect

Permanent displacement of test particles produced by radiation burst

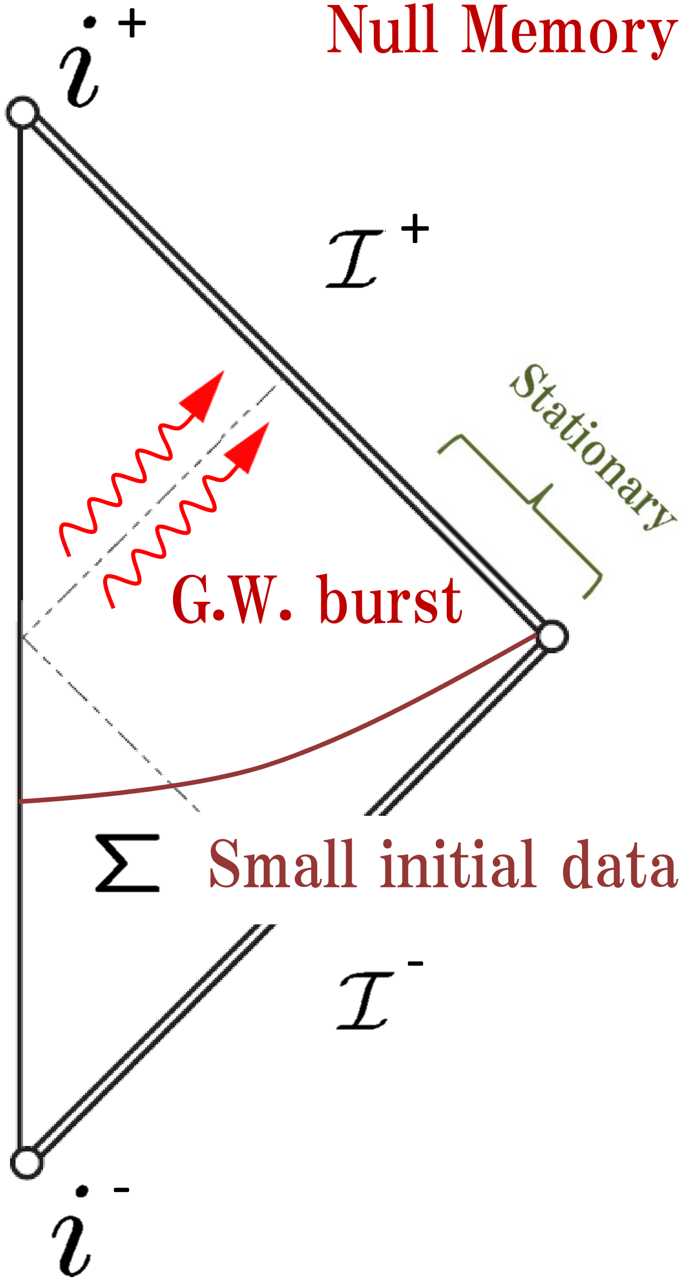


$$\Delta h_{ij}{}^{\mathrm{TT}} = \frac{1}{r}\Delta \sum_a \frac{4M_a}{\sqrt{1-v_a^2}}\left(\frac{v_a^i v_a^j}{1-v_a\cos\theta}\right)^{\mathrm{TT}}$$

Two types of memory effects

Null Memory (c.f. Non-linear memory)

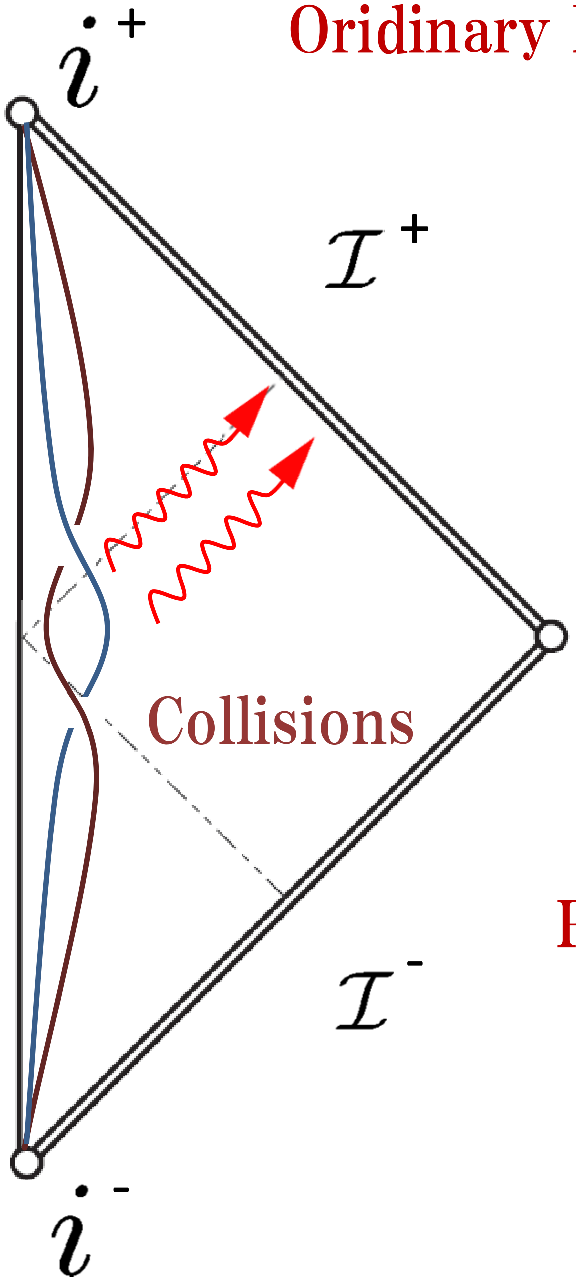
Christodoulou 91



Energy-Momentum is carried mainly by radiation

Oridinary Memory (c.f. Linear memory)

Zel'dovich-Polnarev 74



Most Energy-Momentum is carried by the matter

Particles/matter come in from i^-

go out to i^+

Basic ideas

Perturbations $\quad g_{ab} = \eta_{ab} + h_{ab}$

Einstein equation + gauge condition

$$\nabla^c \nabla_c h_{ab} = 16\pi \underline{S_{ab}}$$

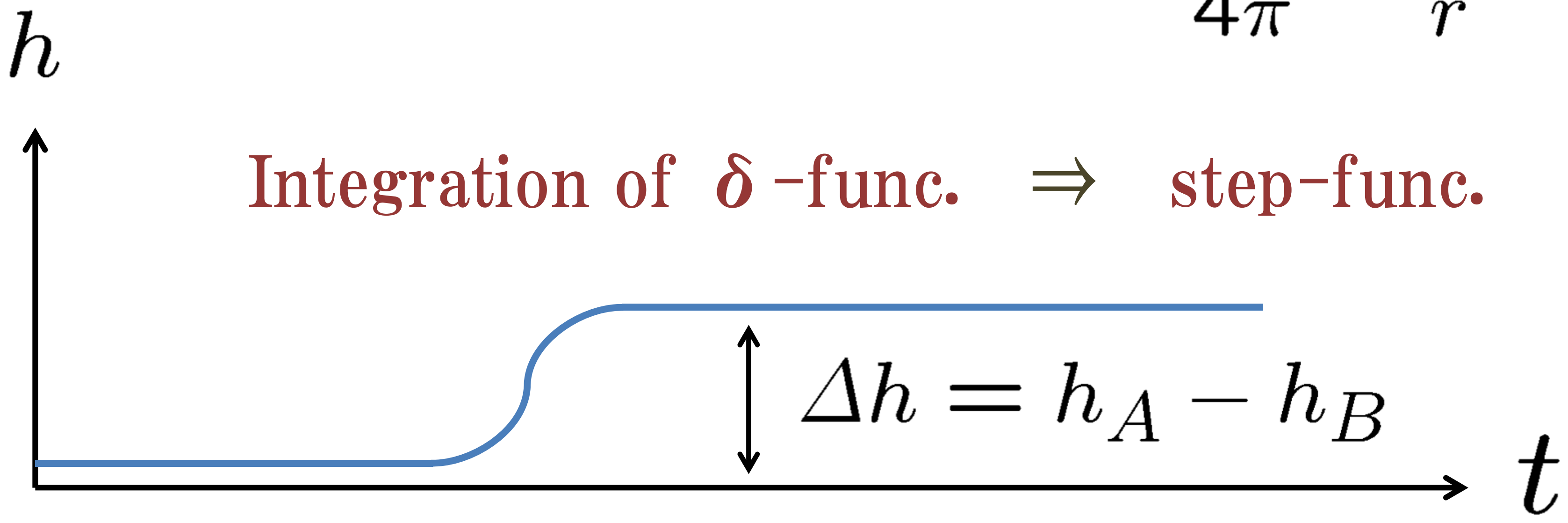
Source

$$h_{ab}(\mathbf{x}) = 16\pi \int d^D\mathbf{x}' \underline{G(\mathbf{x}-\mathbf{x}')} S_{ab}(\mathbf{x}'))$$

Green's function

$D = 4$ Green’s function $G \sim \frac{1}{4\pi}\frac{\delta(t-r)}{r}$

Integration of δ -func. $\Rightarrow$ step-func.



Geodesic deviation $\frac{d^2}{dt^2}D^i = \underline{R_{titj}}D^j$

curvature wrt h_{ab}

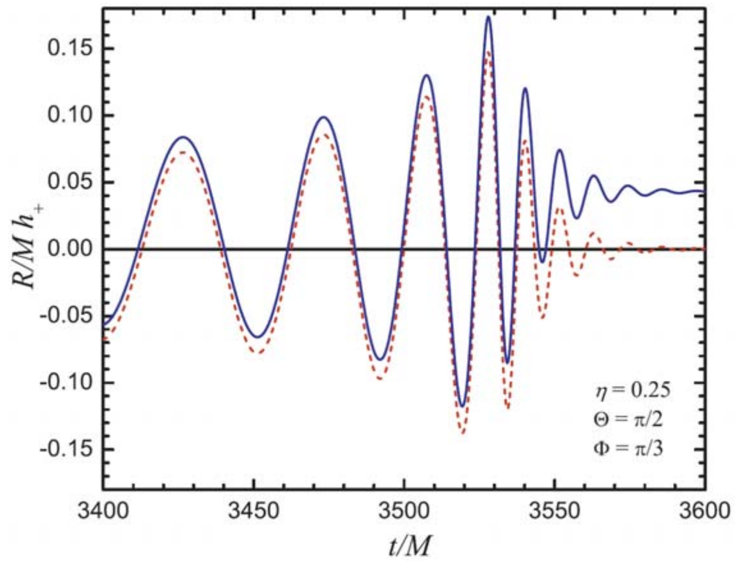
Displacement $\Delta D^i = \frac{1}{2}\Delta h_{ij}{}^{\mathrm{TT}} D^j$

Why interesting?

I. Memory effects $\Rightarrow$ detectable

e.g. LISA

Supermassive BH mergers out to $z \leq 2$



e.g. Favata '10

Why interesting?

II. BMS symmetry and Soft graviton

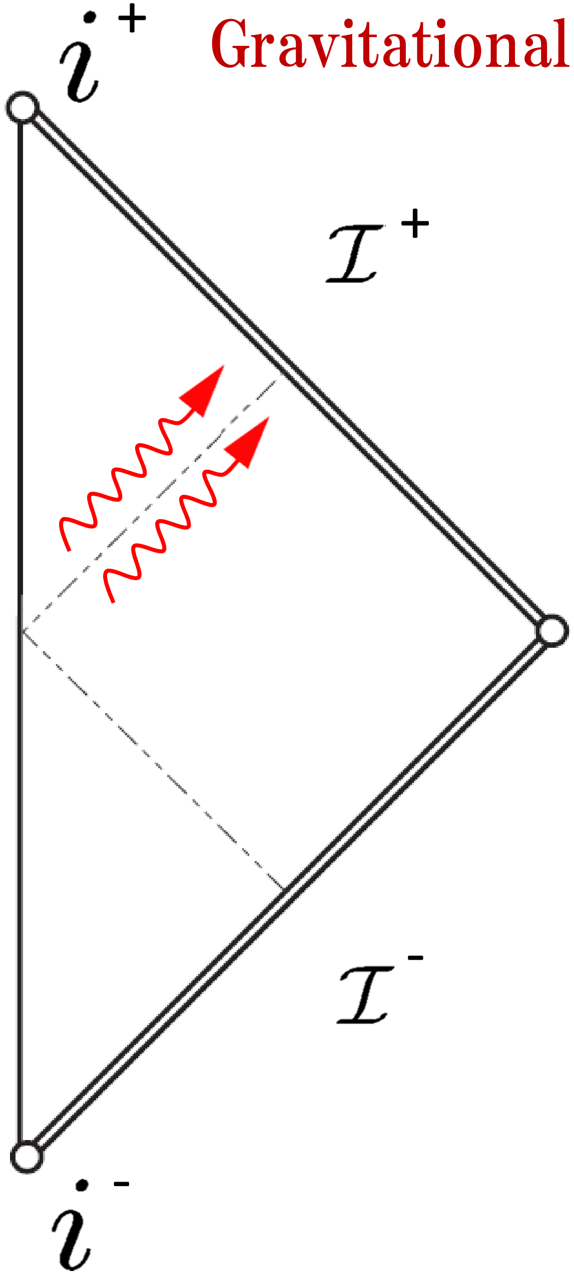
Memory,
BMS supertranslations, and
Soft theorems

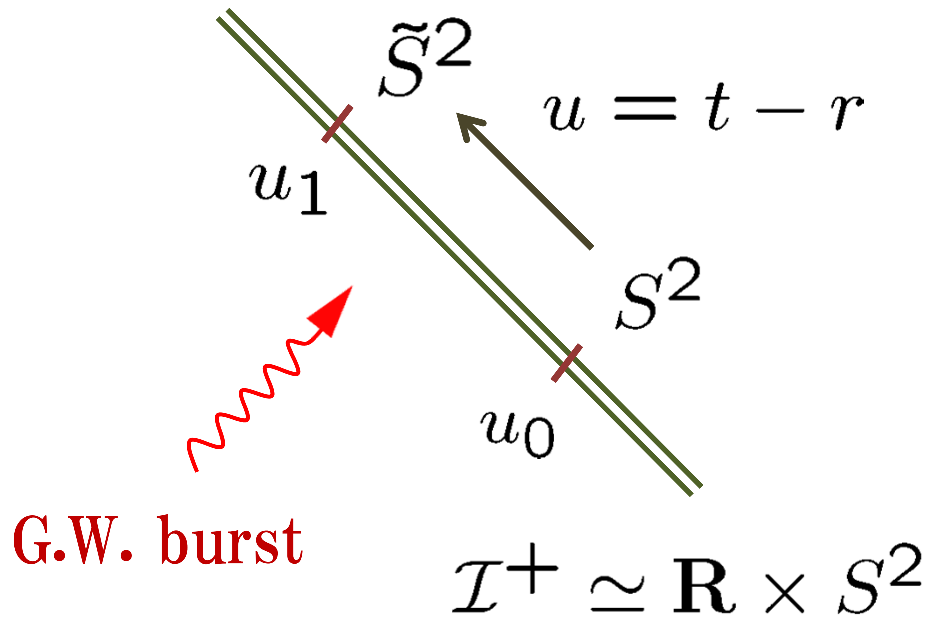
Strominger '14, Strominger-Zhiboedov '15

BMS supertranslations, and
Black hole's soft hair

Hawking-Perry-Strominger '16

Gravitational Wave Memory and BMS Symmetry





$$\tilde{g}_{ab} = \Omega^2 g_{ab}$$

$\Omega \sim \dfrac{1}{r} \to 0$ at null infinity $\mathcal{I}$

Asymptotic Symmetry

e.g. Minkowski space $\tilde{g}_{ab} = \Omega^2 \eta_{ab}$

In physical spacetime	At null infinity
Poincare group 10 parameters	BMS group Infinite dimensions
• Translations 4	• Super translations $\mathbf{X} = T(\theta, \phi) \cdot \frac{\partial}{\partial u} + \cdots$ map generator to generator
• Lorentz transf. 6	• $\sim$ BMS/super translations

BMS Supertranslations ： angle dependent translations

- Generator： $\mathbf{X} = T(z) \cdot \left(\frac{\partial}{\partial u} - \frac{\partial}{\partial r} \right) - \frac{1}{r} \cdot D^A T(z) \cdot \frac{\partial}{\partial z^A}$

$z^A = (\theta, \phi)$ ： angle coordinates on S^2

$T(z)$ ： spherical harmonics on S^2 w/ $\ell \geq 2$

(c.f. $\ell = 1 \Rightarrow$ $\mathbf{X}$ ：Lorentz boost)

- Displacement tensor： $\Delta_A{}^B = -\left(D_A D^B - \frac{1}{2}\delta_A{}^B D^c D_C \right) T(z)$

- Displacement： $\Delta D^B = \frac{1}{r} \Delta_A{}^B D^A$

Memory effects in Higher Dimensions?

Our claim

No Memory in HD General Relativity

1. Explicit calculations in a simple model 1702.00095
2. General analysis 1612.03290

1. Explicit calculations in a simple model

1702.00095
Garfinkle-Hollands-AI-Tolish-Wald

$D = 4$ Green's function $G \sim \frac{1}{4\pi}\frac{\delta(t-r)}{r}$

Integration of δ-func. $\Rightarrow$ step-func.

$D > 4$ (even dimensions)

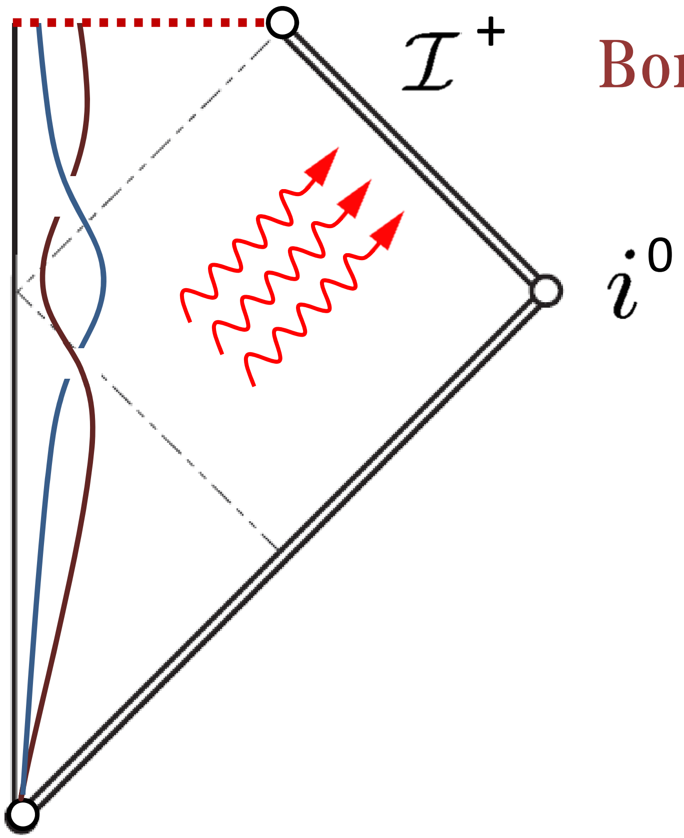
Leading term is given by derivatives of δ-func.

e.g. $D = 6$ $G \sim \frac{\delta'(u)}{r^2} + \cdots$

2. General analysis

1612.03290
Hollands-AI-Wald

Notion of asymptotic flatness and energy



Bondi energy
- defined at null infinity

ADM energy
- defined at spatial infinity

How to define asymptotic flatness?

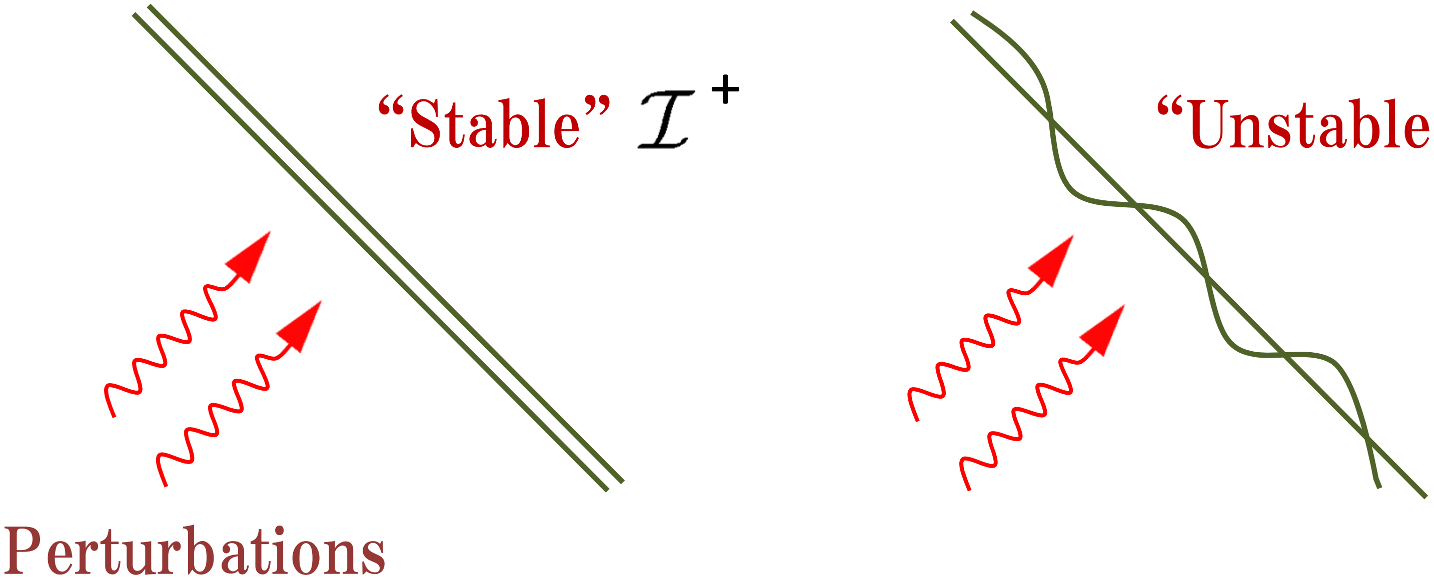
I. The fall off conditions should be **general enough**

→ to include interesting phenomena
e.g. radiating spacetimes

II. ———— ———— should be **strength enough**

→ to allow one to derive physically meaningful expression for energy

Stability of conformal null infinity



- Null infinity $\mathcal{I}$ should remain a regular null surface

Perturbed unphysical metric $\delta\tilde{g}_{ab} = \Omega^2 \delta g_{ab}$

$\Rightarrow$ should be extended to $\mathcal{I}$

$D = 4$ case $\Rightarrow$ stable : Geroch – Xanthopoulos '78

Stability of conformal null infinity

In even dimensioins $D > 4$

Desirable fall-off conditions

$$\delta\tilde{g}_{ab} = O(\Omega^{(d-2)/2})$$

$$\delta\tilde{g}_{ab}\tilde{\nabla}^a\Omega = O(\Omega^{d/2})$$

$$\delta\tilde{g}_{ab}\tilde{\nabla}^a\Omega\tilde{\nabla}^b\Omega = O(\Omega^{(d+2)/2})$$ remain null

$$\tilde{g}^{ab}\delta\tilde{g}_{ab} = O(\Omega^{d/2})$$ particular in $D > 4$

— Linearized version of our general analysis of 1612.03290

Proof : Hollands-AI 05

TT gauge works !

Choice of variables ϕ_α :

$$\tau_{ab} := \Omega^{-(D-2)/2}\delta\tilde{g}_{ab} \qquad \tau_a := \Omega^{-1}\tau_{ab}\tilde{\nabla}^b\Omega$$

Einstein's equations reduce to a hyperbolic system:

$$\tilde{\nabla}^c\tilde{\nabla}_c\phi_\alpha = A_\alpha{}^{\beta a}\tilde{\nabla}_a\phi_\beta + B_\alpha{}^\beta\phi_\beta$$

- can show this possesses a well-posed initial value problem under our fall-off conditions

$\Rightarrow$ ϕ_α has smooth extension on $\mathcal{I}^+$ $\square$

- Difference between $4D$ and $D > 4$

radiations drop off $\sim \dfrac{1}{r^{(D-2)/2}}$

static Coulomb part drops off $\sim \dfrac{1}{r^{D-3}}$

$D = 4$ ⇒ both parts behave $\sim \dfrac{1}{r}$ ⇒ Memory

$D > 4$ ⇒ $1 < \frac{D-2}{2} < D - 3$ ⇒ No Memory

Our fall-off conditions

$\Rightarrow$ guarantee stability of conformal null infinity

$\Rightarrow$ general enough

to allow for radiating spacetimes

$\Rightarrow$ stringent enough

to allow for physically meaningful energy

(i.e. well-defined symplectic current)

……… but NOT allow Supertranslations

Generator of asymptotic symmetry

$$\mathbf{X} = T(\mathrm{angle}) \cdot \frac{\partial}{\partial u} + \cdots$$

$$\begin{cases} D = 4 & T(z) \text{ : arbitrary func. on } S^2 \\ D > 4 & T(z) \text{ : spherical harmonics on } S^{D-2} \end{cases}$$

w/ $\ell = 0, 1$

$\Rightarrow$ $\mathbf{X}$: D -dimensional translations

How to define Bondi energy?

— follow Noether charge method   Wald-Zoupas '00

$$L = \frac{1}{16\pi} R \cdot \epsilon$$

θ : $(D-1)$-form $\delta L = \mathsf{EOM} \cdot \delta g + \mathrm{d}\theta$

ω : symplectic current

$$\omega(\delta_1 g,\, \delta_2 g) = \delta_1 \theta(\delta_2 g) - \delta_2 \theta(\delta_1 g)$$

Under our fall-off conditions,

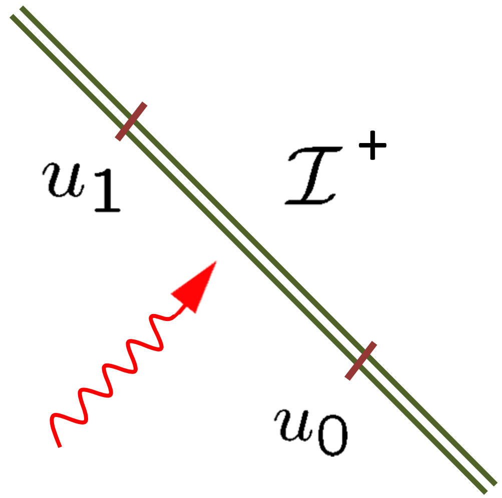
the symplectic current ω is well-defined

News tensor：

$$N_{ab} := \Omega^{-(D-4)/2} \times \text{(pull back of Schouten tensor)}$$

Flux formula：

$$\mathcal{H}_{\mathbf{X}}(u) = -\frac{1}{32\pi}\int_{S^{D-2}(u)} T N^{ab} N_{ab}$$



Grav. Waves

All these are well-defined on our fall-off conditions

Summary：

- $D = 4$

  BMS Supertranslations must be included

  $\Rightarrow$ parametrized by $^{\forall}T(z)$ on S^2

- $T(z)$ acts as "potential" of Memory

- $D > 4$ $\Rightarrow$ need to impose reasonable fall-off conditions in order to guarantee

  stability of null-infinity

  well-definedof symplectic current

$\Rightarrow$ No Supertranslations $\Rightarrow$ No memory