Towards a 2nd Law for Lovelock Theory

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“Towards a second law for Lovelock theories”

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Introduction and Motivation

- Black Holes are "Theoretical Laboratories" for understanding quantum gravity.
- String theory $\Rightarrow$ Despite being successful in some aspects $\Rightarrow$ unresolved issues remaining.
- It is, therefore in principle useful to use *"general principles"* to constrain admissible low energy theories.
- The second law of thermodynamics is one such principle that we can test on low energy solutions of gravity.

Introduction and Motivation

- In Einstein's theory of gravity BH's are thermodynamic objects :
  - They have energy, temperature and entropy.
  - The entropy is given by the area of BH horizon.
  - The entropy satisfies a 1st law of thermodynamics.
  - In dynamical situations the horizon area always increases $\Rightarrow$ The 2nd law
- The proof of 1st and 2nd law in general relativity requires classical EOM's + certain conditions on the matter stress tensor (Null Energy Conditions).

Introduction and Motivation

- The Einstein-Hilbert action can not be the complete story : The low energy limit of any UV complete quantum theory of gravity generate higher derivative corrections to the leading two derivative action :

$$I = \int d^d x \sqrt{-g}\,\left[R + \mathcal{L}_{matter} + \mathcal{L}_{HD}\right]$$

  - Precise form of $\mathcal{L}_{HD}$ depends on particular nature of UV completion.
  - Additional corrections in the action $\Rightarrow$ modifications in the EOM's.
  - However, BH solutions will continue to exist.
  - However, since the EOM's have changed, the earlier proofs of 1st and 2nd law (for two derivative theories of gravity) will no longer be valid.

Introduction and Motivation

- Wald argued that for stationary black hole solutions of higher derivative gravity theories, the entropy is a Noether charge associated with time translations along the horizon generating Killing field.
  - Wald entropy was constructed to explicitly satisfy the first law of thermodynamics, which being an equilibrium statement, can be understood in the stationary solution.
- The 2nd law involves dynamics and thus is a statement beyond equilibrium.
- However, we do not have a general proof that "Wald Entropy" satisfies 2nd law for any higher derivative theories of gravity.
- An open question since then has been whether there is a notion of second law of black hole mechanics in higher derivative gravity.

Introduction and Motivation

- The precise statement of a 2nd law :

  > If perturbing one equilibrium configuration (say eq_1) one can reach another equilibrium configuration (say eq_2) in course of time evolution, then
  > the total entropy of $eq_2 \geq$ the total entropy of eq_1

- How does this translate to gravity :
  - Equilibrium configuration $\Rightarrow$ Metric with Killing horizon
  - Equilibrium entropy $\Rightarrow$ Wald entropy on Killing horizon
  - Suppose, $metric_1$ and $metric_2$ both have Killing horizons and dynamical perturbations around $metric_1$ eventually evolves to $metric_2$.
  - Second law : $\boxed{Wald\ entropy_{metric_2} \geq Wald\ entropy_{metric_1}}$

- We will actually consider a local version of second law, a stronger statement.

Introduction and Motivation

- Our eventual target:

To investigate the 2nd law for any dynamical BH solution in arbitrary higher derivative theories of gravity or To find a concrete counter example

- What we have attempted in this work is an initial small step towards this final goal.
- We have examined whether 2nd law holds for dynamical black holes in Lovelock theories of gravity, one special case of higher derivative theories of gravity.
- More precisely, we have extended the Wald entropy (a notion of equilibrium entropy) to the dynamical setting satisfying a local version of 2nd law for Lovelock theories.

Strategy of the Proof

- Naively, 2nd law is a non-local statement, comparing only the initial and final configuration : $\boxed{S^{final}_{Wald} - S^{initial}_{Wald} \geq 0}$
- To prove this we explicitly construct a function of the system variables, the "entropy function": $\boxed{S_{\text{total}} = S_{Wald} + S_{cor}}$
- The conditions on the entropy function S_{total} will be
  - it reduces to the familiar notion of entropy S_{Wald}, whenever the geometry has a Killing horizon

$$\boxed{(1) \quad S_{\text{total}} \overset{equilibrium}{=} S_{Wald}}$$

  - and it is monotonically increasing under time evolution as along as the time evolution is governed by the higher derivative EOM

$$\boxed{(2) \quad \partial_v S_{\text{total}} \geq 0 \quad \Rightarrow \text{for all } v}$$

Strategy of the Proof

- Existence of such a S_{total} is enough to prove the 2nd law

$$(1)\ S_{\text{total}} \overset{\textit{equilibrium}}{=} S_{\textit{Wald}} \quad \text{and} \quad (2)\ \partial_v S_{\text{total}} \geq 0$$
$$\Rightarrow \quad S_{\textit{Wald}}^{\textit{final}} - S_{\textit{Wald}}^{\textit{initial}} = \int_{\textit{initial}}^{\textit{final}} \partial_v S_{\text{total}}\, dv \geq 0$$

- The steps of the proof

1. Define entropy density : $S_{\text{total}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h}\, \rho_{\text{total}}$
2. Define Θ : $\partial_v S_{\text{total}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h}\, \Theta$
3. We aim to show $\partial_v \Theta \leq 0$
4. We assume : $v \to \infty, \quad \Theta \to 0.$
5. This implies $\Theta \geq 0$ for every finite v.
6. Finally we obtain that : $\partial_v S_{\text{total}} \geq 0$

Strategy and Final result

- Note that our method is a bit indirect : in the sense
  - We don't directly try to prove $\partial_v S_{\text{total}} \geq 0$,
  - Rather a stronger and local version : $\Theta \geq 0$
  - Also there is no claim for uniqueness.

Our Result :

- For Lovelock theory we have been able to construct an S_{total} that works in any time evolution, maintaining spherical symmetry
- Outside spherical symmetry, our construction works provided a very particular total derivative term is always non-negative in the course of evolution.

Basic Set Up

- Higher derivative terms in the action come with dimensionfull coefficients $\Rightarrow \ \alpha_2 \sim {\ell_s}^2 \ \Rightarrow$ length dimension $= 2$

$$I = \int d^d x \sqrt{-g} \left[R + \alpha_2 \left(R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right) \right]$$

- Our initial equilibrium configuration = Stationary BH solution in this theory.
- Perturbing away from equilibrium are parametrized by
  - (a) the amplitude of the departure from equilibrium : $\mathfrak{a}$,
  - (b) the characteristic frequency : ω
- An entropy function should, at the very least, carry information about these three parameters : $\alpha_2, \mathfrak{a}, \omega$

Basic Set Up : Assumptions

- The following summarizes what is known to date:
  - (a) Wald entropy can be constructed for arbitrary α_2 with $\mathfrak{a} = 0$ (Iyer-Wald),
  - (b) Entropy functions are constructed for $f(R)$ theories for finite range of α_2 but arbitrary $\mathfrak{a}, \omega$. (Jacobson-Kang-Myers)
  - (c) Some developements are there for $f(Lovelock)$ and general four derivative theories but for $\mathfrak{a} \ll 1$. (Padmanavan $\cdots$, Wall-Sarkar $\cdots$)
- We aim to construct an entropy function in higher derivative gravity, perturbatively in the couplings $\alpha_2 \ll 1$, and the frequency $\omega \ell_s \ll 1$, but valid for arbitrary amplitudes $\mathfrak{a}$
- We work perturbatively in the higher derivative terms,
  - (a) The corrections to Einstein-Hilbert theory are treated in a gradient expansion,
  - (b) the effective small dimensionless parameter : $\omega \ell_s \sim \sqrt{\alpha_2}\, \partial_v$
  - (c) no assumption about the amplitude.

Basic Set Up : Assumptions

- At horizon higher derivative corrections are suppressed compared to the Einstein-Hilbert term
  $\Rightarrow$ Around horizon, curvature-scales are large compared to fluctuations of the BH horizon $\sim$ determined by $\alpha_2 \sim \ell_s$
- A classical description of gravity is valid, i.e., no loop correction etc. enter our discussion
- Despite the 2-derivative theory dominates on the horizon scale, why do we need to modify the entropy for 2nd law ?

  $\Rightarrow$ Because :

Although the leading area contribution is large, it is possible under evolution, the area variation is anomalously small and overwhelmed by the higher derivative $\mathcal{O}(\omega\ell_s)$ contributions, spoiling the monotonicity of the entropy.

Basic Set Up : Choice of Coordinates

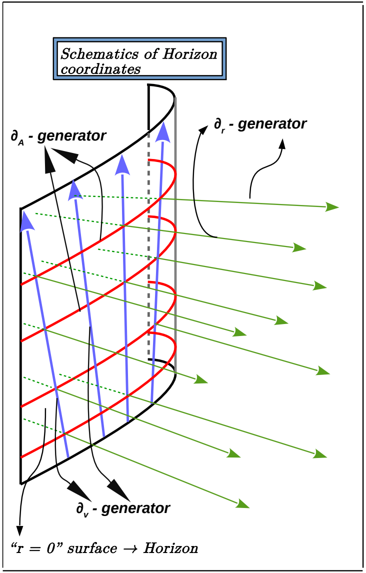
- Around horizon we can choose a coordinate system $\{x^\mu\}$ such that the effect of higher derivative corrections are handled in a derivative expansion $\Rightarrow \omega \ell_s \sim \sqrt{\alpha_2}\, \partial_\mu \ll 1$
- The geometry must have a horizon $\Rightarrow$ a null hypersurface.
- On the horizon $\mathcal{H}$:
  - (a) define coordinate $v \Rightarrow$ affine parameter along the null generators ∂_v,
  - (b) along constant v slices, Σ_v : define the spatial coordinates x_A .

  $\Rightarrow \{v, x^A\}$ are coordinates on $\mathcal{H}$ and $\{x^A\}$ are coordinates on Σ_v
- Away from $\mathcal{H}$: define coordinate $r \Rightarrow$ affinely parametrized along null geodesics ∂_r piercing through the horizon at angle

$$(\partial_v, \partial_r)\Big|_{\mathcal{H}} = 1, \quad (\partial_r, \partial_A)\Big|_{\mathcal{H}} = 0$$

- Choose the origin of the r coordinate such that horizon is at $r = 0$.

Basic Set Up : Choice of Coordinates



Basic Set Up : Choice of Metric

- In our coordinates the metric metric will take the following form

$$\boxed{\begin{gathered} ds^2 = 2dvdr - f(r,v,x^A)dv^2 + 2k_A(r,v,x^A)dvdx^A + h_{AB}(r,v,x^A)dx^A dx^B \\ \text{such that } \; f(r,v,x^A)\Big|_{\mathcal{H}} = k_A(r,v,x^A)\Big|_{\mathcal{H}} = \partial_r k_A(r,v,x^A)\Big|_{\mathcal{H}} = 0 \end{gathered}}$$

- Note that any metric with a horizon could be expressed in the above form

- Our construction will be in terms of explicit derivatives of $f(r,v,x^A)$, $k_A(r,v,x^A)$ and $h_{AB}(r,v,x^A)$

- Following notation would be useful

$$\boxed{\mathcal{K}_{AB} = \frac{1}{2}\partial_v h_{AB}\Big|_{\mathcal{H}} \,, \quad \overline{\mathcal{K}}_{AB} = \frac{1}{2}\partial_r h_{AB}\Big|_{\mathcal{H}}}$$

- $h_{AB}(r=0,v,x^A)$ is the induced metric on $\mathcal{H}$

Gauss-Bonnet Theory

- We consider Gauss-Bonnet theory :

$$I = \int d^d x \sqrt{-g} \left[R + \alpha_2 \left(R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right) \right]$$

- We will add correction to S_{wald}, and ρ and Θ can be written as

$$\boxed{\begin{aligned} S_{\text{total}} &= S_{\text{Wald}} + S_{\text{cor}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h} \left[\rho_{eq} + \rho_{\text{cor}} \right] \\ \partial_v S_{\text{total}} &= \int_{\Sigma_v} d^{d-2}x \sqrt{h} \ \left[\Theta_{eq} + \Theta_{\text{cor}} \right] \\ \Theta_{eq} &= \frac{1}{\sqrt{h}} \partial_v \left(\sqrt{h}\, \rho_{eq} \right), \quad \Theta_{cor} = \frac{1}{\sqrt{h}} \partial_v \left(\sqrt{h}\, \rho_{cor} \right) \end{aligned}}$$

- Finally we want to show : $\partial_v [\Theta_{eq} + \Theta_{\text{cor}}] \leq 0$

- Condition on the correction term : $S_{\text{cor}} \overset{\text{equilibrium}}{=} 0$

Gauss-Bonnet Theory : Final Result

- We consider Gauss-Bonnet theory with Wald entropy

$$I = \int d^d x \sqrt{-g} \left[R + \alpha_2 \left(R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right) \right]$$

$$S_{\text{Wald}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h} \, \underbrace{(1 + 2\,\alpha_2\, \mathcal{R})}_{(=\rho_{eq})}$$

- We add the correction

$$S_{\text{cor}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h}\, \rho_{\text{cor}}\,, \qquad \rho_{\text{cor}} = \alpha_2^2 \sum_{n=0}^{\infty} \kappa_n\, \alpha_2^n\, \partial_v^n \left(\mathfrak{h}_{2(0)}\right)_B^A\, \partial_v^n \left(\mathfrak{h}_{2(0)}\right)_A^B$$

$$\text{where} \quad \left(\mathfrak{h}_{2(0)}\right)_B^A = \delta^{AA_1A_2}_{BB_1B_2}\, \mathcal{K}^{B_1}_{A_1}\, \overline{\mathcal{K}}^{B_2}_{A_2}$$

- We showed that

$$\partial_v[\Theta_{eq} + \Theta_{\text{cor}}] \le 0, \quad \Rightarrow \quad A_n = 2\kappa_n - \frac{\kappa_{n-1}^2}{A_{n-2}}$$

$$\text{for } n = -2, -1, 0, \cdots, \quad \text{and } \kappa_{-2} = -1/2\,, \quad \kappa_0 = -1\,, \quad \kappa_{-1} = -2$$

Gauss-Bonnet Theory : Details of proof

- We consider Gauss-Bonnet theory with Wald entropy

$$I = \int d^d x \sqrt{-g}\left[R + \alpha_2 \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right)\right]$$

$$S_{\text{Wald}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h}\, \underbrace{(1 + 2\,\alpha_2\,\mathcal{R})}_{(=\rho_{eq})}, \quad \text{and} \quad \Theta_{eq} = \frac{1}{\sqrt{h}}\partial_v\left(\sqrt{h}\,\rho_{eq}\right)$$

- Next We compute

$$\partial_v\Theta_{eq} = \text{Term 1} + \text{Term 2} + \text{Term 3} + \text{Term 4} + \text{Term 5}$$

$$\text{Term 1} = -T_{vv}, \quad \text{Term 2} = -\mathcal{K}_{AB}\mathcal{K}^{AB}, \quad \text{Term 3} = \alpha^2\mathcal{K}^A_B\mathcal{K}^{A'}_{B'}\mathcal{M}^{BB'}_{AA'}$$

$$\text{Term 4} = \alpha^2\mathcal{K}^A_B\,\partial_v\left[\delta^{BA_1A_2}_{AB_1B_2}\mathcal{K}^{B_1}_{A_1}\overline{\mathcal{K}}^{B_2}_{A_2}\right], \quad \text{Term 5} = \nabla_A\mathcal{Y}^A$$

- Note that $\mathcal{M}^{BB'}_{AA'}$ is some specific four indexed and two derivative tensor but no ∂_v.

Gauss-Bonnet Theory : Details of proof

$$\partial_v \Theta_{eq} = \text{Term 1} + \text{Term 2} + \text{Term 3} + \text{Term 4} + \text{Term 5}$$

$$\text{Term 1} = -T_{vv}, \quad \text{Term 2} = -\mathcal{K}_{AB}\mathcal{K}^{AB}, \quad \text{Term 3} = \alpha_2^2 \mathcal{K}^A_B \mathcal{K}^{A'}_{B'} \mathcal{M}^{BB'}_{AA'}$$

$$\text{Term 4} = \alpha_2^2 \mathcal{K}^A_B \, \partial_v \left[\delta^{BA_1A_2}_{AB_1B_2} \mathcal{K}^{B_1}_{A_1} \overline{\mathcal{K}}^{B_2}_{A_2} \right], \quad \text{Term 5} = \nabla_A \mathcal{Y}^A$$

- For Einstein theory ($\alpha = 0$) 2-nd law is valid for wald entropy,

$$\partial_v \Theta_{eq} = -T_{vv} - \mathcal{K}_{AB}\mathcal{K}^{AB} \quad \Rightarrow \quad \partial_v \Theta_{eq} \leq 0$$

- Term 1 : Null Energy condition, and Term 2 ≤ 0 .
- Term 3 $\ll$ Term 2 $\quad \Rightarrow \quad \text{Term 2} + \text{Term 3} = \mathcal{K}^A_B \mathcal{K}^{A'}_{B'} \left[\delta^B_A \delta^{B'}_{A'} + \alpha^2 \mathcal{M}^{BB'}_{AA'} \right]$
- Term 4 is naively small compared to Term 2 : But not always true

$$\text{Term 2} + \text{Term 4} = \mathcal{K}^A_B \left[\mathcal{K}^B_A + \alpha^2 \, \partial_v \left[\delta^{BA_1A_2}_{AB_1B_2} \mathcal{K}^{B_1}_{A_1} \overline{\mathcal{K}}^{B_2}_{A_2} \right] \right]$$

$$\text{Term 4} \gtrsim \text{Term 2} \quad \Rightarrow \quad \partial_v \Theta_{eq} \not\leq 0$$

Need to add corrections to Wald entropy to handle it.

Gauss-Bonnet Theory : Details of proof

- We decide the correction term

$$
\begin{aligned}
S_{\text{total}} &= S_{\text{Wald}} + S_{\text{cor}} = \int_{\Sigma_v} d^{d-2}x \sqrt{h}\,[\rho_{eq} + \rho_{\text{cor}}] \\
\partial_v S_{\text{total}} &= \int_{\Sigma_v} d^{d-2}x \sqrt{h}\,[\Theta_{eq} + \Theta_{\text{cor}}] \\
\partial_v \Theta_{eq} &= -T_{vv} - \mathcal{K}_{AB}\mathcal{K}^{AB} + \alpha^2 \mathcal{K}^A_B\, \partial_v H^B_A + \nabla_A \mathcal{Y}^A + \text{ Negligible Terms} \\
H^B_A &= \delta^{BA_1A_2}_{AB_1B_2} \mathcal{K}^{B_1}_{A_1} \overline{\mathcal{K}}^{B_2}_{A_2}
\end{aligned}
$$

- We add S_{cor} to adjust $\partial_v \Theta_{\text{cor}}$ such that

$$
\begin{aligned}
\partial_v \Theta_{\text{cor}} &= \alpha^4\, \gamma\, \partial_v H^B_A\, \partial_v H^A_B \\
\partial_v[\Theta_{eq} + \Theta_{\text{cor}}] &= -\, T_{vv} - \left[\mathcal{K}^A_B - \frac{\alpha^2}{2}\,\partial_v H^A_B\right]^2 - \alpha^4 \left(\gamma - \frac{1}{4}\right) \partial_v H^B_A\, \partial_v H^A_B \\
\Rightarrow \quad & \partial_v[\Theta_{eq} + \Theta_{\text{cor}}] \leq 0 \quad \text{if} \quad \gamma \leq 1/4
\end{aligned}
$$

- We need to impose : $\nabla_A \mathcal{Y}^A = \nabla_A \nabla_A \mathcal{Z}^{AB} = 0$, which is true for spherically symmetric evolution.

Extension to Lovelock Theories

- We extended the analysis beyond Gauss-Bonnet to Lovelock theories

$$\boxed{\begin{gathered} I \equiv \int \sqrt{-g}\,\left[R + \alpha_m\,\ell_s^{2m-2}\,\mathcal{L}_m + \mathcal{L}_{matter}\right] \\ \mathcal{L}_m = \delta^{\mu_1\nu_1\cdots\mu_m\nu_m}_{\rho_1\sigma_1\cdots\rho_m\sigma_m}\,R^{\rho_1}{}_{\mu_1}{}^{\sigma_1}{}_{\nu_1}\cdots R^{\rho_m}{}_{\mu_m}{}^{\sigma_m}{}_{\nu_m} \end{gathered}}$$

$\delta^{\mu_1\nu_1\cdots\mu_m\nu_m}_{\rho_1\sigma_1\cdots\rho_m\sigma_m}$ = determinant of $(n \times n)$ matrix whose (ij)-th element is $\delta^{\mu_i}_{\nu_j}$

- The correction we need to add

$$\boxed{\begin{aligned} S_{\text{total}} &= S_{\text{Wald}} + S_{\text{cor}} = \int_{\Sigma_v} d^{d-2}x\,\sqrt{h}\,[\rho_{eq} + \rho_{\text{cor}}] \\ \rho_{eq} &= \left.\frac{\delta\mathcal{L}_{\text{grav}}}{\delta R^v{}_v{}^r{}_r}\right|_{R\to\mathcal{R}}, \qquad \mathcal{L}_{\text{grav}} = R + \alpha_m\,\ell_s^{2m-2}\,\mathcal{L}_m \\ \rho_{\text{cor}} &= \sum_{n=0}^{\infty}\kappa_n\left[\ell_s^n\partial_v^n\left(\frac{1}{2}\left.\frac{\delta^2\mathcal{L}_{\text{grav}}}{\delta R^A{}_{A_1}{}^{C_1}{}_v\,\delta R^v{}_{B_1}{}^{D_1}{}_B}\right|_{R\to\mathcal{R}}\mathcal{K}^{C_1}_{A_1}\overline{\mathcal{K}}^{D_1}_{B_1}\right)\right]^2 \end{aligned}}$$

- The replacement rule : $R \to \mathcal{R} \;\Rightarrow$ replace all the curvature tensors of the spacetime with those intrinsic to Σ_v

Extension to Lovelock Theories

$$S_{\text{total}} = S_{\text{Wald}} + S_{\text{cor}} = \int_{\Sigma_v} d^{d-2}x\,\sqrt{h}\,[\rho_{eq} + \rho_{\text{cor}}]$$

$$\rho_{eq} = \left.\frac{\delta\mathcal{L}_{\text{grav}}}{\delta R^{v}{}_{v}{}^{r}{}_{r}}\right|_{R\to\mathcal{R}}, \qquad \mathcal{L}_{\text{grav}} = R + \alpha_m\,\ell_s^{2m-2}\,\mathcal{L}_m$$

$$\rho_{\text{cor}} = \sum_{n=0}^{\infty}\kappa_n\left[\ell_s^n\partial_v^n\left(\frac{1}{2}\left.\frac{\delta^2\mathcal{L}_{\text{grav}}}{\delta R^{A}{}_{A_1}{}^{C_1}{}_{v}\,\delta R^{v}{}_{B_1}{}^{D_1}{}_{B}}\right|_{R\to\mathcal{R}}\mathcal{K}^{C_1}_{A_1}\overline{\mathcal{K}}^{D_1}_{B_1}\right)\right]^2$$

- $\partial_v S \geq 0$ in any time evolution maintaining spherical symmetry provided the κ_n's satisfy the following recursive inequality

$$A_n = 2\,\kappa_n - \frac{\kappa_{n-1}^2}{A_{n-2}} \leq 0\,, \qquad \text{for } n = -2, -1, 0, \cdots$$

$$\text{initial condition}: \quad \kappa_{-2} = -\frac{1}{2}\,, \quad \kappa_{-1} = -2\,.$$

- The Obstruction term in the form of a toal derivative is still there.

Conclusions

- For Lovelock theory we have been able to construct an S_{total} satisfying $2nd$ law, surely for spherical symmetry.
- We need to understand the implications of the obstruction term.
- Our construction is not unique, for example
  - Instead of $\partial_v \Theta \leq 0$ we prove $\partial_v (Z\ \Theta) \leq 0$ for some $Z \geq 0$, and it would do the job.
    In fact this is how 2nd law is proved for $f(R)$ theories.
- Field redifinitions and foliation dependence.
- The method is indirect : It is possible that $\partial_v S_{\text{total}}$ not monotonically decreasing. To obtain some constraints on the structure of the higher derivative corrections we need a direct method $\partial_v S_{\text{total}} < 0$
- Possible connection with Holographic Entanglement entropy $\Rightarrow$ Myers, Dong, Camps analysis.

Thank You For Attention